Structural and Dimensional Synthesis of Parallel Manipulator with Two End—Effectors

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Abstract In this paper the methods of structural and dimensional synthesis of a parallel manipulator with two end-effectors are presented. Parallel manipulator with two end-effectors is formed from two serial manipulator by connecting of their links by binary links with two revolute kinematic pairs. Geometrical parameters of the binary link have been determined at three, four and five finitely separated position of the end-effectors.

Keywords Parallel manipulator · End–effector · Dimensional synthesis · Finitely separated positions

1 Introduction

In technological processes with stereotyped motion of the manipulating object it is advisable to use a parallel manipulator (PM) with fewer DOF or with one DOF instead of robot with many DOF. PM is synthesized by given laws of motion of the end-effectors and input links. Therefore PM unlike the serial manipulator operates at certain kinematic diagram and geometrical parameters of its links. There are

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many works on the structural and dimensional synthesis of mechanisms [1–4]. In these works the structural synthesis of mechanisms is carried out separately, and the dimensional synthesis of mechanisms is carried out at the certain kinematic diagram. We propose a method of structural-parametric synthesis of mechanisms and manipulators [5–7]. According to this method any mechanism irrespective of functional purpose (function generation, path generation, body guidance) is formed from the actuating kinematic chain (AKC) and the closing kinematic chain (CKC). A kinematic chain with many DOF which implements the given laws of motions of the end-effectors and the input link is called AKC. A kinematic chain with negative DOF which connects the links of the AKC and forms a mechanism (manipulator) is called CKC. AKC and CKC are the structural modules. In this paper the methods of structural and dimensional synthesis of PM with two end-effectors on the basis of the method of structural-parametric synthesis are presented. Two serial manipulators (AKC) and the binary link with two revolute kinematic pairs (CKC) are the structural modules of the PM with two end-effectors.

2 Structural Synthesis of PM with Two End-Effectors

Consider the problem of manipulating of two objects. Depending on the type of technological operation work the manipulation of two objects may be implemented in two modes: a simultaneous manipulation and a consecutive manipulation.

In the simultaneous manipulation in the initial position of the serial manipulators *ABC* and *DEF* (Fig. 1a) a griping operation of two objects P_1 and P_2 is carried out by the grippers *C* and *F* (Fig. 1a). Then a transference of the objects to the positional point $P_{1(2),N}$ is carried out (Fig. 1b). After positioning of the manipulating objects the grippers back to their initial positions (Fig. 1a).

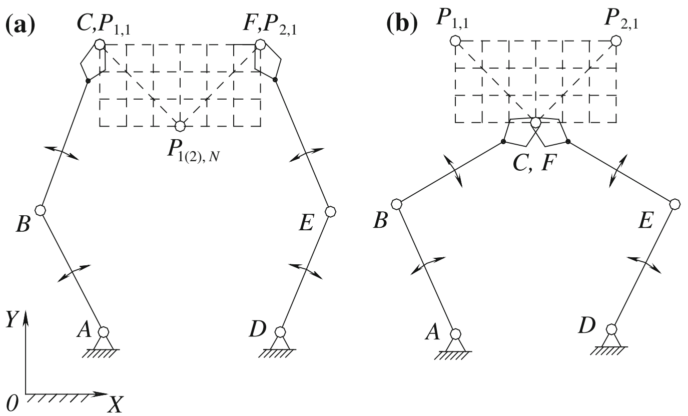

Fig. 1 Two serial manipulators

In the consecutive manipulation in the initial positions of the serial manipulators *ABC* and *DEF* a griping operation of the object P_1 is carried out by the gripper *C* of the first serial manipulator *ABC* (Fig. 1a). Then a transference of the object to an intermediate position $P_{1(2),N}$ is carried out (Fig. 1b). At the same time the gripper *F* of the second manipulator *DEF* simultaneously positioned in the intermediate point. In this intermediate point $P_{1(2),N}$ a re-grasping operation of the object occurs by the second manipulator *DEF*. Then both manipulators back to their initial positions (Fig. 1b).

A novel PM (parallel manipulator) with two end-effectors and with two DOF to manipulate of two objects has been developed (Fig. 2). This PM provides a set of laws of motion of two objects in two modes. Closed kinematic chain of the novel PM increases its load capacity and positioning accuracy. Furthermore, the number of drives is reduced from four to two, and two drives in mobile joints *B* and *E* are excluded which is also an advantage of the novel PM.

