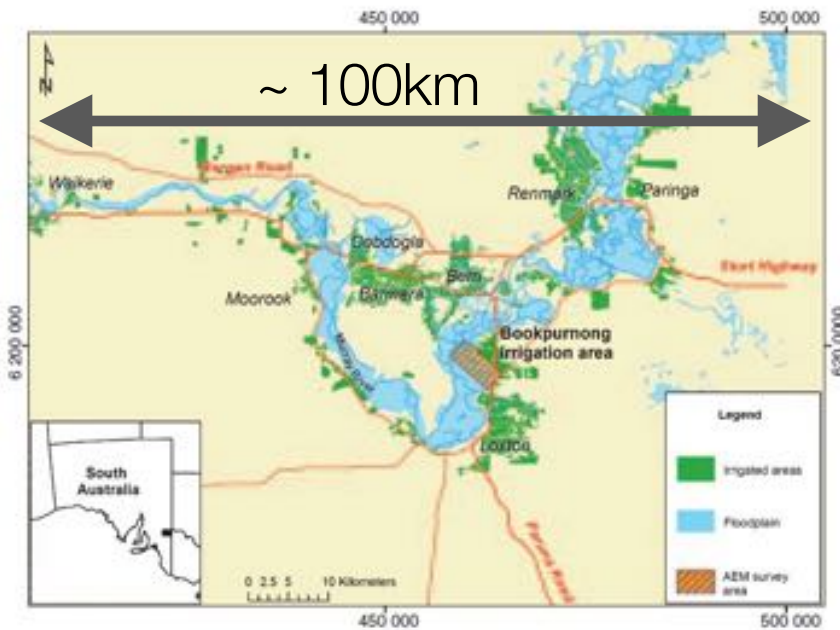


EM Fundamentals



Motivation: applications difficult for DC

Large areas to be covered



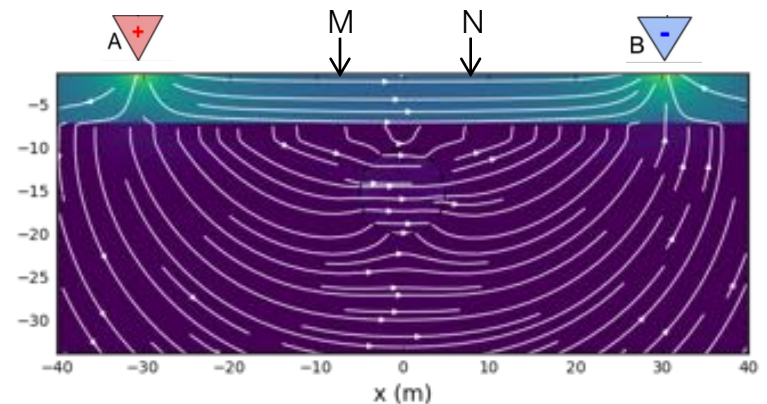
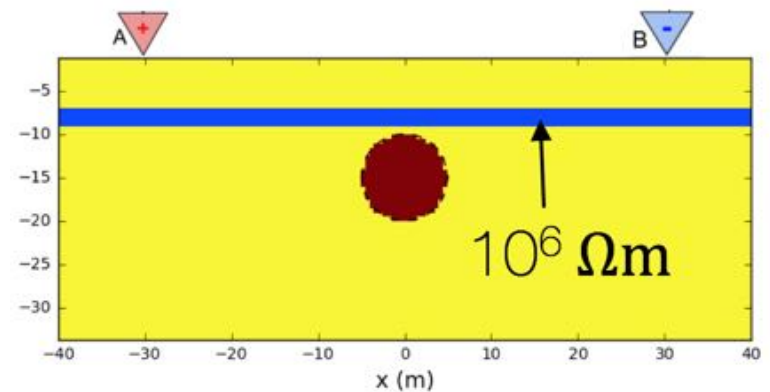
Rugged terrain



Hard to inject



Resistive layer “shields” target

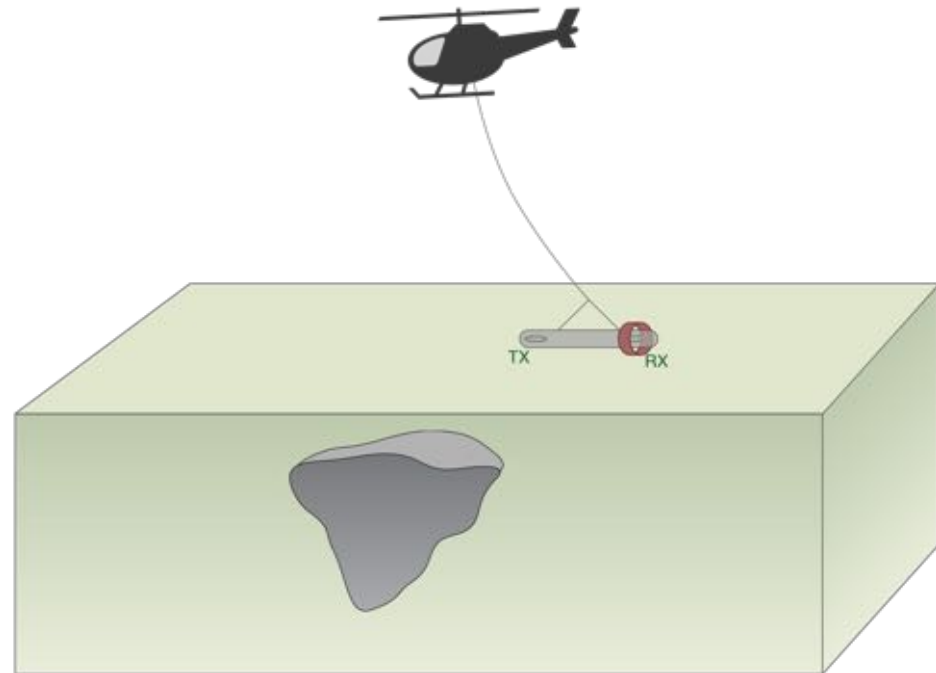


Outline

- Basic Survey
- Ampere's and Faraday's Laws (2-coil App)
- Circuit model for EM induction
- Frequency and time domain data
- Sphere in homogeneous earth
- Cyl code
- Energy losses in the ground

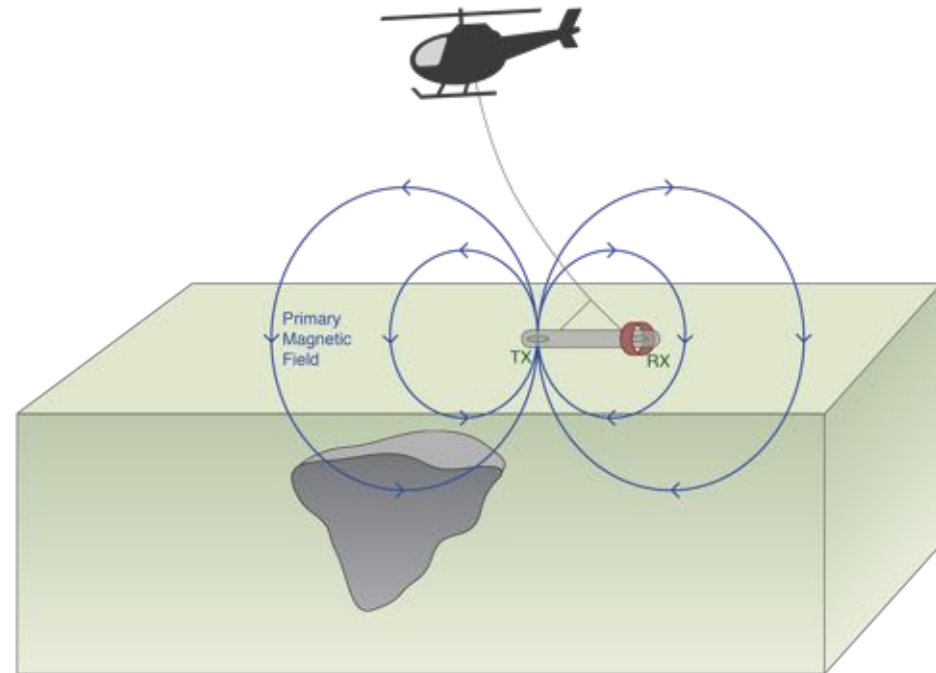
Basic Experiment

- **Setup:**
 - transmitter and receiver are in a towed bird



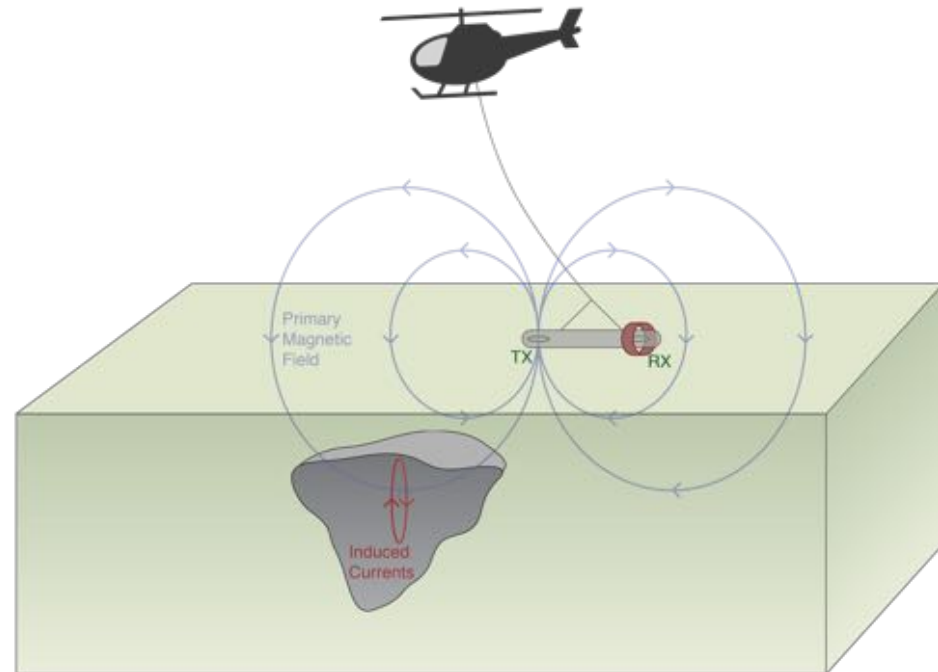
Basic Experiment

- **Setup:**
 - transmitter and receiver are in a towed bird
- **Primary:**
 - Transmitter produces a primary magnetic field



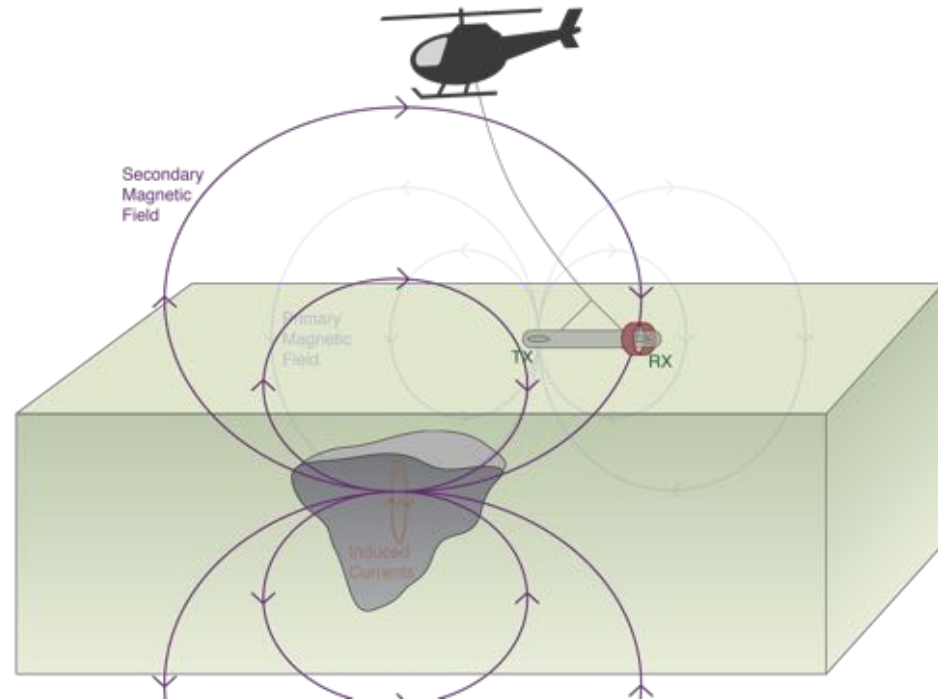
Basic Experiment

- **Setup:**
 - transmitter and receiver are in a towed bird
- **Primary:**
 - Transmitter produces a primary magnetic field
- **Induced Currents:**
 - Time varying magnetic fields generate electric fields everywhere and currents in conductors



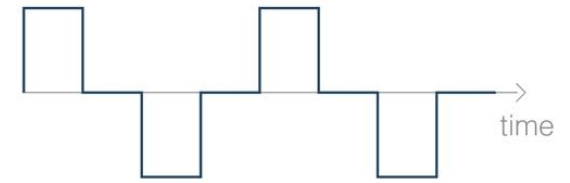
Basic Experiment

- **Setup:**
 - transmitter and receiver are in a towed bird
- **Primary:**
 - Transmitter produces a primary magnetic field
- **Induced Currents:**
 - Time varying magnetic fields generate electric fields everywhere and currents in conductors
- **Secondary Fields:**
 - The induced currents produce a secondary magnetic field.

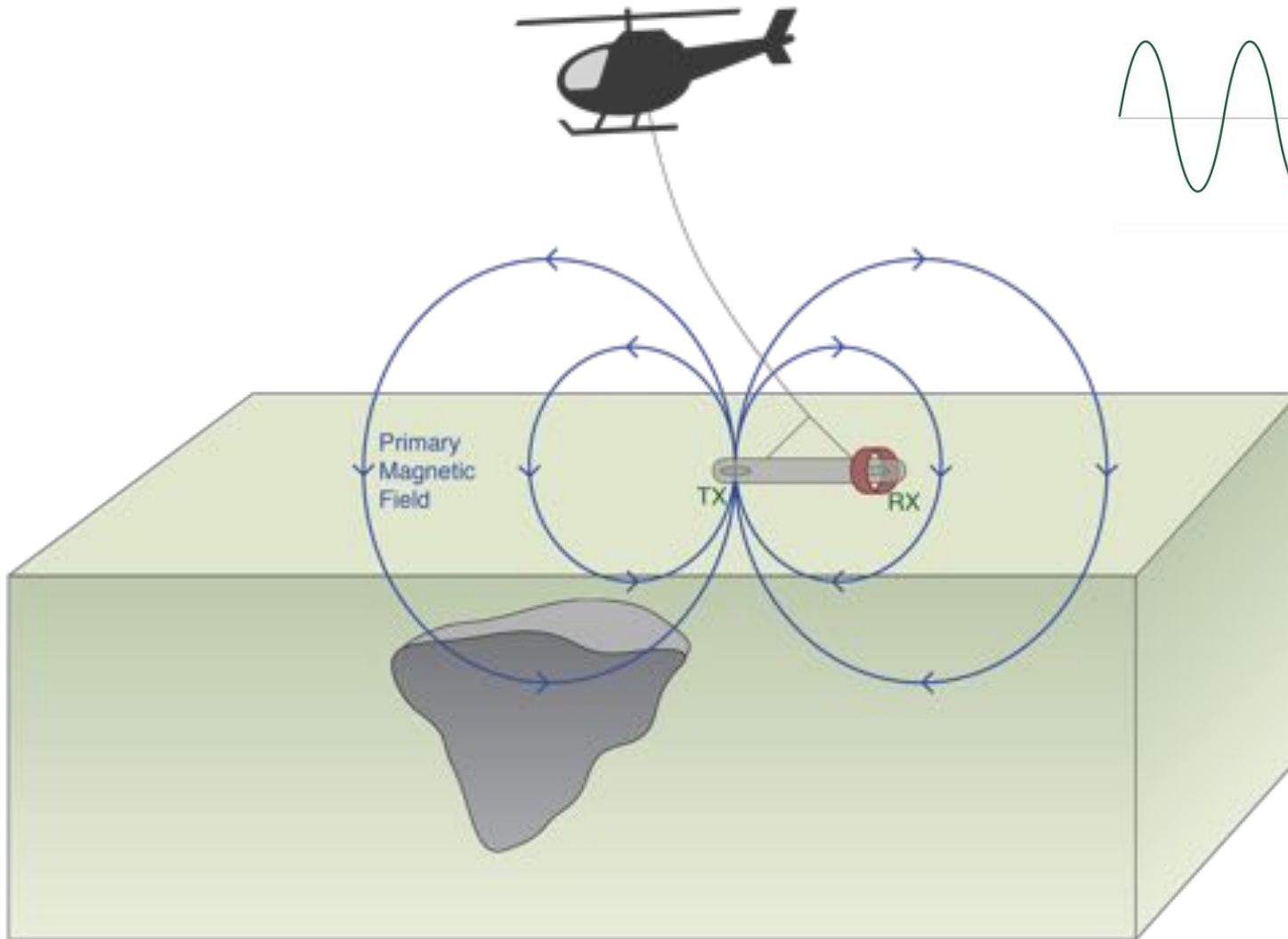
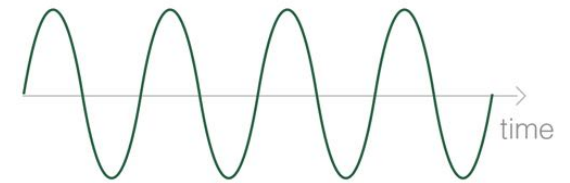


