Re-parameterization of the Cole-Cole model for improved spectral inversion of IP data

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Cole-Cole model

$$\sigma = \underbrace{\frac{1}{F}\sigma_W}_{\text{bulk water}} + \underbrace{\sigma'_{surf}}_{\text{surface conduction}} + \underbrace{i\sigma''_{surf}}_{\text{surface polarization}}$$

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Model parameters: $\{\sigma_0, m_0, \tau_\sigma, C\}$

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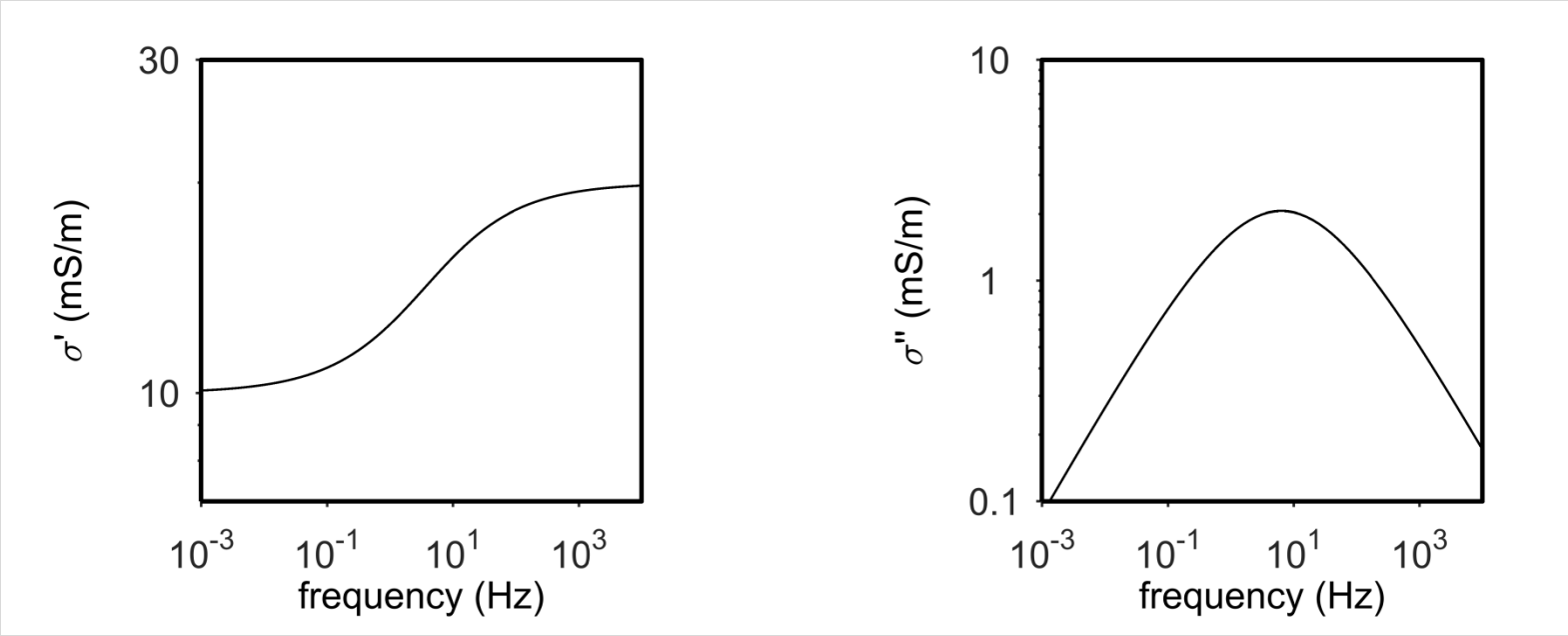

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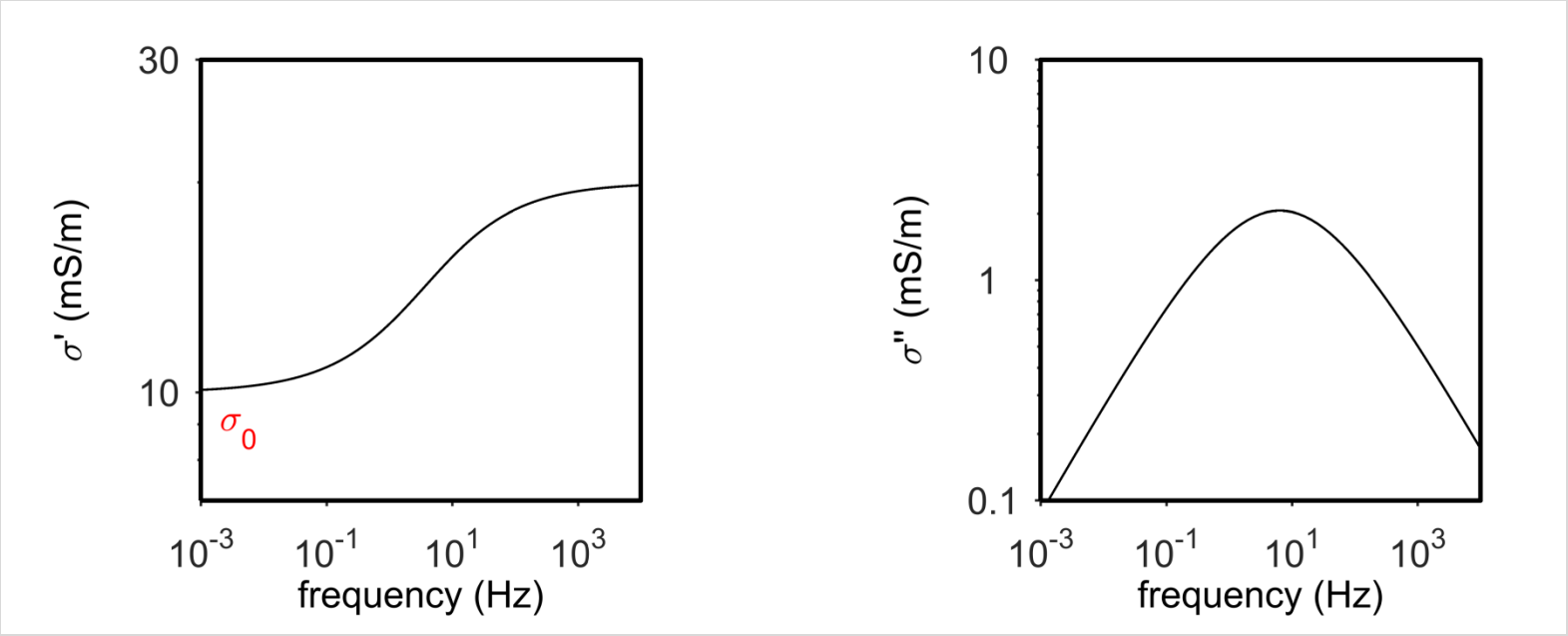

