

Contents

1	Introduction	1
1.1	What This Book Is About	1
1.2	The Structure of a Rule-Based Fuzzy System	7
1.3	A New Direction for Fuzzy Systems	9
1.4	Fundamental Design Requirement	10
1.5	An Impressionistic Brief History of Type-1 Fuzzy Sets and Fuzzy Logic	10
1.6	Literature on Type-2 Fuzzy Sets and Fuzzy Systems	11
1.6.1	Early Literature: 1975–1992	12
1.6.2	Publications that Heavily Influenced the First Edition of This Book	13
1.6.3	Application Papers	14
1.7	Coverage	15
1.8	Applicability Outside of Rule-Based Fuzzy Systems	19
1.9	Computation	19
	References	20
2	Type-1 Fuzzy Sets and Fuzzy Logic	25
2.1	Crisp Sets	25
2.2	Type-1 Fuzzy Sets and Associated Concepts	26
2.2.1	Lotfi A. Zadeh	27
2.2.2	Type-1 Fuzzy Set Defined	28
2.2.3	Type-1 Fuzzy Numbers	32
2.2.4	Linguistic Variables	33
2.2.5	Returning to Linguistic Labels from Numerical Values of MFs	36
2.3	Set Theoretic Operations for Crisp Sets	37
2.4	Set Theoretic Operations for Type-1 Fuzzy Sets	38
2.5	Crisp Relations and Compositions on the Same Product Space	42

2.6	Fuzzy Relations and Compositions on the Same Product Space	44
2.7	Crisp Relations and Compositions on Different Product Spaces.	46
2.8	Fuzzy Relations and Compositions on Different Product Spaces.	49
2.9	Hedges	52
2.10	Extension Principle	54
2.11	α -Cuts	60
2.12	Representing Type-1 Fuzzy Sets Using α -Cuts	62
2.13	Functions of Type-1 Fuzzy Sets Computed by Using α -Cuts	66
2.14	Multivariable MFs and Cartesian Products	69
2.15	Crisp Logic.	70
2.16	From Crisp Logic to Fuzzy Logic.	74
2.17	Mamdani (Engineering) Implications	77
2.18	Remarks	82
	Appendix 1: Properties of Type-1 Fuzzy Sets.	84
	Exercises.	86
	References.	96
3	Type-1 Fuzzy Systems.	101
3.1	Type-1 Fuzzy Systems	101
3.2	Rules.	101
3.3	Fuzzifier	106
3.4	Fuzzy Inference Engine	107
	3.4.1 General Results	107
	3.4.2 Type-1 Rule Partitions.	110
	3.4.3 Fuzzification and Its Effects on Inference	114
3.5	Combining Fired-Rule Output Sets on the Way to Defuzzification	120
	3.5.1 Mamdani Fuzzy System: Combining Using Set-Theoretic Operations	120
	3.5.2 Mamdani Fuzzy System Combining Using a Weighted Combination	121
	3.5.3 Mamdani and TSK Fuzzy Systems Combining During Defuzzification.	123
3.6	Defuzzifier	123
	3.6.1 Mamdani Fuzzy System: Centroid Defuzzifier.	124
	3.6.2 Mamdani Fuzzy System: Height Defuzzifier	125
	3.6.3 Mamdani Fuzzy System: COS Defuzzifier.	125
	3.6.4 TSK Fuzzy System Defuzzifiers	127
3.7	Comprehensive Example.	128
3.8	Fuzzy Basis Functions	131

3.9	Remarks and Insights	139
3.9.1	Layered Architecture Interpretations of a Fuzzy System.	139
3.9.2	Universal Approximation by Fuzzy Systems	141
3.9.3	Continuity of Fuzzy Systems.	142
3.9.4	Rule Explosion and Some Ways to Control It.	144
3.9.5	Rule Interpretability	146
	Exercises.	148
	References.	154
4	Type-1 Fuzzy Systems: Design Methods and Applications	161
4.1	Designing Type-1 Fuzzy Systems.	161
4.2	Some Design Methods	170
4.2.1	One-Pass Methods	170
4.2.2	Least Squares Method	173
4.2.3	Derivative-Based Methods	176
4.2.4	SVD-QR Method.	179
4.2.5	Derivative-Free Methods	181
4.2.6	Iterative Design Methods.	186
4.2.7	Remarks.	186
4.3	Case Study: Forecasting of Time Series	187
4.3.1	Mackey–Glass Chaotic Time Series.	188
4.3.2	One-Pass Design: Singleton Fuzzification	189
4.3.3	Derivative-Based Design: Singleton Fuzzification	191
4.3.4	A Change in the Measurements.	194
4.3.5	One-Pass Design: Non-singleton Fuzzification.	195
4.3.6	Derivative-Based Design: Non-singleton Fuzzification	197
4.3.7	Final Remark.	199
4.4	Case Study: Knowledge Mining Using Surveys	199
4.4.1	Methodology for Knowledge Mining.	200
4.4.2	Survey Results.	202
4.4.3	Determining Type-1 Fuzzy Sets from Survey Results.	203
4.4.4	What Does One Do with a Histogram of Responses?	206
4.4.5	Averaging the Responses: Consensus FLAs	207
4.4.6	Preserving All of the Responses	209
4.4.7	On Multiple Indicators.	210
4.4.8	How to Use a FLA	210
4.4.9	Connections to the Perceptual Computer.	212
4.5	Forecasting of Compressed Video Traffic Using Mamdani and TSK Fuzzy Systems.	214
4.5.1	Forecasting I Frame Sizes: General Information	215