Transmitter

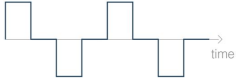
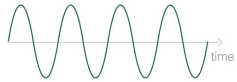
waveform



or



Basic Equations: Quasi-static

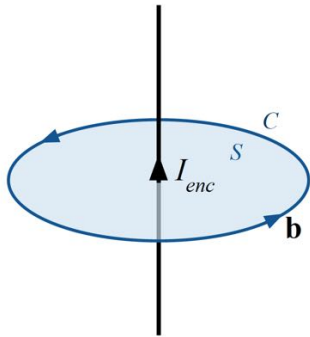
	Time 	Frequency 
Faraday's Law	$\nabla \times \mathbf{e} = - \frac{\partial \mathbf{b}}{\partial t}$	$\nabla \times \mathbf{E} = - i\omega \mathbf{B}$
Ampere's Law	$\nabla \times \mathbf{h} = \mathbf{j} + \frac{\partial \mathbf{d}}{\partial t}$	$\nabla \times \mathbf{H} = \mathbf{J} + i\omega \mathbf{D}$
No Magnetic Monopoles	$\nabla \cdot \mathbf{b} = 0$	$\nabla \cdot \mathbf{B} = 0$
Constitutive Relationships (non-dispersive)	$\mathbf{j} = \sigma \mathbf{e}$ $\mathbf{b} = \mu \mathbf{h}$ $\mathbf{d} = \varepsilon \mathbf{e}$	$\mathbf{J} = \sigma \mathbf{E}$ $\mathbf{B} = \mu \mathbf{H}$ $\mathbf{D} = \varepsilon \mathbf{E}$

* Solve with sources and boundary conditions

Ampere's Law

$$\nabla \times \mathbf{H} = \mathbf{J}$$

Wire



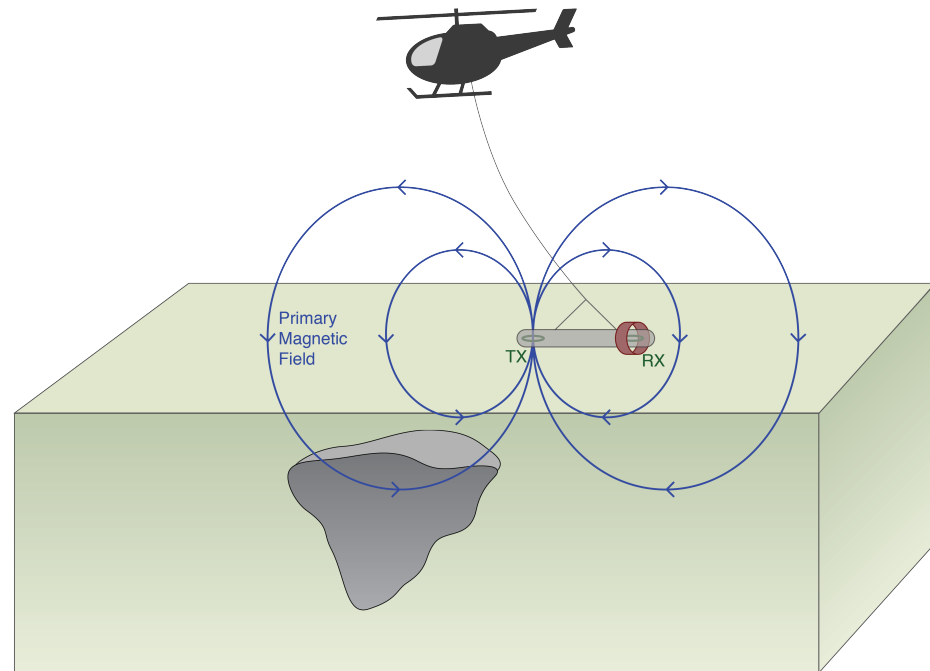
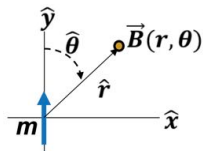
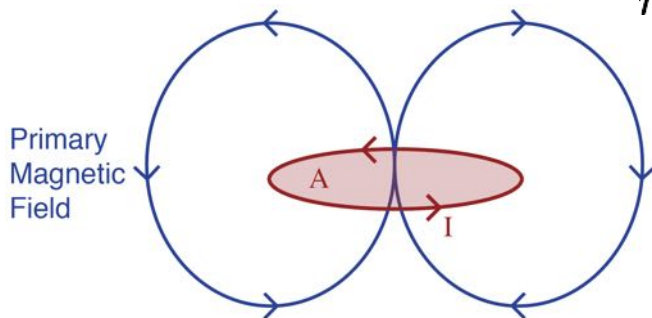
$$\mathbf{B} = \frac{\mu_0 I_{enc}}{2\pi r} \hat{\phi}$$

Right hand rule

Current loop

$$\mathbf{B} = \frac{\mu_0 m}{4\pi r^3} (2 \cos \theta \hat{\mathbf{r}} + \sin \theta \hat{\boldsymbol{\theta}})$$

$$m = IA$$



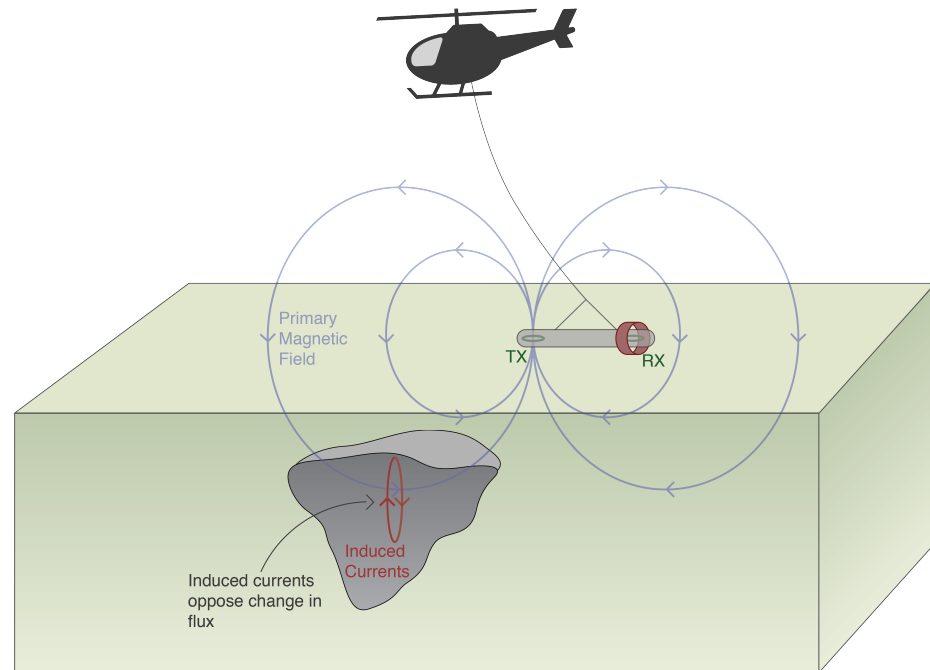
Faraday's Law and Induced Currents

$$\nabla \times \mathbf{e} = - \frac{\partial \mathbf{b}}{\partial t}$$

Lenz'
Law

Ohm's Law

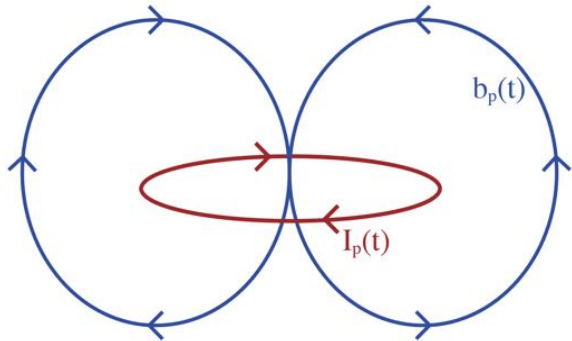
$$\mathbf{j} = \sigma \mathbf{e}$$



Two Coil Example: Harmonic

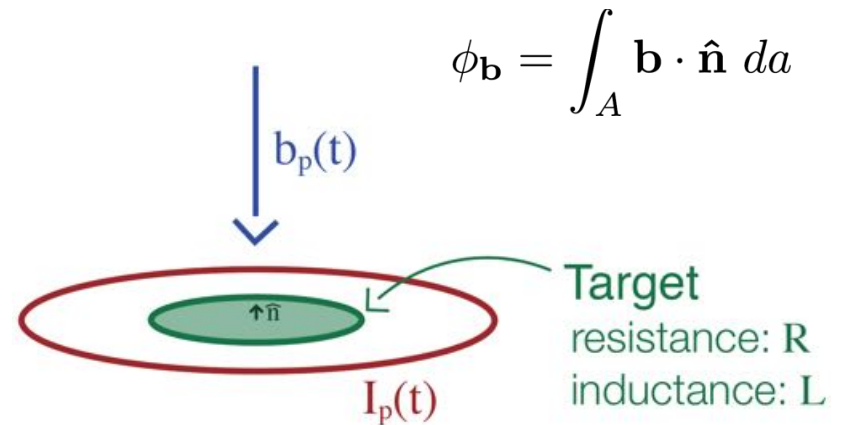
Source (red loop)

- Time varying current $\rightarrow$ Time varying magnetic flux



Target (green loop)

- Time varying magnetic flux



Faraday's Law

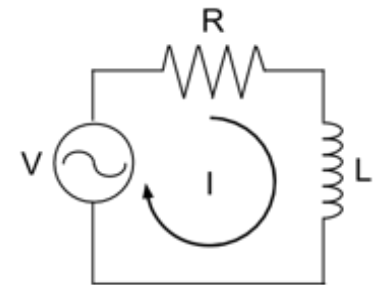
$$\nabla \times \mathbf{e} = -\frac{\partial \mathbf{b}}{\partial t}$$

Ohm's Law

$$\mathbf{j} = \sigma \mathbf{e}$$

EMF (voltage) is related to time rate of change in flux.

$$V = EMF = -\frac{d\phi_b}{dt}$$

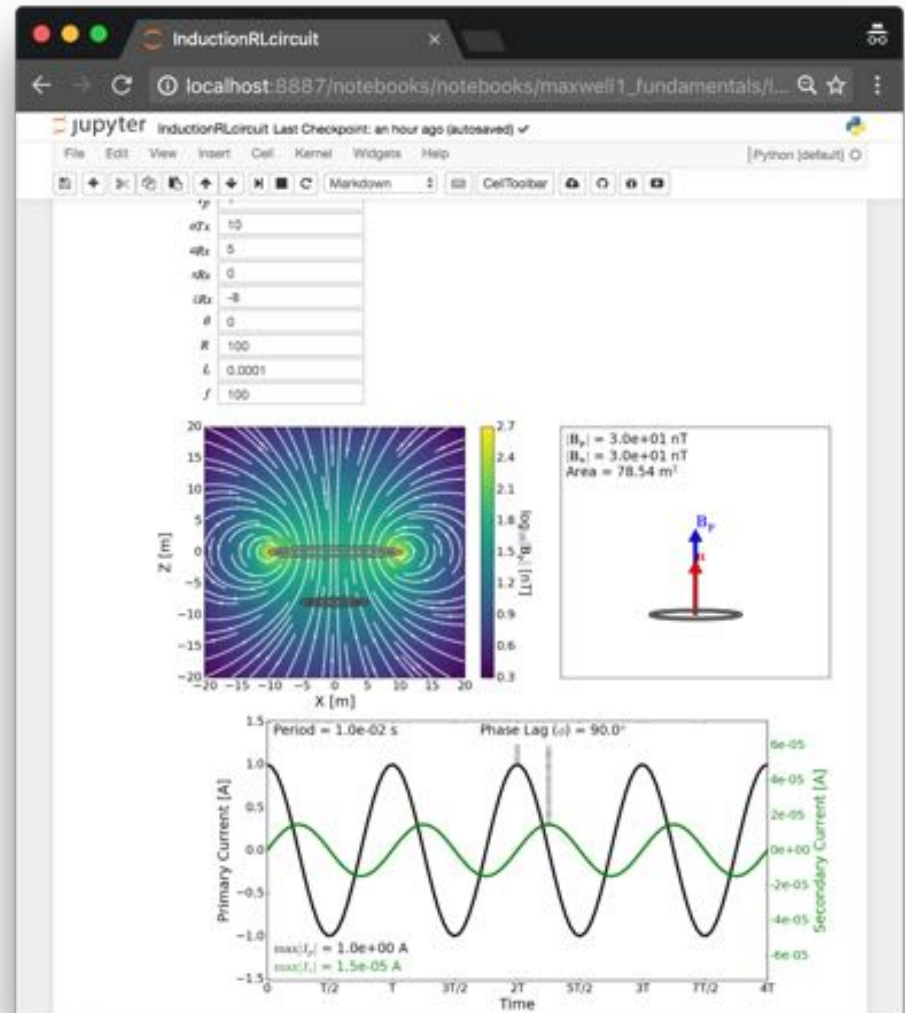
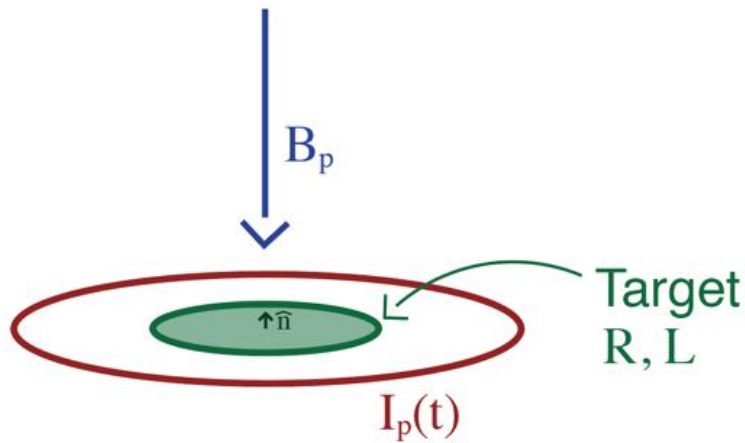


App for Faraday's Law

$$\nabla \times \mathbf{e} = - \frac{\partial \mathbf{b}}{\partial t}$$

Lenz'
Law

$$\mathbf{j} = \sigma \mathbf{e}$$



Two Coil Example: Harmonic

Induced Currents

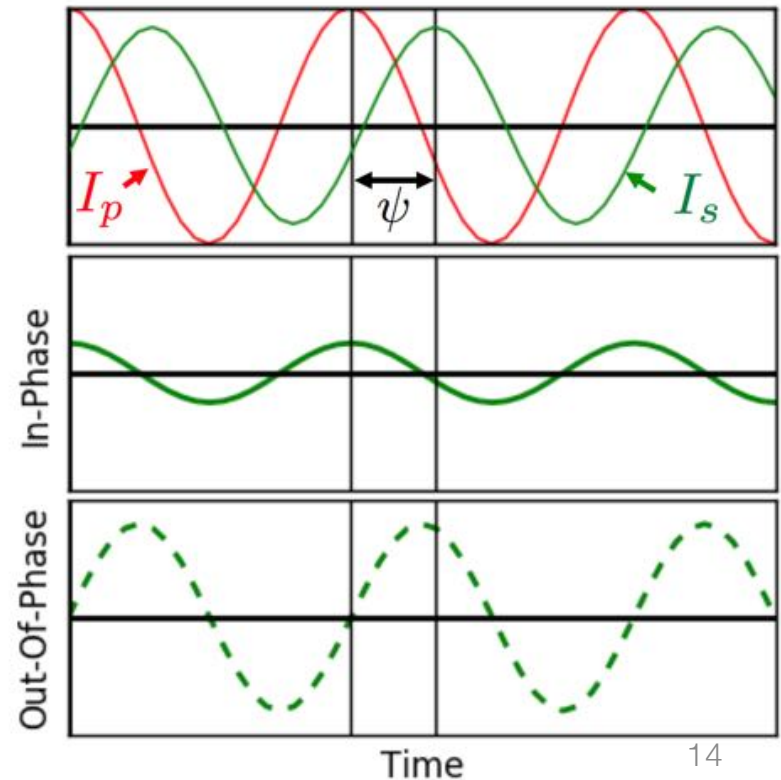
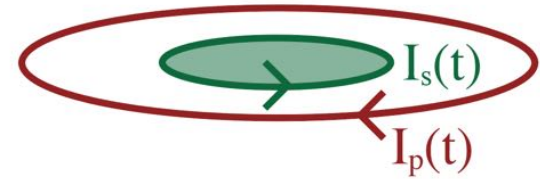
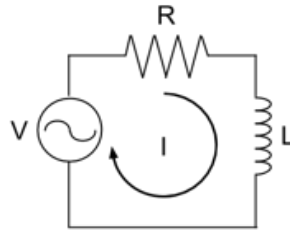
$$I_p(t) = I_p \cos \omega t$$

$$I_s(t) = I_s \cos(\omega t - \psi)$$

$$= \underbrace{I_s \cos \psi \cos \omega t}_{\substack{\text{In-Phase} \\ \text{Real}}} + \underbrace{I_s \sin \psi \sin \omega t}_{\substack{\text{Out-of-Phase} \\ \text{Quadrature} \\ \text{Imaginary}}}$$

