

Chapter 2

Aftershock Cascade of the 3.11 Earthquake (2011) in Fukushima-Miyagi Area

Yoji Aizawa and Satoru Tsugawa

Abstract Details of the aftershock cascade in [35°–40°N, 140°–145°E] are reported from the viewpoint of three empirical laws; the Omori law, the Gutenberg-Richter law and the Weibull law for the interoccurrence times, and the universal relationship among those three empirical laws is theoretically derived under the quasi-stationary condition. The generalization of the Omori law enables us to derive the extrapolation formula of the GR law, and the multi-fractal relation confirmed universally in moving ensembles combines the magnitude distribution and the interoccurrence time distribution. Furthermore, the generalized Omori formula is interpreted in terms of the quasi-stationary interoccurrence time distribution.

2.1 Introduction

Here we give a brief sketch and comments about the empirical laws in seismic statistics, which will be used in the latter analysis.

2.1.1 Aftershock Frequency: Omori Law

The rate of aftershocks is first formulated by Omori in 1894, where the aftershock frequency dN/dt was very well adjusted by,

$$\frac{dN}{dt} \propto t^{-p}, \quad (2.1)$$

Y. Aizawa (✉) • S. Tsugawa

Department of Applied Physics, Advanced School of Science and Engineering,
Waseda University, Shinjuku, Tokyo 169-8555, Japan
e-mail: aizawa@waseda.jp; s.tsugawa@aoni.waseda.jp

Here N stands for the total number of aftershocks and t is the time measured from the main shock. As the p -value is close to unity, the aftershock frequency leads to a logarithmic scaling of N in relatively long time behavior,

$$N(t) \approx \ln(t + c), \quad (2.2)$$

This formula is often called the Omori law [8]. Some generalizations of the Omori formula were pursued to obtain the better prediction of aftershocks. Enya (1901) discussed by the following form [3],

$$\frac{dN}{dt} \propto \ln \left(1 + \frac{1}{t + c} \right), \quad (2.3)$$

and another generalization by Utsu [9] is,

$$\frac{dN}{dt} \propto (c + t)^{-p}, \quad (2.4)$$

where c means the characteristic time in each formula. In these generalizations, it should be noted that the stationary activity, which will be realized at $t \rightarrow \infty$, is discarded in practical treatment. In the latter part of this paper, some refined aspects of aftershocks will be reported based on the generalized form of the Omori law.

Aftershocks are obviously nonstationary phenomena and reveal remarkable clustering where a huge number of aftershocks are directly induced by the main shock. Moreover, the aftereffect of the big main shock remains for long time, for instance, in the case that Omori reported in 1894, the Omori formula is well justified for very long time more than 80 years since the main shock. From these facts we are obliged to be skeptic whether we can admit any stationary statistical laws in the sequence of earthquakes or not. In the present paper, however, we define the stationary regime from a practical point of view, that is to say, where the seismic activity is not in high level but is relatively static one. We assume that the ensemble which describes the stationary regime could be obtained if we consider a very long time series of shocks happened in the definite area. Indeed, some important laws are known in the stationary ensembles, which will be briefly introduced in what follows.

2.1.2 Intensity Distribution: Gutenberg-Richter Law

Gutenberg and Richter [4] suggested, by use of the magnitude m introduced by Richter, that the cumulative number of earthquake $n(m)$ (for the magnitude larger than m) obeys the GR formula,

$$\ln n(m) \approx a - bm, \quad (2.5)$$

where a and b are assumed to be constant. As the GR law was approximately confirmed in many cases of worldwide data, the magnitude became a useful measure which characterizes the intensity of earthquake in stationary regime.

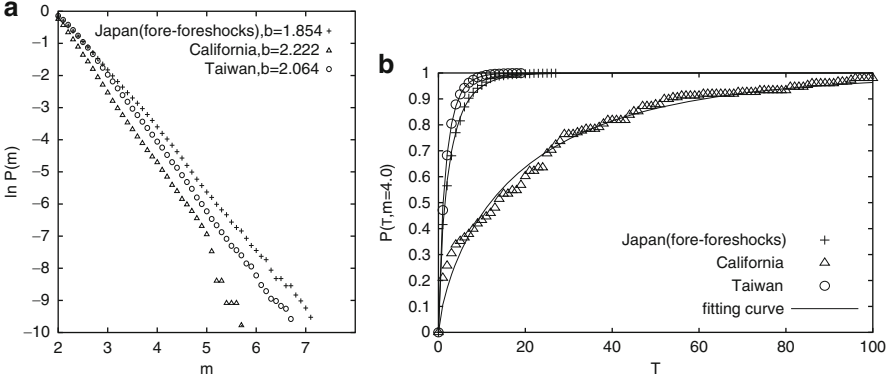


Fig. 2.1 The Gutenberg-Richter law (a) and the Weibull law (b) in stationary regime. The fore-foreshock region for 10 years (2001–2010) in Fukushima-Miyagi area (+) is compared with other results for worldwide data in South California (Δ) and in Taiwan (\circ)

