Complementation of Branching Automata for Scattered and Countable Series-Parallel Posets

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Abstract. We prove the closure under complementation of the class of languages of scattered and countable N-free posets recognized by branching automata. The proof relies entirely on effective constructions.

Keywords: Transfinite N-free posets · Series-parallel posets · SP-rational languages · Automata · Commutative monoids

1 Introduction

Automata over finite words have been widely studied since their introduction by Kleene in the last fifties, because they are a natural model for sequential computation with bounded memory, and they are linked to many other areas, as for example formal logic, coding theory or formal series. The depth of those links and the richness of the results led the community to develop generalizations o f Kleene automata, as for example automata over trees [18], ω-words [6,17], ordinals [7], and more recently, over linear orderings [5].

Among those generalizations, Lodaya and Weil proposed a notion of *branching-automata* that are a natural model for parallel computation with the Fork/Join principle. The Fork/Join principle splits an execution flow f into n concurrent flows $f_1, \ldots, f_n$ and joins $f_1, \ldots, f_n$ before it continues. Divide-and-conquer concurrent programming naturally uses this Fork/Join principle. Traces of execution of programs are in this case finite N-free posets, or equivalently, finite series-parallel posets [19,22]. Lodaya and Weil extended some fundamental results of automata on words to branching-automata, as for example a Kleene-like Theorem or algebraic recognizability [13–16]. Unfortunately, and contrarily to the finite words case, the algebraic counterpart of branching automata may be infinite, leading to difficulties regarding the generalization of fundamental results over finite words to finite N-free posets. Kuske [11,12] extended branching-automata to recognition of ω-N-free posets, and established a connection with monadic second-order logic (MSO[<]) in the particular case of languages of N-free posets with bounded-size antichains. The logical characterization of languages of finite N-free posets recognized by branching automata of Lodaya and Weil is provided in [3] in the general case.

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In [4], branching automata are generalized to N-free posets with finite antichains and countable and scattered chains, and a Kleene-like Theorem is provided. The connection with MSO[<] is established in [2] in the particular case of languages of N-free posets with bounded-size antichains. In this paper, we prove that the class of languages recognized by the generalization of branching automata of [4] is closed under complement. The (effective) proof relies on an algebraic approach of branching automata, on the use of Simon's factorization forests proposed by Colcombet in [9] for regular languages of linear orderings, and on the closure under complementation of the class of rational sets of finitely generated commutative monoids [10].

2 Notation and Basic Definitions

Let E be a set. We denote by $|E|$, $\mathcal{P}(E)$, $\mathcal{P}^+(E)$ and $\mathcal{M}^{>1}(E)$ respectively the cardinality of E, the set of subsets of E, the set of non-empty subsets of E and the set of multi-subsets of E with at least two elements. For any integer n, the set $\{1, \ldots, n\}$ is denoted by $[n]$ and the group of permutations of $[n]$ by S_n.

We start by some basic definitions on linear orderings. We refer to [20] for a survey on the subject. Let J be a set equipped with an order $<$. The ordering J is *linear* if all elements are comparable : for any distinct j and k in J, either $j < k$ or $k < j$. For any linear ordering J, we denote by $-J$ the backward linear ordering obtained from the set J with the reverse ordering. A linear ordering J is *dense* if for any $j, k \in J$ such that $j < k$, there exists an element i of J such that $j < i < k$. It is *scattered* if it contains no dense sub-ordering. The orderings $\omega = (\mathbb{N}, <)$ and $\zeta = (\mathbb{Z}, <)$ are scattered. Ordinals are also scattered orderings. We denote by $\mathcal{O}$ the class of countable ordinals and $\mathcal{S}$ the class of countable scattered linear orderings. An *interval* K of $J \in \mathcal{S}$ is a subset $K \subseteq J$ such that $\forall k_1, k_2 \in K, \forall j \in J$, if $k_1 < j < k_2$ then $j \in K$.

A *poset* $(P, <)$ is a set P partially ordered by $<$. In order to lighten the notation we often denote the poset $(P, <)$ by P. An *antichain* is a subset P' of P such that all elements of P' are incomparable (with $<$). The *width* of P is $\mathrm{wd}(P) = \sup\{|E| : E \text{ is an antichain of } P\}$ where sup denotes the least upper bound of the set. If $x, y \in P$, we denote by $x^- = \{z \in P : z < x\}$, $x^+ = \{z \in P : x < z\}$ and $x \sim_< y$ if $x^- \cup x^+ \cup \{x\} = y^- \cup y^+ \cup \{y\}$. In this paper, we restrict to *countable scattered posets of finite width* which are thus partially ordered countable sets without any dense sub-ordering. Let $(P, <_P)$ and $(Q, <_Q)$ be two disjoint posets. The *parallel composition* of $(P, <_P)$ and $(Q, <_Q)$ is the poset $(P \cup Q, <)$ where $x < y$ if and only if ($x, y \in P$ and $x <_P y$) or ($x, y \in Q$ and $x <_Q y$). The *sum* (or *sequential composition*) $P + Q$ of P and Q is the poset $(P \cup Q, <)$ such that $x < y$ if and only if one of the following three conditions is true: (1): $x \in P$, $y \in P$ and $x <_P y$; (2): $x \in Q$, $y \in Q$ and $x <_Q y$; (3): $x \in P$ and $y \in Q$. The sum of two posets can be generalized to any linearly ordered sequence of pairwise disjoint posets: if J is a linear ordering and $((P_j, <_j))_{j \in J}$ is a sequence of posets, then $\sum_{j \in J} P_j = (\cup_{j \in J} P_j, <)$ such that $x < y$ if and only if ($x \in P_j$, $y \in P_j$ and $x <_j y$) or ($x \in P_j$ and

$y \in P_k$ and $j < k$). The sequence $((P_j, <_j))_{j \in J}$ is called a *J-factorization*, or *factorization* for short, of the poset $\sum_{j \in J} P_j$. A nonempty poset P is *sequential* if it admits a J-factorization where J contains at least two elements, or P is a singleton. It is a *parallel poset* otherwise. The only poset $(\emptyset, <)$ of width 0 is called *empty poset* and is denoted by ε. The class $SP^\diamond$ of *series-parallel* scattered and countable posets is the smallest class of posets containing ε, the singleton and closed under finite parallel composition and sum indexed by countable scattered linear orderings. It has a nice characterization in terms of graph properties: $SP^\diamond$ coincides with the class of scattered and countable N-free posets without infinite antichain (see [4]). We denote by $SP^{\diamond+} = SP^\diamond - \{\varepsilon\}$.