Phase Lag

$$\psi = \frac{\pi}{2} + \tan^{-1} \left(\frac{\omega L}{R} \right)$$



Two Coil Example: Harmonic

Induced Currents

$$I_p(t) = I_p \cos \omega t$$

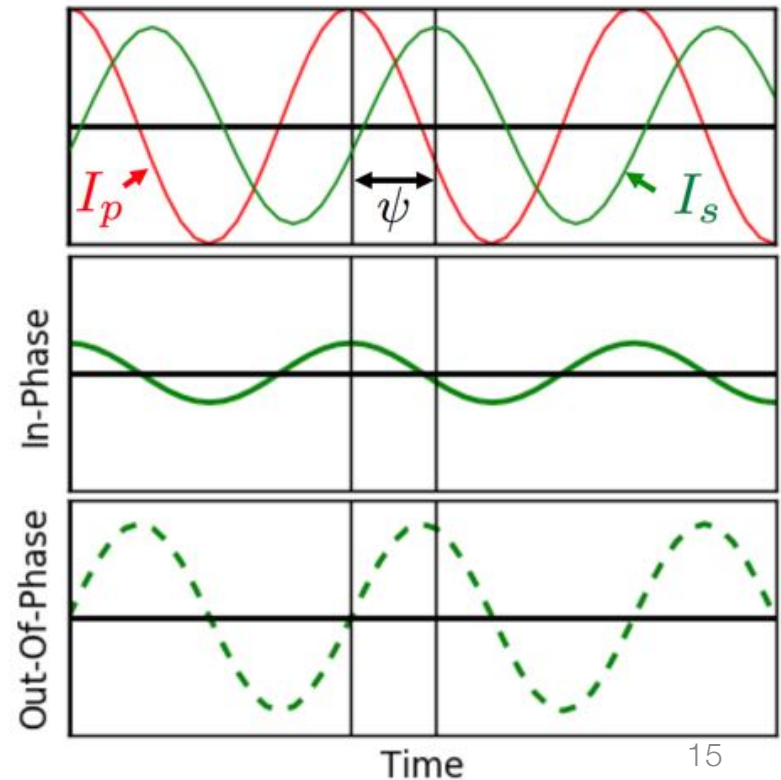
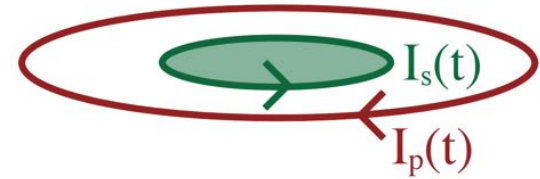
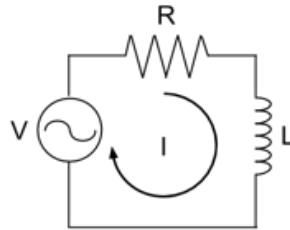
$$I_s(t) = I_s \cos(\omega t - \psi)$$

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Phase Lag

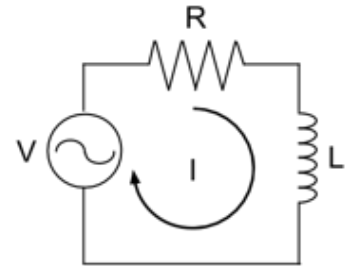
$$\psi = \frac{\pi}{2} + \underbrace{\tan^{-1} \left(\frac{\omega L}{R} \right)}_{\alpha}$$

Induction number

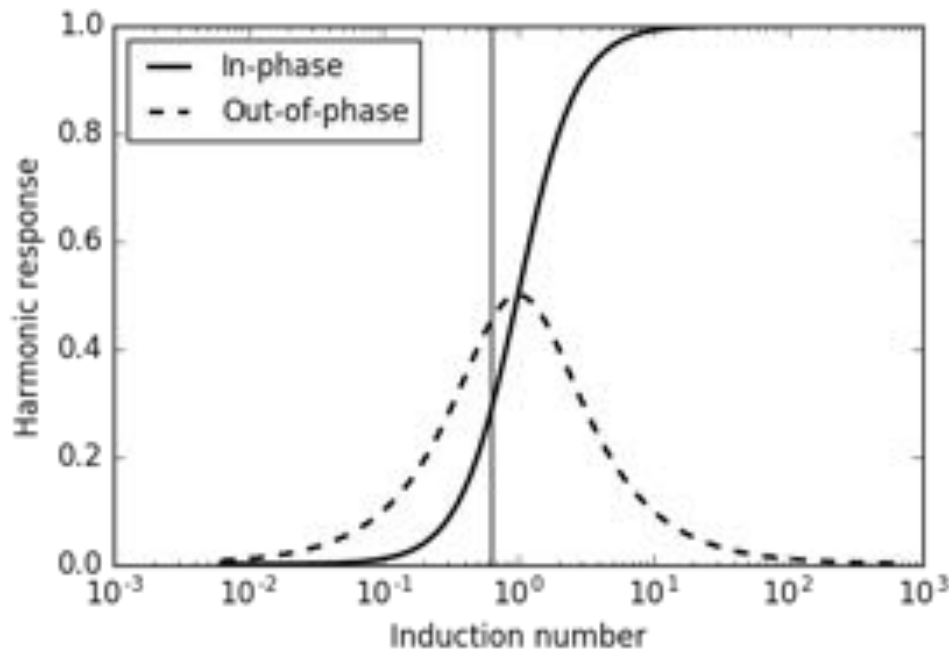


Response Function

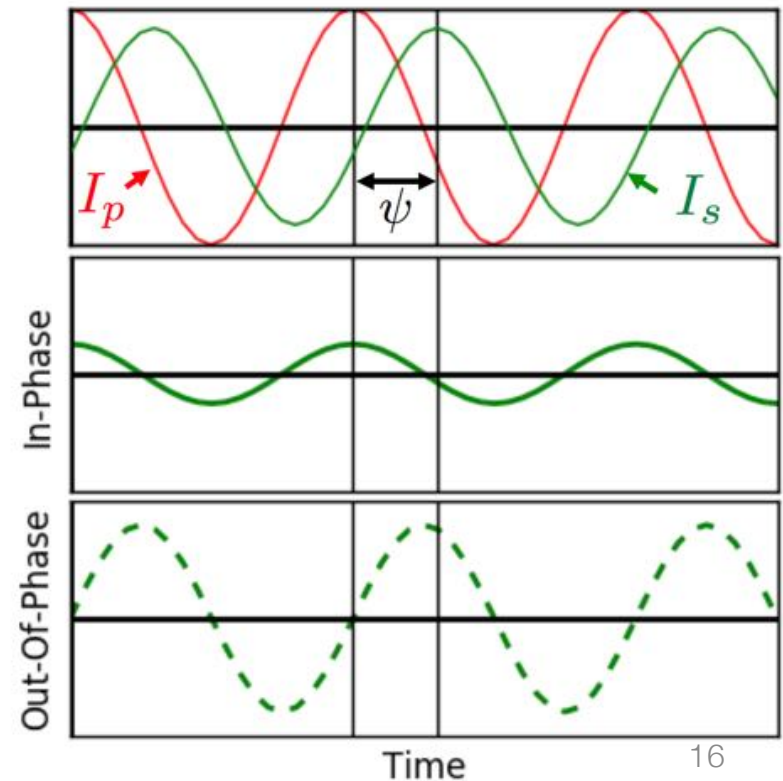
- Quantifies how a target responds to a time varying magnetic field
- Partitions real and imaginary parts



Response Function

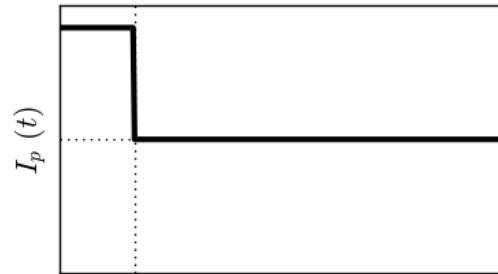


$$\alpha = \frac{\omega L}{R}$$

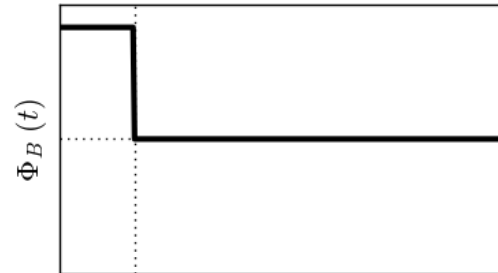


Two Coil Example: Transient

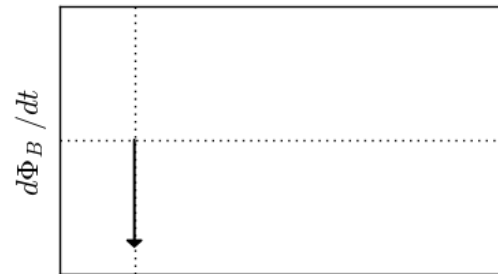
Primary currents



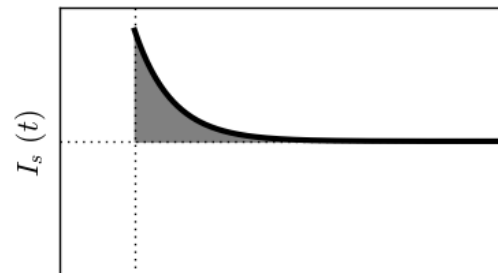
Magnetic flux



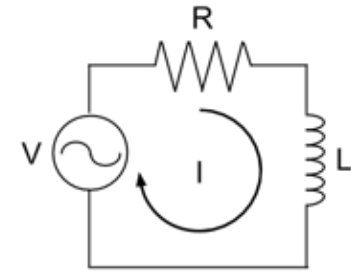
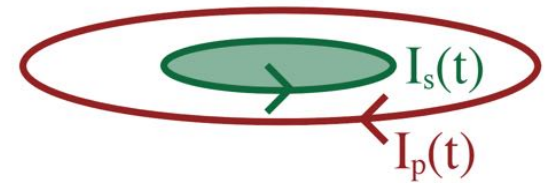
Time-variation of magnetic flux



Secondary currents



Time



$$I_s(t) = I_s e^{-t/\tau}$$

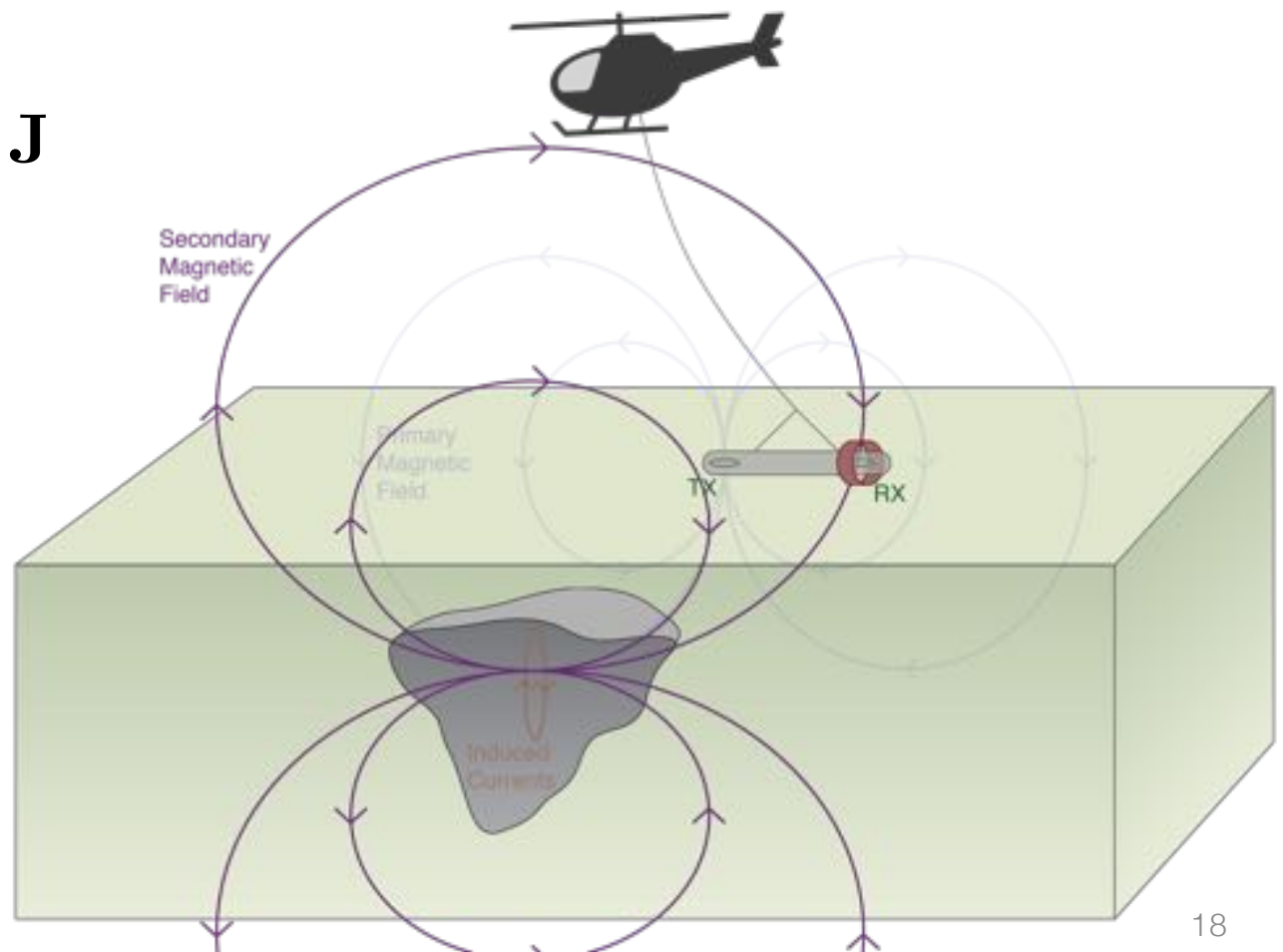
$$\tau = L/R$$

Secondary magnetic fields

Induced currents generate magnetic fields

- Ampere's Law

$$\nabla \times \mathbf{H} = \mathbf{J}$$

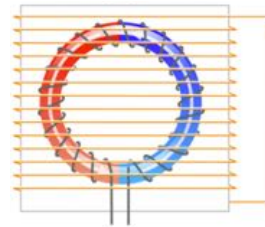


Receiver and Data

Magnetometer

- Measures:
 - Magnetic fields
 - 3 components
- eg. 3-component fluxgate

$\mathbf{b}(t)$

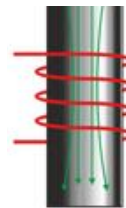


Fluxgate

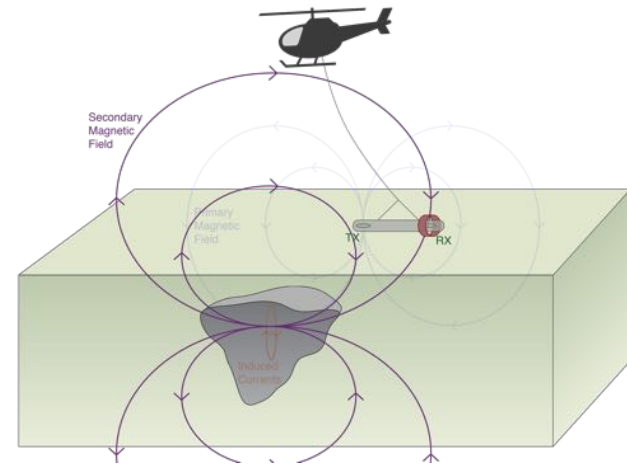
Coil

- Measures:
 - Voltage
 - Single component that depends on coil orientation
 - Coupling matters
- eg. airborne frequency domain
 - ratio of H_s/H_p is the same as V_s/V_p

$\frac{\partial b}{\partial t}$



Coil



Coupling

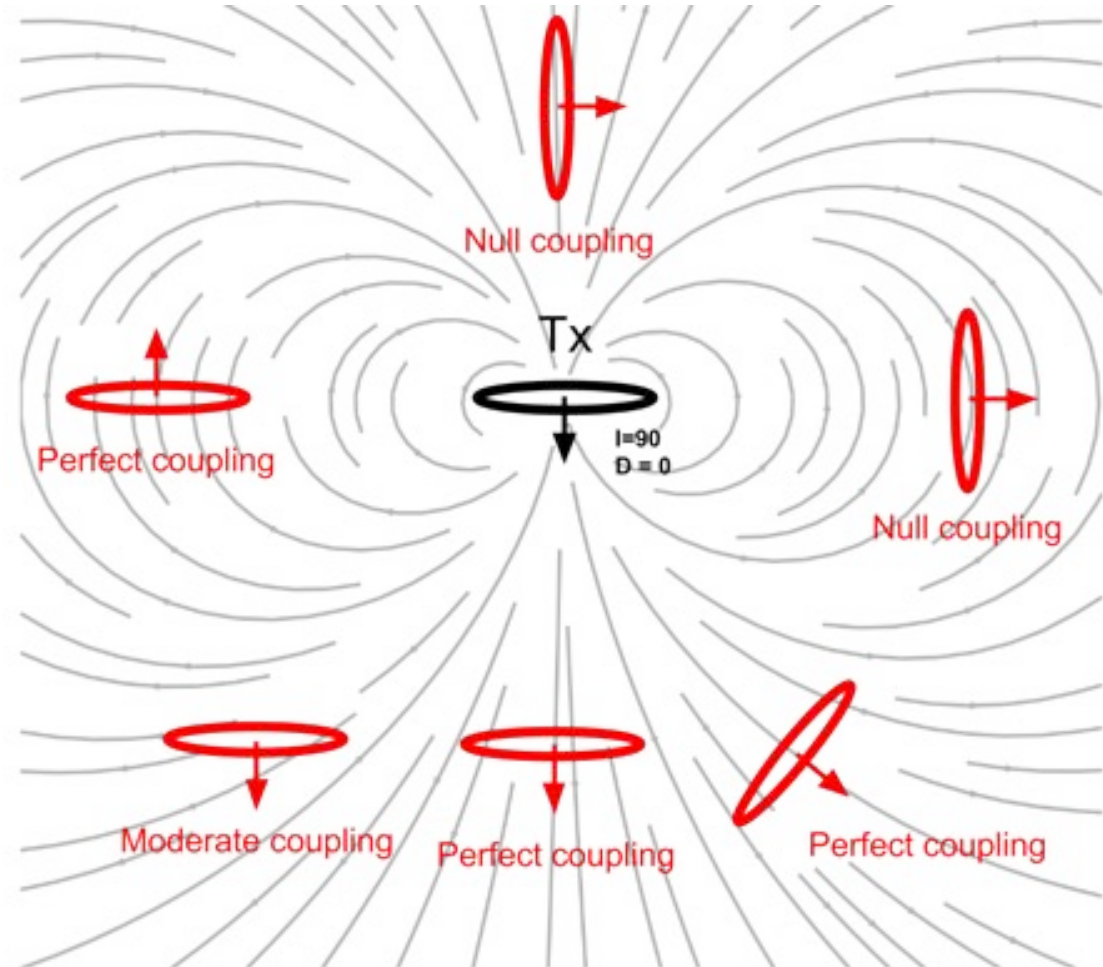
- Transmitter: Primary

$$I_p(t) = I_p \cos(\omega t)$$

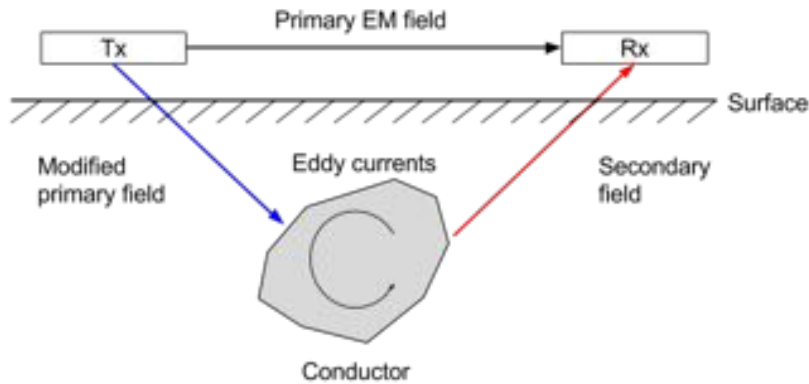
$$\mathbf{B}_p(t) \sim I_p \cos(\omega t)$$