Figure 2.1 shows the GR law realized clearly. To compare with other worldwide data from 2001 to 2010, we showed the GR law for the earthquakes in Taiwan (21° – 26° N, 119° – 123° E) and South California (32° – 37° N, 114° – 122° E). Though the mean magnitudes are different, the GR law seems to be well satisfied in each case.

In the nonstationary regime, the GR formula reveals peculiar deviations from the exponential one as shown in latter sections, where we give the extrapolation formula of the GR law in aftershocks.

2.1.3 Interoccurrence Time Distribution: Weibull Law

Interoccurrence times play the most important role in the prediction theory of earthquakes. When the cutoff magnitude m increases, the corresponding interoccurrence time τ is prolonged in statistical sense. So the interoccurrence time distribution is parameterized by the magnitude m , i.e., $P(\tau; m)$. If we fix the threshold value m , the sequence of interoccurrence times defines a renewal process. The purpose of the interoccurrence time statistics is to determine the functional form of the cumulative probability $P(\tau; m)$ and to find out the universal nature hidden behind those statistical distributions.

Recently, we have shown that the interoccurrence time distribution $P(\tau; m)$ is very well adjusted by the superposition of the Weibull distribution $P_w(\tau; m)$ and the log-Weibull one $P_{lw}(\tau; m)$ for many natural earthquakes in stationary regime [2, 5–7].

$$P(\tau; m) = pP_w(\tau) + (1 - p)P_{lw}(\tau) \quad (2.6)$$

where p is a parameter ($0 \leq p \leq 1$). P_w and P_{lw} are written by,

$$P_w(\tau) = 1 - \exp \left[- \left(\frac{\tau}{\beta_1} \right)^{\alpha_1} \right], \quad (2.7)$$

$$P_{lw}(\tau) = 1 - \exp \left[- \left(\frac{\log(\tau/k)}{\beta_2} \right)^{\alpha_2} \right]. \quad (2.8)$$

Here α_i , β_i , k , and p are parameters depending on the cutoff magnitude m , but when the magnitude m increases, the contribution of the log-Weibull distribution sharply decreases. Furthermore, P_{lw} contributes effectively only in the short time behavior of $P(\tau; m)$, and the dominant part of $P(\tau; m)$ comes from the Weibull distribution (Fig. 2.1). In the paper [2], it is shown that the Weibull fittings well adjust the nonstationary case of aftershocks as well, though the parameters (α, β) depend on the time t .

2.1.4 Multifractal Relation in Stationary Regime

The Weibull parameters (α, β) given by the function of m ,

$$\alpha = f_\alpha(m), \text{ and } \beta = f_\beta(m) \quad (2.9)$$

are called the multi-fractal relations, which characterize the magnitude scales as well as the time-scales in the shock sequence under consideration. The multifractal relations obey the following universal form [1],

$$\beta_m e^{-b(m-m_\mu)} \Gamma \left(1 + \frac{1}{\alpha_m} \right) = e^{-k_{EQ}}, \quad (2.10)$$

and this is applied for many cases [2, 6, 7]; k_{EQ} is a constant that determines the mean interval of two successive shocks, and m_μ the minimum cutoff magnitude in our analysis ($m_\mu = 2$). Figure 2.2 shows the multi-fractal diagram (in rescaled form) in stationary regime, where m_c stands for the reference magnitude satisfying $\alpha = 1$ and $\beta_c = c$ at $m = m_c$.

