Chapter 2
The Theories of Motion in the Middle Ages and in the Renaissance

The period considered in this chapter (about ten centuries long) is that which, in its final part, is close to the time of Galileo and then, also in the light of the theories on the importance of some medieval works, it is essential to make clear in what context Galileo worked. This context includes a complex of theoretical treatments, starting from the earlier reworking and revisions of Aristotle's work in the Dark Ages until the work of the immediate forerunners of Galileo, that we are tempted to define as "the medieval mechanics" as a whole.

At first sight it may seem daring and superficial to give this appellation to the cultural production of a period which includes also the Renaissance. As a justification we can say that the works of Leonard, Tartaglia or even Benedetti makes one think more to an "autumn of the Middle Ages" (in the sense that one has to do with criticisms "from the inside") than to the dawn of a "new science". Obviously, as we say over and over again, our conclusion only concerns dynamics.

2.1 Preliminary Remarks

The cultural revival of the Middle Ages is usually traced back to the great work of translation (in Latin) of the works of Greek philosophers (above all Aristotle) both through their Arabic translations and directly from Greek.

As we know, the works of Aristotle mainly dedicated to natural philosophy are five (**Physics**, **On the Heavens** (**De Caelo**), **Generation and Corruption, Metheorologia** and **The Soul**). The work of translation in Latin of the Aristotelian corpus began starting from Boetius in the VI century, as above said, translating both second hand from Arabic, and directly from the available Greek manuscripts.

Almost all the translations were performed in the XII century. The most important translators from Arabic were Gerard of Cremona in the XII century and Michael Scot in the XIII century. Robert Grosseteste and William of Moerbeke were, in the

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XIII century, the most important translators from Greek. If we look at the list of the works of the natural philosophers of the Middle Ages, apparently we meet always with the same titles; in fact all of them write commentaries to the works of Aristotle (**Commentary on the Physics**, **Commentary on De Caelo**, etc.).

But these commentaries were not a mere comment on Aristotle, in the sense in which we use the term nowadays; they contained discussions and revisions, and new theories that were proposed for correcting or replacing those of Aristotle. In this way, everyone vehiculated his ideas through the commentaries on "the philosopher". The diffusion of the ideas contained in these works obviously occurred through the manuscripts and the lectures that the authors gave in the universities of that time, predominantly in the so called Faculties of the Arts (in the disciplines of the Quadrivium).

Therefore, by using a locution in fashion nowadays, the catchment area was quite restricted and this fact also explained why only a few of these works were printed and made known with the advent of printing.

The historians of science of the XIX century therefore found themselves unaware of their existence. We can add that the same thing had already occurred in the preceding centuries. In fact, it is symptomatic that the work of Bernandino Baldi (1553–1617) **Le Vite de' Matematici** (written, it seems, in the penultimate decade of the XVI century), which is the first attempt of history of mathematical sciences made in the modern age,[1] does not mention the protagonists of the Parisian school of XIV century (Buridan, etc.) nor the English mathematicians of the Merton College. To be precise, Bradwardine and Swineshead (Italianized in Brauardino and Suisseto), besides to be quoted in the **Vite**, are inserted with brief mentions also in the little work **Cronica de Matematici** (printed in Urbino in 1707), wherefrom it can be inferred that it is second hand information; in any case, there is no mention of writings concerning the mechanics.

This circumstance could cast doubt on the supposed "large dissemination" of the theories of the Parisian school in Italy, but we shall come back on this topic. Instead, the "rediscovery" of the medieval mechanics occurred much later starting from the manuscripts preserved in the libraries. The principal "explorer", author of numerous discoveries, was the French physicist (and historian of science) Pierre Duhem (1861–1917) who recasted in thousands of pages his studies on the found medieval manuscripts. As to the part which directly concerns the mechanics, we cite the two works **Les Origines de la Statique**—Paris, Hermann (two volumes, 1905–1906) and **Études sur Léonard de Vinci**—Paris, Hermann (three volumes, 1909–1913). The third volume of this last work has as subtitle **Les Précurseurs Parisiens de Galilée**. The historical thesis of Duhem, that we shall discuss later on, is already expressed in the title. We also cite the monumental work in ten volumes **Le Système du Monde**—Paris, Hermann, 1913–1959, with subtitle **Histoire des Doctrines cosmologiques de Platon à Copernic**, three volumes of which (7°, 8°, 9°) are dedicated to "La Physique parisienne au XIV siècle". The last five volumes of this work were published posthumous from 1954 to 1959.

[1]See Bernardino Baldi: **Le Vite de' Matematici**—Edizione annotata e commentata della parte medievale e rinascimentale a cura di Elio Nenci—Milano, Franco Angeli, 1998.

Coming back to the above assertion concerning the historians of science of the XIX century, the work of Ernst Mach **The Science of Mechanics** (first edition 1883)[2] which—leaving aside the epistemological theses of the author—represents the first and authoritative critical study on the history of mechanics, for what regards the dynamics substantially begins with Galileo, without any references to medieval problems. This structure of the text remained unchanged in all subsequent editions.

In fact, Mach's book had seven editions during the author's lifetime, therefore Mach had the time to follow the first part of the scientific production of Duhem. In the preface of the seventh edition (1912), Mach, after Emil Wohlwill and Giovanni Vailati, thanks Pierre Duhem for his critical observations and in the chapter regarding the statics cites **Les Origines de la Statique**, even if not completely agreeing with the author.

In the chapter on dynamics he quotes, instead, a historical and critical contribution of 1905 of Duhem regarding the accelerated motion (and Galileo).[3] Obviously, he could not have read the third volume of the **Études** published in 1913. Also in this quotation, Mach expressed a partially different opinion on the methodology of Galileo.

In the years between the two world wars, the historical research on the medieval science had a notewhorthy development and originated, astride the fifties of the last century, also a series of works addressed to an audience larger than that of the community of experts.[4]

Duhem's theses were in part accepted and in part refused, but always discussed with great respect for the work of the French scholar. Beyond doubt the most balanced (and nourished of profound culture and erudition) criticisms to Duhem came from the German scholar Anneliese Maier (1905–1971).[5] An exception, if it

[2]See Ernst Mach: **The Science of Mechanics: A Critical and Historical Account of its Development**—Translated by T. J. Mc Cormac—Open Court Classics 1988. (The first edition in German language was published in Prague in 1883).

[3]See P. Duhem: **De l'accelération produite par une force constante—Notes pour servir à l'histoire de la Dynamique**—Congrés international de philosophie (Geneva 1905), p. 859.

[4]One of the first works of this type was the **Histoire de la Mécanique** of René Dugas (Neuchâtel, 1950), which was published with a preface by Louis De Broglie and was translated in English some years later (the English translation was reprinted by Dover in 1988). To this, Dugas added **La mécanique au XVII^{e} siècle—Dès antécédents scolastique à la pensée classique** (Neuchâtel, 1954).

[5]Unfortunately, Mayer's work is almost unknown out of the exclusive circle of the specialists who deal with the history of philosophy and medieval science. Maier took her degree in Germany and then, after having completed her studies at Zurich and Berlin, passed in Italy in 1936 to look for manuscripts of Leibnitz (entrusted with a mandate by the Prussian Academy of Sciences). Starting from 1937 she resided in Italy almost consecutively and there published the greater part of her writings. She wrote always in German and this happened when by that time the scientific literature regarding the subjects she dealt with was for the most part in English language. Her writings were almost all published (in German language) by the Edizioni di Storia e Letteratura—Roma—since the fifties of the last century. Only some decades later an English translation of a selection of her essays on the medieval sciences and Galileo was published: **On the Threshold of Exact Science—Selected Writings of Anneliese Maier on Late Medieval Natural Phylosophy**, edited and translated with an Introduction by Steven D. Sargent—University of Pensilvania Press, 1982. For these reasons, as we said, her name has not crossed the border of the specialistic research and therefore the results of her research are known only through the quotations of the specialists.

can be defined so, was constituted by Antonio Favaro (1847–1922), the author of the national edition of Galileo's works, who always considered himself invested of the "sacred mission" of fighting against those he called "the detractors of Galileo". This tendency to transform in an almost personal matter every criticism to the work of Galileo coming from scholars of various origins in some cases risked to make less admissible his opinion even when it was correctly and scientifically grounded.

Among the detractors of Galileo Favaro included also (and above all) Duhem and opposed his conclusion which credited to the "doctores Parisienses" the authorship of the results universally credited to Galileo.

Rightly, in our opinion, he could conclude «Then, we trust to be able to conclude that the opinion expressed by Duhem is at least susceptible to a revision, in which we would the great precept not to be forgiven: in the critical studies on the theory of the sciences it is necessary to forbear crediting to non-modern authors assertions which do not appear as direct conclusions from the preliminaries they enunciate, or preliminaries necessary for the conclusions at which they arrive».[6]

In other occasions he was less fair, in respect of Duhem.[7] He was also much aggressive towards Raffaello Caverni, who was the first to deal seriously with the results of Galileo concerning the mechanics. Caverni studied the mechanics of Galileo in the fourth volume of his work **History of the Experimental Method in Italy** (1891–1900).[8]

The negative criticism of Favaro was followed by the isolation of the work (in the meantime the author was dead before completing his work) which was substantially excluded, for about half a century, from the debate of Italian scholars and consequently, ignored abroad. Actually, Favaro's opinion on Caverni was practically shared by the most outstanding scholars of that time (Enriques, Marcolongo, Mieli, etc.).[9]

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[6]A. Favaro: **Galileo Galilei e i "doctores parienses"**—Rendiconti della Reale Accademia dei Lincei—Classe di scienze morali, storiche e filologiche. Serie Quinta. Vol. XXVII (1918), pp 139–150.

[7]In 1921, Favaro, on reviewing a work of a French author regarding the Italian though of XVI century, takes the opportunity to address accusations to Duhem. With regard to the studies of Duhem, he says: «… for having an exact enough opinion of the important question of which such studies show only one side, and perhaps the less important, it will not be inopportune to remind its highest origins, even if for this is necessary to unveil the back stage.» In substance, Duhem's studies on the school of Parisian terminists and the consequent opinion on Galileo (only a continuator of theirs) would have been originated (through Cardinal Dechamps and abbot Mercier) by a directive of Pope Lion XIII for promoting the neo-scholasticism. See: A. Favaro—**Galileo Galilei in una rassegna del pensiero italiano nel corso del secolo decimosesto**—Archivio di storia della scienza, 2, 137–146 (1921).

[8]Raffaello Caverni: **Storia del Metodo Sperimentale in Italia** (six volumes)—Firenze, Stabilimento G. Civelli Editore (1891–1900). The work has had two anastatic reprints (Forni, Bologna 1970 and Johnson Reprint Corporation, New York/London, 1972).

[9]See: D. Boccaletti: **Raffaello Caverni and the society for the progress of the sciences: an independent priest criticized by the lay scientists**—Physis—vol. XLVIII (2011–2012) Nuova Serie—Fasc. 1–2.

2.2 The First Substantial Criticisms to Aristotelian Mechanics—Philoponus and Avempace

The Aristotelian theories concerning the motions in general and the fall of heavy bodies in particular were destined to last until the XVI century, but this did not prevented them from meeting with criticisms and oppositions already in the late Antiquity. In fact, the Greek world of the late Antiquity contributed, with an impressive quantity of commentaries to the works of Aristotle, to the development of the natural philosophy. A relevant criticism came from John Philoponus[10] (called the grammatist) who expressed his ideas in the VI century in the commentary to the **Physics** of Aristotle.