- Target: Secondary

$$\begin{aligned} EMF &= -\frac{\partial \phi_{\mathbf{B}}}{\partial t} \\ &= -\frac{\partial}{\partial t} (\mathbf{B}_p \cdot \hat{\mathbf{n}}) A \end{aligned}$$



Circuit model of EM induction

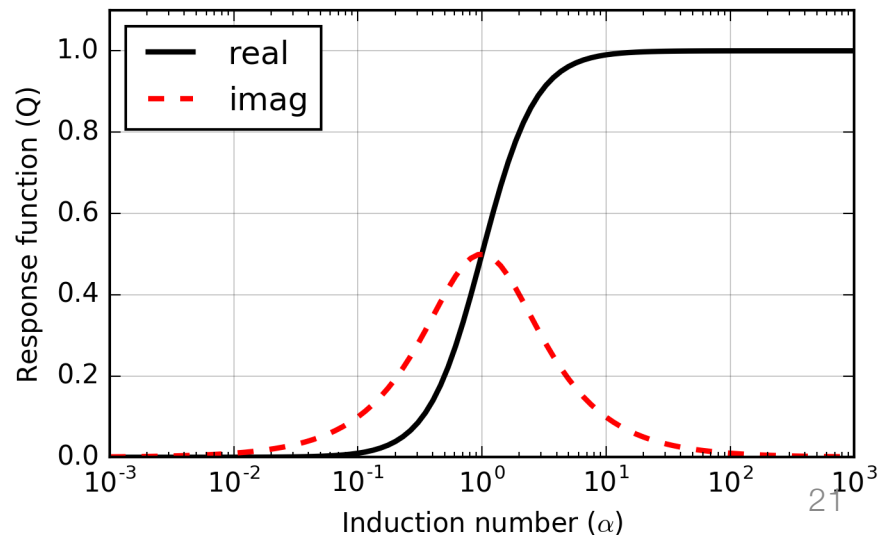


Magnetic field at the receiver

$$\frac{H^s}{H^p} = -\frac{M_{12}M_{23}}{M_{13}L} \underbrace{\left[\frac{\alpha^2 + i\alpha}{1 + \alpha^2} \right]}_Q$$

Induction Number

- Depends on properties of target $\alpha = \frac{\omega L}{R}$



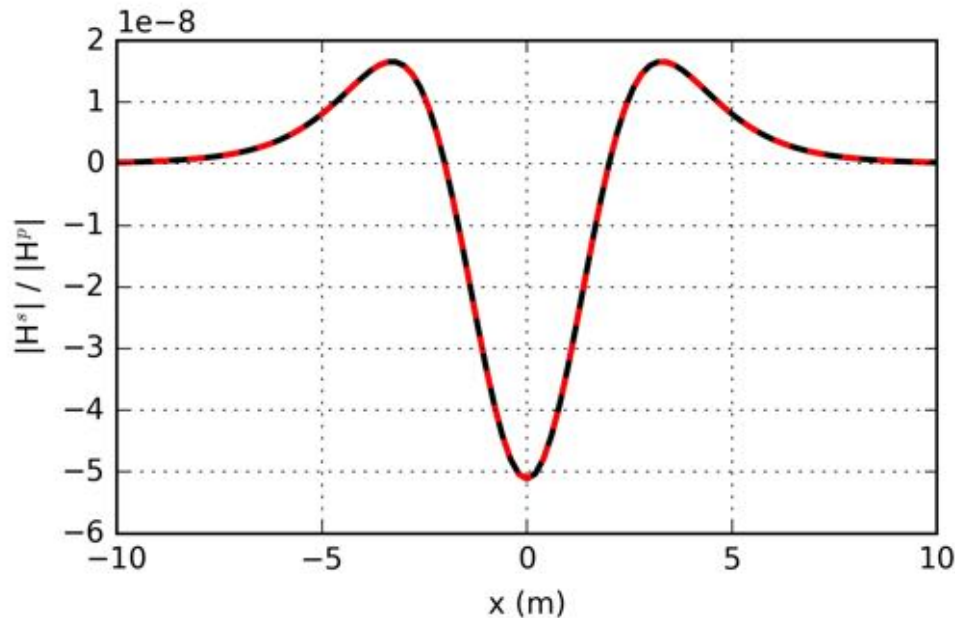
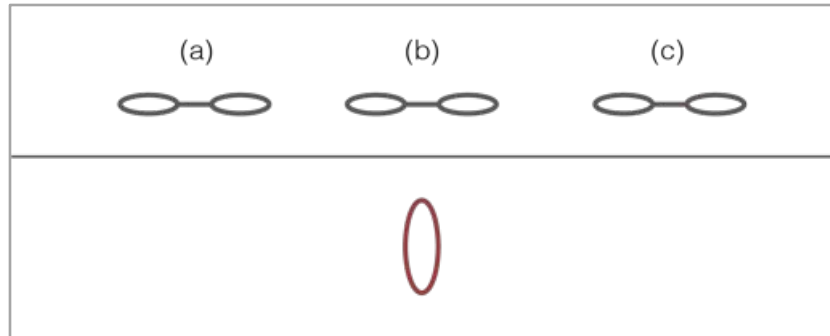
Coupling coefficient

- Depends on geometry

$$M_{12} = \frac{\mu_0}{4\pi} \oint \oint \frac{dl_1 \cdot dl_2}{|\mathbf{r} - \mathbf{r}'|^2}$$

Conductor in a resistive earth: Frequency

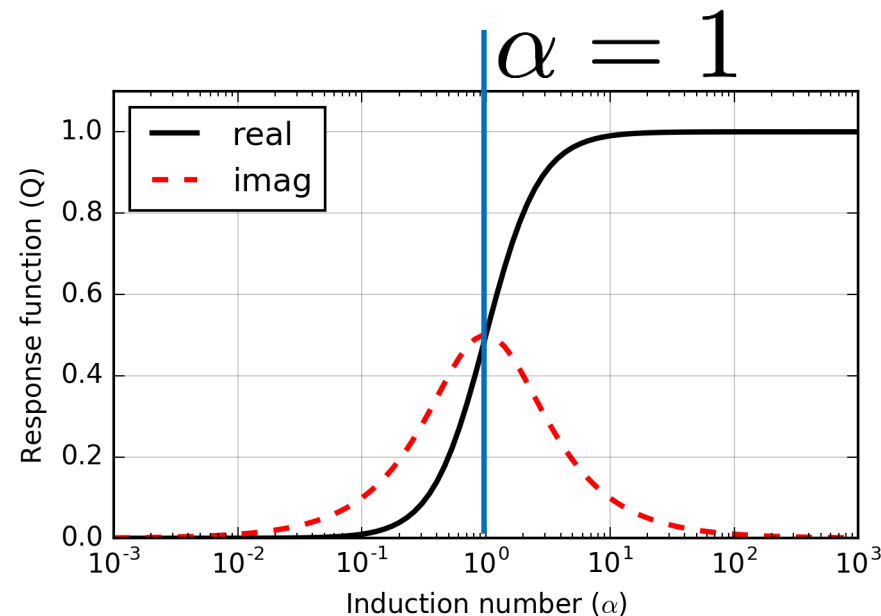
Profile over the loop



- Induction number

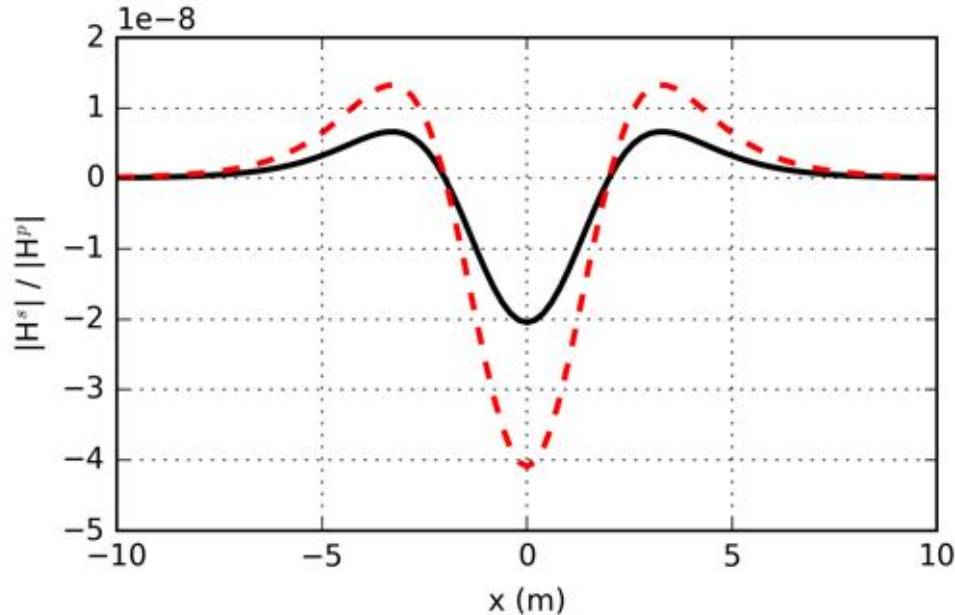
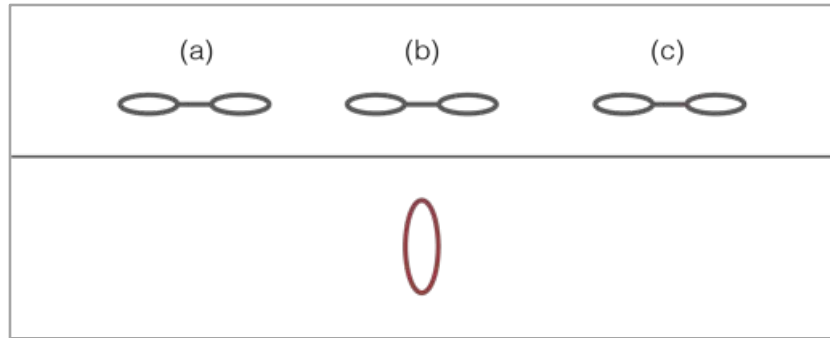
$$\alpha = \frac{\omega L}{R}$$

- When $\alpha = 1$
 - Real = Imag



Conductor in a resistive earth: Frequency

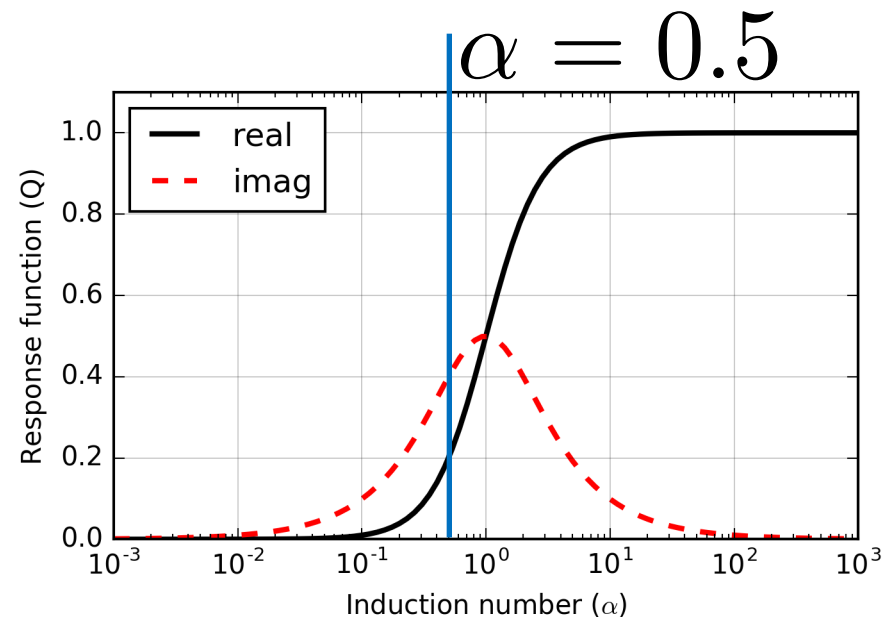
Profile over the loop



- Induction number

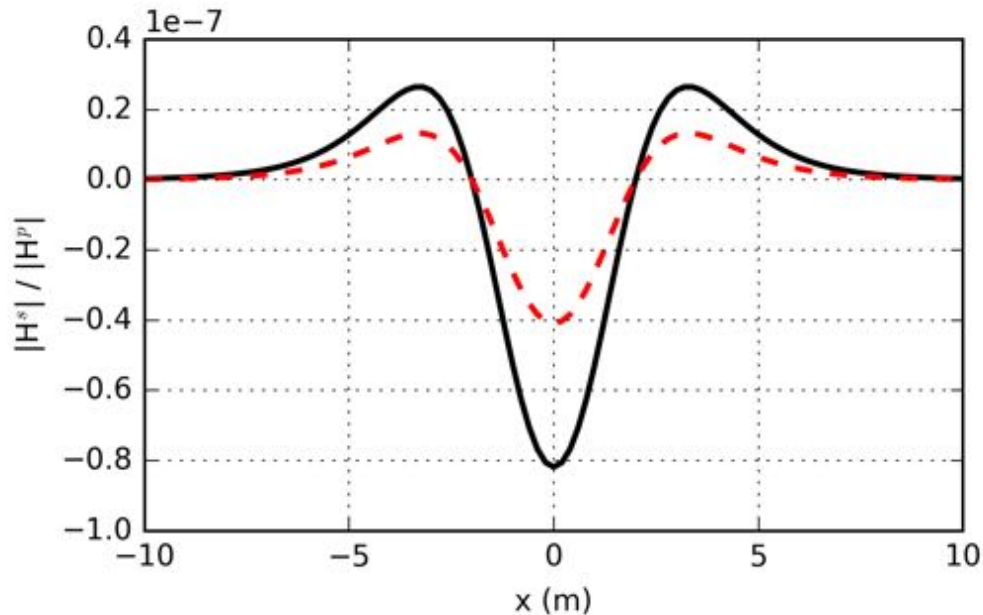
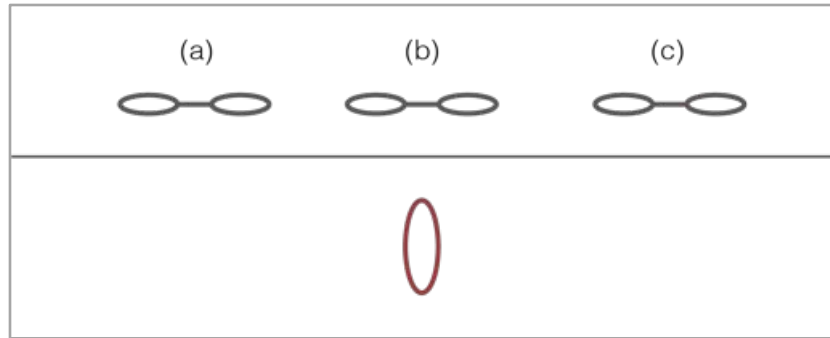
$$\alpha = \frac{\omega L}{R}$$