2.2 Data Analysis Toward Aftershock Statistics

Figure 2.3 shows the time series of the shock sequence m_t before and after the main shock ($M9.0$) on March 11, 2011. One can see that another big shock ($M7.3$) had occurred on March 9. We clearly recognize that there are three regions; (i) aftershock

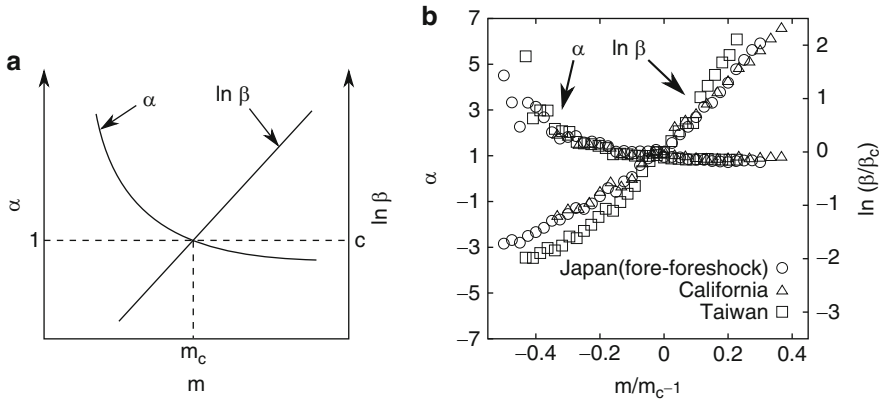


Fig. 2.2 Multi-fractal diagram; (a) theoretical curves, (b) three cases in stationary regime for 10 years (2001–2010) – the fore-foreshock region in Fukushima-Miyagi area (\circ), Taiwan (\square), and South California (\triangle)

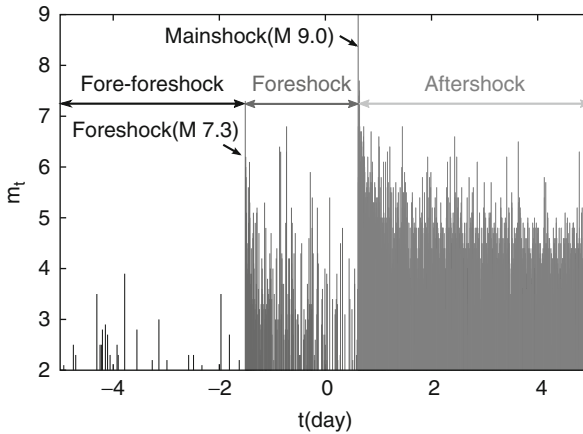


Fig. 2.3 Time series of shocks during 10 days before and after the main shock at March 11, 2011 ($t = 0$) in Fukushima-Miyagi area (JMA database). There are three typical regions; (i) Aftershock region (light gray), (ii) Foreshock region (dark gray), (iii) Fore-foreshock region (black)

region (light gray), (ii) foreshock region (dark gray), and (iii) fore-foreshock region (black). In the fore-foreshock region, seismic activity is nearly stationary and the density of earthquakes is relatively low, but in the foreshock region and aftershock region the density as well as the intensity of shocks are much enhanced. The number of earthquakes in the foreshock region is nearly 470, but in the aftershock region 72,636 shocks occurred for 20 months (3.11, 2011–11.11, 2012) and the aftereffect of the main shock continues still now.

