

# Preface to the Second Edition

Over the last 15 years, there has been a considerable amount of advancements in condensed matter physics: graphene, pnictide superconductors, and topological insulators, to name just a few. The understanding, and to a large degree the very discovery, of these new phenomena required the use of advanced theoretical tools. The knowledge of the basic methods of quantum many-body theory thus becomes more important than ever for each student in the field.

Some of the most challenging current problems stem from the spectacular progress in quantum engineering and quantum computing, more specifically, in developing solid-state based—mostly superconducting—quantum bits and qubit arrays. During this short period, we arrived from the first experimental demonstration of coherent quantum tunnelling in single qubits (which are, after all, quite macroscopic objects) to precise manipulation of quantum state of several qubits, their quantum entanglement over macroscopic distances and, recently, signatures of quantum coherent behaviour in devices comprising hundreds of qubits. The difficulty is that it is impossible to directly simulate such large, partially coherent, essentially nonequilibrium quantum systems, due to the sheer volume of computation—which was the motivation behind quantum computing in the first place. It would seem that one needs a quantum computer in order to make a quantum computer! The hope is that appropriate generalizations of the methods of non-equilibrium many-body theory would provide good enough approximations and keep the research going until the time when (and if) the task can be handed to quantum computers themselves.

Given the above considerations, I did not feel the need to change the scope or the approach of the book. I have, though, added a new chapter, in order to introduce bosonization and elements of conformal field theory. These are beautiful and powerful ideas, especially useful when dealing with low-dimensional systems with interactions, and belong to the essential condensed matter theory toolkit. I have also corrected some typos—hopefully introducing fewer new ones in the process.

In addition to those of my teachers and colleagues, whom I had the opportunity to thank in the preface to the first edition, I would like to express my gratitude to Profs. A. N. Omelyanchouk, F. V. Kusmartsev, Jeff Young, and Franco Nori, and to all my colleagues at the University of British Columbia, D-Wave Systems Inc., RIKEN, and Loughborough University, with whom I had the pleasure and honour

to collaborate during this time. My special thanks to Dr. Uki Kabasawa, who translated the first edition of this book to the Japanese, and whose questions and helpful remarks contributed to improving the book you hold.

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# Preface to the First Edition

This book grew out of lectures that I gave in the framework of a graduate course in quantum theory of many-body systems at the Applied Physics Department of Chalmers University of Technology and Göteborg University (Göteborg, Sweden) in 1992–1995. Its purpose is to give a compact and self-contained account of basic ideas and techniques of the theory from the “condensed matter” point of view. The book is addressed to graduate students with knowledge of standard quantum mechanics and statistical physics. (Hopefully, physicists working in other fields may also find it useful.)

The approach is—quite traditionally—based on a quasiparticle description of many-body systems and its mathematical apparatus—the method of Green’s functions. In particular, I tried to bring together all the main versions of diagram techniques for normal and superconducting systems, in and out of equilibrium (i.e., zero-temperature, Matsubara, Keldysh, and Nambu–Gor’kov formalisms) and present them in just enough detail to enable the reader to follow the original papers or more comprehensive monographs, or to apply the techniques to his own problems. Many examples are drawn from mesoscopic physics—a rapidly developing chapter of condensed matter theory and experiment, which deals with macroscopic systems small enough to preserve quantum coherence throughout their volume; this seems to me a natural ground to discuss quantum theory of many-body systems.

The plan of the book is as follows.

In Chapter 1, after a semi-qualitative discussion of the quasiparticle concept, Green’s function is introduced in the case of one-body quantum theory, using Feynman path integrals. Then its relation to the  $\mathcal{S}$ -operator is established, and the general perturbation theory is developed based on operator formalism. Finally, the second quantization method is introduced.

Chapter 2 contains the usual zero-temperature formalism, beginning with the definition, properties, and physical meaning of Green’s function in the many-body system, and then building up the diagram technique of the perturbation theory.