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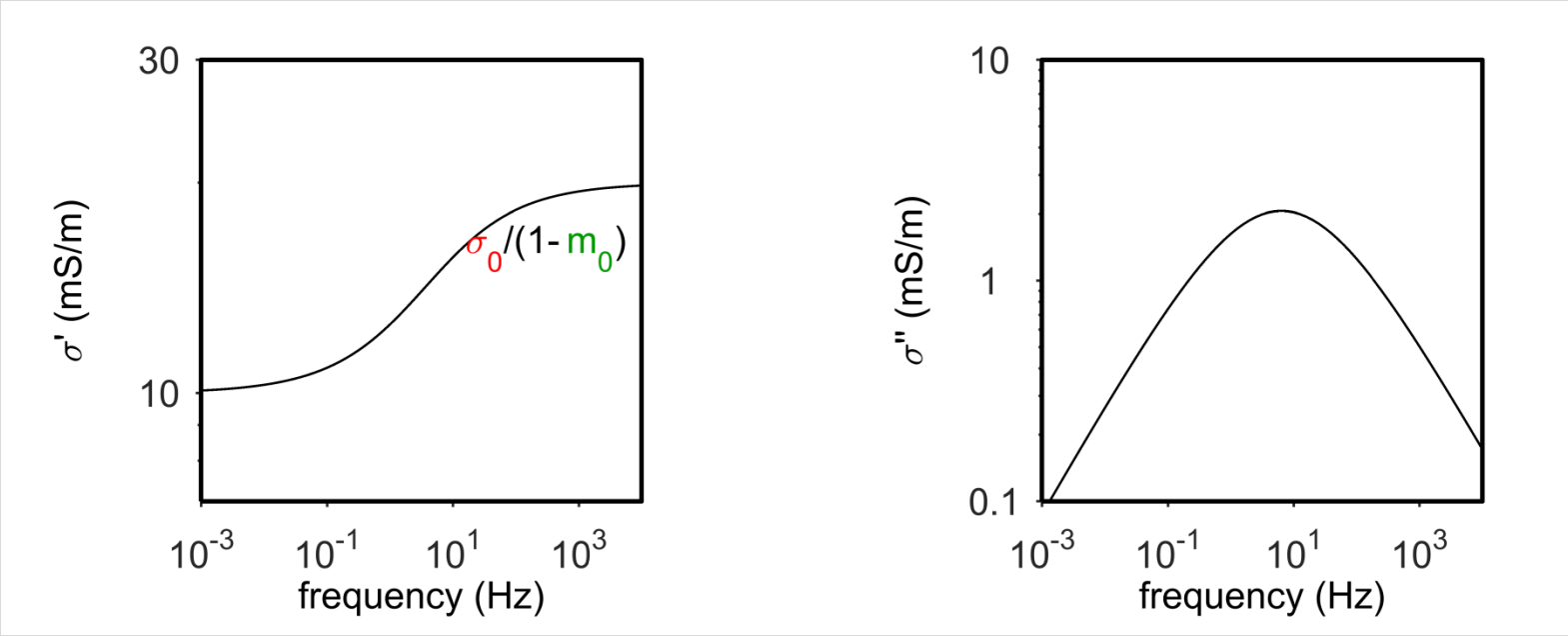