Obviously, we must not look in the commentary of Philoponus for elements which foreshadow the setting in of a new mechanics, but rather the elements of criticism to Aristotle, that is the enfeeblement of the Aristotelian tenets which began in this way to be subjected to a critical revision. A fundamental point, for instance, is this: Philoponus asserts, on the contrary of Aristotle, that the existence of void is possible and then he speaks of motion in the void as well. In the opinion of Philoponus, the fundamental and primary entity that determines the motion is the motive force.

If a body moves in the void, the motive force makes it to walk a certain space in a certain time. If instead the body moves in a certain medium, it meets with a certain resistance which is in direct proportion to the density of the medium and therefore one must add an additional time to the primary time (i.e. that taken to move in the void). Therefore, Philoponus rejects the Aristotelian theory which identifies in the ambient medium the "motor conjunctus" of a "projectum separatum".

By anticipating, in a certain sense, the theory of impetus he asserts that the motive force must be considered as the "motor conjunctus" that the "projector" has imparted to the "projectum" when throwing it. With regard to the fall of heavy bodies, Philoponus agrees with Aristotle that the heavier bodies fall with higher velocity. This happens also in the void:

> «…. And if bodies possess a greater or a lesser downward tendency in and of themselves, clearly they will possess this difference in themselves even if they move through a void. The same space will consequently be traversed by the heavier body in shorter time and by the lighter body in longer time, even though the space be void. The result will be due not to greater or lesser interference with the motion but to the greater or lesser downward tendency, in proportion to the natural weight of the bodies in question…».[11]

The work of Philoponus remained unnoticed in the Latin West until the XVI century (Galileo knew a Latin translation published in 1535—quoted by him many

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[10]John Philoponus (about 490–570) was a Byzantine philosopher of Greek language (Neoplatonist and also Christian) and also director of the School of Alexandria.

[11]See: **A Source Book in Greek Science** by M. R. Cohen and I.E. Drabkin, Harvard University Press—1966, p. 217.

times, but not in relation with the theory of impetus). Sentences, in a certain sense connected to those of Philoponus, can be found in the work of the Spanish Muslim Ibn-Badja, known to the Latin scholastics under the name of Avempace,[12] at a distance of six centuries.

In the opinion of Avempace, on the contrary of Aristotle, the medium is not essential for the natural motion with finite velocity since the velocity of motion is determined by the difference and not by the ratio, between the density of the body and that of the medium. Therefore, $V = F - R$, so that when $R = 0$, $V = F$ (F is the motive power measured by the specific gravity of the body which moves, R the resistance of the medium measured by its specific gravity, V the velocity).

It must be noted that Avempace did not say how the motion, in the magnitudes characterizing it, could be then really measured, at least in the exposition by Averroes who, in his turn, confuted him in his famous commentary to the **Physics** of Aristotle. It does not fall into our purpose to go on to explain the theory of Averroes etc. since this will bring us far from the subject in which we are interested, i.e. the theories which directly foreshadow the work of Galileo.[13]

2.3 The Medieval Kinematics

The concept of motion in the scholastic Philosophy deriving from Aristotle is broader than the simple reference to the change of position of a body in the space, with the relative attributions of velocity etc. as we are used to mean from Galileo on. According to the Scholastics, the motion was a transition from the potentiality to the actuality and vice versa and then regards any case in which one would appeal to the distinction between actual and potential. According to Aristotle

> «…. Again, there is no such thing as motion over and above the things. It is always with respect to substance or to quantity or to quality or to place that what changes changes.»[14]

Here is also important to remind that the philosophy of Aristotle could not be completely accepted in its "original" version by the Christian world. In fact, according to Aristotle, the world existed ever since and therefore could not have been created.

Jointly with discussions, that we can call restricted or particular, about several points of Aristotle's works regarding the philosophy of nature, there was a fundamental "revision" of the Aristotelian philosophy to graft it in the Christian

[12]Ibn-Badja is the first philosopher famous among the Arabs of Spain—he was born in Saragossa at the end of the XI century and died in Fez in 1138.

[13]It is of a great interest, on the discussions regarding the Aristotelian physics in the Middle Ages, the long essay of E. A. Moody: **Galileo and Avempace—The dynamics of the leaning tower experiment**—published in two parts in the Journal of the History of Ideas Vol. 12 (1951. 163–193, 375–422). We shall refer to this essay in Chap. III.

[14]See: Aristotle: **Physics**, (III, 1–200 b 30).

philosophy. One arrived also at a clear distinction between theology and philosophy, but also with the clear statement that the philosophy was “ancilla theologiae”.

The authors of this work of “Christianization” were essentially Albert the Great (Albertus Magnus) (1206–1280), who studied the commentators of Aristotle who preceded him starting from Avicenna, and St. Thomas Aquinas (1225–1274).

After them, with reference to this work, one will ever speak of “Thomistic synthesis”, even if the “synthesis” was not entirely due to Thomas.

In the work of the natural philosophers of the Middle Ages, as on the other hand in the treatments of the motion in the Greek world, the description of the motion and the attributions of their causes are often interconnected, so in these cases it appears hard to disaggregate the kinematics from the dynamics, as in the modern textbooks of rational mechanics.

However, we shall try to deal separately with the two subjects since this will help us in the task of the subsequent comparison with the results obtained by Galileo. A simple case, i.e. of a work which is merely a treatise of knematics, is supplied by the **Liber de Motu** of Gerard of Brussels.

2.3.1 Gerard of Brussels and the Liber de Motu

The manuscript of the **Liber de Motu** was discovered by Duhem into the Latin fund of the French National Library and then briefly summarized in the third volume of his Études.[15] Subsequently, Eneström[16] told of it, and, finally, it was published (for the first time in 1956 and in final edition in 1984) by Marshall Clagett.[17]

We are still out of reliable information on the author, except for the name; the only certain date is a “terminus ante quem” indirectly fixed for the date of composition of the work (1260). Then the work surely goes back to the first half of the XIII century[18] and «it is perhaps the first, certainly one of the most important medieval works dedicated to kinematics» (E. Giusti).

The problem that Gerard deals with is that of the velocity of extended bodies in uniform rotary motions. Although Clagett has emphasized the influence of Euclid and Archimedes on Gerard, the work «to the best of our knowledge, is a completely original creation of the philosopher of Brussels» (Giusti). The **Liber**

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[15] P. Duhem: **Études sur Léonard de Vinci**—Troisième série—1913, pp 292–295.

[16] G. Eneström: **Sur l’auteur d’un traité De Motu au quel Bradwardine a fait allusion en 1328**—Archivio di storia della scienza 2, 133–136, 1921.

[17] M. Clagett: **The Liber de Motu of Gerard of Brussels and the Origin of the Kinematics in the West**-Osiris 12, 73–175, 1956. M. Clagett: **Archimedes in the Middle Ages**, vol. V—Madison Wisc., 1984.

[18] For this and a thorough discussion of the **Liber de Motu**, see E. Giusti: **Alle origini della cinematica medievale: il Liber de Motu di Gherardo da Bruxelles** (Bollettino di Storia delle Scienze Matematiche—vol. XVI, 199–240, 1996).

is divided into three parts (books). The first book deals with the rotation of segments, the second with the rotation of plane figures, the third with the rotation of solid bodies. We refer the reader to the quoted paper of E. Giusti for an exhaustive examination of the cases dealt with by Gerard.

What is important for us is to point out that the work of Gerard arrives at about one thousand four hundred years after the death of Archimedes (212 B. C.), that is, of the last author who had dealt with the uniform motion.

Moreover, we consider important, also prior to a comparison with the kinematics of the Merton College, to point out what separate the definition (really, as we know, it cannot properly be called a definition) of velocity given by Gerard from that of Aristotle. Gerard says:

> «Proportio motuum punctorum est tamquam linearum in eodem tempore descriptarum».[19]

The translation quoted in footnote 19 is that due to Clagett who, on the other hand, agrees with Giusti in translatig the Latin "motus" by "velocity". What is the essential point? Aristotle says in the **Phisics** (VI 2, 232 a):

> «.... it necessarily follows that the quicker of two things traverses a greater magnitude in an equal time, an equal magnitude in less time, and a greater magnitude in less time, in conformity with the definition sometimes given of "the quicker"....».

That is, as we have many times stressed, in the Greek world one was bound to use the proportion which were constructed by ratios of homogeneous magnitudes and these, in the case we are interested in, were distances and times: a ratio between two distances was compared with the ratio between two times. Gerard, for the first time, speaks of velocity as a magnitude by itself, that is, the velocities enter directly in the proportion and are no more indirectly comparable starting from a proportion which does not contain them. This really represents a novelty, even if it will be necessary to wait for more than four centuries to arrive at the concept expressed by the ratio $v = \frac{s}{t}$. Doubtless, it is an advance towards a true definition of velocity, even if overvalued by somebody.[20]

2.3.2 *The Kinematics at Merton College*

We have seen above (cf. footnote 16) that one of the first studies on Gerard of Brussels refers to the citation and subsequent discussion made by Thomas Bradwardine in his

[19]See the final edition of Clagett quoted in footnote 17, p. 64: «The proportion between the velocities of points is the same as that between the lines described in the same time».

[20]See: J. Mazur: **Zeno's Paradox**, Dutton, 2007. J. Mazur asserts that with Gerard one has to do, for the first time, with velocities considered as magnitudes and such an approach marks a turning point in the direction of the modern concept of istantaneous velocity. But neither Clagett, nor Giusti and not even Souffrin (who had dealt thoroughly with the concept of velocity) had made an assessment of this kind of the quoted passage of Gerard. The paper of Souffrin we refer to is the already quoted **Sur l'histoire du concept de vitesse d'Aristote à Galilée**—Medioevo—Rivista di Storia della Filosofia medievale—vol. XXIX (2004) pp 99–133.

Tractatus de Proportionibus (1328).[21] Bradwardine is doubtless the first moving spirit of the group that had developed at the Merton College of the University of Oxford and was working about the half of the XIV century. The components of this group were, besides Bradwardine (about 1300–1349), William Heytesbury (1313–1373), Richard Swineshead (?–1355) and John Dumbleton (?–1349).

The most important works from our point of view (i.e. those which have something to do with mechanics) produced by this group are the already mentioned **Tractatus de Proportionibus** (1328), the **Regule solvendi Sophysmata**[22] of Heytesbury and the **Liber Calculationum**[23] of Swineshead which won for his author the nickname "the calculator".

The biographical data regarding these four authors are scarce and doubtful and there are some doubts also about the other works ascribed to them. For a more detailed discussion we refer the reader to the classical work of Marshall Clagett[24] containing also an extended bibliography (even if it stops at 1959).

The important thing for us is to see what progress had been made by the philosophers of Merton College in the study of mechanics and what footholds they had gained wherefrom the mechanics could have started again. Of course, we must always keep in mind that the results obtained by these authors should not be appraised from the modern (post-Galileian) point of view, i.e. that of estimating how much of "scientific" they contain. The interests of the philosophers of Merton College ranged from theology to logic and mathematics, and kinematics was not the predominant interest. Rather, if we can say so, it came as a particular application of the philosophical problem of how the qualities (or other forms) increase in intensity. In the scholastic terminology it was the problem "de intentione et remissione formarum".

The philosophers of Merton College, among the qualitative variations, considered also the problems of motion in the space and then the variations of velocity. Marshall Clagett summarizes in this way the contributions of the Mertonians to the development of the Mechanics:

1. A clear distinction between *dynamics* and *kinematics*, expressed as distinction between the *causes* and the space-time *effects*.
2. A new approach to the swiftness or velocity, within the ambit of which the idea of an instantaneous velocity was considered, perhaps for the first time,, and the idea of "function" was specified.