The sets of (Dedekind-MacNeille) cuts of a poset P is defined as a generalization of cuts of linear orderings. It is the set of all pairs (A, B), with $A, B \subseteq P$, such that B consists of all the elements of P greater than all the elements of A, and reciprocally, A consists of all the elements of P less than all the elements of B. The cuts are partially ordered with inclusion on the first component, and with the elements of P with $(A, B) < x$ if $x \in B$. The partially ordered set of all cuts of P is denoted $\hat{P}$, and we usually denote by $P \cup \hat{P}$ the partially ordered set consisting of the elements of P with its cuts. Note that an equivalence class of cuts of P for $\sim_<$ is totally ordered. The notation $\hat{P}^{\iota\iota'}$ with $\iota, \iota' \in \{[,]\}$ excludes or not the minimum and maximum elements from $\hat{P}$. We denote also by $\hat{P}^* = \hat{P} - \{(\emptyset, P), (P, \emptyset)\}$. We define the partial ordering $\preceq$ over the cuts of P by $(A, B) \preceq (A', B')$ if and only if $A \cup B = A' \cup B'$ and $A \subseteq A'$.

An *alphabet* is a nonempty set whose elements are called *letters*. In this paper, we use only finite alphabets, thus the term “finite” is omitted. A poset *labeled* by A is a poset $(P, <)$ equipped with a *labeling* map $P \to A$ which associates a letter to any element of P. The notion of a labeled poset corresponds to the notion of a *pomset* in the literature. Also, the finite labeled posets of width 1 correspond to the usual notion of words. In order to shorten the notation, we make no distinction between a poset and a labeled poset, except for operations. The *sequential product* (or *concatenation*, denoted by $P \cdot P'$ or PP' for short) and the *parallel product* (denoted by $P \parallel P'$) of labeled posets are respectively obtained by the sequential and parallel compositions of the corresponding (unlabeled) posets. The class of posets of $SP^\diamond$ labeled by A (or over A) is denoted by $SP^\diamond(A)$. We set $A^\diamond = \{P \in SP^\diamond(A) : \mathrm{wd}(P) \leq 1\}$. Observe that the elements of $A^\diamond$ are precisely the usual words on scattered and countable linear orderings, as defined in [5]. A *language* of $SP^\diamond(A)$ is a subset of $SP^\diamond(A)$. Let A and B be two alphabets and let $P \in SP^\diamond(A)$, $L \subseteq SP^\diamond(B)$ and $\xi \in A$. The labeled poset P in which each occurrence of the letter ξ is non-uniformly replaced by a labeled poset of the language L is denoted by $L \circ_\xi P$. The substitution, sequential and parallel products can be easily extended from labeled posets to languages of posets.

3 Rational Languages and Branching Automata

Let A be an alphabet and $\xi \in A$. Using the definition of substitution $\circ_\xi$, we define the iterated substitution on languages. By the way the usual rational operations on linear orderings are recalled. Let L and L' be languages of $SP^\diamond(A)$:

$$L \circ_\xi L' = \bigcup_{P \in L'} L \circ_\xi P, \qquad L^* = \{\prod_{j \in n} P_j | n \in \mathbb{N}, P_j \in L\}$$

$$L^{*\xi} = \underset{i \in \mathbb{N}}{\cup} L^{i\xi} \text{ with } L^{0\xi} = \{\xi\} \text{ and } L^{(i+1)\xi} = (\underset{j \leq i}{\cup} L^{j\xi}) \circ_\xi L$$

$$L^\omega = \{\prod_{j \in \omega} P_j | P_j \in L\} \qquad L^{-\omega} = \{\prod_{j \in -\omega} P_j | P_j \in L\}$$

$$L^\natural = \{\prod_{j \in \alpha} P_j | \alpha \in \mathcal{O}, P_j \in L\} \qquad L^{-\natural} = \{\prod_{j \in -\alpha} P_j | \alpha \in \mathcal{O}, P_j \in L\}$$

$$L \diamond L' = \{\prod_{j \in J \cup \hat{J}^*} P_j : J \in \mathcal{S} - \{0\} \text{ and } P_j \in L \text{ if } j \in J \text{ and } P_j \in L' \text{ if } j \in \hat{J}^*\}$$

A language $L \subseteq SP^\diamond(A)$ is *rational* if it is empty, or obtained from the letters of the alphabet A using usual rational operators : finite union $\cup$, finite concatenation $\cdot$, and finite iteration $*$, ω and $-\omega$ iterations, iteration and reverse iteration on ordinals $\natural$ and $-\natural$ as well as diamond operator $\diamond$, and using also the rational operators of finite parallel product $\|$, substitution $\circ_\xi$ and iterated substitution ${}^{*\xi}$, provided that the letter $\xi \in A$ appears only inside parallel factors. This latter condition excludes from the rational languages those of the form $(a\xi b)^{*\xi} = \{a^n \xi b^n : n \in \mathbb{N}\}$, for example, which are known to be not Kleene rational. Observe also that the usual Kleene rational languages are a particular case of the rational languages defined above, in which the operators $\|$, $\circ_\xi$ and ${}^{*\xi}$ are not allowed. Note also that the rational expressions are precisely those of Bruyère and Carton [5] over labeled posets on scattered and countable linear orderings, with additional operators $\|$, $\circ_\xi$ and ${}^{*\xi}$ for parallelism and substitution.