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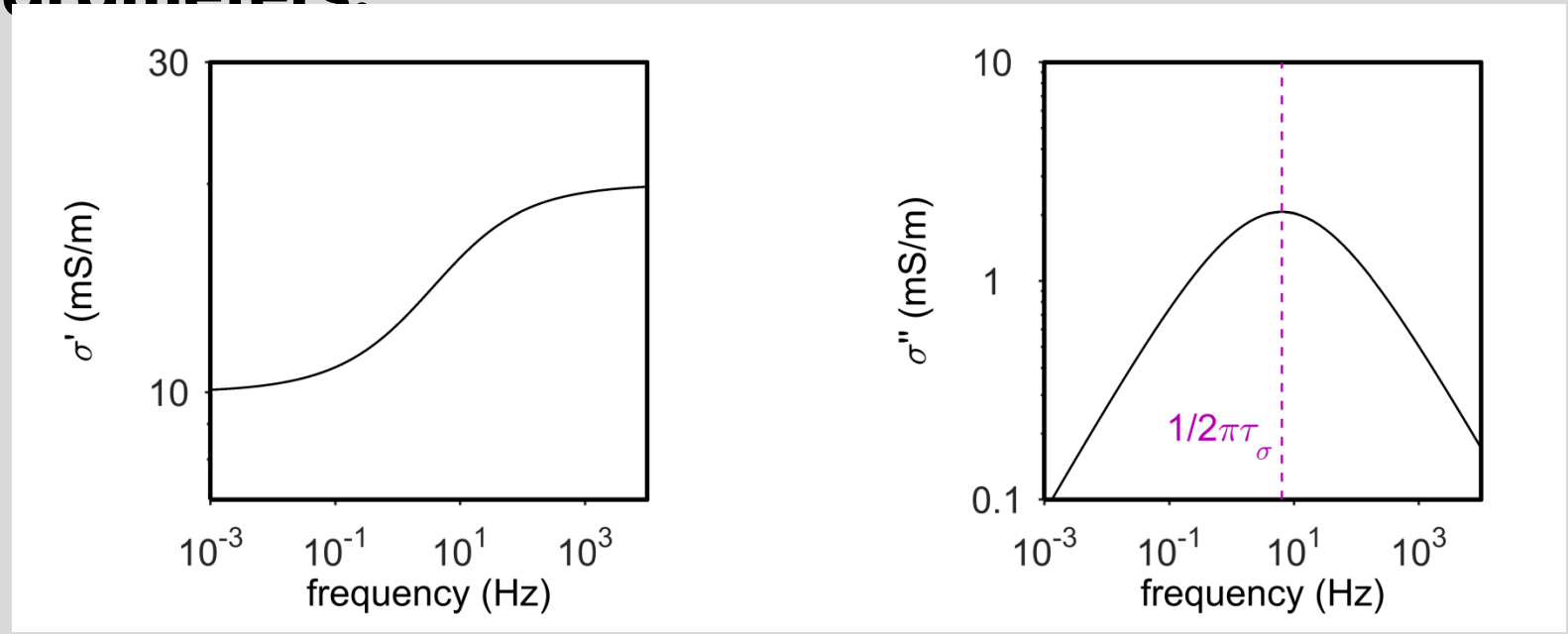

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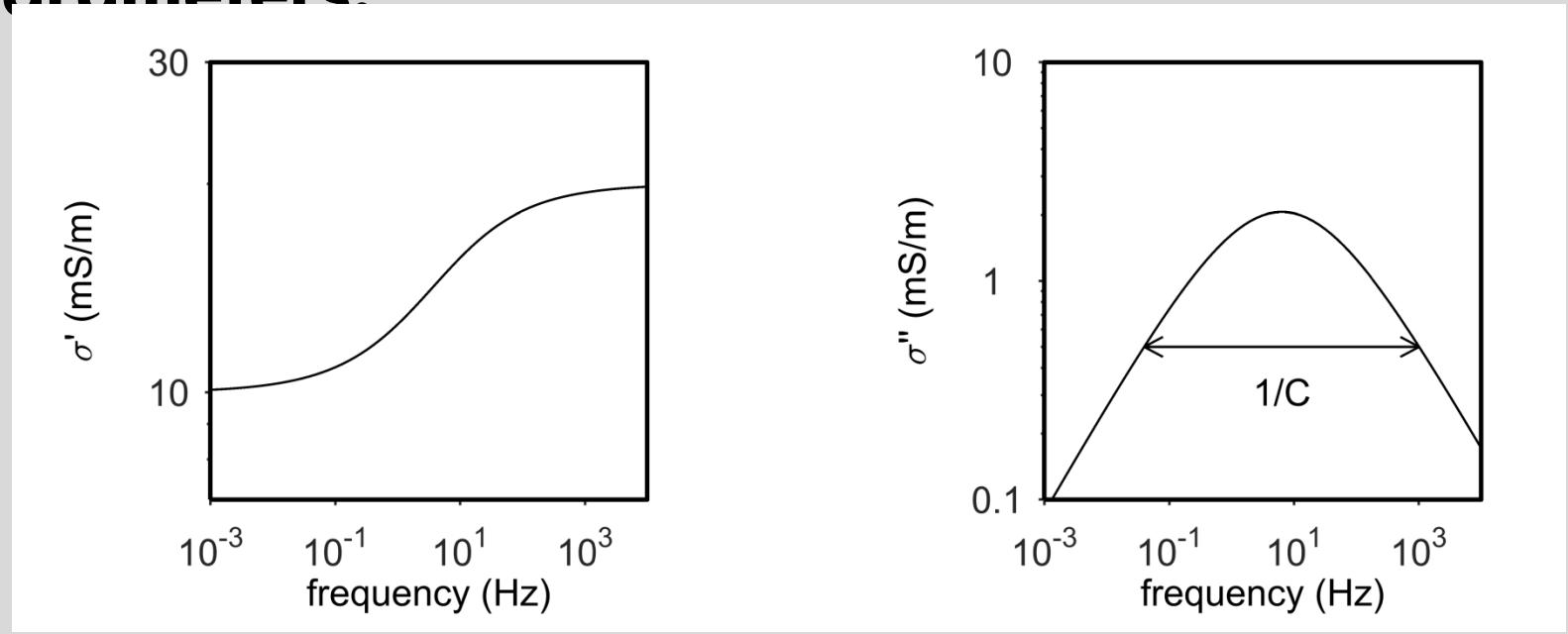

