

Preface

Control Theory and Control Systems Design has a history of over sixty years and covers a wide range of topics, many of which have relevance to Control Engineering practice and the process of education and training. Iterative Learning Control (ILC for short) is an area of control theory and applications that has risen in its relevance and popularity over the past 30 years. It is seen, for example, in applications of control where a machine is required to complete a specific tracking task over a finite time interval $[0, T]$ to a high degree of accuracy. The accuracy required is taken to be beyond the practical capabilities of feedback control. The additional features of the system that makes improved accuracy possible are the assumptions that the task is repeated an indefinite number of times, so that (possibly large) errors are tolerable in the first few repetitions and that information on previously seen tracking errors, inputs and other signals can be measured and stored.

Systems with these properties include manufacturing and rehabilitation robotics and systems for mechanical testing of components and component assemblies in the automotive and related industries. In the automotive arena, this includes the testing of whole car performance using laboratory test rigs to replicate accurately road or track conditions. The words Iterative Control are used to describe the repetitive nature of the system behaviour and the need for control action. The word iterative is chosen by analogy with iteration in numerical analysis where the repetition is used to sequentially (iteration to iteration) reduce the error in obtaining the solution of, say, a set of algebraic equations. The natural choice of the name “repetitive control” was rejected by the community as this has been used previously to describe feedback control design for the tracking of periodic reference signals of known period.

ILC is naturally a branch of control theory and control systems’ design but differs from its more traditional counterparts owing to the need to include both times during the repetition (iteration or trial) and the iteration index in data descriptions. This is done by setting $f_k(t)$ to be the value of a signal f at time “ t ” on iteration “ k ”. The control problem therefore has two independent variables (t, k).

Control of the system not only requires proper and effective control during an iteration (using normal classical criteria for control performance on $[0, T]$) but also includes the need to control performance over successive iterations with indices $k = 0, 1, 2, 3, 4, \dots$. The aim is to improve performance from iteration to iteration. The ultimate aim is to provide highly accurate tracking which, in the mathematical limit as “ k ” tends to infinity becomes *perfect tracking*. Ideally, this property will be retained in the presence of modelling errors.

The mechanism for achieving this perfect tracking is the transfer of data from iteration to iteration. This transfer of data is similar to the process of “learning” from experience. Hence the subject title “Iterative Learning Control”. Control design in ILC is the choice of control algorithm to generate an input signal magnitude $u_k(t)$ at time t on iteration k such that the application of this signal over the interval $t \in [0, T]$ leads to improved tracking as described above. In more details, design requirements include:

1. Stability during each iteration.
2. Asymptotic convergence of the tracking error to zero as $k \rightarrow \infty$.
3. Acceptable behaviour of the tracking error from iteration to iteration by,
 - a. achieving reasonable improvement within a few iterations and
 - b. ensuring acceptable convergence and performance despite plant modelling errors.
4. Implementation options that range from advanced control to simpler forms.

A control theory for ILC is hence at least as rich as that seen in classical control with opportunities for feedback and feedforward structures, consideration of output feedback and state feedback strategies, the possibilities for time domain and frequency domain design tools, adaptive and predictive algorithms, optimal control and many more. This is not to mention the need for linear, nonlinear and hybrid systems versions of the theories! A single text covering the whole spectrum of possibilities is not feasible and, indeed, many of these problems are not yet fully analysed and solved to produce design strategies. For this reason the text examines one particular coherent body of knowledge within the current ILC spectrum with the aim of, first, bringing the work together in an integrated whole for researchers, students and interested users and, second, presenting new results, algorithms and insights to add value to the literature.

More precisely, this text concentrates on one particular paradigm and focusses on linear systems, although it is noted that extensions to cover the case of some nonlinear systems are possible using, for example, linearization-based methodologies. Motivated by the natural desire to systematically reduce the tracking error from iteration to iteration, the chosen paradigm is that of the use of optimization as a tool for algorithm design. The text covers several model types including both single-input, single-output (SISO) and multi-input, multi-output (MIMO)

continuous and discrete time, state-space systems. There is some emphasis on discrete systems as the essentials of the theory and algorithm development are more easily derived and understood in this format. It is also consistent with the likely implementation using digital control hardware and software and notionally provides an insight into the continuous time case by letting the sampling rate become infinite. Those reading the text will understand that the continuous time case introduces many additional mathematical complexities that need to be addressed for technical completeness. Much of this “infinite dimensional” material is included but the author believes that it is neither necessary for an understanding of the basic concepts nor, indeed, relevant to many applications.