Example 1. Let $A = \{a, b, c\}$ and $L = c \circ_\xi (a \parallel (b\xi))^{*\xi}$. Then L is the smallest language containing c and such that if $x \in L$, then $a \parallel (bx) \in L$. Thus we have $L = \{c, a \parallel (bc), a \parallel (b(a \parallel (bc))), \dots\}$. □

Let L be a language where the letter ξ is not used. In order to lighten the notation we use the following abbreviation: $L^\circledast = \{\varepsilon\} \circ_\xi (L \parallel \xi)^{*\xi} = \{\|_{i<n} P_i : n \in \mathbb{N}, P_i \in L\}$ and $L^\oplus = L^\circledast - \{\varepsilon\}$. A subset L of $A^\circledast$ is *linear* if it has the form $L = a_1 \parallel \cdots \parallel a_k \parallel \left(\cup_{i \in I}(a_{i,1} \parallel \cdots \parallel a_{i,k_i})\right)^\circledast$ where the a_i and $a_{i,j}$ are elements of A and I is a finite set. It is *semi-linear* if it is a finite union of linear sets. The class of $\|$-*rational* languages of $A^\circledast$ is the smallest containing the empty set, $\{\varepsilon\}$, $\{a\}$ for all $a \in A$, and closed under finite union, parallel product $\|$, and finite parallel iteration $^\circledast$. The notions of rational, $\|$-rational, linear and semi-linear languages, which are defined over free algebras, also naturally apply to non-free algebras. It is known (see [10]) that the $\|$-rational sets of a commutative monoid M are precisely the semi-linear sets of M. Observe also that when L is a rational language of $SP^{\diamond+}(A)$, then $L \subseteq A^\circledast$ if and only if L is $\|$-rational.

We refer to [4] for a proof of the following Lemma:

Lemma 2 (Lemma 19 of [4]). *Let A be an alphabet and let ξ, X be two new symbols. Let $M \subseteq SP^\diamond(A)$ and let $L \subseteq SP^\diamond(A \cup \{X\}) \setminus SP^\diamond(A)$. Then $M \circ_\xi (\xi \circ_X L)^{*\xi}$ is the unique solution of the equation $X = M + L$.*

Automata on countable, scattered and series-parallel posets are a generalization of automata on finite series-parallel posets [13–16], series-parallel ω-posets [12] and automata on linear orderings [5]. A *branching automaton* over an alphabet A is a tuple $\mathcal{A} = (Q, A, E, I, F)$ where Q is a finite set of *states*, $I \subseteq Q$ is the set of *initial states*, $F \subseteq Q$ the set of *final states*, and E is the set of *transitions* of $\mathcal{A}$. The set of transitions E is partitioned into $E = (E_{\text{seq}}, E_{\text{join}}, E_{\text{fork}})$, according to the different kinds of transitions. The set $E_{\text{seq}} \subseteq (Q \times A \times Q) \cup (Q \times \mathcal{P}^+(Q)) \cup (\mathcal{P}^+(Q) \times Q)$ contains the *sequential* transitions, which are usual transitions (elements of $(Q \times A \times Q)$) or *limit* transitions (elements of $(Q \times \mathcal{P}^+(Q)) \cup (\mathcal{P}^+(Q) \times Q)$). The sets $E_{\text{fork}} \subseteq Q \times \mathcal{M}^{>1}(Q)$ and $E_{\text{join}} \subseteq \mathcal{M}^{>1}(Q) \times Q$ are respectively the sets of *fork* and *join* transitions. Transitions $(p, a, q) \in Q \times A \times Q$ and $(P, q) \in \mathcal{P}^+(Q) \times Q$ are sometimes respectively denoted by $p \xrightarrow{a} q$ and $P \to q$. A path γ from a state p to a state q is either the empty poset (in this case $p = q$), or a non-empty poset labeled by transitions, with a unique minimum and a unique maximum element. The states p and q are respectively called *source* (or *origin*) and *destination* of γ. Two paths γ and γ' are *consecutive* if the destination of γ is also the source of γ'. The paths γ labeled by $P \in SP^\diamond(A)$ and of content $C(\gamma) \in \mathcal{P}^+(Q)$ in $\mathcal{A}$ are defined as follows. For all $p \in Q$ there is an empty path from p to p labeled by ε and of content $\{p\}$. For all sequential transition $t = (p, a, q)$, $\gamma = t$ is a path from p to q labeled by a and of content $\{p, q\}$. For any finite sequence $(\gamma_j)_{j \leq k}$ of paths (with $k \geq 1$) respectively labeled by $P_0, \dots, P_k$, from $p_0, \dots, p_k$ to $q_0, \dots, q_k$, if $t = (p, \{p_0, \dots, p_k\})$ is a fork transition and $t' = (\{q_0, \dots, q_k\}, q)$ a join transition, then $\gamma = t(\|_{j \leq k} \gamma_j)t'$ is a path from p to q, labeled by $\|_{j \leq k} P_j$ and of content $C(\gamma) = \{p, q\}$: observe that $C(\gamma)$ does not depend on the parallel parts $\gamma_0, \dots, \gamma_k$ of γ. Furthermore, if the paths $(\gamma_j)_{j \leq k}$ are consecutive with respective contents $(C(\gamma_j))_{j \leq k}$, then $\prod_{j \leq k} \gamma_j$ is a path labeled by $\prod_{j \leq k} P_j$ from the source of γ_0 to the destination of γ_k, and of content $\cup_{j \leq k} C_j$. Finally, for any sequence $(\gamma_j)_{j \in \omega}$ of consecutive paths respectively labeled by $(P_j)_{j \in \omega}$ and of contents $(C(\gamma_j))_{j \in \omega}$, if $R = \{q \in Q : \forall i \in \omega\ \exists j > i\ q \in C(\gamma_j)\}$, then for any transition $t = (R, q)$, $(\prod_{j \in \omega} \gamma_j)t$ is a path from the source of γ_0 and to q, labeled by $\prod_{j \in \omega} P_j$ and of content $(\cup_{j \in \omega} C_j) \cup \{q\}$. The case $-\omega$ is symmetrical to ω. In $\mathcal{A}$, a path γ from p to q labeled by P of content C is denoted by $\gamma : p \xRightarrow[C,\mathcal{A}]{P} q$. The label, content or automaton can be omitted in the notation of a path when they are implicit or of no interest. A labeled poset is *accepted* by an automaton if it is the label of a successful path leading from an initial state to a final state. The language $L(\mathcal{A})$ is the set of labeled posets accepted by the automaton $\mathcal{A}$.