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[21]The editions in modern languages that one can look up are:

1. H. Lamar Crosby: **Thomas Bradwardine. His Tractatus de Proportionibus. Its Significance for the Development of Mathematical Physics** (Madison, Wis., 1955).
2. Thomas Bradwardine: **Traité des Rapports entre Les Rapidités dans les Mouvements**—suivi de Nicole Oresme: **Sur les Rapports**—Introduction, traduction, et commentaires de Sabine Rommevaux-Paris-Les Belles Lettres, 2010.

[22]To date complete translations in a modern language do not exist. For excerpts, one can see Clagett, op. cit., second part and Curtis A. Wilson: **William Heytesbury-Medieval Logic and The Rise of Mathematical Physics** (Madison, Wis., 1956).

[23]For a significant excerpt (in a modern language), see Clagett, op. cit. Chap. 5.3.

[24]Marshall Clagett: **The Science of Mechanics in the Middle Ages**—University of Wisconsin Press, 1959.

3. The definition of the uniformly accelerated motion considered as that motion in which equal increases of velocity are obtained in equal intervals of time.
4. The formulation and the demonstration of the fundamental kinematic theorem which establishes the equality, with respect to the space covered in a given time, of a uniformly accelerated motion and of a uniform motion whose velocity is equal to the velocity of the accelerated motion at the half of the time of acceleration.

With regard to the first point, the distinction is already clearly expressed in the Prologue (presently ascribed to a scribe) of Bradwardine's treatise where the contents of the four chapters of which it is composed were listed:

> «… The third chapter makes clear the meaning of the ratio between the velocities of motion in comparison with the things moved and of the movers … The fourth chapter investigates the ratio between the velocities of motion in comparison with the quantities of moveable and of the space covered …».

As one can see, also Bradwardine, like the other Mertonians, puts the dynamics before the kinematics, contrary to the modern use. For the second and the third point, the most suitable reference is in a passage of the **Regule solvendi Sophysmata**[25] of Heytesbury where the definitions of uniform velocity, uniform acceleration and instantaneous velocity appear (the non-uniform motion was named "difformis" and that with constant acceleration "uniformiter difformis"). The definition of uniform motion is further specified (that is, the velocity must be the same in any fraction of time however small) in a writing ascribed to Swineshead.[26]

We quote here some excerpts:

> «Of local motions, then, that motion is called uniform in which an equal distance is continuously traversed with equal velocity in an equal part of time. Non-uniform motion can, on the other hand, be varied in an infinite number of ways, both with respect to the magnitude, and with respect to the time.» … «In non-uniform motion, however, the velocity at any given instant will be measured *(attendetur)* by the path which *would* be described by the most rapidly moving point if, in a period of time, it were moved uniformly at the same degree of velocity *(uniformiter illo gradu velocitatis)* with which it is moved in that given instant, whatever [instant] be assigned.» … «With regard to the acceleration *(intensio)* and deceleration *(remissio)* of local motion, however, it is to be noted that there are two ways in which a motion may be accelerated or decelerated: namely, uniformly, or non-uniformly. For any motion whatever is *uniformly accelerated (uniformiter intenditur)* if, in each of any equal parts of the time whatsoever, it acquires an equal increment (*latitudo*) of velocity. And such a motion is uniformly decelerated if, in each of any equal parts of the time, it loses an equal increment of velocity.».

And finally Richard Swineshead was careful to specify that «uniform velocity is to be defined by the traversal of an equal distance in *every (omni)* equal period of time.».

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[25]See Clagett, op. cit. ibidem and also **A Source Book in Medieval Science** (edited by Edward Grant-Harvard University Press, 1974) p. 238.

[26]See Clagett, op. cit., ibidem.

Let us pass now to that named fourth point by Clagett. In this case we must dwell longer since we have to do with the most important result obtained by the Mertonians: the so-called theorem of uniform acceleration or of the mean velocity. Substantially, one has to evaluate the space covered in a uniformly accelerated motion, when starting both from rest and from a point reached with a certain velocity. If this velocity is indicated by v_0 and the final by v_f and a indicates the constant acceleration for passing from v_0 to v_f, we should write for the covered space $s = v_o t + \frac{1}{2}at^2$, where t is the time taken for passing, with uniformly accelerated motion, from initial velocity v_0 to final velocity v_f. We also know that it must be $v_f = at$.

The Mertonians' theorem maintains that the uniformly accelerated motion is equivalent to a uniform motion with a velocity equal to that of the accelerated motion at half of its path. We can conflate all in the modern formula $s = \left[v_o + \left(\frac{v_f - v_0}{2}\right)\right]t$. In the case where the accelerated motion starts from rest ($v_0 = 0$), we shall have $s = \frac{1}{2}v_f \cdot t = \frac{1}{2}at^2$ (having taken into account in the last equality that $v_f = at$). In the case of $v_0 \neq 0$, being $v_f = at + v_o$, we shall have the known formula quoted above: $s = v_o t + \frac{1}{2}at^2$.

Let us see now how Heytesbury expressed what we have said above in his **Regule** (almost certainly it is the oldest enunciation of the theorem):

> «From the foregoing it follows that when any mobile body is uniformly accelerated from rest to some given degree [of velocity], it will in that time traverse one-half the distance that it would traverse if, in that same time, it were moved uniformly at the degree [of velocity] terminating that latitude. For that motion, as a whole, will correspond to the mean degree of that latitude, which is precisely one-half that degree which is its terminal velocity.
>
> It also follows in the same way that when any moving body is uniformly accelerated from some degree [of velocity] (taken exclusively) to another degree inclusively or exclusively, it will traverse more than one-half the distance which it would traverse with a uniform motion, in an equal time, at the degree [of velocity] at which it arrives in the accelerated motion. For that whole motion will correspond to its mean degree [of velocity], which is greater than one-half of the degree [of velocity] terminating the latitude to be acquired; for although a non-uniform motion of this kind will likewise correspond to its mean degree [of velocity], nevertheless the motion as a whole will be as fast, categorematically, as some uniform motion according to some degree [of velocity] contained in this latitude being acquired, and, likewise, it will be as slow.».[27]

Several demonstrations (Probationes) have been given of the above theorem. One can read the text of three of these demonstrations (authors: Heytesbury, Swineshead and Dumbleton) in Chap. 5 of Clagett. In the demonstration of Heytesbury, after the "theorem of the mean velocity", also the so-called "law of the distances" is demonstrated, in which it is maintained that a body which moves with uniformly accelerated motion starting from rest covers, in the second half of the time, a path threefold greater than in the first.

This law will be subsequently generalized by Oresme and we shall find it demonstrated by Galileo in general form.

[27]See Clagett, op. cit., ibidem and **A Source Book in Medieval Science**, op. cit. pp 239–240.

2.3.3 The Kinematics of the Parisian School

The results obtained by the philosophers of Merton College spread fast in the universities of that time until being reworked at the University of Paris by the local masters, the so-called *doctores parisienses*. Here too, we can speak of three authors who, besides belonging as masters of logic to the movement of the *terminists*, have dealt with the mechanics. They were Jean Buridan, latinized Buridanus (about 1295–about 1358), Nicole Oresme (about 1220–1382), and Albert of Saxony (about 1316–1390). We shall deal with the theory of Buridan (the impetus theory) further on, having chosen to put the dynamics after the kinematics.

The most significant and, say, innovative results in the field of kinematics are due to Oresme.[28]

Even if the historians of science have pointed out that Oresme had forerunners in the introduction of a geometric method in the study of kinematics,[29] it is usual to consider him the founder of this method since the clearest and rigorous formulation of the method is due to him. Granted that all work (both in Latin and in French) of Oresme has been left handwritten until the present time and with titles often assigned later by copysts, the two works that are of interest for the subject we are dealing with bear the title **Tractatus de configurationibus qualitatum et motuum**[30] (1350), and **De proportionibus proportionum**,[31] respectively.

The basic idea of the **Tractatus de configurationibus** is the following: the quantity of a quality can be represented by a geometric figure. The old problem *de remissione et retentione formarum* is geometrized.

The extension of quality (for instance, a time interval in the case of a motion) is represented by a part of a horizontal line while the qualitative intensity (for instance, the velocities at the different instants of the above interval of time) are represented by vertical segments perpendicular to the line of extension (see Fig. 2.1).

In the case of a uniform motion (i.e. with constant velocity) it is clear that the segments which represent the intensities will be all equal.

If we represent with segment AB a given interval of time and with AC and BC the equal velocities at the initial and final instants, we shall have a rectangle ABCD where the segments representing the velocities at the different instants will be contained, i.e. the figure represents the whole distribution of the intensities in

[28]For the overall work of Oresme and the scanty biographical data on him we refer to the article of Clagett in Charles Coulston Gillispie—**Dictionary of Scientific Biography**—Scribners, New York 1970—vol. 9, pp 223–230.

[29]See Clagett, op. cit., ibidem and S. Rommevaux, op. cit. in footnote 21, pp LXII–LXVI.

[30]See **Nicole Oresme and the Medieval Geometry of Qualities and Motions**: **A treatise on the Uniformity and Difformity of Intensities known as Tractatus de configurationibus Qualitatum et Motuum**—edited and translated by Marshall Clagett—Madison, Wisc., 1968.

[31]See Nicole Oresme: **De proportionibus Proportionum—ad Pauca Respicientes** (ed. E. Grant)—Madison, Wisc., 1966. See also the edition in French language quoted in footnote 21.

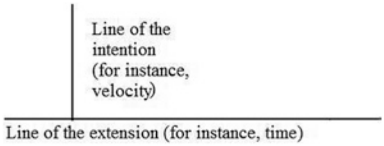

Fig. 2.1

the quality, that is, the quantity of the quality. In the case of motion, it represents the whole space covered in the given interval of time (Fig. 2.2).

In the case of a uniformly accelerated motion (uniformly difform), since the velocity increases at a uniform rate, the relevant figure will be a right-angle triangle if the motion starts from rest (see Fig. 2.3). At this point, the (geometric) demonstration of the Mertonian theorem results quite immediate.

If ABC is the right-triangle representing the uniformly accelerated motion, it is immediate to control that rectangle ABFD, which represents the uniform motion with velocity equal to the velocity of the accelerated motion at half of the taken time (segment EG), has the same area (Fig. 2.4).

Therefore, the space covered in the same time is the same. It is obvious that a mirror image of Fig. 2.4 (which represents a uniformly accelerated motion) represents a uniformly slow motion.

We shall come back later on the use of Fig. 2.4 for representing the fall of heavy bodies. One must remark that Oresme's treatment of the kinematics always remained at an "abstract" level, that is, there has not been on his part any attempt of application to motions existing in nature.

The aforementioned motion of fall of the heavy bodies was interpreted by Oresme by making reference, although not completely in agreement, to the Buridan's theory and then as due to a continuous accumulation of impetus.