Optimization is used as the paradigm for algorithm development because of its proven ability to guarantee, in the absence of modelling errors, monotonic reductions in a norm of the tracking error time series from each iteration k to the next iteration $k + 1$. This reduction has many interpretations, the simplest being the reduction of the “energy” in the tracking error. The energy interpretation is related to the use of the mean square error as the norm. The consequences of this simple observation are the subject of this text which, following a discussion of some of the relevant history and applications of ILC algorithms in the introductory chapter, explores the known consequences and design options available. The rest of the text is divided into several parts:

1. The design of algorithms might be expected to be model dependent. This is true but the construction of algorithms and the derivation of their properties is most easily seen using an operator description of the model and regarding input, output and other signals as elements of appropriate Hilbert spaces. This level of abstraction is analogous to the use of transfer function descriptions of linear systems. As an aid to study, the reader is provided with a chapter of mathematical methods which provides a summary of the essential properties of operators between Hilbert spaces and the geometrical interpretation of signals and their relationships. This is followed by material that looks closely at the structure of discrete time, state space models and the use of the (matrix) *supervector description*.
2. Many of the ideas and algorithms apply widely. The overview and formulation of ILC is constructed using the *language of operator theory* and recursive relationships of the typical form $e_{k+1} = Le_k, k \geq 0$ in a Hilbert space \mathcal{Y} . Convergence of the solution sequence $\{e_k\}_{k \geq 0}$ to a limit is related to the spectrum of the operator $L : \mathcal{Y} \rightarrow \mathcal{Y}$. If \mathcal{Y} is finite dimensional, the results are familiar from matrix theory but the case when \mathcal{Y} is infinite dimensional is more complex. Fortunately, for the purposes of optimization, only the case of L being self adjoint is relevant to this text. This case is however important for continuous time systems for example.

3. Although not obviously connected to optimization concepts, a fairly detailed examination of so-called *inverse model algorithms* is then provided. This approach assumes the existence of a left or right inverse to the plant operator and motivates a set of Iterative Control algorithms that provide simple monotonic error convergence at a rate described by a single gain parameter. This has practical value but its real value in the following optimization approaches lies in robustness analysis. The robustness theory presented in this text underpins the need for *positive real multiplicative modelling errors*. The results can be converted into frequency domain tests when the model has a discrete time, state-space form. By the end of the text, the reader will have made the surprising observation that this robustness theory applies, with suitable modification, to many of the algorithms described.
4. The first step towards the optimization paradigm is made in the consideration of *gradient* or *steepest descent* algorithms motivated by familiar numerical optimization methods. The gradient is naturally described by the adjoint plant operator and provides a link to the *co-state* equations that appear in optimal control theory. Similar algorithm analyses are provided together with a robustness analysis which again has a frequency domain form for discrete time state space systems that reappears for more complex algorithms later in the text.
5. The central section of the text contains the basic concepts of *Norm Optimal Iterative Learning Control* (NOILC) This algorithm has strong connections to linear quadratic optimal control theory and proceeds by minimization of a sequence of quadratic objective functions. For discrete state-space systems, it has a realization using either feedforward computations or familiar feedback structures based on solutions of time dependent matrix Riccati equations. The algorithm is a benchmark algorithm in the sense that monotonic error convergence is always guaranteed with convergence rates being influenced by the relative weighting of error and input terms. Greater insight into the behaviour is obtained using eigenvalue/singular value analysis and approximate eigenvectors constructed from frequency domain considerations. A robustness analysis is presented that has close links to inverse model and gradient results, probably because the NOILC paradigm can be regarded as being both a descent algorithm and an approximation to an inversion process.
6. The power of the NOILC philosophy and the use of operator descriptions are demonstrated by the descriptions of natural extensions of the algorithm. These include *intermediate point control problems*, *tracking for multi-rate sampled systems* and *systems where the initial condition on each iteration can be varied*. The concept of *Multi-task Algorithms* unifies these control problems as a mix of these and similar variations. All of these variations are described by the NOILC relationships but differ in form when converted from the operator description to more familiar state-space equations.
7. In all of these cases, the performance of the control can be enhanced by including one or both of two important features:

- a. by searching for a solution to the tracking problem that also minimizes an additional *Auxiliary Objective Function* representing additional control objectives and/or
- b. improving convergence rates by basing computations on a *multi-model* that enhances performance using optimization problems incorporating predictions of future errors. This resultant *Predictive Norm Optimal Iterative Learning Control* algorithm, when applied to state-space systems, can again be implemented using either feedforward computations or feedback realizations using the Riccati matrix and states of the multi-model.

In all cases, the robustness theory needed has a very similar structure to that of inverse model and gradient algorithms.