Note that branching automata without fork and join transitions are precisely the automata on scattered and countable linear orderings defined by Bruyère and Carton [5]. The same way, if limit transitions are removed, we get branching automata for finite labeled posets of Lodaya and Weil [13–16]. As for finite words, rational languages and branching automata for scattered series-parallel posets are connected with a Kleene-like Theorem:

Theorem 3 [4]**.** *Let $L \subseteq SP^\diamond(A)$. Then L is the language of a branching automaton if and only if it is rational.*

Example 4. The automaton $\mathcal{A} = ([6], \{a, b, c\}, E, \{1\}, \{6\})$ defined by $E_{\text{seq}} = \{(2, a, 4), (3, b, 5), (6, c, 1), (\{1, 6\}, 6), (6, \{1, 6\})\}$, $E_{\text{fork}} = \{(1, \{2, 3\})\}$ and $E_{\text{join}} = \{(\{4, 5\}, 6)\}$ verifies $L(\mathcal{A}) = (a \parallel b) \diamond c$. □

An automaton is *sequentially separated* if, for all pairs (p, q) of states, all labels of paths from p to q are parallel posets, or all labels of paths from p to q are sequential posets. For every automaton $\mathcal{A}$ there is a sequentially separated automaton $\mathcal{B}$ such that $L(\mathcal{A}) = L(\mathcal{B})$. Also, for every pair of states (p, q) of an automaton, it is decidable whether there is a path from p to q or not.

The following Theorem states the main result of this paper:

Theorem 5. *Let A be an alphabet. The class of rational languages of $SP^{\diamond+}(A)$ is effectively closed under complement.*

Section 5 is devoted to a sketch of its proof, which essentially relies on the algebraic approach of automata.

4 Algebras

We now focus on the definitions of algebras for the recognition of languages of $SP^\diamond(A)$, with A an alphabet. Recall that an algebra is finite if it is composed of a finite number of elements. Even if in this paper we deal with infinite algebras, we use notions of universal algebras which are usually defined on finite algebras, and that can be easily generalized to our case. We refer to [1] for the basic algebraic definitions. A semigroup $(S, \cdot)$ is a set S equipped with an associative binary operation $\cdot$ called *product*. A $\parallel$*-semigroup* [13–16] $(S, \cdot, \parallel)$ is an algebra such that $(S, \cdot)$ is a semigroup and $(S, \parallel)$ is a commutative semigroup. In ambiguous contexts, the $\cdot$ and $\parallel$ products are respectively called *sequential* (or *series*) and *parallel*. The $\diamond$*-semigroups* are a generalization of semigroups for the recognition of words of $A^\diamond$ (see [8] for more details): a $\diamond$*-semigroup* $(S, \prod)$ is a set equipped with a map $\prod$ (also called *sequential product*) which associates an element of S to any countable and linearly ordered sequence $s = (s_j)_{j\in J}$ (with $J \in \mathcal{S}$) of elements of S, such that $\prod(t) = t$ for any $t \in S$ and $\prod$ is associative (i.e. for any factorization of the sequence s into a sequence of sequences $(t_j)_{j\in J'}$, $\prod(s) = \prod((\prod t_j)_{j\in J'})$). Finally, a $\parallel$-$\diamond$*-semigroup* $(S, \prod, \parallel)$ is an algebra such that $(S, \prod)$ and $(S, \parallel)$ are respectively a $\diamond-$ and a commutative semigroup. In order to lighten the notation we often denote an algebra by its set of elements: for example, we denote the semigroup $(S, \cdot)$ by S. We denote by S^1 the algebra S if S has an identity 1 for all its operations, $S \cup \{1\}$ otherwise. We also denote by A^+, $SP(A)$ and $A^\diamond$ respectively the free semigroup, $\parallel$-semigroup, and $\diamond$-semigroup over the alphabet A. In this paper we particularly focus on $SP^\diamond(A)$ which is the free $\|$-$\diamond$-semigroup over A. Let S and T be two algebras of the same type. A morphism $\varphi : S \to T$ *recognizes* a subset X of S if $\varphi^{-1}\varphi(X) = X$. We say that T *recognizes* X if there exists a morphism from S into T recognizing X. A subset X of an algebra S is *recognizable* if there exist a *finite* algebra T with the same type as S and a morphism $\varphi : S \to T$ that recognizes X. Recognizable languages of $SP^+(A)$ are rational. However, in general, rational languages of $SP^+(A)$ are not recognizable. As an example, $(a \parallel b)^{\oplus}$ is not recognizable, since its syntactic $\|$-semigroup is isomorphic to $\mathbb{Z}$ (see [13]). Let $(S, \prod, \|)$ be a $\|$-$\diamond$-semigroup. Its sequential product $\prod$ is a *finite projection* if there exists $X \subseteq S$ such that $(X, \prod)$ is a finite $\diamond$-semigroup and $\prod$ maps every sequential product of at least two elements of S to an element of X. By extension of the work of Wilke [23] on ω-words, when $\prod$ is a finite projection, it can be equivalently replaced by an associative binary sequential product $\cdot$ and two maps $\omega : S \to S$ and $-\omega : S \to S$ such that, for all $s, t \in S$, $s \cdot (t \cdot s)^{\omega} = (s \cdot t)^{\omega}$, $(s \cdot t)^{-\omega} \cdot s = (t \cdot s)^{-\omega}$, $(s^n)^{\omega} = s^{\omega}$ and $(s^n)^{-\omega} = s^{-\omega}$ for all $n \in \mathbb{N}^*$. Observe that it suffices to define ω and $-\omega$ over finitely many elements: the idempotents (for the sequential product) of S.

Example 6. Let $A = \{a, b\}$ and $L \subseteq SP^{\diamond+}(A)$ be the language of non-empty posets P such that P has width at most 2 and each letter a that appears into a parallel part of P is incomparable with a b. Let $S = (X, \prod, \|)$ be the finite $\|$-$\diamond$-semigroup defined by $X = \{a, b, ab, p, 0\}$, the following $\|$ commutative product: $a \parallel a = ab \parallel a = 0$, $p \parallel x = 0$ for all $x \in S$, $a \parallel b = ab \parallel b = ab \parallel ab = b \parallel b = p$ and the sequential product $\prod$ such that, for any non-empty sequence $(s_j)_{j \in J}$ $(J \in \mathcal{S} - \{\emptyset\})$ of elements of S, $\prod((s_j)_{j\in J}) = a$ if $(s_j)_{j\in J}$ contains only as, $\prod((s_j)_{j\in J}) = ab$ if $(s_j)_{j\in J}$ contains at least one a and one b, $\prod((s_j)_{j\in J}) = b$ if $(s_j)_{j\in J}$ contains only bs, and $\prod((s_j)_{j\in J}) = p$ if $(s_j)_{j\in J}$ contains only p, a, b, ab, with at least one p. The element 0 is a zero for both $\prod$ and $\|$. Let $\varphi : SP^{\diamond+}(A) \to S$ be the morphism defined by $\varphi(a) = a$ and $\varphi(b) = b$. Then $L = \varphi^{-1}(\{a, b, ab, p\})$. Furthermore, the sequential product of S is a finite projection since S has a finite number of elements. Then S can be equivalently defined by $W = (X, \cdot, \omega, -\omega, \|)$ where $x \cdot x' = \prod(x, x')$ and $x^{\omega} = x^{-\omega} = x$ for all $x, x' \in X$. $\square$

