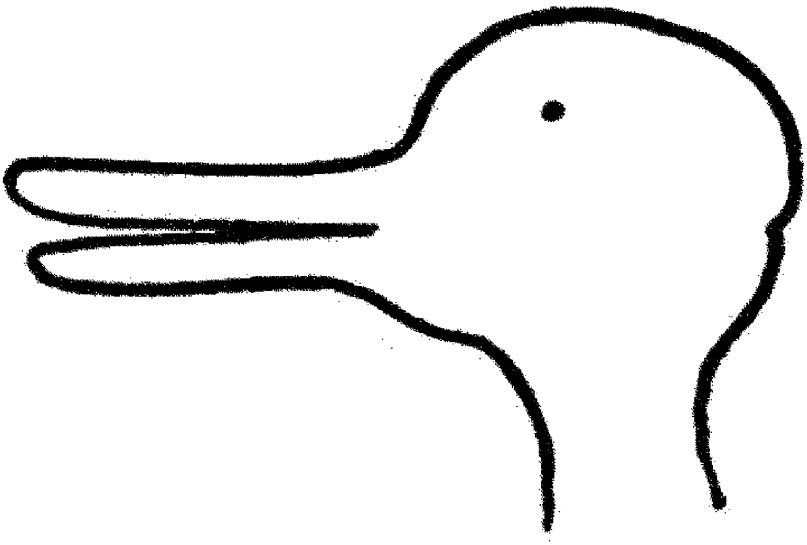
Preface



> ‘Der Kopf, *so* gesehen, hat mit dem Kopf, *so* gesehen, auch nicht die leiseste Ähnlichkeit (...) Der Aspektwechsel. “Du würdest doch sagen, dass sich das Bild jetzt gänzlich geändert hat!” Aber was ist anders: mein Eindruck? meine Stellungnahme? (...) Ich *beschreibe* die Änderung wie eine Wahrnehmung, ganz, als hätte sich der Gegenstand vor meinen Augen geändert.’ (Wittgenstein, *Philosophische Untersuchungen* II, §§127, 129).[1]

As the well-known picture above is meant to allegorize, some physical systems admit a dual description in either classical or quantum-mechanical terms. According to Bohr’s “doctrine of classical concepts”, measurement apparatuses are examples of such systems. More generally—as hammered down by decoherence theorists—the classical world around us is a case in point. As will be argued in this book, the measurement problem of quantum mechanics (highlighted by Schrödinger’s Cat) is *caused* by this duality (rather than *resolved* by it, as Bohr is said to have thought).

[1] ‘The head seen in *this* way hasn’t even the slightest similarity to the head seen in *that* way (...) The change of aspect. “But surely you’d say that the picture has changed altogether now! But what is different: my impression? my attitude? (...) I *describe* the change like a perception; just as if the object has changed before my eyes.’ Translation: G.E.M. Anscombe, P.M.S. Hacker, & J. Schulte (Wittgenstein, 2009/1953, pp. 205–206).

The aim of this book is to analyze the foundations of quantum theory from the point of view of classical-quantum duality, using the mathematical formalism of operator algebras on Hilbert space (and, more generally, C*-algebras) that was originally created by von Neumann (followed by Gelfand and Naimark). In support of this analysis, but also as a matter of independent interest, the book covers many of the traditional topics one might expect to find in a treatise on the foundations of quantum mechanics, like pure and mixed states, observables, the Born rule and its relation to both single-case probabilities and long-run frequencies, Gleason's Theorem, the theory of symmetry (including Wigner's Theorem and its relatives, culminating in a recent theorem of Hamhalter's), Bell's Theorem(s) and the like, quantization theory, indistinguishable particle, large systems, spontaneous symmetry breaking, the measurement problem, and (intuitionistic) quantum logic. One also finds a few idiosyncratic themes, such as the Kadison–Singer Conjecture, topos theory (which naturally injects intuitionism into quantum logic), and an unusual emphasis on both conceptual and mathematical aspects of limits in physical theories.