- When $\alpha < 1$
 - Real < Imag



Conductor in a resistive earth: Frequency

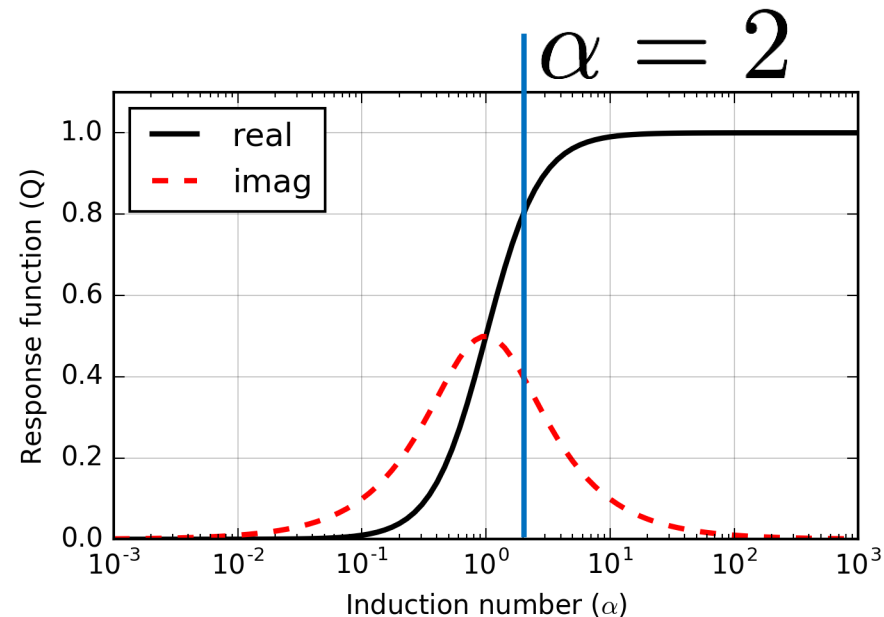
Profile over the loop



- Induction number

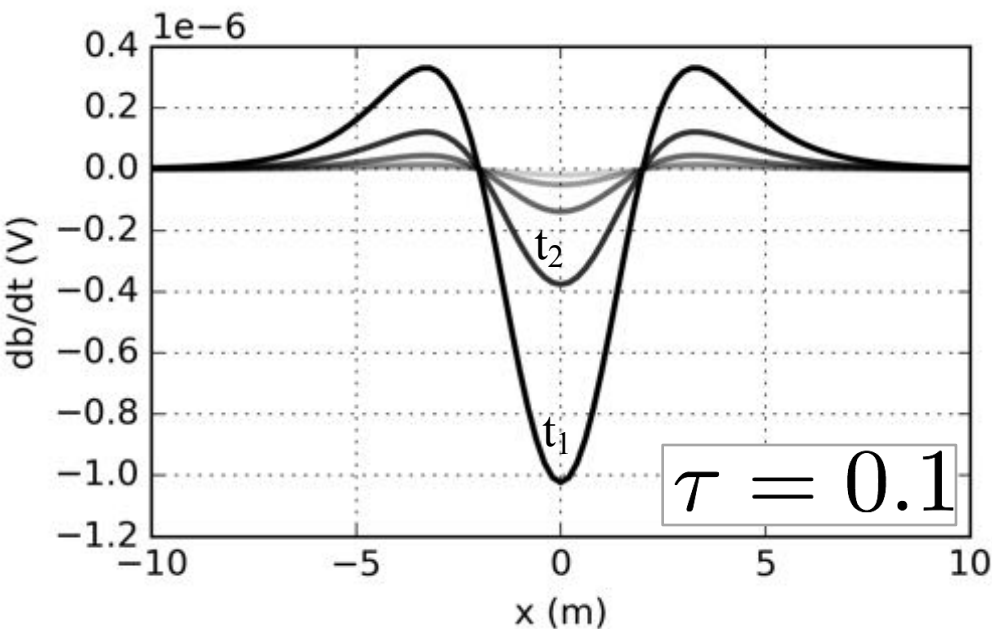
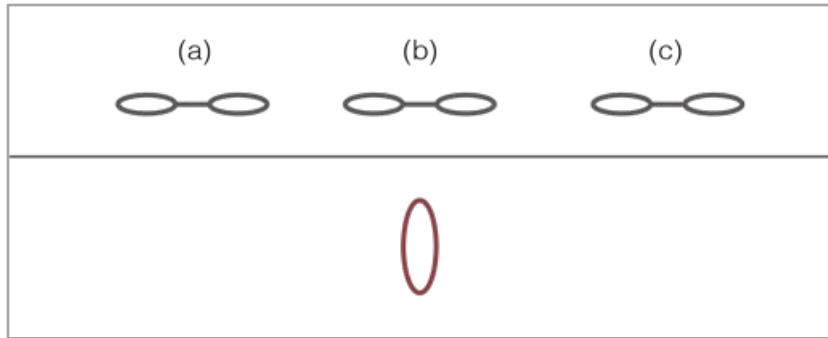
$$\alpha = \frac{\omega L}{R}$$

- When $\alpha > 1$
 - Real > Imag



Conductor in a resistive earth: Transient

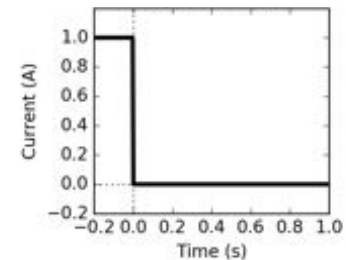
Profile over the loop



- Time constant

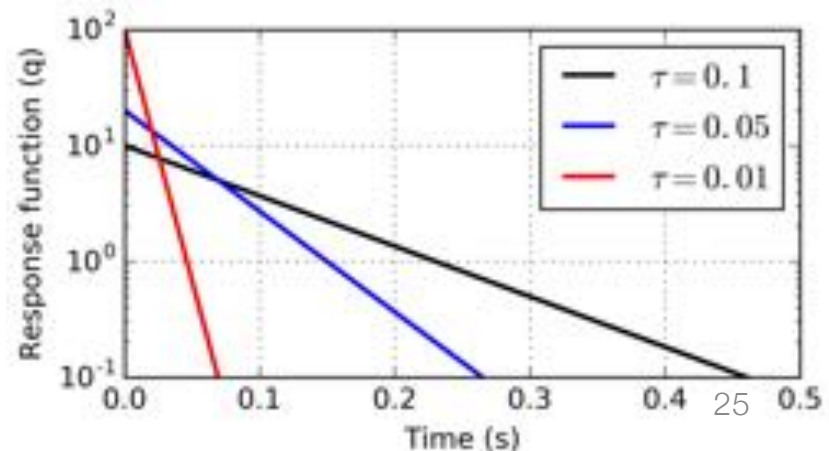
$$\tau = L/R$$

- Step-off current in Tx



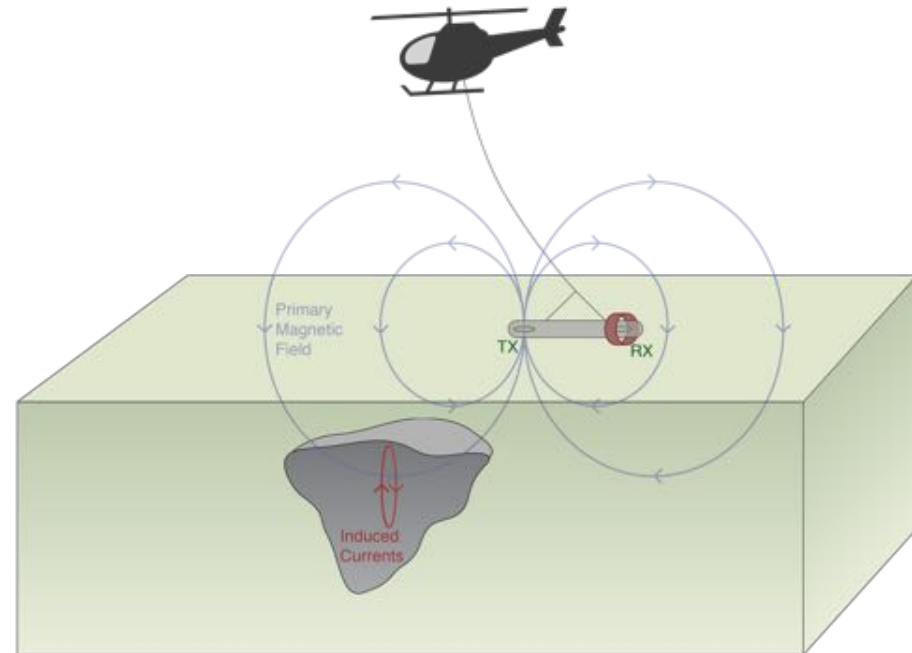
- Response function depends on time, τ

$$q(t) = e^{-t/\tau}$$



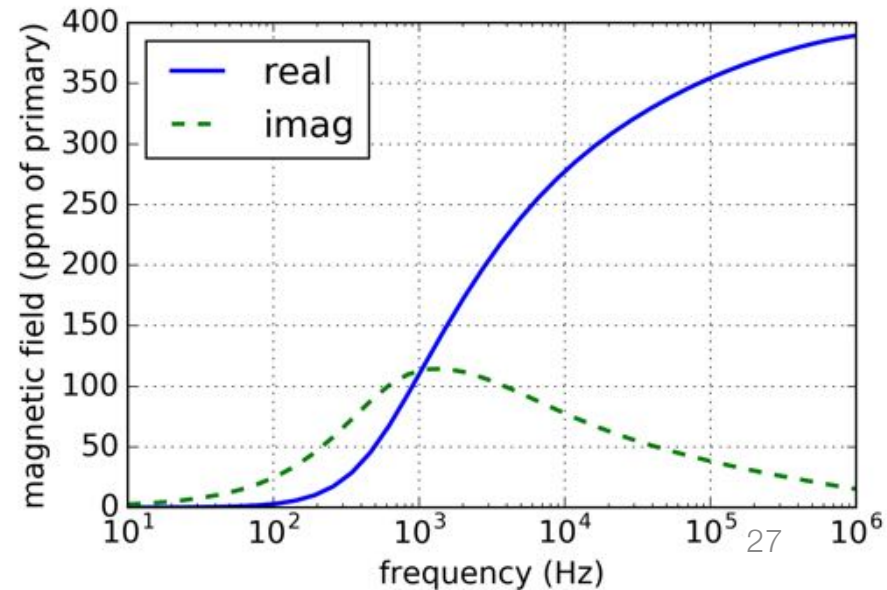
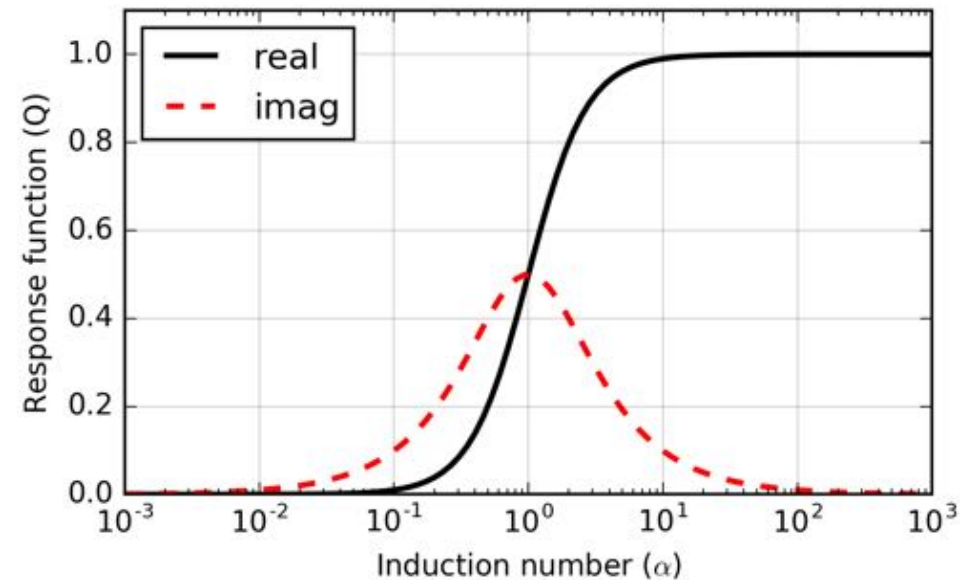
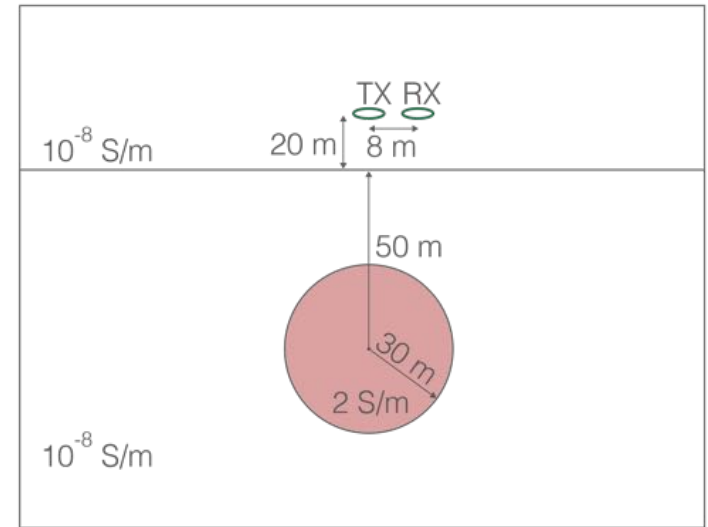
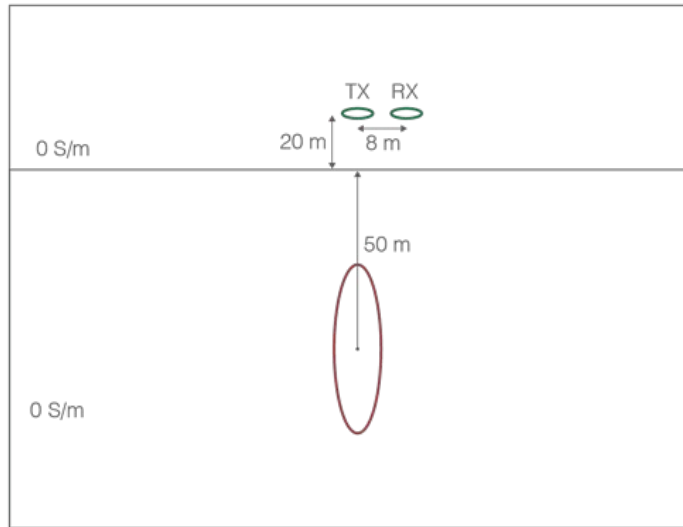
Recap: what have we learned?

- Basics of EM induction
- Response functions
- Mutual coupling
- Data for frequency or time domain systems
- Circuit model provides representative results
 - Applicable to geologic targets?



Sphere in a resistive background

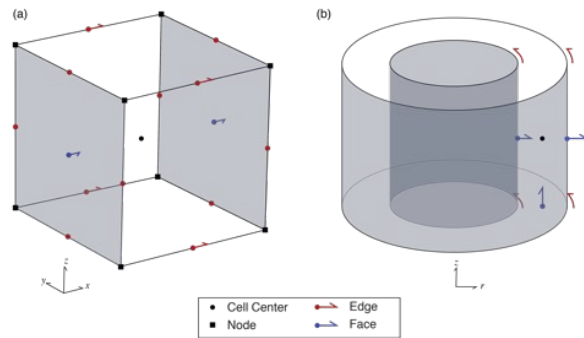
How representative is a circuit model?



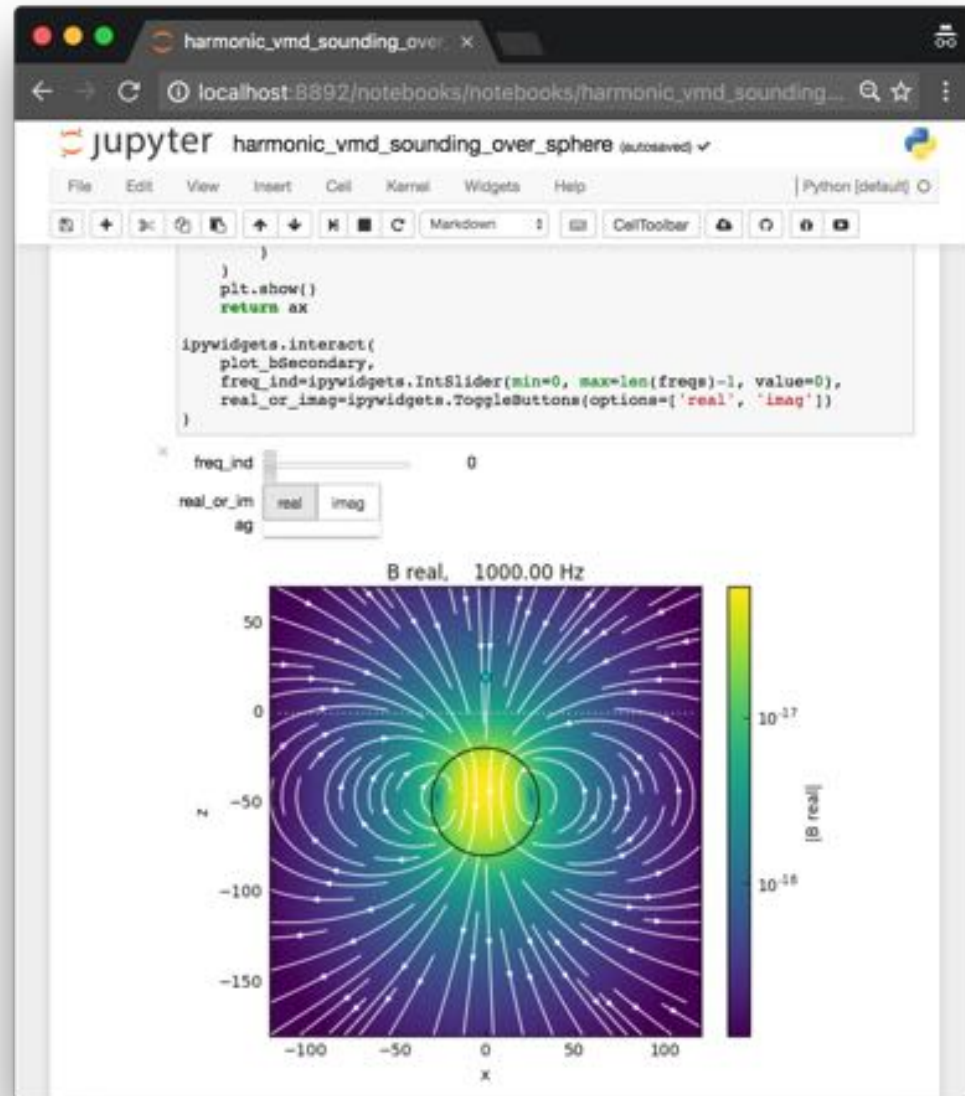
Cyl Code



- Finite Volume EM
 - Frequency and Time



- Built on SimPEG
- Open source, available at:
<http://em.geosci.xyz/apps.html>

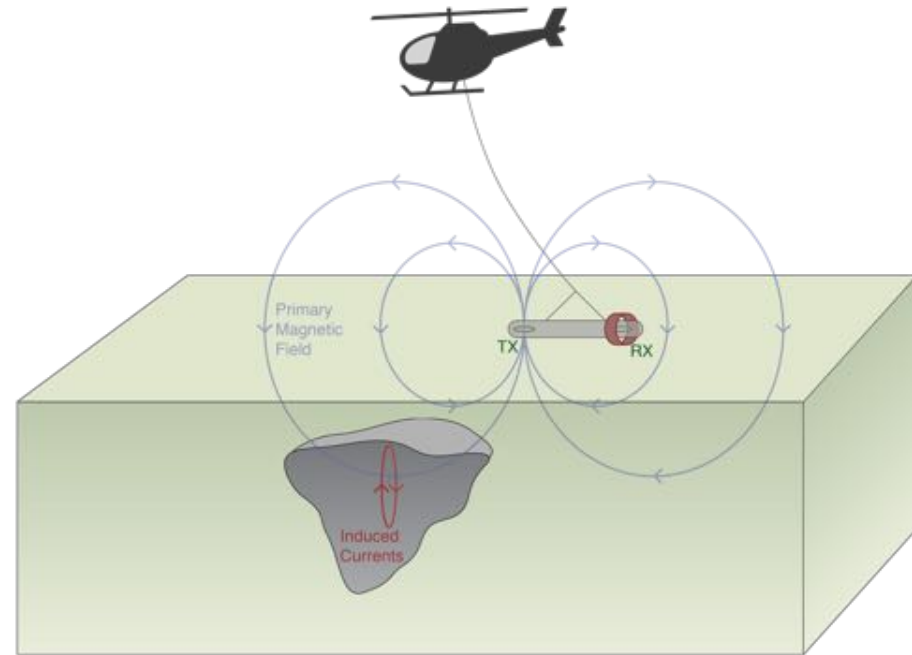


Recap: what have we learned?