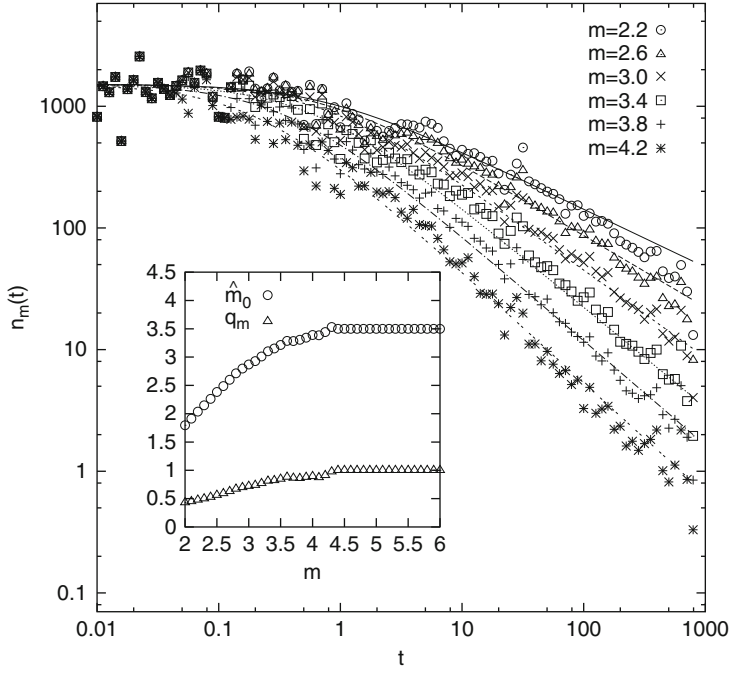


Fig. 2.4 Time courses of the shock frequency $n_m(t)$

In this section, the detailed structure in the Omori law is elucidated, and the statistical aspects in the nonstationary shock sequence are mainly studied by using the moving ensembles, where the interval of each ensemble is defined by the span $[t - \Delta/2, t + \Delta/2]$ at $\Delta = 100$ days (fixed). More details are seen in Ref. [2].

2.2.1 Refined Formula of the Omori Law

Figure 2.4 shows the aftershock frequency (per 1 day) $n_m(t) (= dN_m(t)/dt)$, where $N_m(t)$ is the cumulative number of aftershocks in $[0, t]$ (for the magnitude larger than m). Each curve is well fitted by the following forms,

$$n_m(t) = d(1 + t/c_m)^{-q_m}, c_m = e^{-\hat{b}(m - \hat{m}_0)}. \quad (2.11)$$

$N_m(t)$ is given by the q_m -extension; $N_m(t) = dc_m(1 - q_m)^{-1}((1 + t/c_m)^{1-q_m} - 1)$, and \hat{m}_0 and q_m are monotonically increasing, but they are almost constant for $m > m_0 (\simeq 4.0)$, $q_m \simeq 1$ and $\hat{m}_0 \simeq 3.5$.

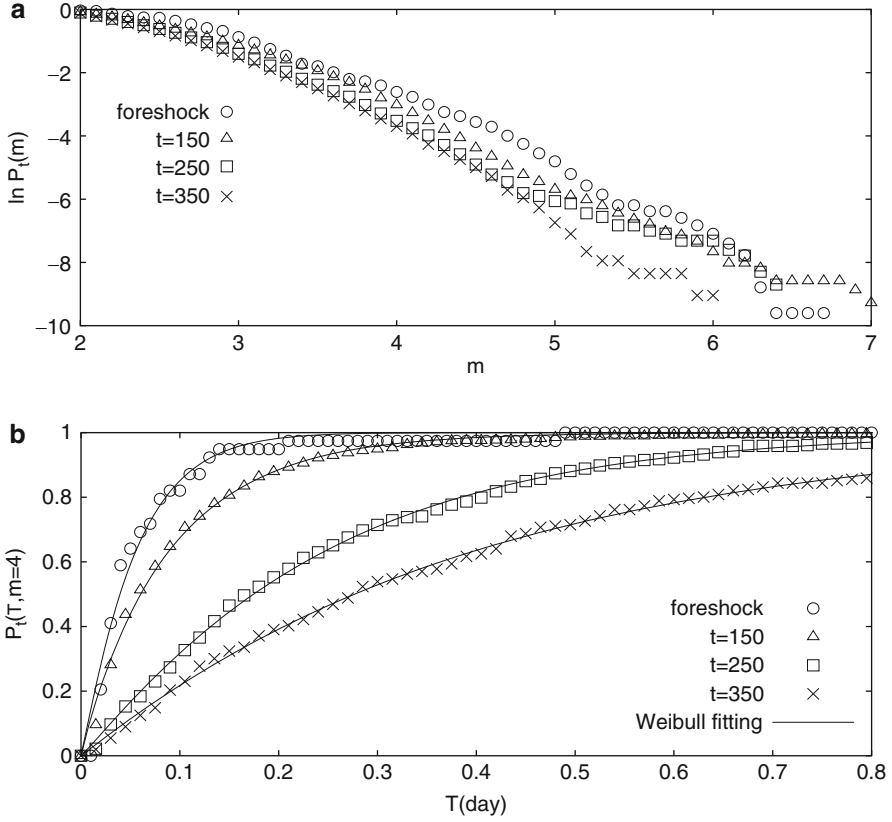


Fig. 2.5 Temporal change of the aftershock statistics in moving ensembles; (a) the Gutenberg-Richter law $P_t(m)$, (b) the interoccurrence time distribution $P_t(\tau, m)$ ($t = 150$ (Δ), 250 (\square) and 350 (\times))