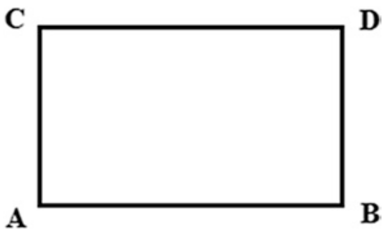

Fig. 2.2

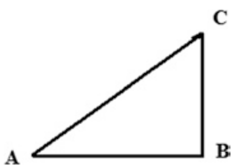

Fig. 2.3

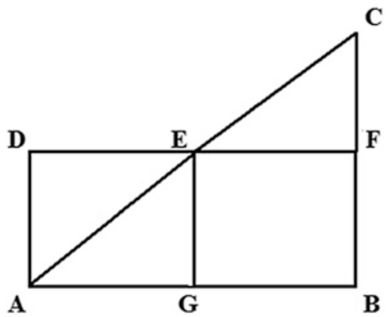

Fig. 2.4

2.4 The Medieval Dynamics

2.4.1 Bradwardine's Dynamics

As we have seen, Bradwardine separated the description of the motion (the kinematics, for us) from the study of the relation between the velocities of the motion and the magnitudes correlated to their causes. Substantially, in his work **Tractatus de proportionibus**, he takes again the question starting from the formulation of Philoponus, made known in those years by the translation from Arabic of the commentaries of Averroes to the works of Aristotle.

Bradwardine refuses a simple arithmetic proportionality of the type $V \propto (F - R)$, like that which can be inferred from the criticism of Philoponus to the theory of Aristotle (expressible through the relations $V \propto \frac{F}{R}$). The last becomes clearly meaningless when both $F = R$ (no motion, but V results different from zero), and $F < R$. Bradwardine's trouble was that of finding a kind of proportionality which could preserve a relation of proportionality between F and R but in the meantime could give vanishing velocities in the case $F = R$.

The solution was that «the proportion of the velocities in the motion follows the proportion of the power of the mover to the power of the thing moved». Said otherwise: the velocity increases arithmetically in correspondence of the geometric increase of the ratio of the force to the resistance. In formula, the relation can be understood, when varying F and R, as

$$^{F_2}\!/_{R_2} = (^{F_1}\!/_{R_1})^n, \quad \text{where}\ ^{F_1}\!/_{R_1} > 1 \text{ and } n = {}^{V_2}\!/_{V_1}.$$

In modern terms, one could express the relation in exponential form as

$$V = \log_a(^{F}\!/_{R}), \quad \text{where } a = {}^{F_1}\!/_{R_1}$$

Obviously, if $F = R$ one has $\log_a 1 = 0$ and then $V = 0$ for any value of a.[32] This result can be obtained anyway also in the rule of Philoponus $\propto (F - R)$.

2.4.2 *Dynamics at the Parisian School and the Impetus—Theory*

Bradwardine's law was largely accepted, except for some partial disagreement as we shall see later, until the beginning of the XVI century. Not even the *doctores parisienses* made an exception: Nicole Oresme and Albert of Saxony dealt with it in their works. Also their master, Buridan, was under the influence of the work of Bradwardine but was noticeable, above all, because of a new theory, generated by a radical criticism to the Aristotelian explanation of the motion of projectiles, gone down in history as the "impetus-theory".

According to Anneliese Maier[33] many people, starting from Duhem, have been interested in the impetus-theory imagining in it an anticipation of the inertia principle (Duhem was particularly insistent on this![34]).

Always quoting the opinion of Maier, «there is no doubt that it prepared the way for the law of inertia. The theory of impetus occupies an important and enduring place in the history of natural philosophy and physics as an independent stage of development between Aristotelianism and classical mechanics.[35]».

Obviously, an "exact analogous" of the inertia principle is beyond dispute. In fact, as we shall see, it was assumed that the uniform motion was due to a particular motive force (the impetus), whereas in the classical mechanics the uniform motion, like the rest, occurs in the absence of applied forces. More, since the universe was considered finite, the (rectilinear) uniform motion cannot last to infinity; only the circular motion of celestial bodies could last eternally. The only common point is: the motion remains uniform until an external force does not intervene.

The statements of Buridan (see further on) seem to confirm this, if isolated from the context. Anyway, the difficulty of interpreting the writings of those who

[32]This interpretation has been suggested by Anneliese Maier in **Die Vorläufer Galileis im 14 Jahrhundert**, Roma 1949, p. 92. We refer the reader to Clagett's work (op. cit. Chap. 7) and to the introduction of Sabine Rommevaux to the already quoted (see footnote 21) translation of Bradwardine's treatise for further widening. Our very swift and schematic synthesis has only the aim of recording which was the conclusion achieved in the criticisms and corrections to the Aristotelian mechanics in the ambit of Mertonians. Swineshead and Dumbleton further elaborated into details the theory of Bradwardine (see Clagett, op. cit. Chap. 7).

[33]See the essay: **The Significance of the theory of Impetus for Scholastic Natural Phylosophy** (in the volume **On the Threshold of Exact Science—Selected Writings of Anneliese Maier on Late Medieval Natural Phylosophy**—University of Pennsylvania Press, 1982)—The first edition of this essay (in German) is of 1955.

[34]Pierre Duhem: **Études sur Léonard de Vinci—Troisième série—Les Précurseurs parisiens de Galilée**—Paris-Hermann, 1913.

[35]In the essay quoted in footnote 33, p. 77.

have theorized on the impetus always lies in the used terminology, which for us is of complex translation and interpretation and therefore is not reproducible through the today's terms which always refer to quite precise definitions. In substance, the impetus is not a force, nor a form of energy, nor a momentum in the modern sense of the term. It shares something with these physical magnitudes but cannot be identified with anyone of them.

At the bottom of the impetus-theory there is always the Aristotelian principle "omne quod movetur ab aliquo movetur", therefore any motion entails the existence of a "virtus" or "vis motrix" as a cause.

The motion continues until the motive force exists and ceases when this vanishes. Of course these limitations immediately pose a number of questions. For instance, what is the cause of the motion of an arrow or a stone in-flight? The force impressed to a body is gradually attenuated by the medium and then the motion will vanish after a certain time because of the resistance met with. Necessarily, the velocity of a body will be directly proportional to the impressed force and in inverse proportion to the resistance. If the resistance remains constant the velocity depends only on the magnitude of the motive force.

Already Albertus Magnus (1206–1280) a century before the diffusion of the impetus-theory, expressed this concept: «**Omnis motus provenit ex virtutis moventis victoria super mobile, et cum illa virtus movet, oportet potentiam passivam eius, quod movetur, sibi esse proportionalem.**».[36]

Obviously, in this way, the velocity is proportional to the force, whereas in the classical mechanics the acceleration is proportional to the force.

The first to formulate the impetus-theory was the Italian Francesco di Marchia[37] (in a series of lectures given in Paris in the academic year 1319–1320). Afterwards, the man who became the principal supporter of the impetus-theory was Jean Buridan (1300– 1358),latinized in Buridanus, who exposed it in his commentary of the Aristotle's **Physics**.

Unlike of the structure essentially philosophical-theological of the work of di Marchia, Buridan refers to experience. Anyway, for both, the theory is as follows: in the instant in which the projector detaches himself from the projectile, this receives a secondary motive force (*impetus*, *vis impressa*, *vis derelicta* according to di Marchia) which is the cause of the subsequent motion. Let us see how Buridan enunciates his ideas on the fall of heavy bodies and their falling velocity.

In Question XII (whether natural motion ought to be swifter in the end than the beginning) in the commentary to book II of Aristotle's **De Caelo**:

> «… one must imagine that a heavy body not only acquires motion unto itself from its principal mover, i.e., its gravity, but that it also acquires unto itself a certain impetus with that motion. This impetus has the power of moving the heavy body in conjunction with

[36]Albertus Magnus, **Physica** VIII, tract. II, cap. 6 (**Opera**, ed. Borgnet, Paris 1890) «Every motion is originated by the victory of the motive force over the moveable and when that force operates, it is necessary that the passive potentiality of the thing moved is proportional to it.»

[37]Di Marchia was a Franciscan and follower of Duns Scoto (the *doctor subtilis)*.

> the permanent natural gravity. And because that impetus is acquired in common with motion, hence the swifter the motion is, the greater and stronger the impetus is. So, therefore, from the beginning the heavy body is moved by its natural gravity only; hence it is moved slowly. Afterwards it is moved by that same gravity and by the impetus acquired at the same time; consequently, it is moved more swiftly. And because the movement becomes swifter, therefore the impetus also becomes greater and stronger, and thus the heavy body is moved by its natural gravity and by that greater impetus simultaneously, and so it will again be moved faster; and thus it will always and continually be accelerated to the end. And just as the impetus is acquired in common with motion, so it is decreased or becomes deficient in common with the decrease and deficiency of the motion. And you have an experiment [to support this position]: If you cause a large and very heavy smith's mill [i.e., a wheel] to rotate and you then cease to move it, it will still move a while longer by this impetus it has acquired. Nay, you cannot immediately bring it to rest, but on account of the resistance from the gravity of the mill, the impetus would be continually diminished until the mill would cease to move. And if the mill would last forever without some diminution or alteration of it, and there were no resistance corrupting the impetus, perhaps the mill would be moved perpetually by that impetus.»[38]

As we can see, in the final part of the quoted excerpt, Buridan also says how the impetus goes out in the case of a circular motion.

And, as regards the violent motion, i.e. of the projectiles, in Question XIII (whether the projectiles move swifter at half-way than at the beginning or at the end):

> «And you see that the projector who moves the projectile is for some time tied with the projectile, continuously pushing the projectile before its ejection; like this, a man who casts a stone moves his hand with the stone, and also in shooting an arrow the string moves for some time with the arrow pushing it; and same also is for the sling which throws the stone, or for the machines which throw much bigger stones. And then, as long as the projector pushes the projectile which exists together with him, the motion is slower at the beginning, since only then the extrinsic mover moves the stone or the arrow; but during the movement an impulse (impetus) is acquired continuously, which combined with the extrinsic mover moves the stone or the arrow, which for this move swifter. But after the ejection from the projector, the projector does not move anymore, but only the acquired impulse (does move), as we shall see elsewhere; and that impulse, because of the resistance of the medium, weakens continuously, for which the motion gets continuously slower. And therefore, one must understand that the violent motions, i.e. those of projectiles, are swifter at the beginning than half-way or at the end, of course excluding that part of motion when the projector is together with the projectile; in fact, considering the remaining motion as a whole, the greatest velocity occurs at the beginning. And in this way the authoritative opinions of Aristotle and of the others must be reconciled. It is true that on this regard I have a doubt, since some say that the arrow thrown by the bow would be more perforating at a distance of twenty foot than at a distance of two foot, and therefore after the ejection from the bow the greatest velocity would be not yet at the beginning. And I have not experienced this, therefore I don't know if it is true; but if it were true, some say that the impetus is not immediately generated by motion, but continuously as a consequence of motion; and then it is not completely generated at the ejection from

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[38] Iohannis Buridani: **Questiones super Libris quattuor de Caelo et Mundo**—edited by E. A. Moody—Cambridge, Massachusset, 1942, Book II, question 12, p. 180. The translation of this excerpt (by M. Clagett) is taken from **A Source Book in Medieval Science**, op. cit. p. 282.

> the bow, but is accomplished in some time, as the rarefaction and the evaporation follow the heating, but not perfectly at once; indeed, once the heating has ceased, for the water is removed from the fire, yet for some time rarefaction and evaporation are seen to continue. And thus it is clear.»[39]

Here the theory of Aristotle is clearly refuted, which maintained that the mover of the projectile (after it has left the projector) is the surrounding air which receives by the projector an impulse which is transmitted from layer to layer, until its exhaustion. According to Buridan, instead, after the projectile has left the projector, the only mover is the acquired impetus.

2.5 The Diffusion in Italy of the Ideas of Mertonians and of the Parisian Masters

In the Middle Ages, the "innovative" ideas in the scientific field were chiefly disseminated by the masters of the universities in their transfers from one place to another. Particularly active was the switch of teachers and students between Italy and France.