- Basics of EM induction
- Response functions
- Mutual coupling
- Data for frequency or time domain systems
- Circuit model is a good proxy

Major item not yet accounted for...

- Propagation of energy from
 - Transmitter to target
 - Target to receiver



How do EM fields and fluxes behave in a
conductive background?

Revisit Maxwell's equations

First order equations

$$\begin{aligned}\nabla \times \mathbf{e} &= -\frac{\partial \mathbf{b}}{\partial t} & \mathbf{j} &= \sigma \mathbf{e} \\ \nabla \times \mathbf{h} &= \mathbf{j} + \frac{\partial \mathbf{d}}{\partial t} & \mathbf{b} &= \mu \mathbf{h} \\ & & \mathbf{d} &= \epsilon \mathbf{e}\end{aligned}$$

Second order equations

$$\nabla^2 \mathbf{h} - \underbrace{\mu \sigma \frac{\partial \mathbf{h}}{\partial t}}_{\text{diffusion}} - \underbrace{\mu \epsilon \frac{\partial^2 \mathbf{h}}{\partial t^2}}_{\text{wave propagation}} = 0$$

In frequency

$$\begin{aligned}\nabla^2 \mathbf{H} + k^2 \mathbf{H} &= 0 \\ k^2 &= \omega^2 \mu \epsilon - i \omega \mu \sigma\end{aligned}$$

Plane waves in a homogeneous media

In frequency

$$\nabla^2 \mathbf{H} + k^2 \mathbf{H} = 0$$

$$k^2 = \omega^2 \mu \epsilon - i \omega \mu \sigma$$

Quasi-static

$$\frac{\omega \epsilon}{\sigma} \ll 1$$

even if...

$$\sigma = 10^{-4} S/m$$

$$f = 10^4 Hz$$

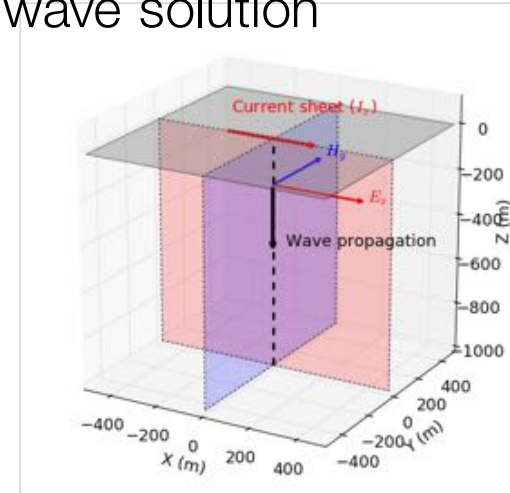
then

$$\frac{\omega \epsilon}{\sigma} \sim 0.005$$

$$k = \sqrt{-i \omega \mu \sigma} = (1 - i) \sqrt{\frac{\omega \mu \sigma}{2}}$$

$$\equiv \alpha - i \beta$$

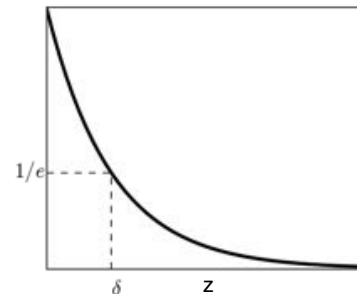
Plane wave solution



$$\mathbf{H} = \underbrace{\mathbf{H}_0 e^{-\alpha z}}_{\text{attenuation}} \underbrace{e^{-i(\beta z - \omega t)}}_{\text{phase}}$$

Skin depth

δ : skin depth



$$\delta = \sqrt{\frac{2}{\omega \mu \sigma}} = 503 \sqrt{\frac{1}{\sigma f}}$$

Plane waves in a homogeneous media

In time

$$\nabla^2 \mathbf{h} - \mu\epsilon \frac{\partial^2 \mathbf{h}}{\partial t^2} - \mu\sigma \frac{\partial \mathbf{h}}{\partial t} = 0$$

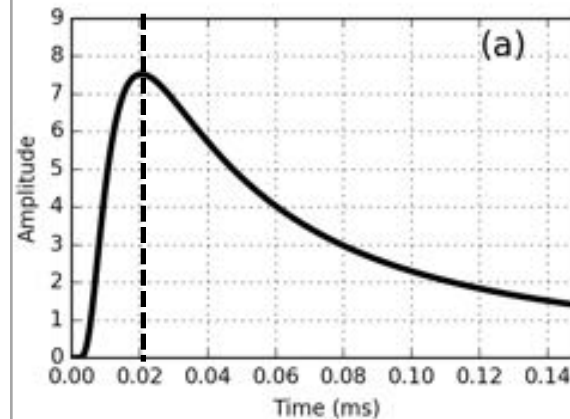
$$\mathbf{h}(t = 0) = \mathbf{h}_0 \delta(t)$$

Solution for quasi-static

$$\mathbf{h}(t) = -\frac{(\mu\sigma)^{1/2} z}{2\pi^{1/2} t^{3/2}} e^{-\mu\sigma z^2/(4t)}$$

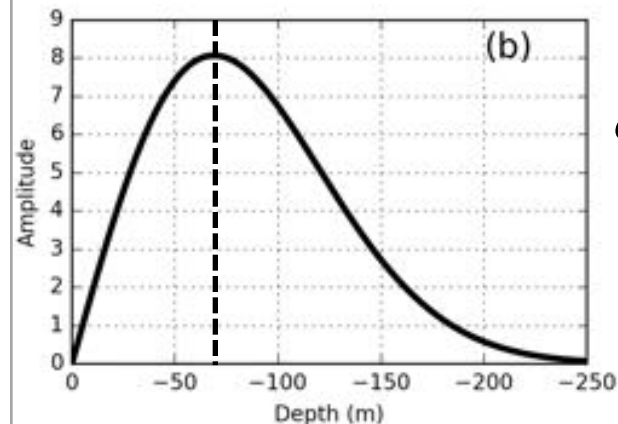
z : depth (m)

Peak time:



$$t_{max} = \frac{\mu\sigma z^2}{6}$$

Diffusion distance

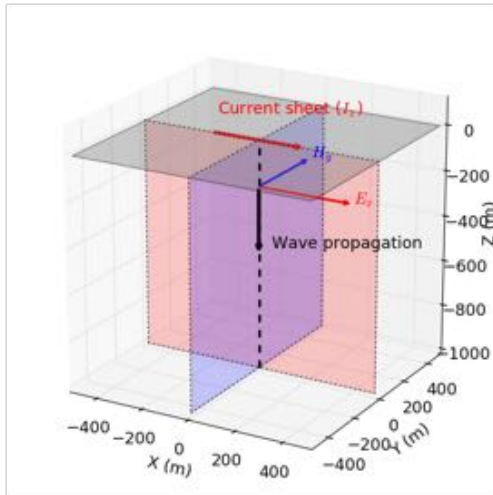


$$d = \sqrt{\frac{2t}{\mu\sigma}}$$

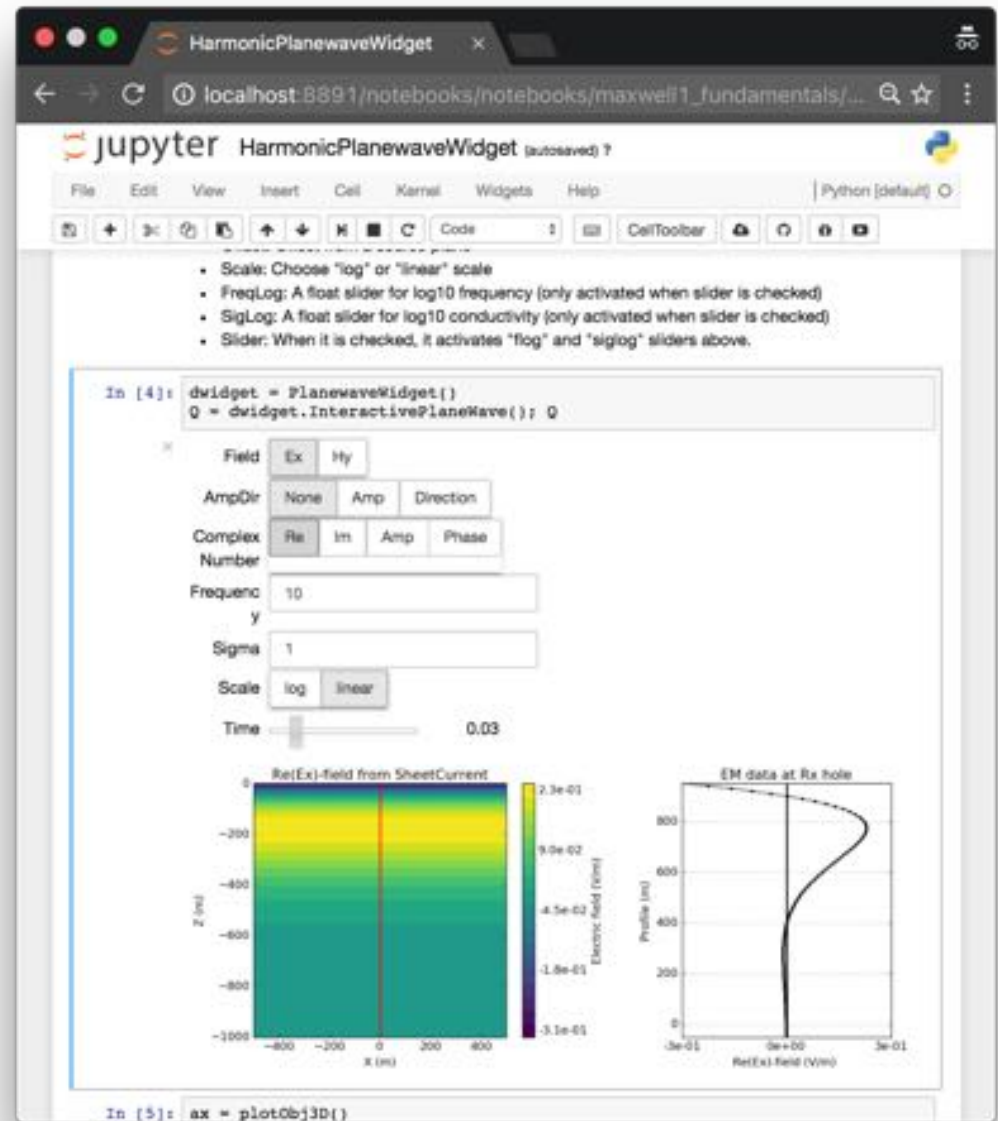
$$\approx 1260 \sqrt{\frac{t}{\sigma}}$$

Frequency Domain App: Plane waves

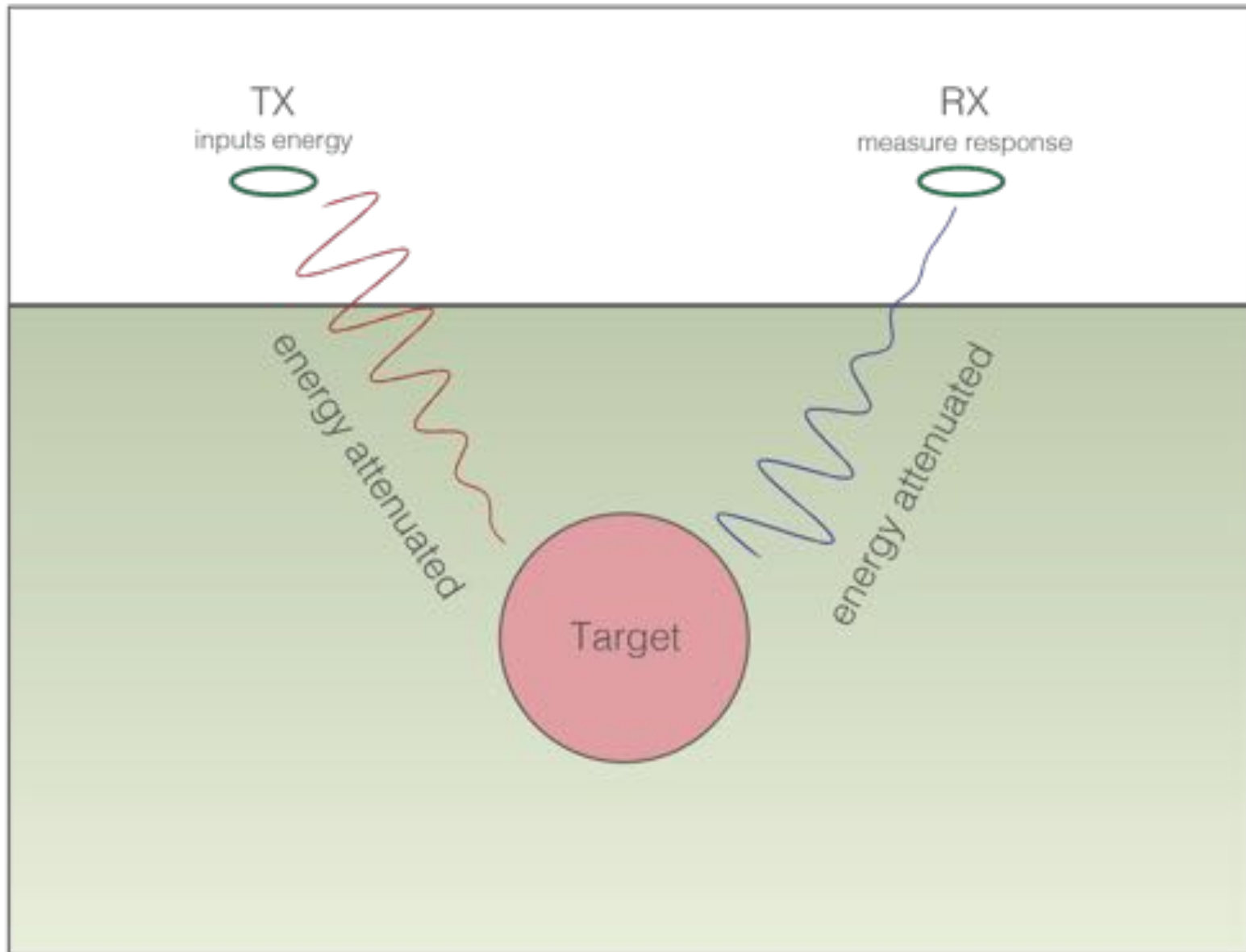
- Plane wave



$$\mathbf{H} = \underbrace{\mathbf{H}_0 e^{-\alpha z}}_{\text{attenuation}} \underbrace{e^{-i(\beta z - \omega t)}}_{\text{phase}}$$



Effects of background resistivity



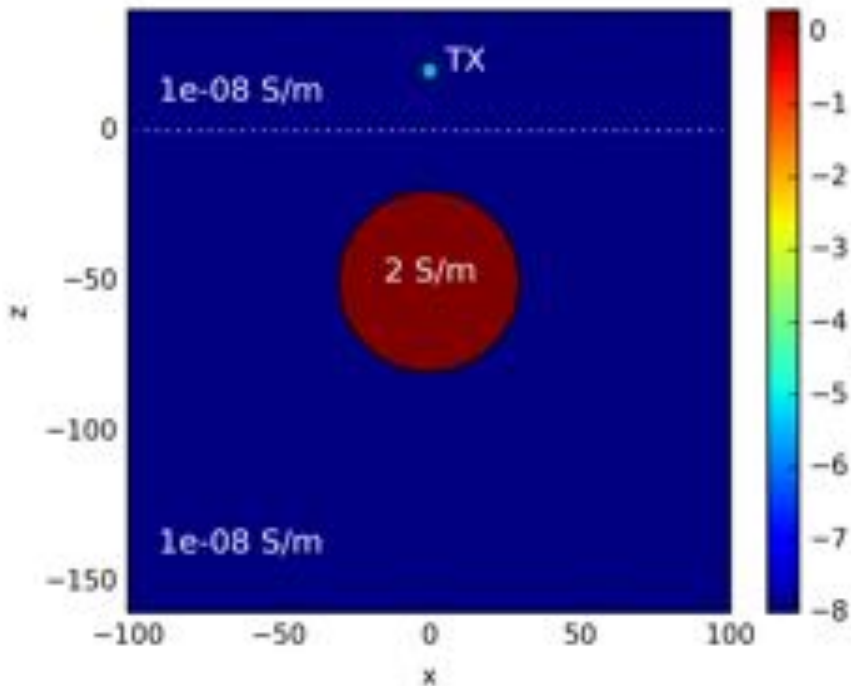
Effects of background resistivity: Frequency