2.2.2 Generalization of Multi-fractal Relation in Moving Ensembles

Figure 2.5 displays the magnitude distribution $P_t(m)$ and the interoccurrence time distribution $P_t(\tau : m)$ for three moving ensembles, which enable us to derive the multi-fractal relation in each time span. One of the remarkable points is that the Weibull law is well satisfied, and that the GR law is a convex function of which fitting curves are given in the next section. The universal aspect discussed in the previous section appears even in the aftershock region, and the rescaled multi-fractal universality is given in Fig. 2.6. Only difference from the stationary case is that the earthquake constant k_{EQ} is not a constant, but is a certain function

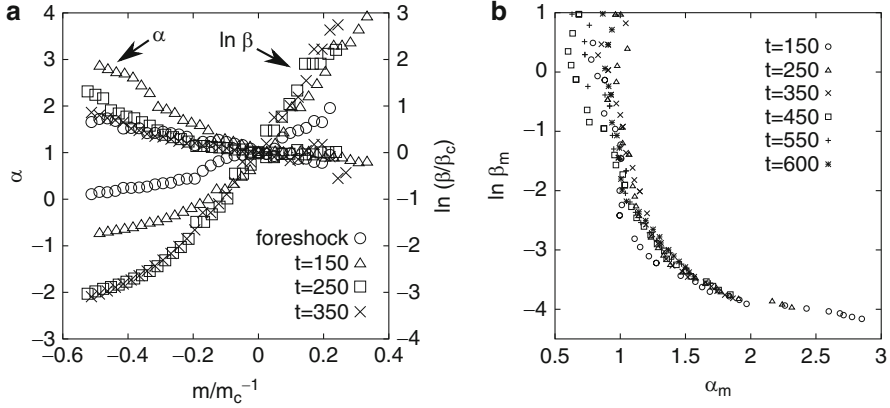


Fig. 2.6 Temporal change of rescaled multi-fractal diagrams (a) and the $(\alpha, \beta)_t$ diagram (b) for moving ensembles; (i) $k_{EQ} = 4.66$ at $t = 150$, (Δ), (ii) $k_{EQ} = 4.07$ at $t = 250$, (\square), (iii) $k_{EQ} = 3.81$ at $t = 350$, (\times). The universal correlation between $\alpha_m(t)$ and $\beta_m(t)$ is confirmed

of time $k_{EQ}(t)$, though $k_{EQ}(t)$ does not depend on the magnitude m except for small tolerable errors. Furthermore, the $(\alpha, \beta)_t$ diagrams suggest that the universal correlation exists between two multifractal forms $\{f_\alpha(m, t), f_\beta(m, t)\}$ even in the nonstationary regime of aftershocks.

2.3 Unified Formulae and Cascade in Aftershocks

Aftershocks are non-stationary process, but the results shown in the previous section demonstrate that the aftershock-sequence obeys some regular statistical rules in each moving ensemble. Here we theoretically consider the temporal change of the statistical laws in the aftershock-sequence under the assumption that the distribution functions are slowly varying in contrast to the characteristic time scale of shock-intervals. This is the quasi-stationary assumption in aftershock statistics, and then the real process of natural aftershocks can be understood as the mean behaviors of the quasi-stationary distributions. Here we theoretically study the detailed mechanism in the aftershock cascade of the 3.11 EQ.

2.3.1 Extrapolation Formula of the Gutenberg-Richter Law

Denote the magnitude distribution function at the time t by $P_t(m)$, then the generalized form of the Omori law (Eq. (2.11)) leads to,

$$\begin{aligned}
P_t(m) &= \text{Probability}(\text{the magnitude} \geq m) \\
&= e^{-\hat{b}(m-m_\mu)} \left(1 - \frac{c_m - c_{m_\mu}}{t + c_m} \right), \tag{2.12}
\end{aligned}$$

where m_μ stands for the minimum cutoff magnitude and $q_m \simeq 1$ is assumed for the sake of simplicity. When t goes to large enough, the exponential formula of the GR law in stationary case is exactly recovered as $c_m \doteq e^{-\hat{b}(m-\hat{m}_0)}$.