4.5.2	Forecasting I Frame Sizes: Using the Same Number of Rules.	216
4.5.3	Forecasting I Frame Sizes: Using the Same Number of Design Parameters.	217
4.6	Rule-Based Classification of Video Traffic	218
4.6.1	Selected Features	220
4.6.2	MFs for the Features	220
4.6.3	Rules and Their Parameters	221
4.6.4	Computational Formulas for the RBC	222
4.6.5	Optimization of Rule Design Parameters	223
4.6.6	Testing the FL RBC	224
4.6.7	Results and Conclusions	225
4.7	Case Study: Fuzzy Logic Control	226
4.7.1	Early History of Fuzzy Control	226
4.7.2	What Is a Type-1 Fuzzy Logic Controller (FLC)?	227
4.7.3	Fuzzy PID Control.	229
	Appendix 1: Proof of Theorem 4.1	234
	Exercises.	236
	References.	239
5	Sources of Uncertainty	245
5.1	Uncertainties in a Fuzzy System.	245
5.1.1	Uncertainty: General Discussions.	245
5.1.2	Uncertainties and Sets	246
5.1.3	Uncertainties in a Fuzzy System	247
5.2	Words Mean Different Things to Different People	250
	Exercises.	256
	References.	257
6	Type-2 Fuzzy Sets	259
6.1	The Concept of a Type-2 Fuzzy Set	259
6.2	Definitions of a General Type-2 Fuzzy Set and Associated Concepts.	262
6.3	Definitions of an IT2 FS and Associated Concepts.	273
6.4	Examples of Two Popular FOU's	279
6.5	Interval Type-2 Fuzzy Numbers	282
6.6	Different Kinds of T2 FSs: Hierarchy.	284
6.7	Mathematical Representations for T2 FSs.	287
6.7.1	Vertical Slice Representation.	287
6.7.2	Wavy Slice Representation	289
6.7.3	Horizontal Slice Representation	290
6.7.4	Which Representations Are Most Useful for Optimal Design Applications?	294

6.8	Representing Non T2 FSs as T2 FSs	297
6.9	Returning to Linguistic Labels for T2 FSs	298
6.10	Multivariable Membership Functions	300
	Exercises	301
	References	304
7	Working with Type-2 Fuzzy Sets	307
7.1	Introduction and Guide for the Reader	307
7.2	Set-Theoretic Operations for GT2 FSs Computed Using the Extension Principle	308
7.2.1	Union of GT2 FSs	309
7.2.2	Intersection of GT2 FSs	314
7.2.3	Complement of a GT2 FS	320
7.2.4	Remarks	322
7.3	Set-Theoretic Operations for IT2 FSs	325
7.3.1	Union of IT2 FSs	326
7.3.2	Intersection of IT2 FSs	328
7.3.3	Complement of an IT2 FS	329
7.4	Set-Theoretic Operations for GT2 FSs Computed by Using Horizontal Slices	331
7.4.1	Union of GT2 FSs	332
7.4.2	Intersection of GT2 FSs	336
7.4.3	Complement of a GT2 FS	340
7.4.4	Historical Remarks	340
7.5	Observations About Set Theory Computations	341
7.6	Relations in General	341
7.7	Type-2 Relations and Compositions on the Same Product Space	344
7.8	Type-2 Relations and Compositions on Different Product Spaces	347
7.9	Compositions of a T2 FS with a Type-2 Relation	348
7.10	Type-2 Hedges	350
7.11	Extension Principle for T2 FSs	351
7.11.1	Extension Principle for IT2 FSs	351
7.11.2	Extension Principle for GT2 FSs	352
7.12	Functions of GT2 FSs Computed Using α -Planes	353
7.13	Cartesian Product of T2 FSs	353
7.14	Implications	354
	Appendix 1: Properties of T2 FSs	355
	Appendix 2: Proofs	362
	Exercise	372
	References	381