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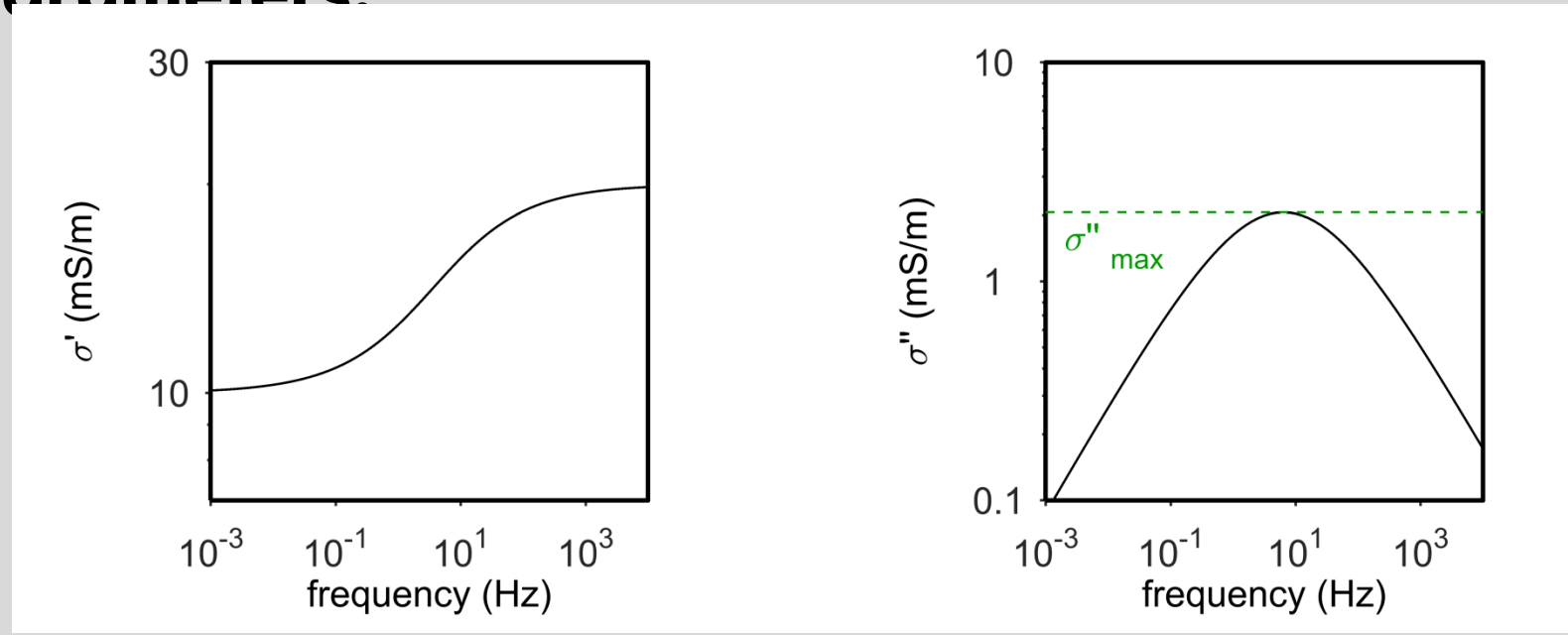
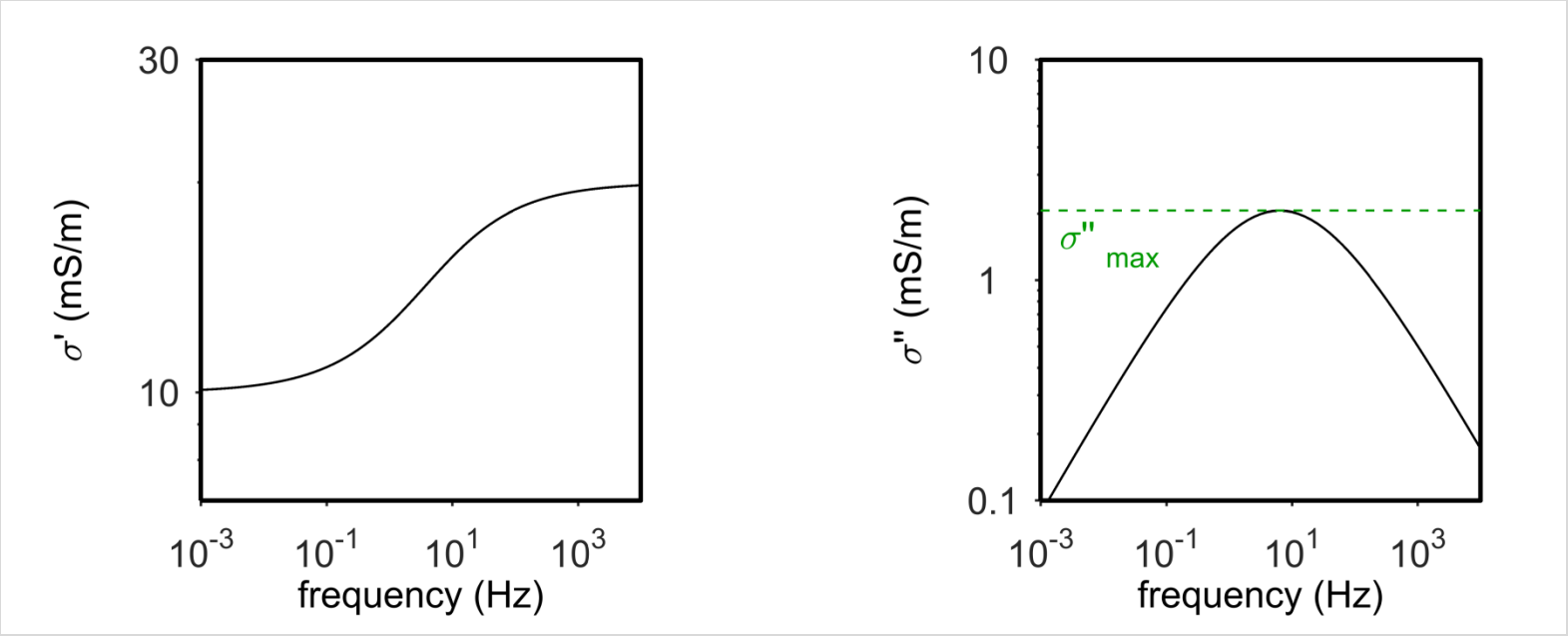