Structural synthesis of the novel PM with two end-effectors is made as follows. Two links *BC* and *EF* of the two serial manipulators *ABC* and *DEF* are connected by binary link *GH* with two revolute kinematic pairs. As a binary link with two revolute kinematic pairs has one negative DOF and it imposes one geometrical constraint to the relative motions of the links we obtain a kinematic chain *ABGHED* with three DOF. If we connect the link *GH* of this kinematic chain with a frame by a binary link *IK* with two revolute kinematic pairs then we obtain a kinematic diagram of the PM with two end-effectors and two DOF.

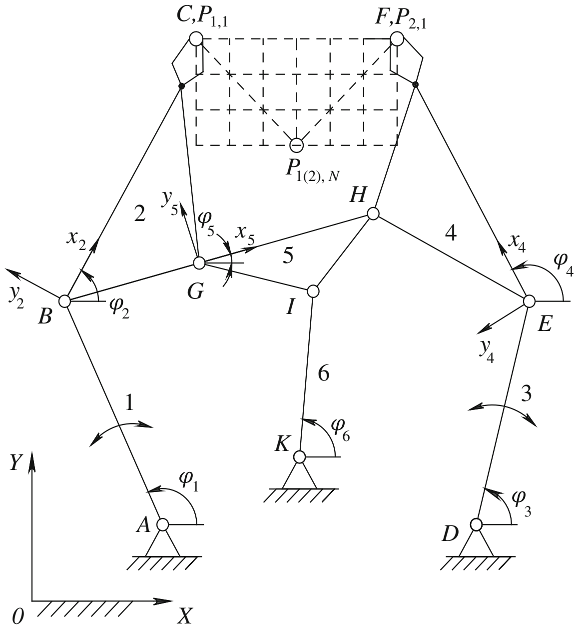

Fig. 2 PM with two end-effectors

Number of DOF of PM is determined by Chebyshev's formula [8]

$$W = 3n - 2p_5, \tag{1}$$

where n is number of mobile links, p_5 is number of kinematic pairs of the fifth class. For the considered PM $n = 6$ and $p_5 = 8$ then $W = 3 \cdot 6 - 2 \cdot 8 = 2$. For the binary link with two revolute kinematic pairs $n = 1$ and $p_5 = 2$ then $W = 3 \cdot 1 - 2 \cdot 2 = -1$.

3 Dimensional Synthesis of PM with Two End-Effectors

The end-effectors C and F have been moved from their initial positions $P_{1,1}$ and $P_{2,1}$ to the ultimate position $P_{1(2),N}$ along the segments $P_{1,1}$ $P_{1(2),N}$ and $P_{2,1}$ $P_{1(2)N}$ (Fig. 2). Set the coordinates $X_{P_{1,i}}$, $Y_{P_{1,i}}$ and $X_{P_{2,i}}$, $Y_{P_{2,i}}$ of the points P_1 and P_2 along these segments in the absolute coordinate system OXY. Also set the coordinates X_A, Y_A and X_D, Y_D of the fixed joints A and D, the lengths of links $l_{AB,}$ l_{BC} and $l_{DE,}$ l_{EF} of two serial manipulators ABC and DEF. Determine the angles φ_1, φ_2 and φ_3, φ_4

$$\begin{aligned} \varphi_{1i} &= \varphi_{ACi} + \cos^{-1}\frac{l_{AB}^2 + l_{ACi}^2 - l_{BC}^2}{2l_{AB}l_{ACi}}, \\ \varphi_{3i} &= \varphi_{DFi} - \cos^{-1}\frac{l_{DE}^2 + l_{DFi}^2 - l_{EF}^2}{2l_{DE}l_{DFi}}, \end{aligned} \tag{2}$$

$$\varphi_{2i} = \mathrm{tg}^{-1}\frac{Y_{P_{1,i}} - Y_{B_i}}{X_{P_{1,i}} - X_{B_i}}, \quad \varphi_{4i} = \mathrm{tg}^{-1}\frac{Y_{P_{2,i}} - Y_{E_i}}{X_{P_{2,i}} - X_{E_i}}, \tag{3}$$

where

$$\begin{bmatrix} X_{Bi} \\ Y_{Bi} \end{bmatrix} = \begin{bmatrix} X_A \\ Y_A \end{bmatrix} + l_{AB} \cdot \begin{bmatrix} \cos\varphi_{1i} \\ \sin\varphi_{1i} \end{bmatrix}, \begin{bmatrix} X_{Ei} \\ Y_{Ei} \end{bmatrix} = \begin{bmatrix} X_D \\ Y_D \end{bmatrix} + l_{DE} \cdot \begin{bmatrix} \cos\varphi_{3i} \\ \sin\varphi_{3i} \end{bmatrix}. \tag{4}$$