Among the Italians who enjoy a great reputation both as scholars and as critics of the new theories the Parmesan Biagio Pelacani (about 1359–1416), also called Biagio da Parma[40] must be undoubtedly mentioned. We have to do with a typical figure of an intellectual of that time, though at the greatest level of interdisciplinarity (physician, philosopher, mathematician and also astrologer): he has both studied and teached in Paris, in addition to Padua, Pavia, Bologna and Florence. As astrological consultant of princes and lords of that time he deserved from his contemporaries the cognomen of *doctor diabolicus*.

Because of this activity of him, he was up before the bishop of Pavia who ordered him to reprocess some of his theses (by luck of Biagio, the Counter-Reformation was yet to come).

We have mentioned this aspect of Biagio's activity since «… his astrological doctrine, if on the one hand assured him great honours and successes at his time (because of his fame of infallibility) on the other hand discredited him in the eyes of the positivist scholars of the ninenteenth century, though they exalted his writings of optics, statics and astronomy.»[41]

[39]**Questiones super Libris quattuor de Caelo et Mundo**, op. cit. Book II, question 13, pp 183–184. (Our translation).

[40]On Biagio Pelacani, besides Clagett op. cit., see also the articles of F. Barocelli and G. Federici Vescovini in **Filosofia Scienza e Astrologia nel trecento europeo**, a cura di Graziella Federici Vescovini e Francesco Barocelli—Il Poligrafo, Padova, 1992–.

[41]Ibidem. G. Federici Vescovini is the most authoritative scholar of the work of Biagio; she has dealt with it for several occasions, in books and articles.

Biagio manifested a great interest both in the new mechanics of the Mertonians and in the phisics of Buridan. Limiting ourselves, according to our choice, to these subjects, we mention two works

1. **Quaestiones super tractatum de proportionibus Tomae Bradwardini.**
2. **Quaestiones de latitudinibus formarum**.
   The first of these works has had two different draftings (Florence 1388–1389 and Pavia 1389–1407). While in the first he accepts Bradwardine's rule of motion, in the second he challenged it for both mathematical and physical reasons.

In the **Quaestiones de latitudinibus formarum**, Biagio geometrically demonstrates the Mertonian theorem of the mean velocity in a way very similar to the demonstration of Heytesbury; but he follows Oresme by representing the lines of intensity by vertical instead of horizontal lines.

Besides Biagio Pelacani, the historians mention the presence in the universities, particularly of Padua and Pavia, of other philosophers interested in the doctrines both of the Mertonians and of the Parisian School. John of Casale and Franciscus of Ferrara flourished in the middle of the XIV century.

As regards John, he was supposed to have anticipated Oresme in the introduction of the use of the geometrical method in the study of the motions. Other names often mentioned are those of Angel of Fossanbruno and Jacopo of Forlì.

In short, one can anyway conclude that both Biagio and the others did not add elements of novelty to the theories of Mertonians and of Parisians, such as to modify their structure.

As a marginal note, we mention that the contributions of these philosophers in the field of mechanics (as of the other disciplines) circulated through copies of the manuscripts in the world of the universities (then in a restricted and exclusive community) and therefore were subject to getting out of the circuit following the disappearance of the authors.

Of the works we have mentioned before, only the **Liber Calculationum**[42] of Swineshead and the **Questiones de Latitudininibus** of Biagio Pelacani remained in circulation and intercepted the development of the art of printing, consequently having several editions in the last two decades of the XV centuries and the two first decades of the XVI. This circumstance, in our opinion, must be considered and accurately valued in the particular cases when one wants to credit forerunners for scientific results obtained in the subsequent centuries.

[42]With regard of what could have been the influence of this work in the Study of Padua, see the interesting paper of Christopher J. T. Lewis: **The Fortunes of Richard Swineshead in the time of Galileo**—Annals of Science, 33 (1976), 561–584.

2.6 The Theory of Motion in the XVI Century

Having as primary aim that of looking at the birth and the development of the theories of motion, starting from the Greek antiquity for arriving at Galileo, one is disappointed in seeing what happens in the XVI century. In fact, from this point of view the XVI century gives an image of itself various enough, if not confused. We mean that, in the case of that part of the mechanics regarding the theory of motion, one is not able to notice a continuous flourishing as in the literature, in the figurative arts, in the architecture and also in the technology (all what, i.e., delivered the idea of Renaissance in the Burckhardt's meaning), and not discerns an underlying theme which characterizes a tendency towards something of definite.

Maybe this impression comes from the fact that, so to say, one wants to begin from the end, that is, to single out the path (a mix of elaborations and results) which leads to the Galileian enunciations.

As it is known, starting from the last years of XV century (in parallel with the development of the art of printing), in Italy, and later on in the rest of Europe, several translations and commentaries of the works of the great Greek mathematicians (Euclid, Achimedes, Hero, etc.) have been published. Translations start to appear not only in Latin, but also in vernacular Italian and in other national languages.

The development of the technology motivates its professionals (who are not able to read the Latin—we remember that even Leonard styles himself as "omo sanza lettere") to demand mathematical texts, also those regarding machines.

Then, the diffused works are **On the Equilibrium of plane Figures** and **On floating Bodies** of Archimedes, the **Elements** of Euclid, and the **Mechanics** (also called **Questions of Mechanics**), work at that time attributed to Aristotle, but actually still of controversial attribution.

Also the writings **De Ponderibus** of the medieval mathematician Jordanus of Nemore (perhaps one had to do with several persons under this name) reappeared to a new life.

Stillman Drake[43] distinguishes among the scholars of the XVI century who engaged in mechanics several traditions: four going back to the Greek classics and two to the medieval authors. At the last, he divides the Italian authors in two groups geographically separate. In the North: Niccolò Tartaglia (1500–1557), Girolamo Cardano (1501–1576), Giovan Battista Benedetti (1530–1590); in the Centre: Federico Commandino (1509–1575), Bernardino Baldi (1533–1617), Guidobaldo Del Monte (1545–1607).

Limiting the range of our interests to those who have dealt with the motion of bodies in an appreciable way, we shall restrict ourselves to Tartaglia and Benedetti. We shall speak further of Guidobaldo Del Monte, when we shall deal with the curve of projectiles.

[43]See **Mechanics in Sixteenth—Century Italy**—Selection from Tartaglia, Benedetti, Guido Ubaldo & Galileo—Translated & Annotated by Stillman Drake & I.E. Drabkin—The University of Wisconsin Press, 1969—pp. 5–16.

2.6.1 Niccolò Tartaglia (1500–1557)—His Life and His Works

Niccolò Tartaglia was born in Brescia about the year 1500, became fatherless when six years old and, being of a poor family, had difficulty in continuing the school. According to what he himself says,[44] he remained in half alphabet. When he was twelve years old he was wounded in the face by a soldier of the French army which had occupied the town slaughtering the citizens.

Niccolò had been given for dead, but instead he survived thanks to his mother's care; however he remained maimed, because of the wound to lips and jaw, and having a great difficulty in speaking.

The cognomen Tartaglia (it seems that the true name were Fontana) derived from this speech impediment. The young Niccolò studied alone with determined zeal and, autodidact, became an expert of mathematics and even translated some classical works from Latin, among which also the **Elements** of Euclid. His passion for the mathematics led him to be interested in Algebra and in the most important problem at that time: the solution of the equations of third degree.

We cannot speak at length of his activity of algebraist (that, on the other hand, is what gave him an international fame) and of the mathematical challenges with Cardano and others, since this would bring us out of our field of investigation.[45] We are interested, instead, in those works where he deals with the motion of projectiles and the fall of heavy bodies.

In this regard, we remember that in the XVI century, thanks to some progress made in the fabrication of guns, it was beginning the request of precise rules for the shots of artillery by the artillerymen themselves. The military technology needed the help of mathematicians. Tartaglia devoted himself to this study and, as we shall see later, with regard to the motion of projectiles he was the first to obtain certain results, still correct nowadays. The works which we shall deal with are two: The **Nova Scientia** and the **Quesiti et Inventioni diverse**.

2.6.1.1 The Nova Scientia and the Quesiti et Inventioni Diverse

The first work was published in 1537,[46] but had several editions and reprints (1550, 1581, 1583). Conceived in five chapters (books), as the author stated in the preface, consisted in only three in all the subsequent editions. «Divided into five

[44] Most of the biographical notes about him come from autobiographical hints scattered in his works, particularly from **Quesiti et Inventioni diverse** (see the bibliographic reference further on).

[45] The reader can refer to the book (in Italian): Fabio Toscano—**La formula segreta. Tartaglia, Cardano e il duello matematico che infiammò l'Italia del Rinascimento**—Sironi, 2009. A French translation (Belin, Belin edition) is also available.

[46] **Nova Scientia** inventa da Nicolo Tartalea—in Vinegia, per Stephano da Sabio, ad instantia di Nicolo Tartalea brisciano, MDXXXVII.

books: In the First is demonstrated theoretically the nature and effects of uniformly heavy bodies in the two contrary motions that may occur in them, and their contrary effects. In the Second is geometrically proved and demonstrated the similarity and proportionality of their trajectories in the various ways that they can be ejected or thrown forcibly through the air, and likewise the [proportionality] of their distances….»

Then, the third deals with the determination of the distances through the observations and the calculation. On the whole one has to do with a work all devoted to the trajectories of the cannonballs, (nowadays we would say a treatise of artillery), which seems to have originated by the questions put to Tartaglia by an artilleryman of Verona.

Tartaglia dedicates his work to the Duke of Urbino, who later on will ask him for professional advice.

Of course, studying the trajectories of a cannonball means studying the motion of a heavy body tossed with a certain force; since after a certain time the projectile will fall at ground due to its gravity, one must face the problem of considering the compresence of two kinds of motion, the violent motion and the natural motion, according to the theory of Aristotle. And it is just within the Aristotelian scheme, completely ignoring the impetus-theory, that Tartaglia moves on in his treatment.

Let us look at its fundamental elements. On the acceleration in the fall of heavy bodies:

> «FIRST PROPOSITION
>
> Every uniformly heavy body in natural motion will go more swiftly the more it shall depart from its beginning or the more it shall approach its end.[47]»

A uniformly heavy body is

> «FIRST DEFINITION
>
> A body is called uniformly heavy which, according to the weight of the material and its shape, is apt not to suffer noticeable resistance from the air in any motion.»,[48]

that is a heavy body according to the everyday language. From the enunciated proportion, one immediately infers that the fundamental element is the distance covered and, then, from this one can deduce that at any instant the velocity of the moveable is proportional to the distance covered.

Notwithstanding the frontispiece of the **Nova Scientia** is constituted by an allegorical picture which represents the ensemble of the sciences as a blockhouse whose entrance is defended by Euclid (that is, in the blockhouse of the science one can enter only if he knows the mathematics), Tartaglia, when describing the

[47]**Nova Scientia**, f. 4r. (The translation is taken from **Mechanics in Sixteenth—Century Italy**, op. cit. p. 74).