- Buried, conductive sphere
- Vary background conductivity

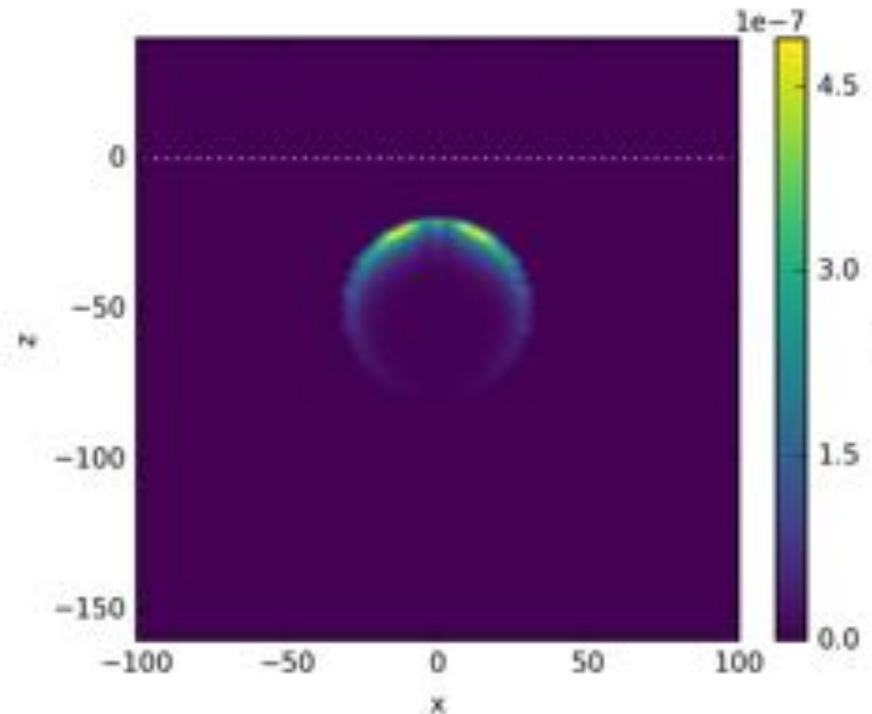
- Frequency: 10^4 Hz



10^{-8} S/m background



Current Density



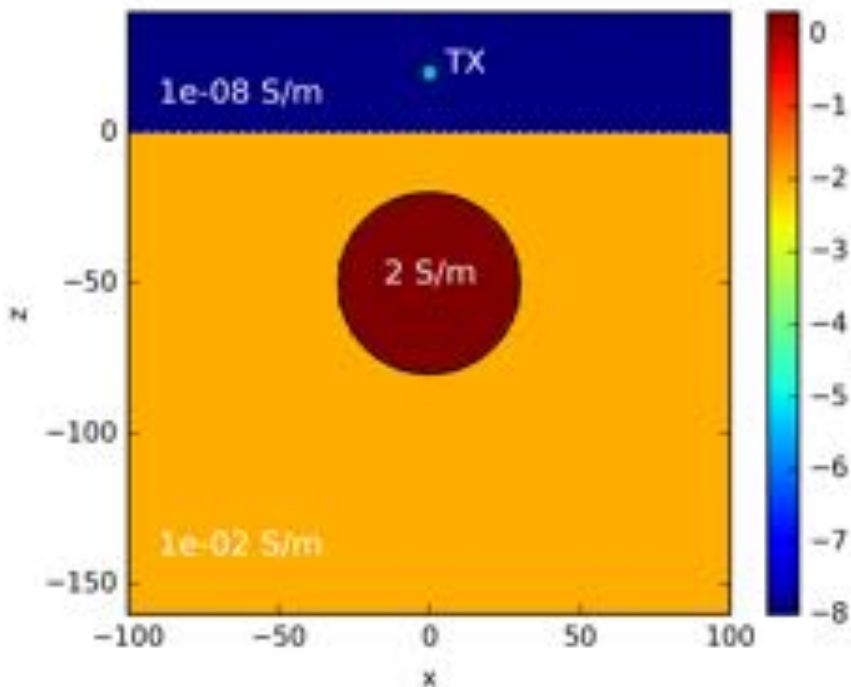
Effects of background resistivity: Frequency

- Buried, conductive sphere
- Vary background conductivity

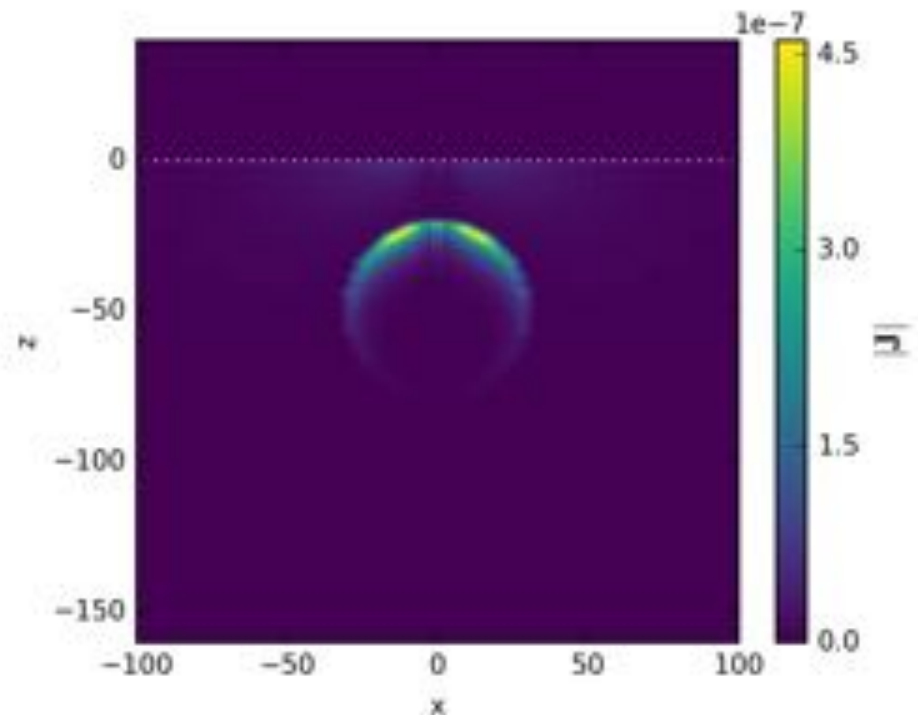
- Frequency: 10^4 Hz



10^{-2} S/m background



Current Density



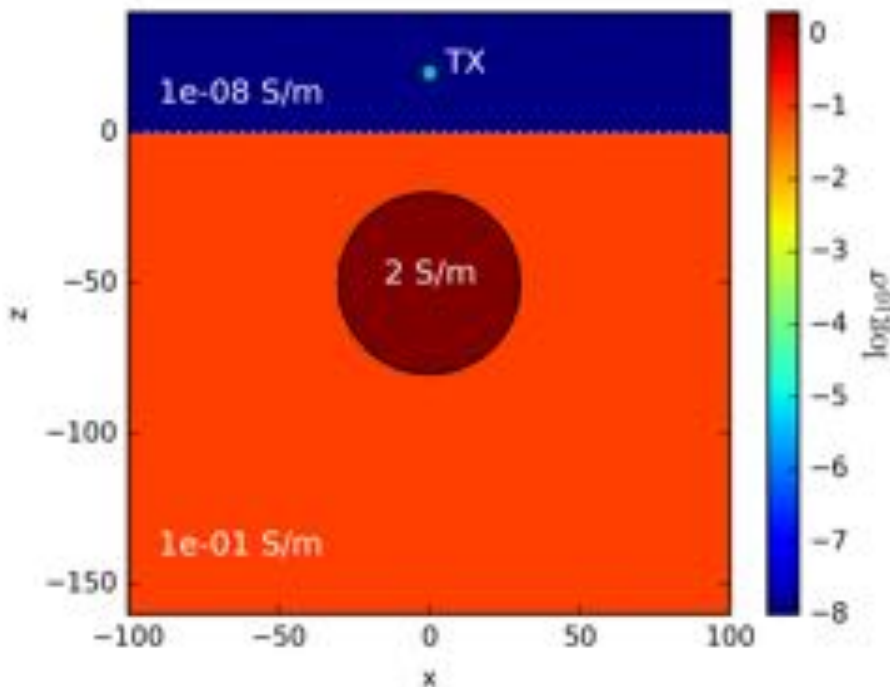
Effects of background resistivity: Frequency

- Buried, conductive sphere
- Vary background conductivity

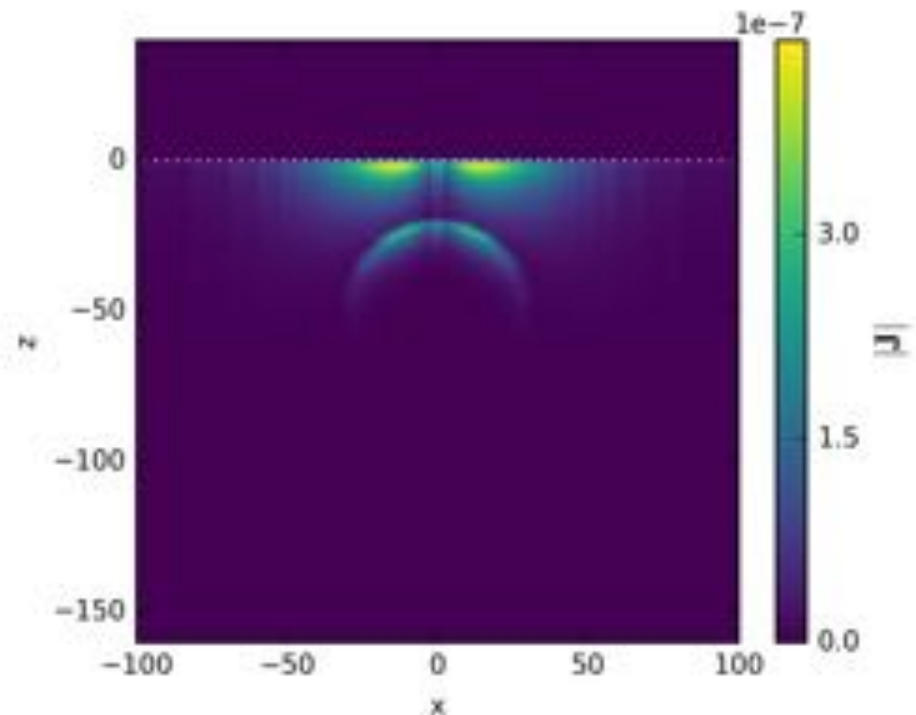
- Frequency: 10^4 Hz



10^{-1} S/m background



Current Density



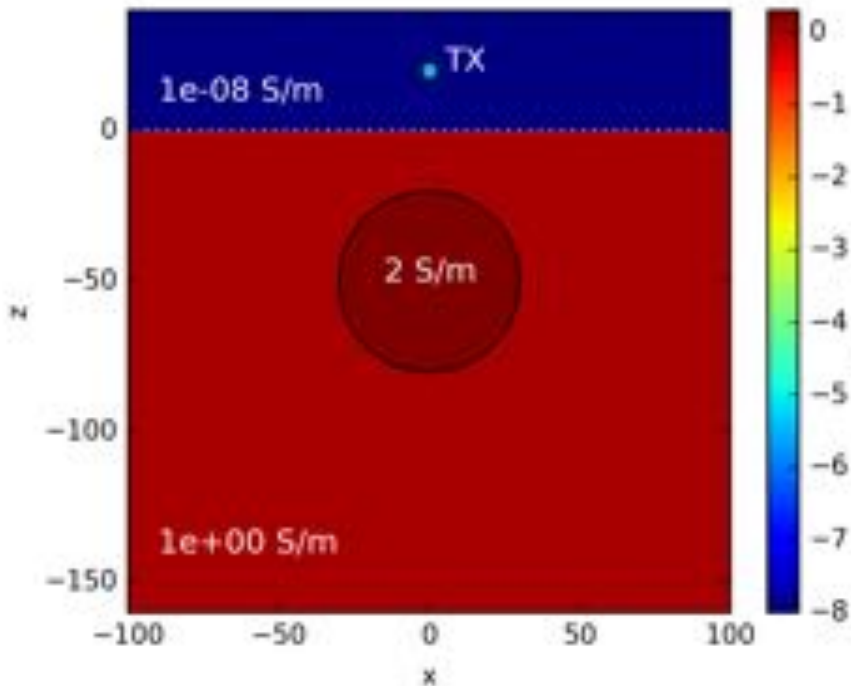
Effects of background resistivity: Frequency