The following notions are adapted from [9]. Let P be a partially ordered set and S a semigroup. A mapping σ from ordered pairs $(x, y) \in P^2$ such that $x \sim_< y$, to S, is an *additive labeling from P to S* if $\sigma(x, y)\sigma(y, z) = \sigma(x, z)$ for all $x < y < z$ in P. From a morphism of semigroups $\varphi : (SP^{\diamond}(A), \cdot) \to S$ and $P \in SP^{\diamond}(A)$, one can build an additive labeling $\varphi_P : (\hat{P}, \preceq) \to S$ with $\varphi_P((A, B), (A', B')) = \varphi(B \cap A')$. A *split of height* n of P is a mapping $s : P \to [n]$ ($n = 0$ is possible; in this case $P = \emptyset$). Two elements x, y such that $x \sim_< y$ and $s(x) = s(y) = k$ are *k-neighbors* if $s(z) \geq k$ for all $z \in [x, y]$ with $z \sim_< x$. Note that k-neighborhoodness is an equivalence relation over the elements of P. Let σ be an additive labeling from P to a semigroup S. Then a split s of P is *Ramseyan* for σ if for every equivalence class C for k-neighborhoodness there exists an idempotent e such that $\sigma(x, y) = e$ for all $x < y$ in C.

The notion of a *finite projection* of a semigroup S is self-understanding from its definition on $\|$-$\diamond$-semigroups. Theorem 4 of [9] can be reformulated for posets as follows:

Theorem 7. *For every poset $P \in SP^\diamond$, every semigroup S with a finite projection $fp(S)$ and additive labeling σ from P to S, there exists a Ramseyan split of P for σ of height at most $2|fp(S)|+1$.*

5 Sketch of the Proof of Theorem 5

Let A be an alphabet, $\mathcal{A} = (Q, A, E, I, F)$ a branching automaton, and $L = L(\mathcal{A})$. When $X \subseteq SP^{\diamond+}(A)$, we denote by $Seq(X)$ the set of sequential posets of X. Denote also by $L_{p,q}$ (resp. $L_{p,q,C}$ with $C \in \mathcal{P}^+(Q)$) the set of non-empty labels of paths from state p to state q (resp. of content C) in $\mathcal{A}$.

The proof of Theorem 5 consists in constructing a rational expression e for $SP^{\diamond+}(A) - L$. When $\phi : SP^{\diamond+}(A) \to S$ is a morphism of $\|$-$\diamond$-semigroups and $D \in \mathcal{P}(Q^2 \times \mathcal{P}^+(Q))$, denote by $\Delta_D^\phi = \{\phi(P) : p \underset{C,\mathcal{A}}{\overset{P}{\Longrightarrow}} q \text{ iff } (p,q,C) \in D\}$. The first step is to construct a $\|$-$\diamond$-semigroup from $\mathcal{A}$, by a generalization of the usual technique used to construct a finite semigroup from a Kleene automaton on finite words. This consists in defining a congruence $\sim_\mathcal{A}$ of $\|$-$\diamond$-semigroups over the posets of $SP^{\diamond+}(A)$, by $P \sim_\mathcal{A} P'$ if and only if P can be substituted by P' in any part of any path $\gamma : p \underset{C,\mathcal{A}}{\overset{R}{\Longrightarrow}} q$ of $\mathcal{A}$ in order to build another path $\gamma' : p \underset{C,\mathcal{A}}{\overset{R'}{\Longrightarrow}} q$ of same source, destination and content and whose label R' is R in which some occurrences of P have been replaced by P'. The natural morphism $\varphi_{\sim_\mathcal{A}} : SP^{\diamond+}(A) \to SP^{\diamond+}(A)/\!\sim_\mathcal{A}$ which associates to each poset $P \in SP^{\diamond+}(A)$ its equivalence class in $SP^{\diamond+}(A)/\!\sim_\mathcal{A}$ recognizes $L_{p,q,C}$ for each $p,q,C \in Q^2 \times \mathcal{P}^+(Q)$, and L. Note that $SP^{\diamond+}(A)/\!\sim_\mathcal{A}$ may be infinite, as it is illustrated by the following example.

Example 8. Consider the automaton $\mathcal{A}$ of Fig. 1 of language $L(\mathcal{A}) = (a \parallel b)^\oplus \diamond c$. For all $k_1, k_2, k_3, k_4 \in \mathbb{N}$ such that $k_1 - k_2 = k_3 - k_4$ and $k_2, k_4 > 0$ we have $a^{\|k_1} \parallel b^{\|k_2} \sim_\mathcal{A} a^{\|k_3} \parallel b^{\|k_4}$. Also, $P \sim_\mathcal{A} P'$ for all $P, P' \in (a \parallel b)^\oplus \diamond c - (a \parallel b)^\oplus$.