All of this is held together by what we call ***Bohrification***, i.e., the mathematical interpretation of Bohr's classical concepts by *commutative* C*-algebras, which in turn are studied in their quantum habitat of *noncommutative* C*-algebras.

Thus the book is mostly written in mathematical physics style, but its real subject is *natural philosophy*. Hence its intended readership consists not only of mathematical physicists, but also of philosophers of physics, as well as of theoretical physicists who wish to do more than 'shut up and calculate', and finally of mathematicians who are interested in the mathematical and conceptual structure of quantum theory.

To serve all these groups, the native mathematical language (i.e. of C*-algebras) is introduced slowly, starting with finite sets (as classical phase spaces) and finite-dimensional Hilbert spaces. In addition, all advanced mathematical background that is necessary but may distract from the main development is laid out in extensive appendices on Hilbert spaces, functional analysis, operator algebras, lattices and logic, and category theory and topos theory, so that the prerequisites for this book are limited to basic analysis and linear algebra (as well as some physics). These appendices not only provide a direct route to material that otherwise most readers would have needed to extract from thousands of pages of diverse textbooks, but they also contain some original material, and may be of interest even to mathematicians.

In summary, the aims of this book are similar to those of its peerless paradigm:

> 'Der Gegenstand dieses Buches ist die einheitliche, und, soweit als möglich und angebracht, mathematisch einwandfreie Darstellung der neuen Quantenmechanik (...). Dabei soll das Hauptgewicht auf die allgemeinen und prinzipiellen Fragen, die im Zusammenhange mit dieser Theorie entstanden sind, gelegt werden. Insbesondere sollen die schwierigen und vielfach noch immer nicht restlos geklärten Interpretationsfragen näher untersucht werden.' (von Neumann, *Mathematische Grundlagen der Quantenmechanik*, 1932, p. 1).[2]

[2] 'The object of this book is to present the new quantum mechanics in a unified presentation which, so far as it is possible and useful, is mathematically rigorous. (...) Therefore the principal emphasis shall be placed on the general and fundamental questions which have arisen in connection with this theory. In particular, the difficult problems with interpretation, many of which are even now not fully resolved, will be investigated in detail.' Translation: R.T. Beyer (von Neumann, 1955, p. vii).

Two other quotations the author often had in mind while writing this book are:

> 'And although the whole of philosophy is not immediately evident, still it is better to add something to our knowledge day by day than to fill up men's minds in advance with the preconceptions of hypotheses.' (Newton, draft preface to *Principia*, 1686).[3]

> 'Juist het feit dat een genie als DESCARTES volkomen naast de lijn van ontwikkeling is blijven staan, die van GALILEI naar NEWTON voert (...) [is] een phase van den in de historie zoo vaak herhaalden strijd tusschen de bescheidenheid der mathematisch-physische methode, die na nauwkeurig onderzoek de verschijnselen der natuur in steeds meer omvattende schemata met behulp van de exacte taal der mathesis wil beschrijven en den hoogmoed van het philosophische denken, dat in één genialen greep de heele wereld wil omvatten (...).' (Dijksterhuis, *Val en Worp*, 1924, p. 343).[4]

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Finally, it is a pleasure to dedicate this book to Jeremy Butterfield, in recognition of his ideas, as well as of his unrelenting support and friendship over the last 25 years.

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[3] Newton (1999), p. 61.

[4] 'The very fact that a genius like Descartes was completely sidelined in the development leading from Galilei to Newton (...) represents a phase in the struggle—that has so often been repeated throughout history—between the modesty of the approach of mathematical physics, which after precise investigations attempts to describe natural phenomena in increasingly comprehensive schemes using the exact language of mathematics, and the haughtiness of philosophical thought, which wants to comprehend the entire world in one dazzling grasp.' Translation by the author.

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