Model re-parameterization

Classic Cole-Cole: $\{\sigma_0, m_0, \tau_\sigma, C\}$

Maximum imaginary conductivity (MIC): $\{\sigma_0, \sigma''_{max}, \tau_\sigma, C\}$



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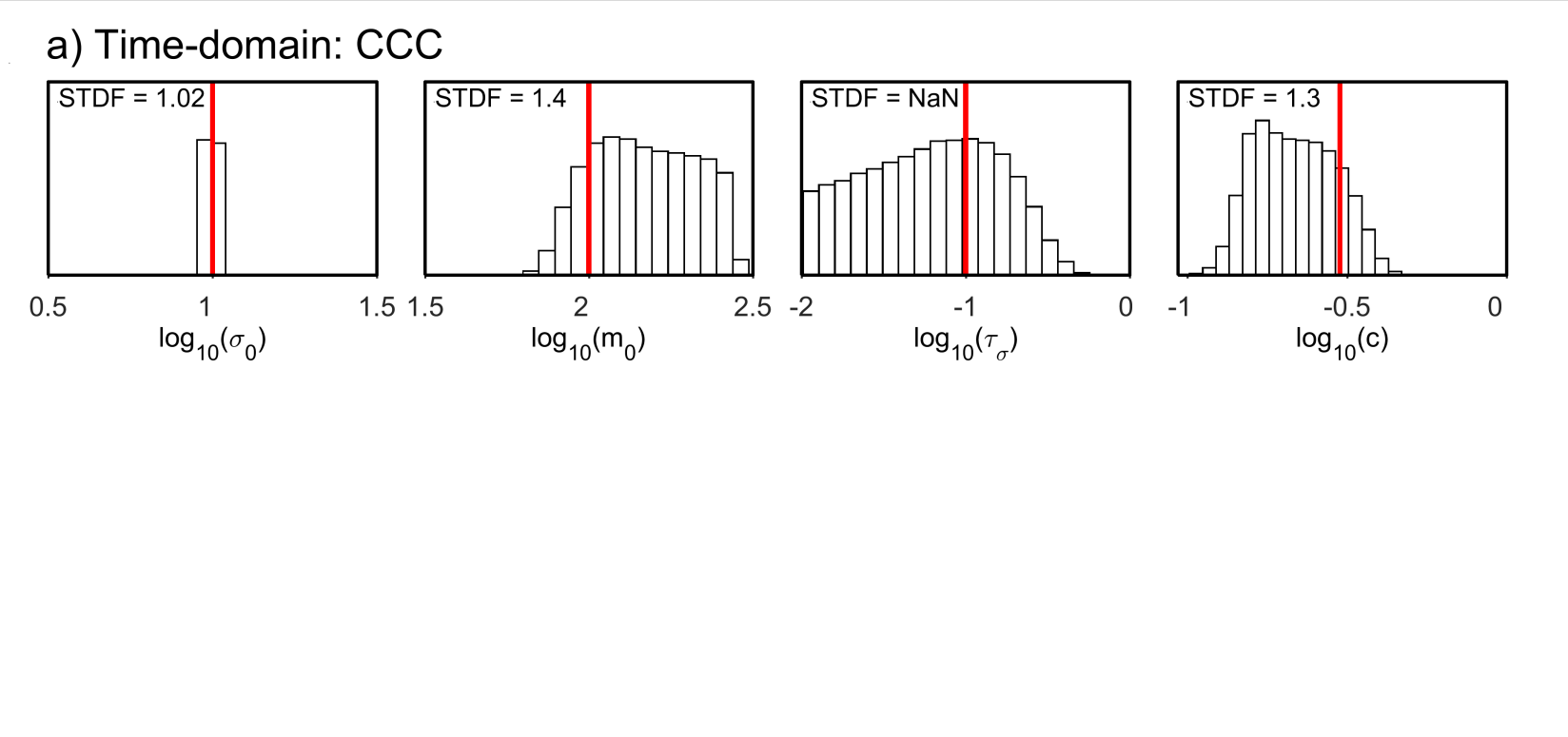