Let $S = \mathbb{Z} \cup \{a, b, c, 0c, c0, 0c0, \perp\}$. Equip S with a commutative parallel product with $z \parallel z' = z +_\mathbb{Z} z'$, $a \parallel z' = 1 \parallel z'$, $b \parallel z' = -1 \parallel z'$ for all $z, z' \in \mathbb{Z}$, $a \parallel b = 0$, and all other parallel product are sent to $\perp$. Equip also S with a sequential product such that for all sequence $s = (s_i)_{i \in I}$ of elements of S, $I \in \mathcal{S} - \{0,1\}$, $\prod_{i \in I} s_i = 0c0$ if $s \in 0 \diamond c$, $c0c = c$, $z^2 = zx = xz = c^2 = \perp^2 = \perp$ for all $z \in \mathbb{Z} \cup \{a, b\}$, $x \in S$. As the sequential product of S is a finite projection and the idempotents for the sequential product are $0c, c0, \perp$, it can equivalently be defined by the binary product as above and $(0c)^\omega = 0c0 = (c0)^{-\omega}$, $(0c)^{-\omega} = 0c$, $(c0)^\omega = c0$. Note that $(S, \|)$ is finitely generated by $\{-1, 1, a, b, c, 0c, c0, 0c0, \perp\}$. Let $\varphi : SP^{\diamond+}(A) \to S$ defined by $\varphi(x) = x$ for all $x \in A$. Then $L = \varphi^{-1}(\{0, 0c0\})$. Furthermore, $SP^{\diamond+}(A)/\!\sim_\mathcal{A}$ is isomorphic to S. $\square$

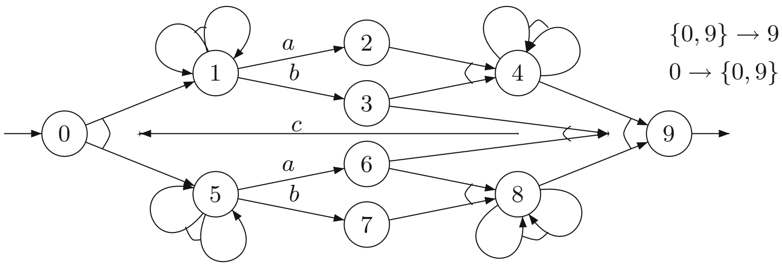

Fig. 1. An automaton $\mathcal{A}$ for $(a \parallel b)^{\oplus} \diamond c$. Fork transitions are $(0,\{1,5\})$, $(5,\{5,5\})$ and $(1,\{1,1\})$, join transitions are $(\{2,3\},4)$, $(\{6,7\},8)$, $(\{4,8\},9)$, $(\{4,4\},4)$, $(\{8,8\},8)$ and $(\{3,6\},9)$

However, $(SP^{\diamond+}(A)/\sim_{\mathcal{A}}, \parallel)$ is finitely generated by $\varphi_{\sim_{\mathcal{A}}}(Seq(SP^{\diamond+}(A)))$, $\varphi_{\sim_{\mathcal{A}}}(L_{p,q})$ is a $\parallel$-rational set of $SP^{\diamond+}(A)/\sim_{\mathcal{A}}$ for all $p, q \in Q$, and thus, so is $\varphi_{\sim_{\mathcal{A}}}(L)$. We also have $\varphi^{-1}_{\sim_{\mathcal{A}}}(\Delta_D^{\varphi_{\sim_{\mathcal{A}}}}) = \Delta_D^{\mathrm{id}}$ for all $D \in \mathcal{P}(Q^2 \times \mathcal{P}^+(Q))$. Recall that the $\parallel$-rational sets of a commutative monoid M form a boolean algebra [10, Theorem 3], which is effective when M is finitely generated (as emphasized in [21]). As a consequence, $\Delta_D^{\varphi_{\sim_{\mathcal{A}}}}$ is a $\parallel$-rational set of $SP^{\diamond+}(A)/\sim_{\mathcal{A}}$ for all D.

As $SP^{\diamond+}(A) - L = \cup_{\substack{D \in \mathcal{P}(Q^2 \times \mathcal{P}^+(Q)) \\ D \cap I \times F \times \mathcal{P}^+(Q) = \emptyset}} \Delta_D^{\mathrm{id}}$, it suffices to show that $\varphi^{-1}_{\sim_{\mathcal{A}}}(\Delta_D^{\varphi_{\sim_{\mathcal{A}}}})$ is a rational set of $SP^{\diamond+}(A)$ for all D. We translate the problem into a $\parallel$-$\diamond$-semigroup $\mathbb{N}^{k*}$ with more properties than $SP^{\diamond+}(A)/\sim_{\mathcal{A}}$. Very informally speaking, denote by $G = \{g_1, \ldots, g_k\}$ the finite generator of $(SP^{\diamond+}(A)/\sim_{\mathcal{A}}, \parallel)$. We may suppose that $\mathcal{A}$ is sequentially separated. Thus that the elements of G are indecomposable with respect to the parallel product, that is to say, each $g_i \in G$ can not be written $g_i = s \parallel s'$ with $s, s' \in SP^{\diamond+}(A)/\sim_{\mathcal{A}}$. We are going to define a morphism $\mu : SP^{\diamond+}(A) \rightarrow \mathbb{N}^{k*}$ that enables, for every $P \in SP^{\diamond+}(A)$ whose maximal parallel factorization is $P = P_1 \parallel \cdots \parallel P_n$, the count of all i, $i \in [n]$, such that $\varphi_{\sim_{\mathcal{A}}}(P_i) = g_j$, for every $j \in [k]$. Also, every language recognized by $SP^{\diamond+}(A)/\sim_{\mathcal{A}}$ is recognized by $\mathbb{N}^{k*}$.