Determine the geometrical parameters $x_G^{(2)}, y_G^{(2)}, x_H^{(4)}, y_H^{(4)}, l_{GH}$ of the binary link GH, where $x_G^{(2)}, y_G^{(2)}$ and $x_H^{(4)}, y_H^{(4)}$ are the coordinates of the joints G and H in the local systems of coordinates Bx_2y_2 and Ex_4y_4 of the links 2 and 4 respectively, l_{GH} is the length of the link GH. Consider the inverse motion of the system of coordinate Ex_4y_4 relative to the system of coordinate Bx_2y_2. Then the joint H moves along the circular arc with center at the joint G and with radius l_{GH}. Write the equation of the circle

$$(x_{Hi}^{(2)} - x_G^{(2)})^2 + (y_{Hi}^{(2)} - y_G^{(2)}) = l_{GH}^2, \quad i = 1, 2, \ldots, N, \tag{5}$$

where $x_{Hi}^{(2)}$ and $y_{Hi}^{(2)}$ are the coordinates of the joint H in the system of of coordinates Bx_2y_2 of the link 2; N is number of given positions of the systems of coordinates Bx_2y_2 and Ex_4y_4.

Coordinates $x_{Hi}^{(2)}$ and $y_{Hi}^{(2)}$ of the joint H in the system of coordinates Bx_2y_2 of the link 2 are defined by the equation

$$\begin{bmatrix} x_{Hi}^{(2)} \\ y_{Hi}^{(2)} \end{bmatrix} = \begin{bmatrix} \cos\phi_{2i} & \sin\phi_{2i} \\ -\sin\phi_{2i} & \cos\phi_{2i} \end{bmatrix} \cdot \begin{bmatrix} X_{Hi} - X_{Bi} \\ Y_{Hi} - Y_{Bi} \end{bmatrix}, \tag{6}$$

where X_{Hi}, Y_{Hi} and X_{Bi}, Y_{Bi} are the coordinates of the joints H and B in the absolute system of coordinates OXY which are defined by the Eq. (4) and the equation

$$\begin{bmatrix} X_{Hi} \\ Y_{Hi} \end{bmatrix} = \begin{bmatrix} X_{Ei} \\ Y_{Ei} \end{bmatrix} + \begin{bmatrix} \cos\phi_{4i} & -\sin\phi_{4i} \\ \sin\phi_{4i} & \cos\phi_{4i} \end{bmatrix} \cdot \begin{bmatrix} x_H^{(4)} \\ y_H^{(4)} \end{bmatrix}. \tag{7}$$

Substituting the expressions (6) and (7) into the Eq. (5) we obtain

$$(x_{Hi}^{(2)} \cdot x_G^{(2)} + y_{Hi}^{(2)} \cdot y_G^{(2)}) + \frac{1}{2}(l_{GH}^2 - x_G^{(2)^2} - y_G^{(2)^2}) = \frac{1}{2}(x_{Hi}^{(2)^2} + y_{Hi}^{(2)^2}). \tag{8}$$

Denote

$$h = \frac{1}{2}(l_{GH}^2 - x_{Gi}^{(2)^2} - y_{Gi}^{(2)^2}), \quad r_{Hi}^{(2)^2} = x_{Hi}^{(2)^2} + y_{Hi}^{(2)^2}. \tag{9}$$

Then the Eq. (8) takes the form

$$x_{Hi}^{(2)} \cdot x_G^{(2)} + y_{Hi}^{(2)} \cdot y_G^{(2)} + h = \frac{1}{2} r_{Hi}^{(2)^2}, \; i = 1, 2, \ldots, N. \tag{10}$$

The resulting system of Eq. (10) is linear of the coordinates $x_G^{(2)}, y_G^{(2)}$ of the center G of the circle and its radius l_{GH} expressed through the parameter h.