The same idea is extended to the moving ensemble $[t - \frac{\Delta}{2}, t + \frac{\Delta}{2}]$, and the magnitude distribution function $P_{t,\Delta}(m)$,

$$\begin{aligned}
P_{t,\Delta}(m) &= \frac{N_m(t + \Delta/2) - N_m(t - \Delta/2)}{N_{m_\mu}(t + \Delta/2) - N_{m_\mu}(t - \Delta/2)} \\
&= \frac{c_m}{c_{m_\mu}} \frac{\ln \left(1 + \frac{\Delta}{c_m + t - \Delta/2} \right)}{\ln \left(1 + \frac{\Delta}{c_{m_\mu} + t - \Delta/2} \right)}, \tag{2.13}
\end{aligned}$$

Here the exponential formula of the GR law is also recovered as t goes to large, but transient behaviors depend on the interval of the ensemble Δ . In the case of the generalization by the q_m -extension,

$$P_{m,\Delta}(t) = \frac{c_m^{q_m}}{c_{m_\mu}^{q_{m_\mu}}} \frac{1 - q_{m_\mu}}{1 - q_m} \frac{(t + c_m + \Delta/2)^{1-q_m} - (t + c_m - \Delta/2)^{1-q_m}}{(t + c_{m_\mu} + \Delta/2)^{1-q_{m_\mu}} - (t + c_{m_\mu} - \Delta/2)^{1-q_{m_\mu}}} \tag{2.14}$$

The GR parameter is modified by q_m , though the exponential form is realized when t goes to large. The GR law in Fig. 2.5 is well explained by Eq. (2.14).

2.3.2 Multi-fractal Relation and the Interoccurrence Time Distribution

Consider the case for small Δ ($\doteq 1$ day), and denote the interoccurrence time distribution at the time t for the cutoff magnitude m by $P_{t,m}^W(\tau)$, where τ is the successive shock-interval. The mean interval $\langle \tau \rangle_{m,t}$ is related to the shock frequency $n_m(t)$ at arbitrary value of m ,

$$\langle \tau \rangle_{m,t} n_m(t) = 1 \tag{2.15}$$

Therefore, if we use the Weibull parameters $\alpha_m(t)$ and $\beta_m(t)$, the general form (Eq. (2.11)) of the Omori law leads us to,

$$\beta_m(t) \Gamma \left(1 + \frac{1}{\alpha_m(t)} \right) \left(1 + \frac{t}{c_m} \right)^{-q_m} = d^{-1} \tag{2.16}$$

When t goes to large enough, this ensures the previous universal relation in the stationary ensemble, and that the earthquake constant $k_{EQ}(t)$ is connected to the coefficient d and other parameters (\hat{b}, \hat{m}_0).

In the case of moving ensembles $[t - \frac{\Delta}{2}, t + \frac{\Delta}{2}]$, the Weibull parameters ($\alpha_{m,\Delta}(t)$ and $\beta_{m,\Delta}(t)$) satisfy the following time-dependent relation (in re-scaled form) for $m > \hat{m}_0$,

$$\left(\frac{\beta_{m,\Delta}(t)}{\beta_{m_c,\Delta}(t)} \right) \Gamma \left(1 + \frac{1}{\alpha_{m,\Delta}(t)} \right) \frac{\ln(1 + \frac{\Delta}{t+c_m-\Delta/2})}{\ln(1 + \frac{\Delta}{t+c_{m_c}-\Delta/2})} e^{-\hat{b}(m-m_c)} = 1. \quad (2.17)$$

In this paper we consider only the Weibull distribution for $P_{t,m}^W$, but the more consistent way to derive the interoccurrence time distribution is given by the quasi-stationary assumption. Here we discuss only the idea toward the theoretical unification between the Omori law and the generalized interoccurrence time statistics. Consider the interoccurrence time distribution density $p_{t,m}(\tau) (= \frac{dP_{t,m}(\tau)}{d\tau})$, then the expectation value of the renewal event in $[t, t + \tau]$ is determined by using the convolution of the density $p_{t,m}(\tau)$, i.e., $\sum_{r=1}^{\infty} P_{t,m}^{r*}$, where $P_{t,m}^{r*}$ denotes the r -th convolution. The number of events in $[t, t + \tau]$ is surmised to obey the generalized formula Eq. (2.11) if the aftershock sequence is quasi-stationary,

$$N_m(t + \tau) - N_m(t) = \sum_r P_{t,m}^{r*}(\tau). \quad (2.18)$$

This implies that the interoccurrence time distribution can be derived only from the aftershock statistics $N_m(t)$. The distribution density is given by the inverse transformation $\mathcal{L}_\tau p_{t,m}(\tau) = \hat{p}_{t,m}(s)$, and the interoccurrence time distribution is subordinate (for small τ) to the Omori law,

$$\hat{P}_{t,m}(s) = 1 - \frac{1}{1 + ac_m \hat{n}_m(as)}, \quad (2.19)$$

where $\hat{n}_m(s)$ is the Laplace transformation of $n_m(\tau)$ and $a = 1 + t/c_m$.