Denote by $(\mathbb{N}^{k*}, +)$ the commutative semigroup whose elements are k-tuples of non-negative integers, without $(0, \ldots, 0)$. It is generated by the k-tuples with all components set to 0, except one which is set to 1. For short we denote by 1^i the element of the generator of $\mathbb{N}^{k*}$ with the i^{th} component set to 1. The (parallel) product $+$ of $(\mathbb{N}^{k*}, +)$ is the sum componentwise. Define a surjective morphism of commutative semigroups $\psi : (\mathbb{N}^{k*}, +) \rightarrow (SP^{\diamond+}(A)/\sim_{\mathcal{A}}, \parallel)$ by $\psi(1^i) = g_i$ for all $i \in [k]$. As $\psi^{-1}(g_i) = \{1^i\}$ for all $i \in [k]$, $\psi^{-1}(ss')$ is a singleton for all $s, s' \in SP^{\diamond+}(A)/\sim_{\mathcal{A}}$. Now we equip $(\mathbb{N}^{k*}, +)$ with a structure of $\parallel$-$\diamond$-semigroup by setting, for all $n, n_1, n_2 \in \mathbb{N}^{k*}$, $n_1 n_2 = \psi^{-1}(\psi(n_1)\psi(n_2))$, $n^{\omega} = \psi^{-1}((\psi(n))^{\omega})$ and $n^{-\omega} = \psi^{-1}((\psi(n))^{-\omega})$. This sequential product is a finite projection. We define a surjective morphism of $\parallel$-$\diamond$-semigroups $\mu : SP^{\diamond+}(A) \rightarrow \mathbb{N}^{k*}$ by $\mu(a) = \psi^{-1}\varphi_{\sim_{\mathcal{A}}}(a)$ for all $a \in A$. The diagram of Fig. 2 sums up the situation. For all $s \in SP^{\diamond+}(A)/\sim_{\mathcal{A}}$, $\varphi^{-1}_{\sim_{\mathcal{A}}}(s) = \mu^{-1}\psi^{-1}(s)$. We also have $\psi^{-1}(\Delta_D^{\varphi_{\sim_{\mathcal{A}}}}) = \Delta_D^{\mu}$ and $\mu^{-1}(\Delta_D^{\mu}) = \Delta_D^{\mathrm{id}}$ for all $D \in \mathcal{P}(Q^2 \times \mathcal{P}^+(Q))$. According to [10, Corollary III.2] Δ_D^{μ} is a $\parallel$-rational set of $\mathbb{N}^{k*}$, and thus semi-linear: it has the form $\Delta_D^{\mu} =$

$$\begin{tikzcd} SP^{\diamond +}(A) \arrow[r, "\varphi_{\sim_{\mathcal{A}}}"] \arrow[d, "\mu"'] & SP^{\diamond +}(A)/{\sim_{\mathcal{A}}} \\ \mathbb{N}^{k*} \arrow[ur, dashed, "\psi"] & \end{tikzcd}$$

Fig. 2. The morphisms between the $\|$-$\diamond$-semigroups. Full arrows represent morphisms of $\|$-$\diamond$-semigroups, and dashed arrows morphisms of commutative semigroups

$\cup_{i \in I_D}(a_{D,i} + B_{D,i}^{\circledast})$ for some finite set I_D, $a_{D,i} \in \mathbb{N}^{k*}$, $B_{D,i}$ some finite part of $\mathbb{N}^{k*}$. For all $i \in I_D$ set $\Delta^{\mu}_{D,i} = a_{D,i} + B^{\circledast}_{D,i}$. We may assume that all the $\Delta^{\mu}_{D,i}$ are pairwise disjoint [10, Theorem IV]. Setting $B_{D,i} = \{b_{D,i,1}, \dots, b_{D,i,l_{D,i}}\}$ it holds $\mu^{-1}(\Delta^{\mu}_{D,i}) = \mu^{-1}(a_{D,i}) \parallel \{\mu^{-1}(b_{D,i,1}) \cup \dots \cup \mu^{-1}(b_{D,i,l_{D,i}})\}^{\circledast}$, so it just remain to show that

Lemma 9. *For all $n \in \mathbb{N}^{k*}$, $\mu^{-1}(n)$ is a rational set of $SP^{\diamond +}(A)$.*

Proof. (Sketch of) Let $\varphi : SP^{\diamond +}(A) \to S$ be a morphism of $\|$-$\diamond$-semigroups. For each j, M non-negative integers, $x \in S$, and $\iota, \iota' \in \{[,]\}$, define $S^{\iota\iota'}_{j,M}(x)$ (or $S^{\iota\iota'}_{j}(x)$ for short) as the posets P of $\varphi^{-1}(x)$ such that $\hat{P}^{\iota\iota'}$ admits a Ramseyan split s, for φ_P, of height M; and s is also a Ramseyan split of $\{(A,B) \in \hat{P}^{\iota\iota'} : A \cup B = P\}$, for φ_P, of height j.

Considering linear orderings only, Colcombet [9] expressed $S^{\iota\iota'}_{j+1}(x)$ with an equality that depends only of the $S^{\iota''\iota'''}_{j}(s)$, $s \in S$, $\iota'', \iota''' \in \{[,]\}$ and that uses only the rational operators for linear orderings:

$$\begin{aligned} S^{\iota\iota'}_{j+1}(x) = S^{\iota\iota'}_{j}(x) &+ \sum_{yz=x} S^{\iota[}_{j}(y) S^{]\iota'}_{j}(z) + \sum_{\substack{yez=x\\ e^2=e}} S^{\iota[}_{j}(y) C_{e,j+1} S^{]\iota'}_{j}(z) \\ &+ \sum_{\substack{ye^{\omega}z=x\\ e^2=e}} S^{\iota[}_{j}(y) C^{\omega}_{e,j+1} S^{[\iota'}_{j}(z) + \sum_{\substack{ye^{-\omega}z=x\\ e^2=e}} S^{\iota]}_{j}(y) C^{-\omega}_{e,j+1} S^{]\iota'}_{j}(z) + \sum_{\substack{ye^{\zeta}z=x\\ e^2=e}} S^{\iota]}_{j}(y) C^{\zeta}_{e,j+1} S^{[\iota'}_{j}(z) \end{aligned}$$

with $\varphi^{-1}(x) = S^{[]}_{2|S|}(x)$, $S^{][}_{0}(x) = \varphi^{-1}(x) \cap A$, $S^{[[}_{0}(x) = S^{]]}_{0}(x) = \varphi^{-1}(x) \cap \{\varepsilon\}$, $S^{[]}_{0}(x) = \emptyset$, and $C_{e,j+1}$, $C^{\omega}_{e,j+1}$, $C^{-\omega}_{e,j+1}$, $C^{\zeta}_{e,j+1}$ rational sets that depend only of the languages of the form $S^{\iota''\iota'''}_{j}(s)$, $s \in S$, $\iota'', \iota''' \in \{[,]\}$.