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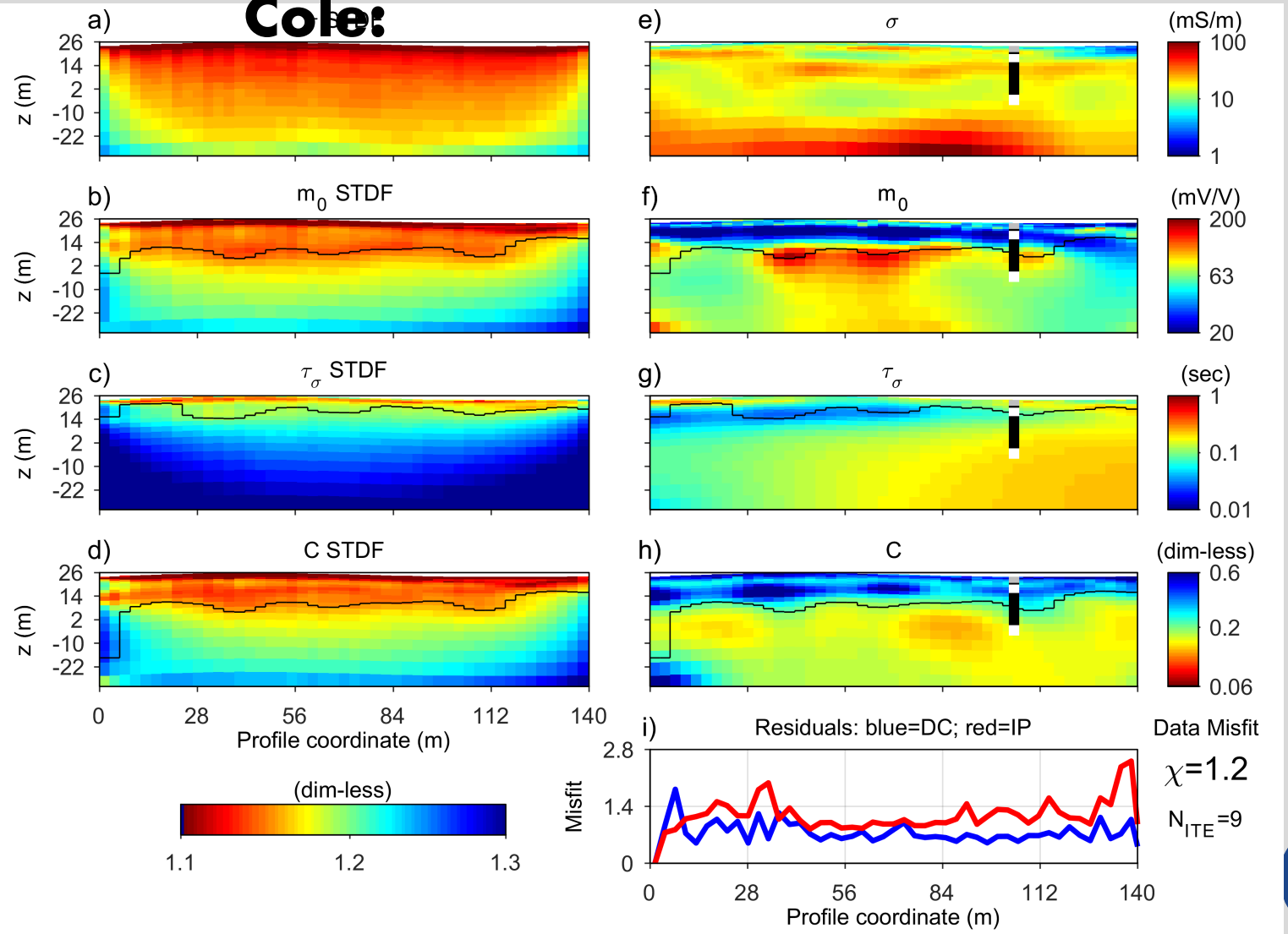