[48]Ibidem. f. 1r. (**Mechanics …** op. cit. p. 70).

accelerated motion of a heavy body in free fall does not appeal as much to the mathematical rigor as to a simile in the literary tradition:

> «This same is also verified in anything that goes toward a desired place, for the more closely it approaches the said place, the more happily it goes, and the more it forces its pace, as appears in a pilgrim that comes from a distant place; for when he nears his country, he naturally hastens his pace as much as he can, and the more so, the more distant the land from which he comes. Therefore the heavy body does the same thing in going toward its proper home, which is the center of the earth; and when it comes from farther from that center, it will go so much the more swiftly approaching it.»[49]

In the subsequent edition of 1550,[50] at this point Tartaglia adds:

> «The opinion of many is that if there were a tunnel that penetrated diametrically through the whole earth, and through this there were let go a uniformly heavy body, as said above, then that body when it arrived at the center of the world would immediately stop there. But I say that this opinion (that it would stop immediately upon arriving there) is not true; instead, by the great speed which would be found in it there, it would be forced to pass by with very violent motion, running much beyond the said center toward the sky of our subterranean hemisphere; and thereafter it would return by natural motion toward the same center, and arriving there it would pass by once more, for the same reasons, with violent motion toward us; and yet again it would return by natural motion toward the same center, and pass it still again with violent motion, thereafter returning by natural motion, and so it would continue for a time, passing with violent motion and returning by natural motion, continually diminishing in speed, and then finally it would stop at the said center. By which it is a manifest thing that violent motion is caused by natural motion, and not the reverse; that is, natural motion is never caused by violent motion, but is rather its own cause.»[51]

Therefore, Tartaglia states that the body in free fall in the tunnel which crosses diametrically the Earth is subject to a damped oscillatory motion which at the end will be extinguished in the centre of the Earth itself.

An analogous statement had been already expressed two centuries before, both by Oresme and by Albert of Saxony. One could suppose a direct reading of the work of the later by Tartaglia since, at that time, a Venetian edition of the commentary of Albert to the **De Caelo** of Aristotle already existed.[52]

Stillman Drake substantially espoused this thesis, by attributing to Tartaglia the adoption of the impetus-theory in the version of Albert,[53] that is, with a continuous decrease of the impetus in the course of motion.

Tartaglia, however, limits himself to statements which copy those of Albert, without ever speaking of impetus. This problem will be reconsidered by Galileo in the **Dialogue**, but with the conclusion that the motion lasts up to infinity. We shall have the opportunity of coming back on the subject.

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[49]Ibidem, f. 4r. (Ibidem, p. 75).

[50]**La Nova Scientia**, Stampata in Venetia per Nicolo de Bascarini a instantia de l'Autore. 1550. See the anastatic reprint by Arnaldo Forni Editore, 1984.

[51]See **La Nova Scientia**, op. cit. f. 4r,v. (**Mechanics …** op. cit. pp 75–76).

[52]See: Albert od Saxony—**Questiones subtilissime in libros de celo et mundo Aristotelis**—Venice 1492, (ff. 32r–33v)—The quotation is taken from Clagett, op. cit. chap. 9.

[53]See: **Mechanics in Sisteenth—Century Italy**, op. cit., p. 76.

In the second proposition, Tartaglia adds:

«All similar and equal uniformly heavy bodies leave from the beginning of their natural movements with equal speed, but, coming to the end of their movements, that which shall have passed through a longer space will go more swiftly.»[54]

That is, the dependence of the final velocity on the covered distance is reaffirmed. Benedetti, later on, will point out that it is not necessary that the bodies of which we study the fall have the same weight.

On the contrary of what happens for the natural motion:

«A uniformly heavy body in violent motion will go more weakly and slowly the more it departs from its beginning or approaches its end. (Proposition III).»[55]

Moreover, he specifies:

«All similar and equal uniformly heavy bodies, coming to the end of their violent motions, will go with equal speed; but, from the beginning of such movements, that which shall have to pass through the longer space will leave more swiftly.» (Proposition IV).»[56]

And, by insisting on the fact that the two kinds of motion must be considered separately (according to Aristotle),

«No uniformly heavy body can go through any interval of time or of space with mixed natural and violent motion.» (Proposition V).[57]

He further explains, with an example, the impossibility of the coexistence of the two kinds of motion in any point of the trajectory with the fact that the violent motion is decelerated (see the third proposition) while the natural motion is accelerated (see the second proposition).

As a consequence, since a motion cannot be in the same time accelerated and decelerated, it comes that the trajectory of a projectile must be composed by rectilinear tracts and curved tracts. Let us see as he solves the problem:

«Every violent trajectory or motion of uniformly heavy bodies outside the perpendicular of the horizon will always be partly straight and partly curved, and the curved part will form part of the circumference of a circle. » (Supposition II of the second book).[58]

But he is forced to admit that this is an approximate solution. In fact, he continues by saying about a motion of a heavy body which occurs out of the vertical:

«Truly no violent trajectory or motion of a uniformly heavy body outside the perpendicular of the horizon can have any part that is perfectly straight, because of the weight residing in that body, which continually acts on it and draws it toward the center of the world. Nevertheless, we shall suppose that part which is insensibly curved to be straight, and that which is evidently curved we shall suppose to be part of the circumference of a circle, as they do not sensibly differ.»[59]

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[54]**La Nova Scientia**, op. cit. f. 5r. (**Mechanics …** op. cit. pp 75–76).

[55]Ibidem, f. 5v. (**Mechanics …** op. cit. p. 78).

[56]Ibidem, f. 6v. (**Mechanics …** op. cit. p. 79).

[57]Ibidem, f. 7r. (**Mechanics …** op. cit. p. 80).

[58]Ibidem, f. 10v. (**Mechanics …** op. cit. p. 84).

[59]Ibidem, f. 11r. (**Mechanics …** op. cit. pp 84–85).

Then, contrary to what asserted by some authors in the past, Tartaglia was quite conscious that the trajectory of a projectile consists of a continuous curve of which the representation with rectilinear segments and arcs of circle is only an approximation.

On the other hand, he does not give an explanation of the use of the circle as a curve for approximating the “curved tracts”. As it is known, the (uniform) circular motion had always been considered the motion of celestial bodies, even if considered exclusively from a geometric point of view in order “to save the phenomena” by composing a suitable number of circular motions.

In this case, instead, an arc of circle directly shapes a tract of the real trajectory. As we shall see, it will take Guidobaldo and Galileo to realize that the whole trajectory of a projectile consists of a parabola.

An important further result which already appears in the **Nova Scientia** is that, if one changes the elevation of the cannon, the maximum gun-range is obtained for an elevation of 45°. In fact he says in the eighth proposition of the second book:

> «If the same motive power shall eject or shoot similar and equal uniformly heavy bodies in different ways violently through air, that shot which shall have a trajectory elevated 45° above the horizon will make its effect farther from its beginning on the plane of the horizon than one elevated in any other way.»[60]

This result is already proudly enunciated by Tartaglia in the dedication of the Duke of Urbino, together with the fact that is possible to reach a target with two different elevations:

> «Further, since by evident reasons I knew that a cannon could strike in the same place with two different elevations or aimings, I found the way of bringing about this event, a thing unheard of and not thought by any other, ancient or modern.»

On the motion of the heavy bodies and the trajectory of the projectiles Tartaglia will come back in the subsequent work **Quesiti et inventioni diverse**,[61] even dedicated to Henry VIII (to the Merciful and Invincible Henry VIII, by Grace of God King of England, of France, and of Ireland, etc.).

The work consisted of nine books, but only the first (30 questions on the artillery shots), the seventh (7 questions on the principles of the **Mechanica** of Aristotle) and the eighth (42 questions on the theory of the heavy bodies) concern the subject we are interested with. The first book contains three questions from the Duke of Urbino (dated 1538, i.e. a year after the publication of the **Nova Scientia**). From the point of view of the theory of the motion of bodies, the book does not present new elements, while there are many improvements concerning the technique of the artillery.

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[60] Ibidem, ff. 16v, 17r. (**Mechanics …** op. cit. p. 91).

[61] **Quesiti et inventioni diverse** de Nicolo Tartalea Brisciano in Venetia per Venturino Ruffinelli, ad instantia et requisitione, et a proprie spese da Nicolo Tartalea, autore, 1546. The first edition of 1546 was followed by others until the definitive edition **Quesiti et inventioni diverse de Nicolo Tartaglia**—in Venetia per Nicolo de Bascarini, ad instantia et requisitione, et a proprie spese de Nicolo Tartaglia Autore. Nell’anno di nostra salute. MDLIIII. An anastatic reprint of this edition is available with introduction and notes by Arnaldo Masotti—La nuova cartografica, Brescia 1959.

2.6.2 The Mechanics of Giovan Battista Benedetti

The work of Giovan Battista Benedetti, nowadays considered the most important of the immediate forerunners of Galileo, essentially happened in the second half of the XVI century, and has been pointed out to the future historians for the first time by the mathematician and historian of science Guglielmo Libri (1802–1869) in his work **Histoire des sciences mathématiques en Italie**.[62]

Libri devoted several pages of the third tome of his work to commenting the mathematical results obtained by Benedetti and, with regard to the theory of motion, expresses the following judgment:

> « … On l'aurait admiré davantage si l'on avait compris, à cette époque, toute l'importance de sa théorie de la chute des graves, dont on n'a jamais parlé, et qui mérite cependant une place distingué dans l'histoire des sciences.»[63]

and

> « … Benedetti, dont le nome est à la peine prononcé aujourd'hui en Italie, doit être placé au premier rang des savans du seizieme siécle.»[64]

He will be followed by Caverni who starts in the first tome of his **History** with the judgment:

> «The science of the motion, made impossible by the errors of Aristotle, remained, one can say, stationary in the books of the ancient Archimedes. Our Benedetti was one of the most valuable in promoting it, by confuting with reasonable arguments those Aristotelian errors, many of which had been shared even by Niccolò Tartaglia himself, for both the natural and the violent motions.»[65]

Caverni was even more eulogistic in the fourth tome where, comparing Benedetti with his forerunners, says:

> «But in Giovan Battista Benedetti, from whom a new science begins, the words have a very different sound.»[66] and «The doors of the truth, remained bolted by the peripatetic advices for so many centuries, once so happily made free should lead Benedetti to deliver by his own fair hand to Galileo himself the key for going into the most hidden vestibules of the temple.»[67]

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[62]Guillaume Libri: **Histoire des sciences mathématiques en Italie, depuis la renaissance des lettres jusqu'à la fin du XVII^{e} siècle** (Paris 1838–41).

[63]Libri, op. cit. tome troisième, p. 123.

[64]Ibidem, p. 131.

[65]Caverni, op. cit., tomo I, p. 103.

[66]Caverni, op.cit. tomo IV, p. 97.

[67]Ibidem.

Finally, we quote from Vailati, who at the end of the XIX century devoted to Benedetti an important essay:

> «Among those who most efficiently contributed to preparing and making possible that great scientific revolution which is marked by the discoveries of the fundamental laws of motion, Giovanni Benedetti … occupies a special place. The role he had in the first elaboration of the theory and the concepts, which are at the basis of the modern Dynamics, represents a contribution of an entirely different nature than that which, at the constitution of the new science, was brought by the other immediate forerunners of Galileo.»[68]

2.6.2.1 The Life

The biographical data on Giovan Battista Benedetti (Venice 1530–Turin 1590) are exceedingly scanty: some of them inserted by himself in a work of him,[69] some others supplied by his contemporary Luca Gaurico (1475–1558) in his **Tractatus astrologicus …** (Venice, 1552).

According to that is told, one knows that he did not have preceptors and after he was seven years old he did not attend any school and, as he himself remembers, only Niccolò Tartaglia taught him the first four books of Euclid.