2.3.3 Birth and Death Cascade in Aftershocks

We have not yet succeeded to derive the theoretical multi-fractal relation (Fig. 2.6b) in nonstationary regime, but the data-analysis for the 3.11 EQ (2011) shows that there exist clear hierarchical time-dependent structures among different magnitude-scales. Here, the regularity hidden behind the nonstationary aftershock sequence will be formulated and the birth and death cascade in aftershocks is discussed based on the generalized Omori law at $q_m \simeq 1$ for the simplicity sake.

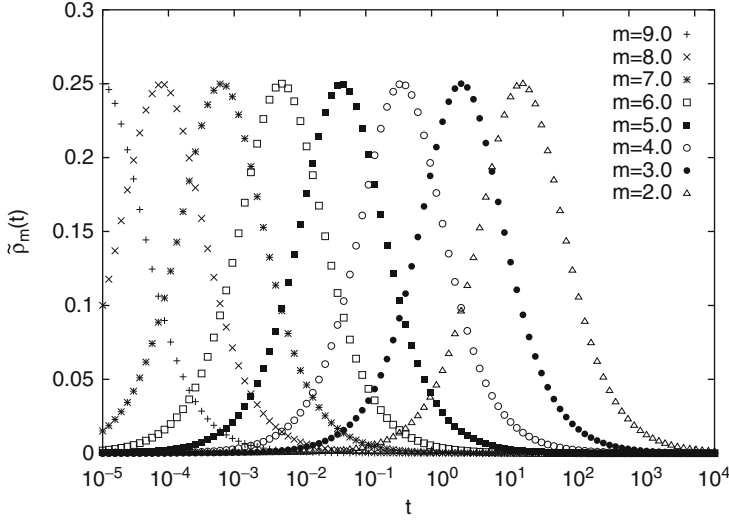


Fig. 2.7 The solitary wave propagation in the space of magnitude m ; $\tilde{\rho}_m(t) = (d\hat{b})^{-1}\rho_m(t)$

We define the magnitude density $\rho_m(t)$ that describes the number of shocks with the magnitude m ,

$$\begin{aligned}\rho_m(t) &= \left| \frac{dn_m(t)}{dm} \right| \\ &= d\hat{b}(t/c_m)(1 + t/c_m)^{-2}, \quad (\ln c_m = \hat{b}(\hat{m}_0 - m))\end{aligned}\quad (2.20)$$

By introducing a scaled variable $z = t/c_m$, it is known that the magnitude density obeys a universal behavior ($\rho_m(t) = \rho(z)$), in all magnitude classes. This indicates that the magnitude scale controls the time scale of shocks, and vice versa. The control mechanism is formulated in the following by using new scaled variables, $\tilde{z}(= \ln z = \tau + \hat{b}m)$ and $\tau(= \ln t - \hat{b}m_0)$, i.e.,

$$\begin{aligned}n_m(t) &= \frac{d}{1 + e^{\tilde{z}}}, \text{ and} \\ \rho_m(t) &= \frac{d\hat{b}e^{\tilde{z}}}{(1 + \tilde{z})^2} = -\hat{b}\frac{dn_m(t)}{d\tilde{z}} \\ &= -\hat{b}n_m(1 - n_m/d)\end{aligned}\quad (2.21)$$

One can see that these solutions represent the typical nonlinear wave (kink and soliton) in m -space, of which traveling velocity is determined by $\hat{b}^{-1}(=dm/d\tau)$. Figure 2.7 shows the cascade process of $\rho_m(t)$ in (m, τ) space, which corresponds to the soliton-propagation in m -space.