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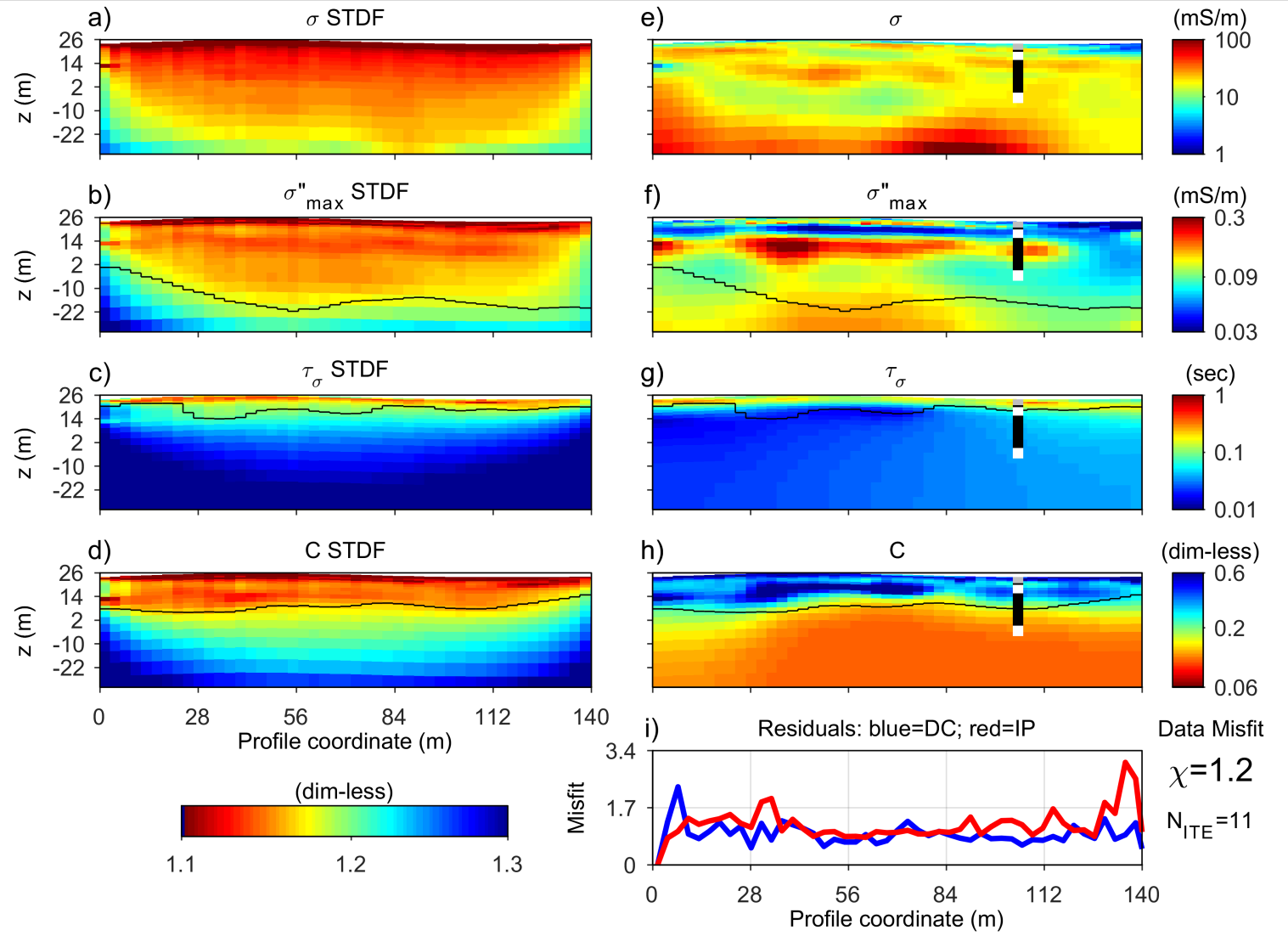
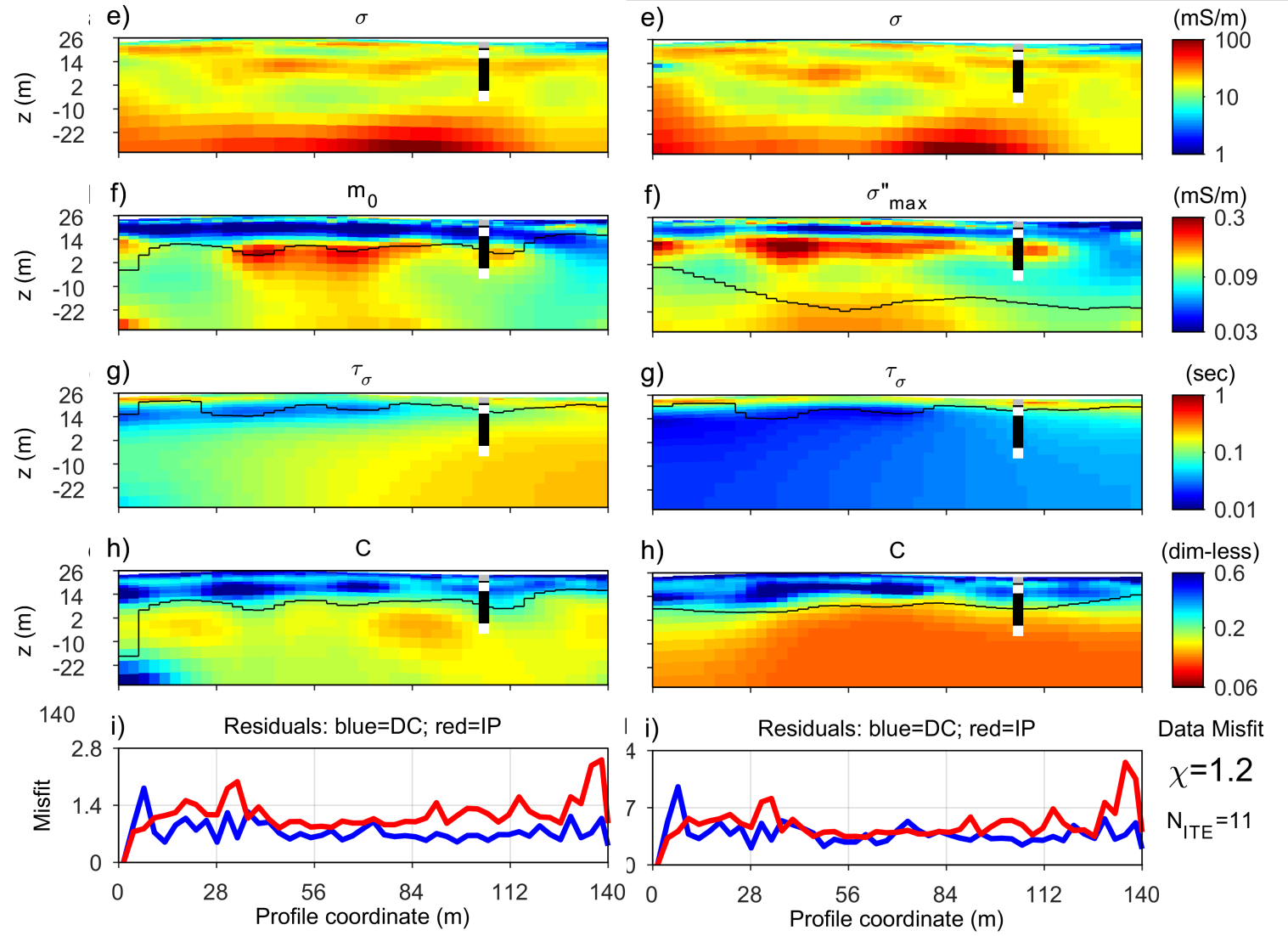