Let the number of given positions $N = 3$. Then we obtain a system of three linear equations with three unknowns $x_G^{(2)}$, $y_G^{(2)}$, h the matrix form which has a form

$$\begin{bmatrix} x_{H_1}^{(2)} & y_{H_1}^{(2)} & 1 \\ x_{H_2}^{(2)} & y_{H_2}^{(2)} & 1 \\ x_{H_3}^{(2)} & y_{H_3}^{(2)} & 1 \end{bmatrix} \cdot \begin{bmatrix} x_G^{(2)} \\ y_G^{(2)} \\ h \end{bmatrix} = \frac{1}{2} \begin{bmatrix} r_{H_1}^{(2)^2} \\ r_{H_2}^{(2)^2} \\ r_{H_3}^{(2)^2} \end{bmatrix} \tag{11}$$

or

$$\boldsymbol{A}\boldsymbol{x} = \boldsymbol{b}, \tag{12}$$

where

$$\boldsymbol{A} = \begin{bmatrix} x_{H_1}^{(2)} & y_{H_1}^{(2)} & 1 \\ x_{H_2}^{(2)} & y_{H_2}^{(2)} & 1 \\ x_{H_3}^{(2)} & y_{H_3}^{(2)} & 1 \end{bmatrix}, \ \boldsymbol{x} = \left[x_G^{(2)} y_G^{(2)} h\right]^T, \ \boldsymbol{b} = \frac{1}{2}\left[r_{H_1}^{(2)} r_{H_2}^{(2)} r_{H_3}^{(2)}\right]^T.$$

Vector $\boldsymbol{x}$ of the binary link GH parameters is determined by the equation

$$\boldsymbol{x} = \mathbf{A}^{-1}\boldsymbol{b} \tag{13}$$

when $\det(A) \neq 0$. If $\det(A) = 0$ then the radius l_{GH} of the circle is equal to ∞, i.e. the point G will move in a straight line. Consequently, the joint G should be transformed into slider.

Thus, when $N = 3$ we can find a point (joint) G on a plane of the link BC and length l_{GH} of the link GH for all points lying on a plane of the link EF in condition $\det(A) \neq 0$.

Let number of given positions $N = 4$. Then we obtain a system of four linear equations from (10). This system of equations has a solution when the determinant of the expanded matrix is zero

$$w_{1,2,3,4} = \begin{vmatrix} x_{H_1}^{(2)} & y_{H_1}^{(2)} & 1 & \frac{1}{2} r_{H_1}^{(2)^2} \\ x_{H_2}^{(2)} & y_{H_2}^{(2)} & 1 & \frac{1}{2} r_{H_2}^{(2)^2} \\ x_{H_3}^{(2)} & y_{H_3}^{(2)} & 1 & \frac{1}{2} r_{H_3}^{(2)^2} \\ x_{H_4}^{(2)} & y_{H_4}^{(2)} & 1 & \frac{1}{2} r_{H_4}^{(2)^2} \end{vmatrix} = 0. \tag{14}$$

If substitute into the determinant (14) the values of $x_{Hi}^{(2)}$, $y_{Hi}^{(2)}$, $r_{Hi}^{(2)}$, (i = 1, 2, 3, 4) defined by the Eqs. (6), (7) and (9) then we obtain the equation of the fourth order which is a curve of circular points. The points on this curve describe the arc of the circle. If we select a point on the curve of circular points it is sufficiently to determine the center of a circle and its radius from any three equations of the system (10). Then this circle passes through the fourth position.

Let the number of given position $N = 5$. Then we obtain a system of five linear equations from (10). This system of five linear equations has a solution if the rank of expanded matrix of this system is three. Consequently, all the minors of the fourth order of this matrix is equal to zero

$$w_{1,2,3,4} = 0, w_{1,2,3,5} = 0, w_{1,2,4,5} = 0, w_{2,3,4,5} = 0, w_{1,3,4,5} = 0. \tag{15}$$

Each of determinants (15) has a view similar to (143). Substituting to the determinants the values of $x_{Hi}^{(2)}$, $y_{Hi}^{(2)}$, $r_{Hi}^{(2)}$, (i = 1, 2,…, 5) defined by the Eqs. (6) and (9) we obtain the curves equations of the fourth order. Solutions of the system (15) is determined by the points of intersection of any two curves of this system, for example by the points of intersections of the curves $w_{1,2,3,4}$ and $w_{1,2,3,5}$. These points are circular points each of which has five positions on one circle. Other three curves also pass through the circular points.