8	Type-Reduction	385
8.1	Introduction	385
8.2	Interval Weighted Average (IWA)	386
8.2.1	Formulation of the IWA	386
8.2.2	Computing the IWA	388
8.2.3	KM Algorithms	391
8.2.4	Enhanced KM Algorithms	396
8.2.5	Enhanced Iterative Algorithm with Stopping Condition (EIASC)	399
8.2.6	Remarks	403
8.3	Type-Reduction for IT2 FSs and Fuzzy Systems	403
8.3.1	Centroid Type-Reduction for IT2 Fuzzy Sets	404
8.3.2	Centroid Type-Reduction in an IT2 Fuzzy System	411
8.3.3	Height Type-Reduction in an IT2 Fuzzy System	412
8.3.4	Center-of-Sets (COS) Type-Reduction in an IT2 Fuzzy System	414
8.3.5	Type-Reduction Example	416
8.3.6	Remarks and Insights	417
8.4	Type-Reduction for GT2 FSs and Fuzzy Systems	420
8.4.1	Centroid Type-Reduction for GT2 Fuzzy Sets	422
8.4.2	Centroid Type-Reduction in a GT2 Fuzzy System	431
8.4.3	COS Type-Reduction in a GT2 Fuzzy System	431
	Appendix 1: A Wavy-Slice Approach to Type-Reduction	431
	Appendix 2: Type-Reduction Properties	434
	Exercises	441
	References	445
9	Interval Type-2 Fuzzy Systems	449
9.1	Introduction	449
9.2	Rules	450
9.3	Fuzzifier	452
9.4	Fuzzy Inference Engine	453
9.4.1	General Results	454
9.4.2	Fuzzification and Its Effects on Inference for IT2 Fuzzy Systems	458
9.5	Combining Fired Rule Output Sets on the Way to Defuzzification	479
9.5.1	Combining Using Set Theoretic Operations in an IT2 Mamdani Fuzzy System	479
9.5.2	Combining During Defuzzification in an IT2 Mamdani Fuzzy System	481

9.6	Type-Reduction + Defuzzification.	481
9.6.1	Centroid Type-Reduction + Defuzzification for an IT2 Mamdani Fuzzy System	481
9.6.2	Height Type-Reduction + Defuzzification for an IT2 Mamdani Fuzzy System	482
9.6.3	COS Type-Reduction + Defuzzification for an IT2 Mamdani Fuzzy System	483
9.6.4	Type-Reduction + Defuzzification for an IT2 TSK Fuzzy System.	484
9.6.5	Novelty Partitions	488
9.7	Comprehensive Example.	489
9.8	Approximate Type-Reduction + Defuzzification (Wu–Mendel Uncertainty Bounds) for IT2 Mamdani Fuzzy Systems	494
9.9	Direct Defuzzification	497
9.9.1	Nie–Tan (NT) Direct Defuzzification.	498
9.9.2	Biglarbegian–Melek–Mendel (BMM) Direct Defuzzification.	500
9.10	Summary	500
9.11	Comprehensive Example Continued	502
9.12	IT2 Fuzzy Basis Functions	504
9.13	Remarks and Insights	507
9.13.1	Layered Architecture Interpretations of an IT2 Fuzzy System	508
9.13.2	Fundamental Differences Between T1 and IT2 Fuzzy Systems	510
9.13.3	Universal Approximation by IT2 Fuzzy Systems.	510
9.13.4	Continuity of IT2 Fuzzy Systems	511
9.13.5	Rule Explosion and Some Ways to Control It.	515
9.13.6	Rule Interpretability	516
9.13.7	Historical Notes	516
	Exercises.	519
	References.	524
10	Interval Type-2 Fuzzy Systems: Design Methods and Applications.	529
10.1	Designing IT2 Fuzzy Systems	529
10.2	Some Design Methods	537
10.2.1	IT2 WM Method	537
10.2.2	Least-Squares Method	538
10.2.3	Derivative-Based Methods	540
10.2.4	SVD-QR Method.	544
10.2.5	Derivative-Free Methods	546