In Chapter 3, I present equilibrium Green’s functions at finite temperature, and then the Matsubara formalism. Their applications are discussed in relation to linear response theory. Then Keldysh technique is introduced as a means to handle essentially nonequilibrium situations, illustrated by an example of quantum

conductivity of a point contact. This gives me an opportunity to discuss both Landauer and tunneling Hamiltonian approaches to transport in mesoscopic systems.

Finally, Chapter 4 is devoted to applications of the theory to the superconductors. Here the Nambu–Gor’kov technique is used to describe superconducting phase transition, elementary excitations, and current-carrying state of a superconductor. Special attention is paid to the Andreev reflection and to transport in mesoscopic superconductor–normal metal–superconductor (SNS) Josephson junctions.

Each chapter is followed by a set of problems. Their solution will help the reader to obtain a better feeling for how the formalism works.

I did not intend to provide a complete bibliography, which would be far beyond the scope of this book. The original papers are cited when the results they contain are either recent or not widely known in the context, and in a few cases where interesting results would require too lengthy a derivation to be presented in full detail (those sections are marked by a star\*). For references on more traditional material, I have referred the reader to existing monographs or reviews.

For a course in quantum many-body theory based on this book, I would suggest the following tentative schedule<sup>1</sup>:

Lecture 1 (Sect. 1.1); Lecture 2 (Sect. 1.2.1); Lecture 3 (Sect. 1.2.2, 1.2.3); Lecture 4 (Sect. 1.3); Lecture 5 (Sect. 1.4); Lecture 6 (Sect. 2.1.1); Lecture 7 (Sect. 2.1.2); Lecture 8 (Sect. 2.1.3, 2.1.4); Lecture 9 (Sect. 2.2.1, 2.2.2); Lecture 10 (Sect. 2.2.3); Lectures 11–12 (Sect. 2.2.4); Lecture 13 (Sect. 3.1); Lecture 14 (Sect. 3.2); Lecture 15 (Sect. 3.3); Lecture 16 (Sect. 3.4); Lecture 17 (Sect. 3.5); Lecture 18 (Sect. 3.6); Lecture 19 (Sect. 3.7); Lecture 20 (Sect. 4.1); Lecture 21 (Sect. 4.2); Lecture 22 (Sect. 4.3.1, 4.3.2); Lecture 23 (Sect. 4.3.3, 4.3.4); Lecture 24 (Sect. 4.4.1, 4.4.2); Lectures 25–26 (Sect. 4.4.3–5); Lecture 27 (Sect. 4.5.1); Lecture 28 (Sect. 4.5.2–4); Lecture 29 (Sect. 4.6).

## Acknowledgments

I am deeply grateful to Professor R. Shekhter, collaboration with whom in preparing and giving the course on quantum theory of many-body system significantly influenced this book.

I wish to express my sincere thanks to the Institute for Low Temperature Physics and Engineering (Kharkov, Ukraine) and Professor I. O. Kulik, who first taught me what condensed matter theory is about; to the Applied Physics Department of Chalmers University of Technology and Göteborg University (Göteborg, Sweden) and Professor M. Jonson, and to the Physics and Astronomy

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<sup>1</sup> Based on a “two hours” (90 min) lecture length.

Department of the University of British Columbia (Vancouver, Canada) and Professor I. Affleck for support and encouragement.

I thank Drs. S. Gao, S. Rashkeev, P. Hessling, R. Gatt, and Y. Andersson for many helpful comments and discussions.

Last, but not least, I am grateful to my wife Irina for her unwavering support and for actually starting this project by the comment, “Well, if you are spending this much time on preparing these handouts, you should rather be writing a book,” and to my daughter Ekaterina for her critical appreciation of the illustrations.

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Quantum Theory of Many-Body Systems  
Techniques and Applications

Zagoskin, A.

2014, XVI, 280 p. 154 illus., Hardcover

ISBN: 978-3-319-07048-3