8. The apparent “perfection” of the NOILC approach is, however, misleading as proofs of convergence do not provide full information about rates of convergence. This is convincingly demonstrated by a consideration of *non-minimum-phase discrete, state-space systems*. For inverse model control, practical problems will then occur as the inverse system is unstable. For gradient and NOILC algorithms, the problem appears as a plateauing/flatlining effect where, after a period of fast error norm reduction, a long period of extremely slow convergence (represented by infinitesimal changes in error and error norm) can occur. In practical terms, the algorithm may fail to achieve acceptable tracking accuracy in the desired number of iterations despite the theoretical proof of ultimate convergence. The magnitude of the problem is assessed and shown to depend on the time interval length and the structure of the initial error e_0 .
9. The operator theory approach has great power but does not easily make possible, for example, the inclusion of constraints. The chapters on *Successive Projection* methodologies provide ways forward. They are equivalent to NOILC and its variants in the absence of constraints but, being based on projection onto closed, convex sets, allow many *convex constraints to be included* whether they be input and output constraints or constraint objectives for auxiliary variables. The geometry of the approach is used to *create accelerated algorithms* using extrapolation mechanisms. An interesting interpretation of the ideas is given in the section on *Loop Management Algorithms* where, rather than using ILC as an automated control algorithm, it can be used as a decision support aid for the human operators who are supervising the iteration process.
10. One, particularly interesting, algorithm that further demonstrates the potential of successive projection is the introduction of the new *Notch Algorithm* which uses successive projection onto sets defined by modified plant dynamics. The modification is parameterized by a single parameter σ^2 that can be related to a property of *Approximate Annihilation* of spectral components in the error close to that value. By varying σ^2 from iteration to iteration, different parts of the spectrum are reduced substantially producing rapid convergence in a systematic

way that resembles an approximate inverse algorithm. Robustness may however be reduced if the choices of σ^2 are not constrained.

11. The final chapter examines the idea of *Parameter Optimal Iterative Learning Control* (POILC) by replacing optimization over input signals by optimization over the parameters in a linear input update equation containing only known error data and a finite number of free parameters. This reduces the dimension of the optimization problem and defines the required parameter values using formulae and off-line computation. The approach has close links to NOILC (when the number of parameters is “large”) but, more generally, retains the property of monotonic error norm reduction and simplifies both the control implementation and its associated computations. The choice of parameterization is free for the user to choose but can be suggested by the previous chapters in the form of approximations to inverses, gradients and norm optimizers. An examination of the inverse model and other cases suggests that the approach will be robust provided that gains are “low” and a positive real condition is again satisfied.

The material in the text has been chosen and ordered to *tell a story* rather than reflect the historical development of the ideas. Emphasis is placed on providing rigour with understanding with the aim of informing and preparing readers for their own studies, research or applications. The text has many sections that hopefully help readers find, quickly, the issues that interest them. The reader should note the following declarations:

1. Although based on optimization concepts, many of the results and algorithms presented are new to the literature. For those algorithms that have been published in the open literature, the text provides a more detailed analysis of their properties and includes new, previously unreported robustness characterizations and methodologies.
2. The author has aimed for consistency in notation but has had to deal with the finite nature of the alphabet and the need for extensive use of subscripts and superscripts. For example, the subscript y_j will typically denote the signal on iteration j but it may also denote the j th element in the column matrix y . It is the author’s view that the context of the use of the symbol will provide the right interpretation.
3. The reader should understand that the text is motivated by the perceived practical needs of the subject when used in applications but also has the important objective of addressing what are thought to be important scientific issues associated with the topic, some of which are speculative. Applications of the ideas in controlled laboratory conditions has, to date, provided evidence that the ideas can translate into practically convergent procedures and this positive outcome has been supported by a number of industrial tests (subject to commercial, in confidence, agreements). However, it must be recognized that real-world systems present a whole spectrum of problems that the theory has not considered. No attempt will be made to list these problems exhaustively but they clearly include

the presence of severe nonlinearity, substantial noise, unrepeatable/unpredictable disturbances and time variation of parameters. In addition, hardware issues of a poor experimental setup or combinations of plant dynamics, parameter choices and reference signals that maximize the sensitivity of the approaches to such problems will also often need to be addressed. In view of these comments, the reader uses the ideas and algorithms at his or her own risk. The author has made every effort to ensure that the text is both accurate and understandable. He believes that the methods have both scientific and engineering merit but, given the uncertainties, he accepts no responsibility for any unacceptable outcomes arising from the use of the ideas or algorithms described in this text.

Sheffield, UK
April 2015

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<http://www.springer.com/978-1-4471-6770-9>

Iterative Learning Control

An Optimization Paradigm

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2016, XXVIII, 456 p., Hardcover

ISBN: 978-1-4471-6770-9