> «… Besides, since one must give back to everyone what is his own, for it is right and legitimate, Niccolò Tartaglia read to me only the first four books of Euclid, all the rest was investigated by my private study and work: in fact nothing is difficult to be learned by a willing man.»[70]

Benedetti remained in Venice until about 1558 and then passed to Parma called there by Duke Ottavio Farnese as a professor (*Lettore*) of philosophy and mathematics. He remained at Parma about eight years also dealing with astronomic studies. He took leave from Parma at the beginning of 1567 and passed to Turin invited there by the Duke of Savoy Emanuele Filiberto. Here, besides holding the office of the mathematician of the Court, was also teacher at the University; he remained in Turin, even after the death of Emanuele Filiberto, until his own death in 1590.

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[68] Giovanni Vailati: **Le speculazioni di Giovanni Benedetti sul moto dei gravi**—Atti della R. Accademia delle Scienze di Torino, vol. XXXIII, 1898 (Reprinted in: Giovanni Vailati—**Scritti** a cura di Mario Quaranta—Arnaldo Forni Editore, 1987—vol. II, pp 143–160).

[69] **Resolutio Omnium Euclidis Problematum Aliorumque ad hoc necessario inventorum una tantummodo circini data apertura, Per Joannem Baptistam De Benedictis inventa**. Venetiis MDLIII.

[70] Taken from the eighth page (the pages are not numbered) of the dedication to abbot Gabriele Guzman of the **Resolutio**: «…ceterum quia cuique quod suum est reddi debet, nam pium et iustum est, Nicolaus Tartaleas mihi quattuor primos libros Euclidis solos legit, reliqua omnia privato labore et studio investigavi: volenti namque scire nihil est difficile».

2.6.2.2 The Works and the Successes of the Ideas on the Mechanics

Exhausted these scanty biographical data,[71] we think it is important, and particularly meaningful, to account for (besides the works) also how his ideas were received by his immediate contemporaries, in order to be able to weight later on the impact that the work of Benedetti has had on those who succeeded to him in the scientific milieu.

Benedetti has dealt with many subjects, although sometimes only occasionally, but we are only interested in what regards the mechanics.[72]

In the long (21 pages) dedication of his first work,[73] to abbot Gabriel Guzman, Benedetti anticipates his criticisms to Aristotle, with regard to the laws of fall of heavy bodies, which he shall develop in the two subsequent editions of the work **Demonstratio Proportionum Motuum Localium contra Aristotelem et Omnes Philosophos** (Venetiis MDLIIII).

The anticipation of his ideas on the motion of heavy bodies has been attributed to the fear to be expropriated of his ideas by other people. Indeed, this happened with the content of the **Demonstratio**. In fact, while there is no evidence of the diffusion of the work among the contemporaries in the immediacy, eight years after a plagiarism of a certain Taisner[74] was published.

It was a work which contained inside, and announced in the subtitle, no less than the **Demonstratio proportionum motuum localium, contra Aristotelem & alios Philosophos**.

In this way Benedetti's ideas went into the scientific debate through that plagiarism which had a spread greater than the original. In fact, Stevin himself (by quoting their false author) reported them in his work **Elementa hydrostatica** (1586).

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[71]Still today, the most complete study on this regard is given by the memoir of Giovanni Bordiga: **Giovanni Battista Benedetti filosofo e matematico veneziano del secolo XVI**—Atti dell'Istituto Veneto di Scienze, Lettere ed Arti, Tomo LXXXV, Parte seconda, pp 585–754 (1925–1926)—reprinted in 1985, with a bibliographic updating by Pasquale Ventrice on the occasion of the workshop on «Giovan Battista Benedetti e il suo tempo». In addition one can consult the entry of him (by Stillman Drake) in the vol. 1 (1970), pp. 604–609, of the **Dictionary of Scientific Biography** op. cit.

[72]With regard to this, the most important studies are due to Carlo Maccagni (of whom we quote **Le speculazioni giovanili "de motu" di Giovan Battista Benedetti** (Pisa, Domus Galilaeana, 1967) and to Enrico Giusti (of whom we quote **Gli scritti «de motu» di Giovan Battista Benedetti**—Bollettino di Storia delle Scienze Matematiche, vol. XVII (1997) fasc. 1, pp 51–103.

[73]See footnote 69.

[74]For this, see: C. Maccagni **Le speculazioni giovanili** «**de motu**» … op cit. pp XXVII–XXXII. The book of Maccagni also contains excerpts of the dedicatory letter of the **Resolutio omnium Euclidis problematum** and the text of the two editions of the subsequent work **Demonstratio …**.

Let us begin to see which are the ideas on the motion that Benedetti introduced in the dedication to abbot Guzman attributing to him the invitation to do it:

> «… Once, when we were still together, you eagerly begged and besought me to write something on the subject of natural motions based on sound theory and also, as far as possible, supported with mathematical demonstrations.»[75]

Benedetti absolutely wants to have assured to him the priority of the ideas where one demonstrates that the theory of Aristotle which maintains that the velocity of a falling heavy body is proportional to the weight of the body itself and in inverse proportion with the density of the medium is groundless.

Benedetti, in his exposition, refers to both the fifth book of Euclid's **Elements** and the work **On Floating Bodies** of Archimedes. He begins, as already Tartaglia had done in the **Quesiti et Inventioni diverse**, by establishing, for the uniform and homogeneous bodies, the proportionality between weight and volume. The relevant demonstration is done for a body which is three times another: almost always the demonstrations are done on particular examples and not in general. Then he moves on to deal with the fundamental subject that is the fall of heavy bodies. Substantially, he maintains the proportionality between the falling velocity and the density (or better, the specific weight), having deducted the buoyancy.[76] Using modern symbols, given two bodies A and B, of densities d_A e d_B, which fall in a medium of density d, and called V_A e V_B their velocities, one has[77]

$$\frac{V_A}{V_B} = \frac{d_A - d}{d_B - d}.$$

It must be remembered that Benedetti, as obviously all the people of that time, cannot think to measure the velocity in the sense that we attribute to these words nowadays. The essential element was the fall time: the body which covered the same distance in a smaller time, or took the same time to cover a greater distance, was the swifter.

Giusti says with regard to the **Resolutio**: «In it, essential conceptual novelties occur: a new law for the motion of heavy bodies which, contrary to the generally accepted Aristotelian law, accounts for the fact that a body with its specific weight equal to that of the medium does not move neither downward nor upward and leads to the isochronism of the fall of the homogeneous bodies.»[78]

As we have already mentioned, the **Demonstratio** had two editions at close range one from the other, starting from Carlo Maccagni—to whom their bibliological study is due—denoted as *__Demonstratio__ and **__Demonstratio__.

[75]«*Olim cum adhuc una essemus, magno me opere orasti obsecratusque es aliqua de motibus naturalibus speculatione sollicita conscriberem, idem quantum possibile est Mathematicis demonstrationibus muniens.*»

[76]The demonstration, based on the Archimedes' principle had been perhaps suggested by the reading of the first treatise of Archimedes translated by Tartaglia in 1551.

[77]E. Giusti, op. cit. in 72) p. 60.

[78]Ibidem, p. 69.

In the ***Demonstratio**, which is an arrangement of the ideas anticipated in the **Resolutio**, it is particularly pointed out that in the void the isochronism in the fall is not limited to homogeneous bodies, but is valid for all heavy bodies, independently of their density. In the ****Demonstratio**, which, according to Giusti, has at last a less general formulation than ***Demonstratio**, the densities are no more alluded and only the weights appear (deducted of the buoyancy). If A and B are two equal bodies of different species, which fall into two media m_1 and m_2, one has

$$\frac{V_A}{V_B} = \frac{P_A - P_1}{P_B - P_2},$$

where P_1 e P_2 are the weights of an equal volume of the two media.[79]

In the ****Demonstratio**, the fact that the resistance of the medium is not only due to buoyancy, but depends on the surface of the body as well, is also reported. This indicated that the equality of the velocities of fall for homogeneous bodies of the same material should hold only in the void. Obviously, for dealing with this fact, Benedetti has not at his disposal the indispensable mathematical technique, therefore he should limit himself to deal only with particular cases.

After the **Resolutio** and the two **Demonstratio**, Benedetti went back to be concerned with the motion of heavy bodies in his last work published in Turin in 1585: **Diversarum Speculationum Mathematicarum & Physicarum Liber**—Taurini MDLXXXV.

The work is organized in six parts of which the third (**De Mechanicis**), the fourth (**Disputationes de quibusdam placitis Aristotelis**) and, in part, the sixth (**Physica Mathematica responsa per Epistolas**) regard the mechanics.[80]

In this work, Benedetti proposes that the velocity of fall of the heavy bodies were proportional to their weight in the medium and in inverse proportion to the intrinsic resistance, that is, their surface.

As Giusti remarks,

> «The path of Benedetti then ends with a position substantially Aristotelian. The proposition he has many times rejected, according to which the velocity of fall was proportional to the weight and in inverse proportion with the resistance of the medium, constitutes the last achievement of his researches. The only difference is: where Aristotle spoke of the absolute weight, Benedetti considered the weight in the medium; and whereas according the philosopher of Stagira the resistance of the medium was proportional to its density, in the opinion of the Venetian mathematicians it does not depend on the greater or smaller density of the medium, but only on the surface of the body which moves onto it. The differences are lesser than what the vehemence of the polemic could suggest».[81]

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[79]Ibidem, p. 75.

[80]Excerpts of this work and also of the **Resolutio**, and the text of ****Demonstratio** are translated in English in the book **Mechanics in Sixteenth—Century Italy** by Stillman Drake & I.E. Drabkin—The University of Wisconsin Press, 1969.

[81]E. Giusti, op. cit. p. 94.

Always remaining on the subject of the motion of heavy bodies, we must add in conclusion an important remark of Benedetti, obviously still in opposition to Aristotle. In chap. XXIV of the **Disputationes de quibusdam placitis Aristotelis**, with regard to the acceleration of the motion of fall of the bodies Benedetti says:

> «Now Aristotle should not have said *(De Caelo* I, Ch. 8) that the nearer a body approaches its terminal goal the swifter it is, but rather that the farther distant it is from its starting point the swifter it is. For the impression is always greater, the more the body moves in natural motion. Thus the body continually receives new impetus since it contains within itself the cause of motion, which is the tendency to go toward its own proper place, outside of which it remains only by force.».[82]

Therefore, the acceleration of falling bodies is sustained by increases of the impetus subsequently supplied ad infinitum. But Benedetti did not give a mathematical formulation of this.

Caverni, Wohlwill and Vailati said words of great appreciation for what we have recalled above, to the extent that Wohlwill described Benedetti as "der bedeutendste"[83] among the immediate forerunners of Galileo.

There are not evidences that he was aware of the medieval developments of the kinematics, but this has not prevented Duhem from considering him as an epigone of the Parisian school.[84] This opinion was shared by Koyré as well,[85] although the text quoted from him induces to think more to the use of an expression generally accepted (the impetus) than to the conscious acceptance of a particular theory:

> «First, every heavy body, when moved either naturally or by force, receives on itself an impression and impetus of motion, so that, even if separated from the motive force, it moves by itself for some length of time. (Indeed, if it is set in *natural* motion, it will always increase its velocity: for then the impetus and impression [of motion] are always being increased, since the motive force is always joined to the body.) Thus, if we move the wheel with our hand and then remove the hand from it, the wheel will not immediately come to rest but will turn for some length of time.»[86]

[82]**Diversarum Speculationum**, op. cit. p. 184—«Aristoteles 8 cap. primi lib. De coelo, dicere non deberet quanto propius accedit corpus ad terminum ad quem, tanto magis fit velox; sed potius, quanto longius distat a termino a quo tanto velocius existit quia tanto maior sit semper impressio, quanto magis movetur naturaliter corpus, et continuo novum impetum recipit cum in se motus causam contineat, quae est inclinatio ad locum suum eundi, extra quem per vim consistit. » (Translation from **Mechanics …** op. cit. p. 217).