As a result of synthesis of the binary link *GH* we obtain a mechanism *ABGHED* with three DOF. If we connect the link *GH* with a frame by a binary link *IK* then we obtain a kinematic diagram of the PM with two DOF.

For synthesis of the binary link *IK* we write the equation of circle in absolute motion

$$(X_{I_i} - X_K)^2 + (Y_{I_i} - Y_K)^2 = l_{IK}^2, \tag{16}$$

where

$$\begin{bmatrix} X_{I_i} \\ Y_{I_i} \end{bmatrix} = \begin{bmatrix} X_{G_i} \\ Y_{G_i} \end{bmatrix} + \begin{bmatrix} \cos\varphi_{5_i} & -\sin\varphi_{5_i} \\ \sin\varphi_{5_i} & \cos\varphi_{5_i} \end{bmatrix} \cdot \begin{bmatrix} x_I^{(5)} \\ y_I^{(5)} \end{bmatrix}. \tag{17}$$

Substituting the expression (17) into the Eq. (16) we obtain

$$(X_{I_i} \cdot X_K + Y_{I_i} \cdot Y_K) + \frac{1}{2}(l_{IK}^2 - X_K^2 - Y_K^2) = \frac{1}{2}(X_{I_i}^2 + Y_{I_i}^2). \tag{18}$$

Denote

$$H = \frac{1}{2}(l_{IK}^2 - X_K^2 - Y_K^2), \quad R_{H_i}^2 = X_{I_i}^2 + Y_{I_i}^2. \tag{19}$$

Then the Eq. (18) takes the form

$$X_{I_i} \cdot X_K + Y_{I_i} \cdot Y_K + H = \frac{1}{2}R_{H_i}^2, \quad i = 1, 2, \ldots, N. \tag{20}$$

The resulting system of Eq. (20) is a linear system with respect to the coordinates $X_{K,}$ Y_K of the center *K* of the circle and its radius l_{IK} expressed through the parameter *H*. Further we determine the coordinates $x_I^{(5)}$, $y_I^{(5)}$ of circular point *I*, the coordinates $X_{K,}$ Y_K of the center *K* and the radius l_{IK} of the circle similarly the synthesis of the binary link *GH*.

Table 1 The coordinate values of the points P_1 and P_2

i	1	2	3	4
X_{P_1}	20.0	24.0	28.0	32.0
Y_{P_1}, Y_{P_2}	68.0	64.0	60.0	56.0
X_{P_2}	44.0	40.0	36.0	32.0

4 Numerical Example

Let the grippers C and F of PM move from their initial positions $P_{1,1}$ and $P_{2,1}$ to the positional position $P_{1(2),4}$ on straight segments $P_{1,1}$ $P_{1(2),4}$ and $P_{2,1}$ $P_{1(2),4}$ in the absolute system of coordinates OXY. The coordinate values of the points are given in Table 1. It is necessary to synthesize PM with two end-effectors.

Set the coordinates $X_A = 16.0; Y_A = Y_D = 9.5; X_D = 54.5$ of the fixed joints A, D and the lengths of the links l_{AB}, $l_{DE} = 30.0; l_{BC}$, $l_{EF} = 35.0$ of two serial manipulators ABC and DEF and calculate the positions of links and coordinates by the Eqs. (2)–(4). On the base of the above developed method the geometrical parameters of the binary links GH and IK are calculated: $x_G^{(2)} = 12.348; y_G^{(2)} = -12.512;$ $x_H^{(4)} = 18.107$, $y_H^{(4)} = 12.014$; $l_{GH} = 22.170; x_I^{(5)} = 12.503$; $y_I^{(5)} = -7.134$; $X_K = 32.701$; $Y_K = 17.502$; $l_{IK} = 20.062$. Results of kinematic analysis have been presented in [9].

5 Conclusions

The methods of structural and dimensional synthesis of the PM with two end-effectors are developed on the base of structural-parametric synthesis. According to these methods PM is formed from the actuating and closing kinematic chains (AKC) and (CKC) which are the structural modules. Two serial manipulators (AKC) and the binary link with two revolute kinematic pairs (CKC) are the structural modules of PM. Geometrical parameters of the serial manipulator's links are given or varied and geometrical parameters of the binary link are determined from the equation of geometrical constraint imposed to the relative motions of the AKC links. The geometrical parameters of the binary link are determined at three, four and five finitely separated positions of the end-effectors.

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