It is difficult to derive the nonlinear wave equation uniquely for the aftershock cascade only from the above special solution, but we can surmise the essential mechanism leading to the solitary wave mentioned above. From Eq. (2.11), $\rho_m(\tau)$ obeys,

$$\frac{\partial \rho_m}{\partial \tau} = \rho_m \{n_m/d - (1 - n_m/d)\}, \quad (2.22)$$

in other words, by using the relations $n_m = \int_{m_\mu}^{m_M} \rho_m dm$, and $\int_{m_\mu}^{m_M} \rho_m dm = d$,

$$\frac{\partial \rho_m}{\partial \tau} = \frac{1}{d} \rho_m \left\{ \int_m^{m_M} \rho'_m dm' - \int_{m_\mu}^m \rho'_m dm' \right\}, \quad (2.23)$$

where m_M and m_μ indicate the maximum and the minimum magnitude in practical analysis respectively, and $m_M = \infty$ and $m_\mu = -\infty$ are assumed in the present treatment. The 1st and the 2nd terms of Eq. (2.23) show the growth and decay effects of $\rho_m(\tau)$. One can check easily that Eq. (2.21) is the particular solution of Eq. (2.23).

The interaction between two shocks is not known clearly, but the interaction obtained in Eq. (2.23) seems to give us a hint, which may enlighten on the hidden coupling mechanism among many shocks with different magnitude-scales. As an approximation, let us consider the birth and death model described by the generalized transition probability $W_{m,m'}$,

$$\frac{d\rho_m}{d\tau} = \int_{m_\mu}^{m_M} W_{m,m'} \rho'_m dm' - \int_{m_\mu}^{m_M} W_{m,m'} \rho_m dm'. \quad (2.24)$$

If we assume $W_{m,m'} \propto \rho_m(m' \geq m)$ and $W_{m,m'} = 0(m' < m)$, Eq. (2.23) is recovered and the birth and death cascade shown in Fig. 2.7 is obtained again as a particular solution; as a matter of course, there are many other possible solutions in Eq. (2.24). The details studied in this section will be reported in the next paper by using much longer data of the aftershocks.

2.4 Discussions and Prospects

The aftershock cascade reveals a very clear regularity not only in the birth and death process but also in the statistical aspects. In this paper, some parts of the regularity are confirmed even in the short time region $t \simeq 10^{-2}$ (days). The more precise studies immediately after the big shock seem to be important to elucidate the precursive mechanism leading to the main shock. The onset time $t = 0$ is the critical point in statistical seismology, but some analytical continuations beyond the singular point must be pursued to obtain the information in the prestage of big

shocks ($t \leq 0$). The birth and death model may give the hint for this end, where some latent variables, for instance, the stress accumulated in the plate interface, should be taken into account. These subjects are still open and in our future challenge.

References

1. Aizawa Y (2011) Foundations of earthquake statistics in view of non-stationary chaos theory. *BusseiKenkyu* (Kyoto University) 97(3):309
2. Aizawa Y, Hasumi T, Tsugawa S (2013) Seismic statistics: universality and interim report on the 3. 11 Earthquake (2011) in Fukushima-Miyagi Area. *Int J Nonlin Phen Comp Sys* 16(2):116–130
3. Enya O (1901) Comments on the aftershocks of earthquakes. *Rep Imp Earthq Investig Comm* 35:35. (in Japanese)
4. Gutenberg B, Richter CF (1956) Magnitude and energy of earthquakes. *Ann Geofis* 9:1
5. Hasumi T, Chen C, Akimoto T, Aizawa Y (2010) The Weibull/Log Weibull transition of interoccurrence time for synthetic and natural earthquakes. *Tectonophys* 485:9
6. Hasumi T, Chen C, Akimoto T, Aizawa Y (2012) The Weibull/Log Weibull transition of interoccurrence time of earthquake. In: D'Amico S (ed) *Earthquake Research and Analysis*. InTech, Croatia, Chap 1, pp 3–24
7. Hasumi T, Chen C, Akimoto T, Aizawa Y (2013) Statistical seismicity in view of complex systems. In: Konstantinou K (ed) *Earthquakes: triggers, environmental impact and potential hazards*. Nova Science Publisher, New York, Chap 5, pp 109–163
8. Omori F (1894) On the aftershocks of earthquakes. *J Coll Sci Imp Univ Tokyo* 7:111
9. Utsu T (1961) Statistical study on the occurrence of aftershocks. *Geophys Mag* 30:521

Nonlinear Phenomena in Complex Systems: From Nano
to Macro Scale

Matrasulov, D.; Stanley, E.H. (Eds.)

2014, XIII, 310 p. 130 illus., 92 illus. in color., Softcover

ISBN: 978-94-017-8707-9