[83]"The most authoritative".

[84]P. Duhem: **Études sur Léonard De Vinci**—troisième série—op. cit. pp 214–227.

[85]Alexandre Koyré: **À l'Aube de la science classique**, Paris, Hermann—1939, pp 41–54.

[86]«Nempe omne corpus grave, aut sui natura, aut vi motum, in se recipit impressionem et impetum motus, ita ut separatum a virtute movente per aliquod temporis spatium ex seipso moveatur. nam si secundum naturam motu cieatur, suam velocitatem semper agebit, cum in eo impetus et impressio semper ageantur, quia coniuctam habet perpetuo virtutem moventem. Unde manu movendo rotam, ab eaque eam removendo rota statim non quiescet, sed per aliquod temporis spatium circumverteretur.» **Diversarum Speculationum** … op. cit. pp 286–287. (**Mechanics** … op. cit. p. 230).

This passage is excerpted from the last section (**Physica Mathematica Responsa per Epistolas**) of the **Diversarum Speculationum** where there are some letters of Benedetti answering questions regarding physical subjects asked by important persons of whom he had made the acquaintance.

The passage quoted above belongs to one of the three letters addressed to Giovanni Paolo Capra, a gentleman of Novara; in this case, the question was related to the motion (rotation) of the well-wheel on its axis.

Benedetti explains the causes because of which the motion of rotation of wheel caused by hand is extinguished after a certain time that the hand has left the wheel.

The same problem had been already dealt with in chap. XIV of the section **De Mechanicis** and to that part Duhem refers (see footnote 84), praising Benedetti for having continued and completed the work of Buridan and Albert of Saxony.

Indeed, the whole work of Benedetti gives the impression of having been written more as an opposition to the Aristotelian traditions than as a criticism, or continuation of what maintained by his more or less immediate predecessors. It is quite true that, in that time, it was not usual to quote the authors when using or discussing their results, except when one had to do with a direct and contingent polemic (in this way also Galileo will behave), the only explicit references being to the ancient authors.

In the case under examination, considering the unceasing and exclusive reference to the "philosopher" it seems more correct to exclude a reference to Buridan. Also the fact that Buridan himself had previously dealt with the problem of the motion of the wheel (see quotation in footnote 39) seems more simply due to the circumstance that quite that problem was one of those which more usually occurred to those which were studying the motion of bodies.

For a demonstration that, if not the history, at least a certain propensity of the historians recurs, we must note that, in the case of Benedetti, to the obsession of Duhem of wanting him to be a follower of the Parisian school, the doggedness of his successors succeeded of attributing to Galileo a direct dependence on the work of Benedetti.

The action begins, we can say, with the **History** of Caverni, who, by forcing the (attested) historic truth, heavily fictionalizes some little facts the testimonies of which have been left over.

Let us make a digression to introducing the subject.

In 1589 (at the express wish of Grand Duke Ferdinando Dei Medici) Jacopo Mazzoni of Cesena, one of the most famous humanists of that time in Italy, renowned above all for his studies on Dante, was called as a professor at the University of Pisa. Mazzoni, in Pisa, gave ordinary lectures of Aristotelian philosophy and extraordinary lectures of Platonic philosophy (we are using the locutions of that time).

In the same year, also the twenty-four years old Galileo was arrived at Pisa, as a professor of mathematics. As far as it is known, the young Galileo entered the intellectual community that had been formed around the forty-years-old Mazzoni, a successful and prestigious intellectual and, as we would say nowadays, an interdisciplinary scholar. Of the association and the friendship with Mazzoni, Galileo talked both to his father and to Guidobaldo Del Monte.[87]

The stay of Galileo at Pisa lasted only three years; in fact, he moved to Padua in September 1592. In 1597, Mazzoni published the first volume (destined to remain the sole) of a work where the philosophies of Plato and Aristotle were compared.[88]

Galileo, as received the book in Padua, wrote a long letter to the old colleague in which, besides the compliments for the book he promises to continue to read, he shows «a greatest satisfaction and comfort» in seeing Mazzoni «in some of those questions that in the first years of our friendship we discussed together with great pleasure, (now) to incline to that part believed true by me, and the contrary by You ….».

Those discussions were on the Copernican system and Galileo takes the opportunity for devoting almost the whole letter to that subject. In fact he ends by saying «… I do not want to trouble you anymore, but only to ask you to tell me, if you agree, if in this matter it is possible to save Copernicus. I'm tired of writing and you of reading: so by removing all the slownesses of ceremonies I shall end by kissing your hands …».[89]

At this point, we must say that Mazzoni in his work quotes in different parts the **Diversarum Speculationum** of Benedetti considering it as a reference text for the physical questions.

It is this circumstance that has sparked off all considerations on a possible indoctrination of Galileo by Mazzoni by using the book of Benedetti as a textbook. All this should have taken place in the triennial 1589–1592, in the Pisan period of Galileo.

Let us see how Caverni reconstructs the facts: «There was, among the young auditors in Pisa in those times, also Galileo, and since Mazzoni recognized to him a peculiar attitude of the mind to penetrate the science of motion, recommended to him the book of Benedetti and privately explained its speculations. The young disciple, from those words of the Master and from the reading he suggested, felt instilling in him the first ineffable taste of freedom in thinking, and since the fervent advices and efficacious example had driven him to no more trust in the Aristotle's

[87]From Galileo to his father Vincenzio (… I'm keeping very well and attend to study and to learn with Signor Mazzoni, who greets you. And not having anything also to tell, I end. From Pisa, the ninth of October 1590…). From Guidobaldo Del Monte to Galileo (… I rejoice at the fact that you are getting along well with Signor Mazzoni, not without envy from me, who would sometimes be with both and take pleasure of his talks: give my best regards and a long hand-kissing to Signor Mazzoni …. From Monte Barroccio, the eighth of December 1590 …) E. N. X, pp 44–45.

[88]Jacopo Mazzoni: **In Universam Platonis, et Aristotelis Philosophiam Praeludia, sive de comparatione Platonis & Aristotelis**—Venetiis, MCXCVII. There is a critical edition of this book edited by Sara Matteoli and with an introduction of Anna De Pace—M. D'Auria Editore—2010.

[89]E. N. II, pp 193–202.

authority, then, he concluded, not even in that of other philosopher, not excepted Benedetti himself, if he too should be considered to deviate from the rectitude of the natural truths.»[90]

We shall reopen the question later on, when we shall deal with the writing of Galileo of the Pisan period left handwritten and later called with the name **De Motu antiquiora**. In any case, it must be remarked that Caverni, in his free historical reconstruction, takes care to give notice that in the Pisan period Galileo's ideas were not a slavish repetition of those of Benedetti.

2.7 Galileo and the Engineers of the Renaissance

The title of this short section follows that of a book famous for having been one of the first, if not the first, to deal with the development of the several techniques in the Renaissance, trying to give of them an overall outline.[91]

However, we do not really want to deal with the world of the "engineers". The subject concerns us here only for what deals with a possible influence that the variegate world of the "mechanical arts" has exerted on Galileo and his work. Therefore we anticipate here some considerations which would have equally find place in the course of the subsequent chapters. But we prefer to join them to the discourse done with regard to Benedetti and, above all, of Tartaglia. As we know, Galileo, after his installation at the University of Padua, was obliged to deal also with subjects which had nothing to do with the "pure science", but rather with the military architecture and connected problems.[92]

Between the XV and the XVI century in Italy (but, in part, also in the rest of Europe) an environment had been developed constituted by artists, designers of various sorts, inventors, who can be grouped (following Gille) under the generic label of "engineers". It is clear that the first name which springs to mind is that of Leonard, but we could add Francesco di Giorgio Martini, Leon Battista Alberti, preceded by Lorenzo Ghiberti, Filippo Brunelleschi etc. In Germany, we can mention Albrecht Dürer.

In the period we generally denote as Renaissance, besides a rebirth of the "humanae litterae" also an exceptional development of the technique had happened, at the beginning also favored by the translations of the ancient works (on the machines and their use) passed from the Greek to the Arabic world. Later on, new treatises, written in Italian, were printed, full of pictures representing new machines. The authors of these treatises did not restrict themselves to describe machines really constructed, but more often, were projects not actually carried out, but elaborated in the pictures in minute details. It's enough to browse, for instance, a copy of the **Diverse et artificiose macchine** of Agostino Ramelli (1588).

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[90] R. Caverni: **La Storia del Metodo Sperimentale in Italia**, op. cit. Tomo IV, p. 275.

[91] Bernard Gille: **Les ingénieurs de la Renaissance** (Hermann, Paris, 1964).

[92] See, in E. N. II, the **Breve instruzione all'architettura militare** and the **Trattato di fortificazione**.

The protagonists of this development of the technique did know neither the Greek nor the Latin, i.e. were, as at that time were named, men without letters.

The problem of the use of the "vernacular Italian" (or Tuscan) had received a great interest also by the men of letters (see the **Dialogo delle lingue** of Sperone Speroni—1549) and therefore the Italian was legitimate anywhere, except for the academic teaching, where the Latin remained the official language.

Nonetheless, the first work that Galileo personally saw into print in the Paduan period (**Le Operazioni del Compasso geometrico et militare**—1606),[93] as "lecturer of mathematics in the Study of Padua", as he presented himself in the title page, was written in Italian.

It is evident that he had taken into account the final-users to whom the work was addressed. On the other hand, the high regard in which he held the world of the technicians clearly appears at the beginning of his last work, the **Discourses**:

> «*Salviati.* Frequent experience of your famous arsenal, my Venetian friends, seems to me to open a large field to speculative minds for philosophizing, and particularly in that area which is called mechanics, inasmuch as every sort of instrument and machine is continually put in operation there. And among its great number of artisans there must be some who, through observations handed down by their predecessors as well as those which they attentively and continually make for themselves, are truly expert and whose reasoning is of the finest.
>
> *Sagredo.* You are quite right. And since I am by nature curious, I frequent the place for my own diversion and to watch the activity of those whom we call "key men" *[Proti]* by reason of a certain pre-eminence that they have over the rest of the workmen. Talking with them has helped me many times in the investigation of the reason for effects that are not only remarkable, but also abstruse, and almost unthinkable.»[94]

The first of the quoted passages is the beginning of the dialogue in the role of Salviati, alter ego of Galileo, whereas the second is the intervention into the dialogue by the Venetian gentleman Giovanfrancesco Sagredo, but Favaro says «… we are sure that Galileo wanted to refer to himself what that adds to the said things …».[95]

All this for anticipating the observation that Galileo shall ever choose the language in which to express himself taking into account both of the addressee and the diffusion he shall want to have. He was conscious of moving in a world which was not anymore the world of Benedetti.

Galileo could talk also to the "engineers".

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[93]Besides in E. N. II, see **Il Compasso geometrico e militare di Galileo Galilei** edited by Roberto Vergara Caffarelli, Edizioni ETS, 1992 and Galileo Galilei—**Operations of the Geometric and military compass**—Smithsonian Institute Press, 1978. (Facsimile reprint translated with an introduction by S. Drake).

[94]E. N. VIII (**Discorsi e Dimostrazioni matematiche intorno a due nuove scienze**), p. 49. (Drake p. 11).

[95]Antonio Favaro: Galileo **Galilei e lo studio di Padova, II**, reprint of the original work of 1883, Editrice Antenore—Padova, 1996, p. 70.

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