We adapt this to the case where $P \in SP^{\diamond +}(A)$, replacing φ by $\mu : SP^{\diamond +}(A) \to \mathbb{N}^{k*}$. There are several points to consider. First, technically the empty poset is not taken into consideration in the framework of posets. Second, $\hat{P}^{\iota\iota'}$ admits a Ramseyan split for φ_P and of height j if and only if $C = \{(A,B) \in \hat{P}^{\iota\iota'} : (A,B) = P\}$ admits a Ramseyan split for φ_P and of height j, and, for each P_i between two consecutive elements of C with $|P_i| > 1$ (thus $P_i = \|_{j \in J_i} P_j$ for some $|J_i| > 1$ and nonempty P_j), each $\hat{P}_j^{[]}$ admits itself a Ramseyan split for φ_P and of height $|2k+1|$. And third, as $\mathbb{N}^{k*}$ is infinite but partitioned into finitely many $\Delta^{\mu}_{D,i}$ in which all elements are equivalent regarding to the sequential product, we need to replace any occurrence of some x involved in a sequential product in the right member of the equality above by some $\Delta^\mu_{D,i}$. The set $S^{\iota\iota'}_{j+1}(\Delta^\mu_{D,i})$ is composed of all the sequential posets of $S^{\iota\iota'}_{j+1}(g)$ for all $g \in G \cap \Delta^\mu_{D,i}$, and, if $\iota\iota' =][$, all the parallel posets and letters of $\mu^{-1}(\Delta^\mu_{D,i})$. For simplicity we write $\Delta^\mu_{D,i}x = y$ (resp. $x\Delta^\mu_{D,i} = y$) when $zx = y$ (resp. $xz = y$) for all $z \in \Delta^\mu_{D,i}$. With the help of Theorem 7, the equalities of Colcombet above can be rewritten in $\mathbb{N}^{k*}$ as

$$\begin{aligned} S^{\iota\iota'}_{j+1}(x) &= S^{\iota\iota'}_{j}(x) + \sum_{\Delta^\mu_{D,i}\Delta^\mu_{D',i'}=x} S^{\iota[}_{j}(\Delta^\mu_{D,i}) S^{]\iota'}_{j}(\Delta^\mu_{D',i'}) \\ &+ \sum_{\substack{\Delta^\mu_{D,i}e\Delta^\mu_{D',i'}=x \\ e^2=e}} S^{\iota[}_{j}(\Delta^\mu_{D,i}) C_{e,j+1} S^{]\iota'}_{j}(\Delta^\mu_{D',i'}) + \sum_{\substack{\Delta^\mu_{D,i}e^\omega\Delta^\mu_{D',i'}=x \\ e^2=e}} S^{\iota[}_{j}(\Delta^\mu_{D,i}) C^\omega_{e,j+1} S^{[\iota'}_{j}(\Delta^\mu_{D',i'}) \\ &+ \sum_{\substack{\Delta^\mu_{D,i}e^{-\omega}\Delta^\mu_{D',i'}=x \\ e^2=e}} S^{\iota]}_{j}(\Delta^\mu_{D,i}) C^{-\omega}_{e,j+1} S^{]\iota'}_{j}(\Delta^\mu_{D',i'}) + \sum_{\substack{\Delta^\mu_{D,i}e^\zeta\Delta^\mu_{D',i'}=x \\ e^2=e}} S^{\iota]}_{j}(\Delta^\mu_{D,i}) C^\zeta_{e,j+1} S^{[\iota'}_{j}(\Delta^\mu_{D',i'}) \end{aligned}$$

where $C_{e,j+1}$, $C^\omega_{e,j+1}$, $C^{-\omega}_{e,j+1}$, $C^\zeta_{e,j+1}$ are rational sets that depend only of the languages of the form $S^{\iota''\iota'''}_{j}(\Delta^\mu_{D,i})$ and can be obtained precisely as in the case of linear orderings (see [9, Proof of Theorem 6]), $D \in \mathcal{P}^+(Q)$, $i \in I_D$, $\iota'', \iota''' \in \{[,]\}$, and with

$$S^{\iota\iota'}_{j+1}(\Delta^\mu_{D,i}) = \begin{cases} S^{\iota\iota'}_{j+1}(a_{D,i}) + S^{\iota\iota'}_{j}(\Delta^\mu_{D,i}) & \text{if } \Delta^\mu_{D,i} = a_{D,i} + B^{\circledast}_{D,i} \\ (\sum_{b \in B_{D,i}} S^{\iota\iota'}_{j+1}(b)) + S^{\iota\iota'}_{j}(\Delta^\mu_{D,i}) & \text{if } \Delta^\mu_{D,i} = B^{\circledast}_{D,i} \end{cases} \tag{1}$$

$$S^{][}_0(\Delta^\mu_{D,i}) = \begin{cases} S^{][}_0(a_{D,i}) + S^{[]}_{2k+1}(a_{D,i}) \parallel (\sum_{b \in B_{D,i}} S^{[]}_{2k+1}(b))^{\oplus} \\ \qquad \text{if } \Delta^\mu_{D,i} = a_{D,i} + B^{\circledast}_{D,i}, \\ (\sum_{b \in B_{D,i}} S^{][}_0(b)) + (\sum_{b \in B_{D,i}} S^{[]}_{2k+1}(b)) \parallel (\sum_{b \in B_{D,i}} S^{[]}_{2k+1}(b))^{\oplus} \\ \qquad \text{if } \Delta^\mu_{D,i} = B^{\circledast}_{D,i} \end{cases} \tag{2}$$

$$S^{][}_0(x) = (\mu^{-1}(x) \cap A) \sum_{y+z=x} S^{[]}_{2k+1}(y) \parallel S^{[]}_{2k+1}(z) \tag{3}$$

$$S^{[[}_0(x) = S^{]]}_0(x) = S^{[]}_0(x) = S^{[[}_0(\Delta^\mu_{D,i}) = S^{]]}_0(\Delta^\mu_{D,i}) = S^{[]}_0(\Delta^\mu_{D,i}) = \emptyset \tag{4}$$

Note that the choices for y, z in (3) are finite since we are in $\mathbb{N}^{k*}$. This gives a finite system of equations, where recursion occurs only in parallel parts, and whose solution is rational with the help of Lemma 2. As $\mu^{-1}(n) = S^{[]}_{2k+1}(n)$ then $\mu^{-1}(n)$ is rational for all $n \in \mathbb{N}^{k*}$.

Immediately, $\mu^{-1}(\Delta_D^{\mu})$ and $\varphi_{\sim_{\mathcal{A}}}^{-1}(\Delta_D^{SP^{\diamond+}(A)/\sim_{\mathcal{A}}})$ are rational sets of $SP^{\diamond+}(A)$ for all $D \in \mathcal{P}(Q^2 \times \mathcal{P}^+(Q))$. Note that all the constructions are effective.

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