Model re-parameterization

Classic Cole-Cole



Model re-parameterization

Classic Cole-Cole: $\{\sigma_0, m_0, \tau_\sigma, C\}$

Maximum imaginary conductivity (MIC): $\{\sigma_0, \sigma''_{max}, \tau_\sigma, C\}$

MIC compared to classic Cole-Cole

- Smaller parameter correlations
- Resolution of σ''_{max} much better than m_0
- Deeper DOI
- Closer link to Archie's law

Model re-parameterization

Classic Cole-Cole: $\{\sigma_0, m_0, \tau_\sigma, C\}$

Maximum imaginary conductivity (MIC): $\{\sigma_0, \sigma''_{max}, \tau_\sigma, C\}$

MIC compared to classic Cole-Cole

- Smaller parameter correlations
- Resolution of σ''_{max} much better than m_0
- Closer link to Archie's law

But

- Bulk conduction and surface conduction are not separated

$$\sigma_0 = \sigma_{bulk} + \sigma'_{surf}(\omega = 0) \quad \sigma_{bulk} := \frac{1}{F}\sigma_W$$

Model re-parameterization: link between σ' and σ''

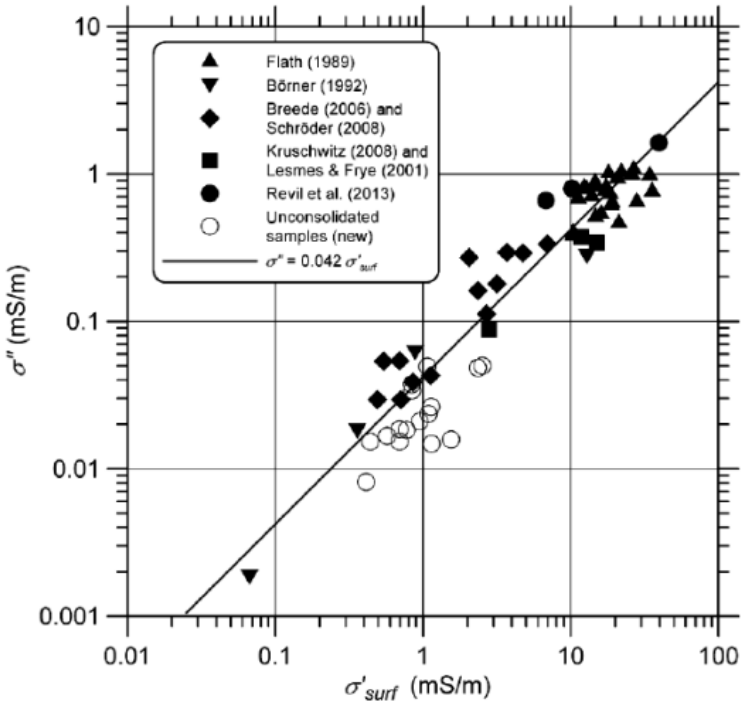


Figure 3. The dependence of imaginary conductivity (σ'') on surface conductivity (σ'_{surf}) for the entire database shown in Table 1. The value of σ'' at 1 Hz was used here. The best-fit line shows the single linear fit to the entire data set where $l = 0.042$ with $R^2 = 0.911$.

$$\sigma''_{surf} = l \cdot \sigma'_{surf} = 0.042 \cdot \sigma'_{surf}$$

$$l = 0.042$$

Weller et al. (2013)

Model re-parameterization

Classic Cole-Cole: $\{\sigma_0, m_0, \tau_\sigma, C\}$

Maximum imaginary conductivity (MIC): $\{\sigma_0, \sigma''_{max}, \tau_\sigma, C\}$

Bulk & imaginary conductivity (BIC): $\{\sigma_{bulk}, \sigma''_{max}, \tau_\sigma, C\}$

$$\sigma_{bulk} := \frac{1}{F}\sigma_W \qquad \sigma_0 = \sigma_{bulk} + \sigma'_{surf}(\omega = 0)$$

$$\sigma''_{surf}\left(\omega = \frac{1}{\tau_\sigma}\right) = l \cdot \sigma'_{surf}\ (\omega = 0)$$

Model re-parameterization

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Maximum imaginary conductivity (MIC): $\{\sigma_0, \sigma''_{max}, \tau_\sigma, C\}$

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BIC compared to MIC

- **Similar resolution of parameters**
- **Disentangled bulk and surface conduction**
- **High σ''_{max} => High σ_0**
- **Perfectly suited for permeability estimation**