10.2.6	Iterative Design Methods	550
10.2.7	Remarks.	551
10.3	Case Study: Forecasting of Time-Series	554
10.3.1	Forecasting of Time Series When the Measurement Noise Is Stationary	554
10.3.2	Forecasting of Time Series When the Measurement Noise Is Nonstationary	558
10.4	Case Study: Knowledge Mining Using Surveys	567
10.4.1	Determining the IT2 FSs for the Vocabulary.	567
10.4.2	What Does One Do with a Histogram of Responses?	569
10.4.3	IT2 Consensus FLAs.	570
10.4.4	Remark	576
10.4.5	How to Use the IT2 FLA	577
10.4.6	Connections to the Perceptual Computer	579
10.5	Forecasting of Compressed Video Traffic Using IT2 Mamdani and TSK Fuzzy Systems.	582
10.5.1	Forecasting I Frame Sizes: Using the Same Number of Rules.	583
10.5.2	Forecasting I Frame Sizes: Using the Same Number of Design Parameters.	585
10.5.3	Conclusion.	585
10.6	IT2 Rule-Based Classification of Video Traffic.	586
10.6.1	FOUs for the Features	587
10.6.2	Rules and Their Parameters	587
10.6.3	Fuzzifiers	588
10.6.4	Computational Formulas for the IT2 RBCs	588
10.6.5	Optimization of the Rule Design Parameters	589
10.6.6	Results and Conclusions	590
10.7	Equalization of Time-Varying Nonlinear Digital Communication Channels	590
10.7.1	Preliminaries for Channel Equalization	592
10.7.2	Why an IT2 FAF Is Needed	593
10.7.3	Designing the IT2 FAFs	596
10.7.4	Simulations and Conclusions	597
10.8	IT2 Fuzzy Logic Control	600
10.8.1	What Is an IT2 Fuzzy Logic Controller (FLC)?	600
10.8.2	IT2 Fuzzy PID Control	601
10.8.3	Simulation Results (IT2-FPID Versus T1-FPID and PID)	602
10.9	Other Applications	608
	Exercises.	609
	References.	614

11	General Type-2 Fuzzy Systems	617
11.1	Introduction	617
11.2	Rules	620
11.3	Fuzzifier	622
11.4	Fuzzy Inference Engine	622
11.5	Combining Fired Rule Output Sets on the Way to Defuzzification	627
11.5.1	Combining Using Set Theoretic Operations in a WH GT2 Mamdani Fuzzy System	628
11.5.2	Combining During Defuzzification in a WH GT2 Mamdani Fuzzy System	629
11.6	Type-Reduction	629
11.6.1	Centroid Type-Reduction for a WH GT2 Mamdani Fuzzy System	630
11.6.2	Center-of Sets Type-Reduction for a WH GT2 Mamdani Fuzzy System	631
11.6.3	Type-Reduction for a WH GT2 TSK Fuzzy System	632
11.7	Defuzzification	634
11.7.1	Approximation and Defuzzification	634
11.7.2	End-Points Defuzzification	635
11.7.3	Average of End-Points Defuzzification	636
11.8	Summary	637
11.8.1	WH GT2 Mamdani Fuzzy System that Uses Centroid Type-Reduction + Average of End-Points Defuzzification	638
11.8.2	WH GT2 Mamdani Fuzzy System that Uses COS Type-Reduction + Average of End-Points Defuzzification	639
11.8.3	Unnormalized A2-C0 WH GT2 TSK Fuzzy System	639
11.8.4	Normalized A2-C0 WH GT2 TSK Fuzzy System	640
11.9	Comprehensive Example	640
11.10	Direct Defuzzification	647
11.10.1	Proposed WH-NT Direct Defuzzification	647
11.10.2	Proposed WH-BMM Direct Defuzzification	649
11.11	Comprehensive Example Continued	650
11.12	GT2 Fuzzy Basis Functions	651
11.13	Remarks and Insights	655
11.14	Designing WH GT2 Fuzzy Systems	657
11.15	Applications	664
11.16	Case Study: WH GT2 Fuzzy Logic Control	665

11.16.1	What Is a GT2 FLC?	665
11.16.2	System Description	666
11.16.3	Controller Designs	667
11.16.4	Simulation Results (WH GT2 FPID Versus IT2 FPID Versus T1 FPID and PID)	668
	Exercises	670
	References	673
Index	675

<http://www.springer.com/978-3-319-51369-0>

Uncertain Rule-Based Fuzzy Systems

Introduction and New Directions, 2nd Edition

Mendel, J.M.

2017, XXII, 684 p. 215 illus., 192 illus. in color.,

Hardcover

ISBN: 978-3-319-51369-0