Preface

Hardy's interest in inequalities (in both discrete and continuous forms) was during the period 1906–1928. As a result of his work, the subject was changed radically, and what had previously been a collection of isolated formulas became a systematic discipline. The classical book *Inequalities* by Hardy et al. [77] contains two chapters devoted to Hardy- and Hilbert-type inequalities and the growth of Hardy-type inequalities in the literature stimulated this book.

The book is devoted to dynamic inequalities of Hardy type and extensions and generalizations via convexity on a time scale $\mathbb{T}$. In particular, the book contains the time scale versions of classical Hardy-type inequalities, Hardy- and Littlewood-type inequalities, Hardy-Knopp-type inequalities via convexity, Copson-type inequalities, Walsh-type inequalities, Liendeler-type inequalities, Levinson-type inequalities and Pachpatte-type inequalities, Bennett-type inequalities, Chan-type inequalities, and Hardy type inequalities with two different weight functions. These dynamic inequalities contain the classical continuous and discrete inequalities as special cases when $\mathbb{T} = \mathbb{R}$ and $\mathbb{T} = \mathbb{N}$ and can be extended to different types of inequalities on different time scales such as $\mathbb{T} = h\mathbb{N}$, $h > 0$, $\mathbb{T} = q^{\mathbb{N}}$ for $q > 1$, etc.

The book consists of seven chapters and is organized as follows. In Chap. 1, we present preliminaries, basic concepts, and the basic inequalities that will be needed in the book. Next, we present time scale versions of classical Hardy-type inequalities and Hardy- and Littlewood-type inequalities. We also prove extensions of Hardy-type inequalities and its general form via convexity on time scales. Chapter 2 is devoted to Copson-type inequalities on an arbitrary time scale $\mathbb{T}$. In particular, we consider classical forms of Copson-type inequalities and their converses which are extensions of Hardy-type inequalities. We also discuss the extension of Copson inequalities obtained by Walsh on discrete time scales. In Chap. 3, we present Leindler-type inequalities and their extensions on an arbitrary time scale $\mathbb{T}$. We also consider the dual of these inequalities and their converses in this chapter. Chapter 4 is devoted to Littlewood-type inequalities on an arbitrary

time scale $\mathbb{T}$. First, we consider the generalized form of Littlewood-type inequalities with decreasing functions. Next, we consider a generalization of Littlewood-type inequalities that was considered by Bennett, and we end the chapter with the sneak-out principle on time scales. Chapter 5 is concerned with weighted Hardy-type inequalities on an arbitrary time scale $\mathbb{T}$. The results can be considered as extensions of the results due to Copson, Bliss, Flett and Bennett, Leindler, Chen, and Yang. In Chap. 6, we discuss Levinson-type inequalities on time scales. Also we include some dynamic inequalities on time scales of Chan and Pachpatte type. The proofs of the main results include the definition of the logarithmic function on time scales and its delta derivative and the application of Jensen's inequality. Chapter 7 is devoted to Hardy-Knopp-type inequalities on an arbitrary time scale $\mathbb{T}$. A one-dimensional, two-dimensional, and multidimensional versions of Hardy-Knopp inequalities are considered and extended on time scales via convexity. The refinement inequalities of Hardy-Knopp type which depends on the applications of superquadratic functions and the corresponding refinement Jensen' s inequality will also be discussed.

In this book, we followed the history and development of these inequalities. Each section is self-contained, and one can see the relationship between the time scale versions of the inequalities and the classical ones. To the best of the authors' knowledge, this is the first book devoted to Hardy-type inequalities and their extensions on time scales.

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