- Buried, conductive sphere
- Vary background conductivity

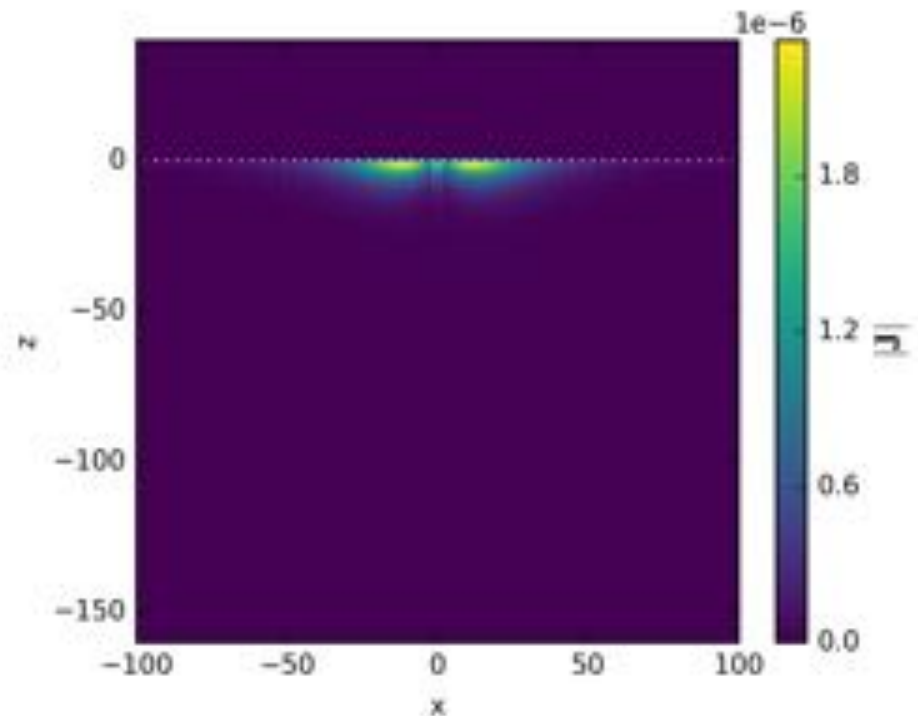
- Frequency: 10^4 Hz



1 S/m background

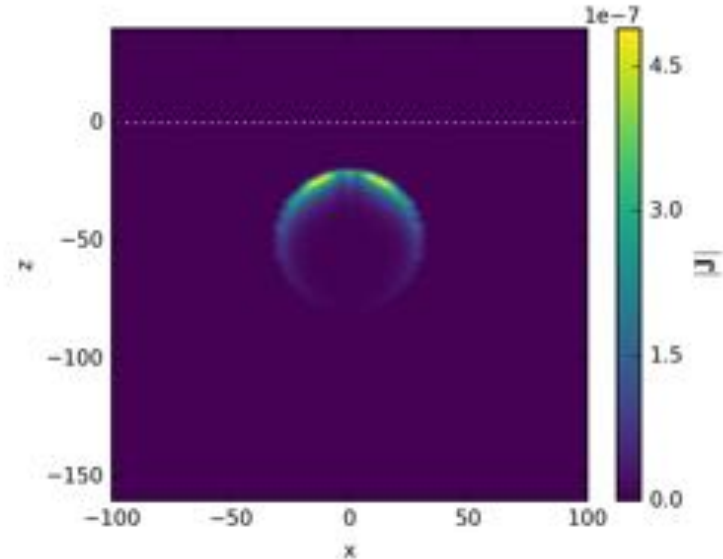


Current Density

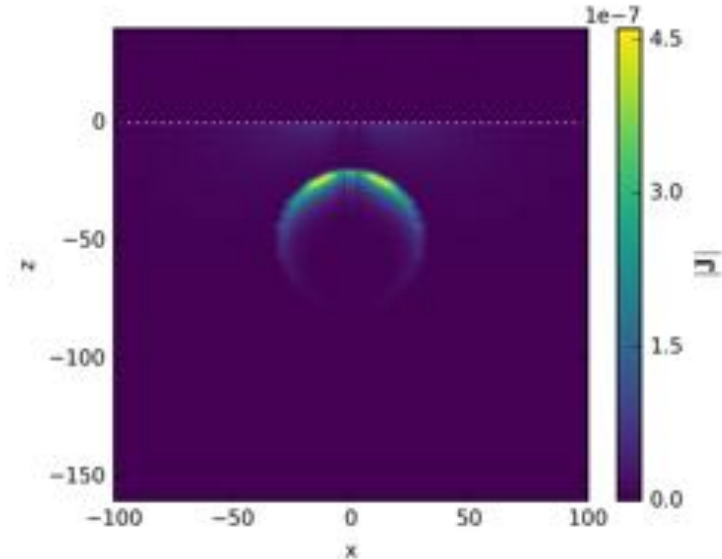


Effects of background resistivity: Frequency

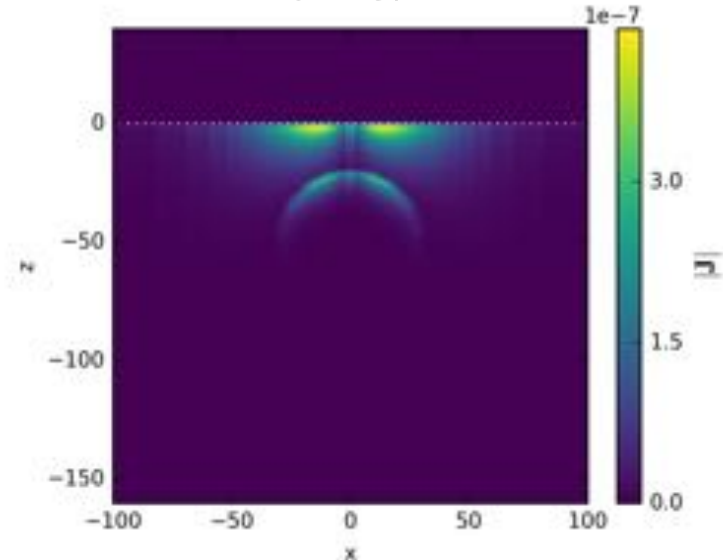
10^{-8} S/m



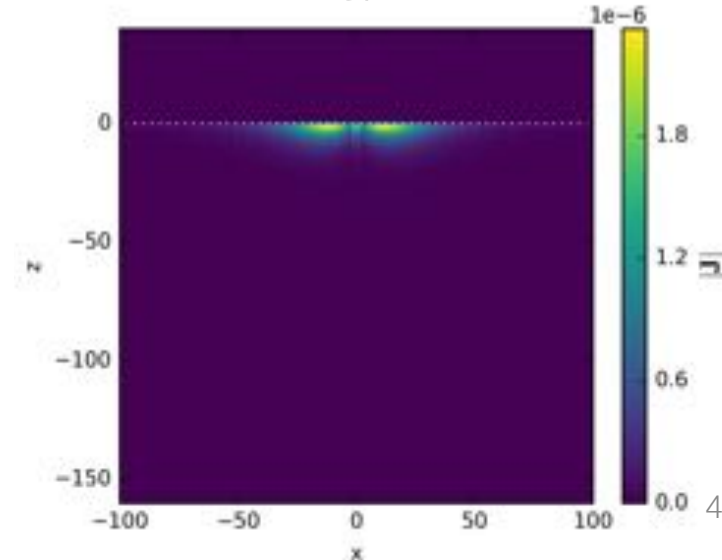
10^{-2} S/m



10^{-1} S/m

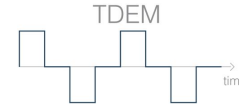


1 S/m

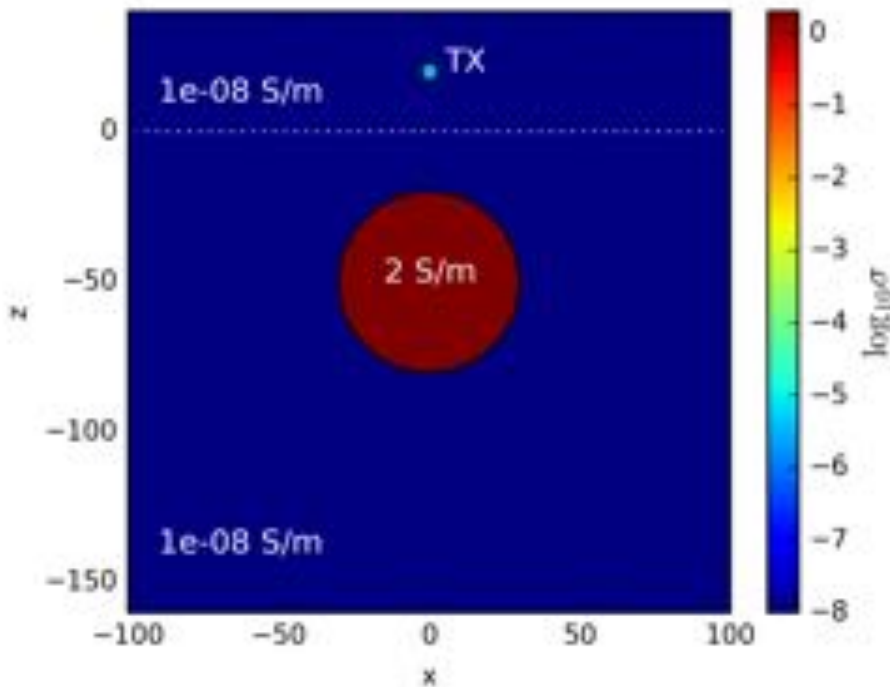


Effects of background resistivity: Time

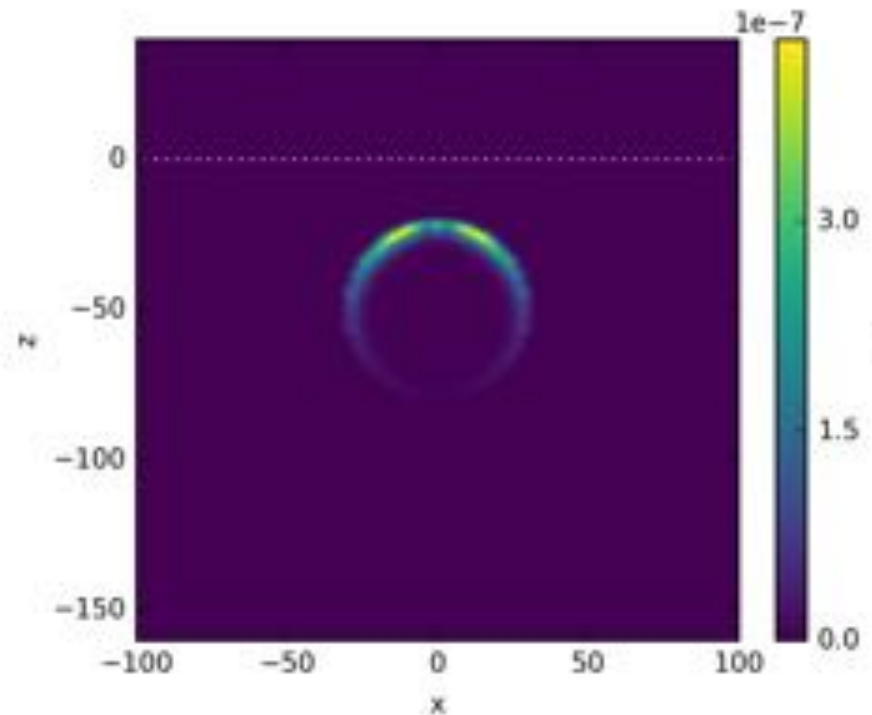
- Buried, conductive sphere
- Vary background conductivity
- Time: 10^{-5} s



10^{-8} S/m background

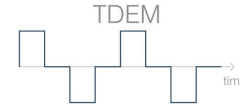


Current Density

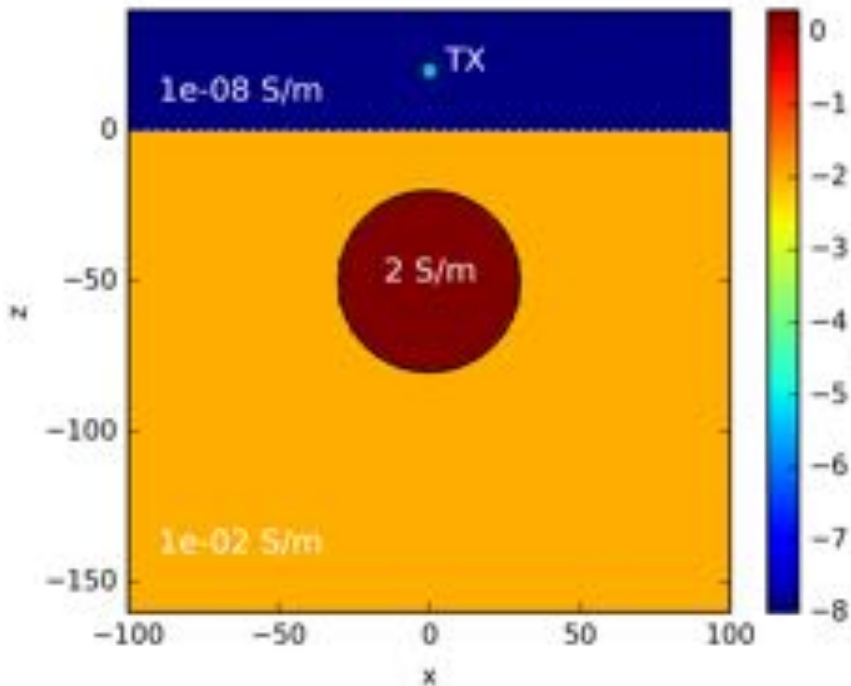


Effects of background resistivity: Time

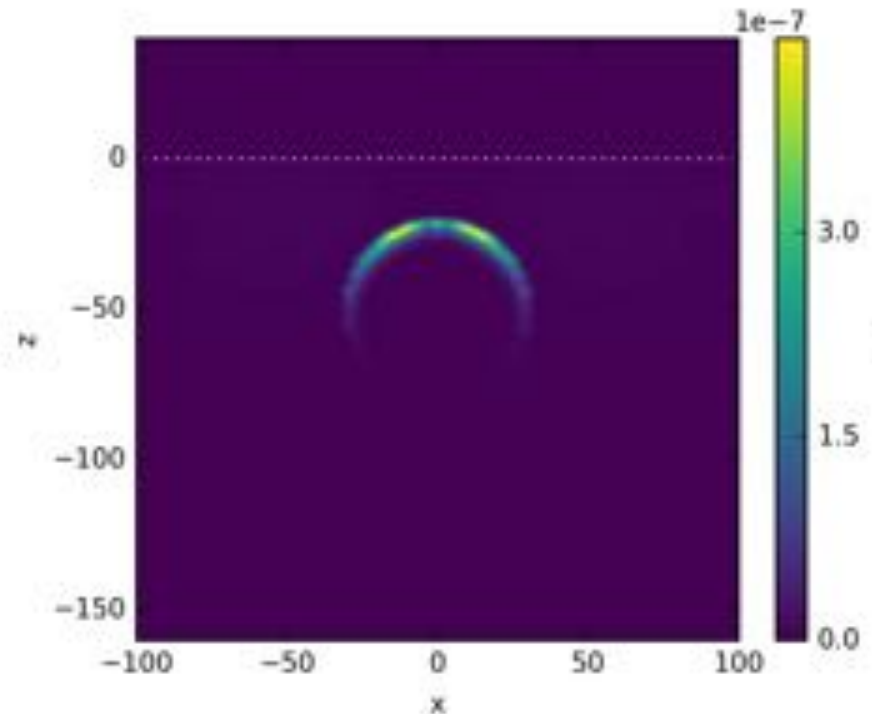
- Buried, conductive sphere
- Vary background conductivity
- Time: 10^{-5} s



10^{-2} S/m background

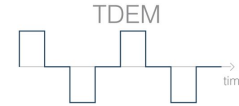


Current Density

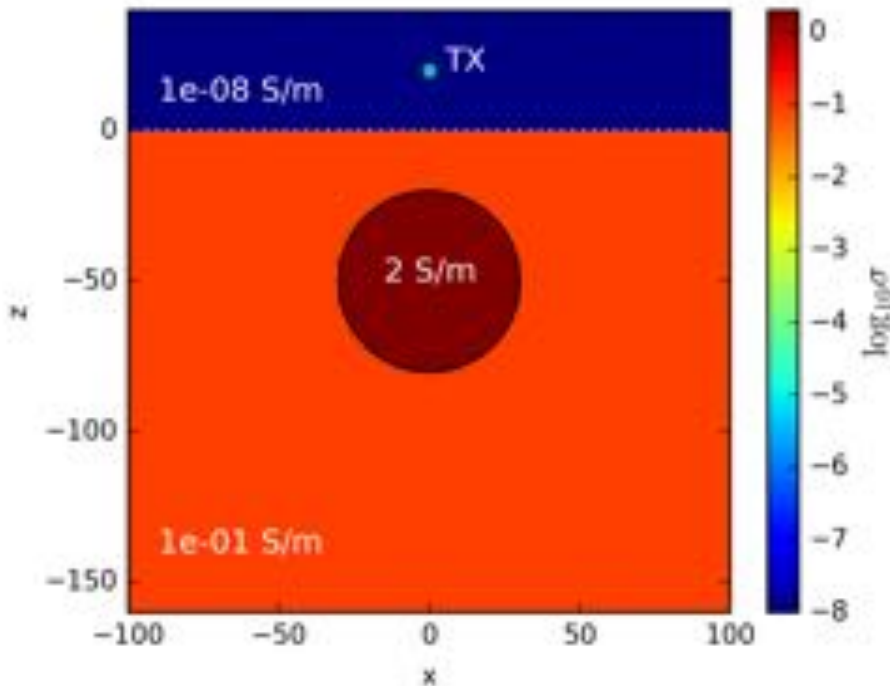


Effects of background resistivity: Time

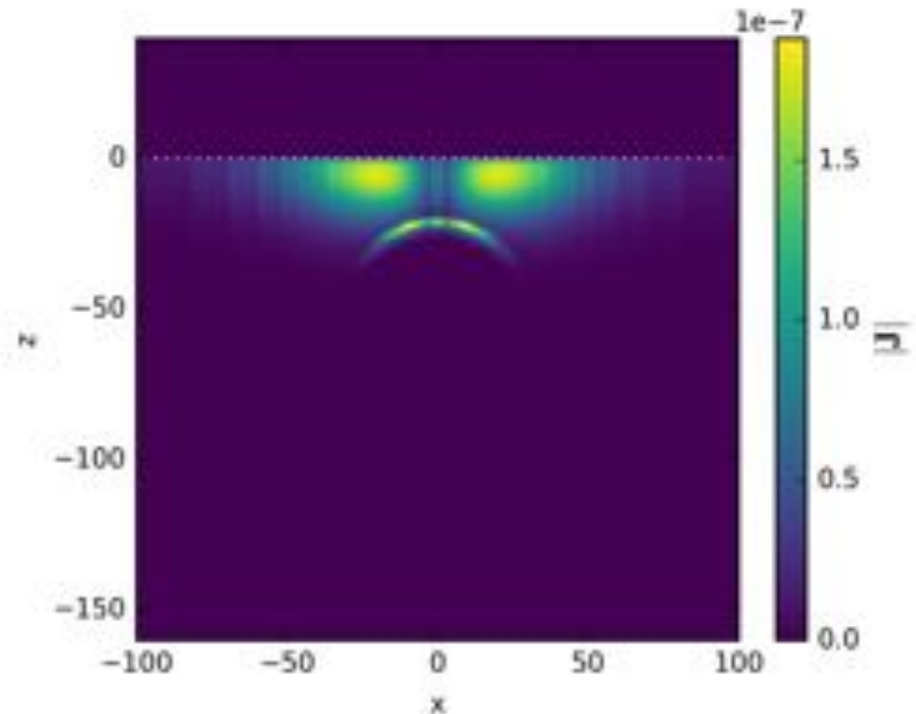
- Buried, conductive sphere
- Vary background conductivity
- Time: 10^{-5} s



10^{-1} S/m background

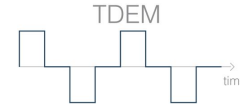


Current Density

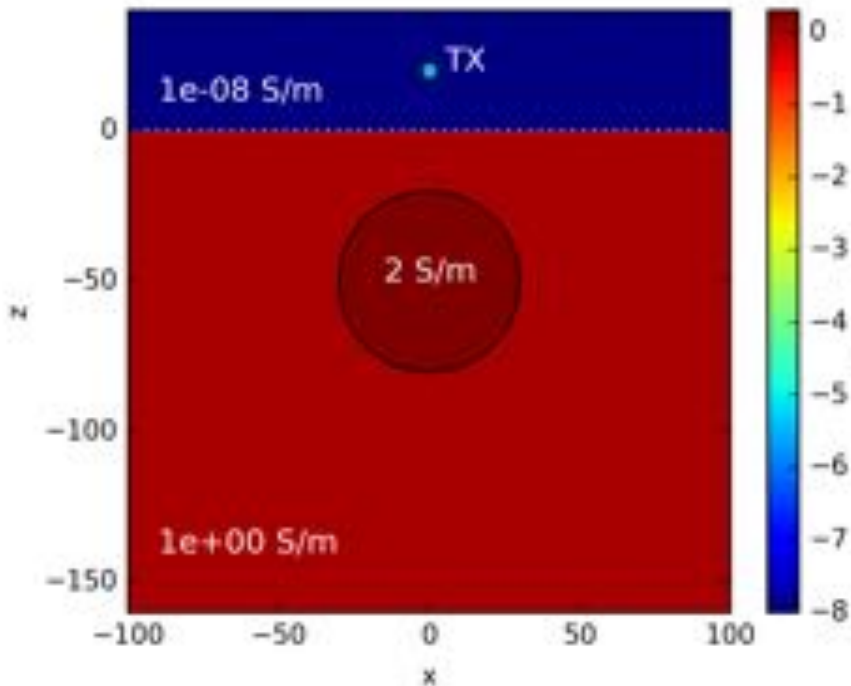


Effects of background resistivity: Time

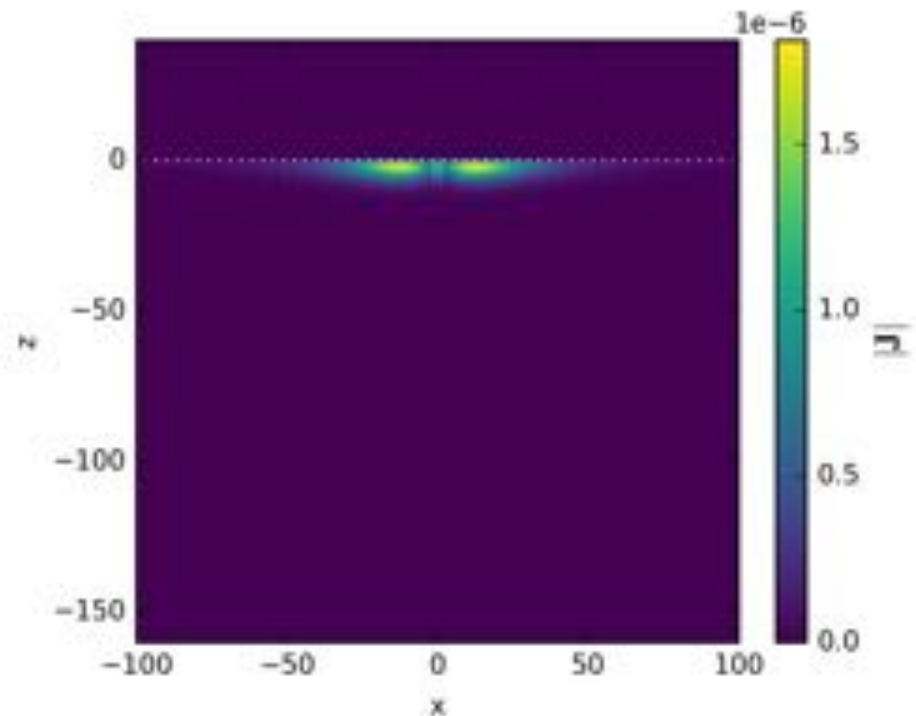
- Buried, conductive sphere
- Vary background conductivity
- Time: 10^{-5} s



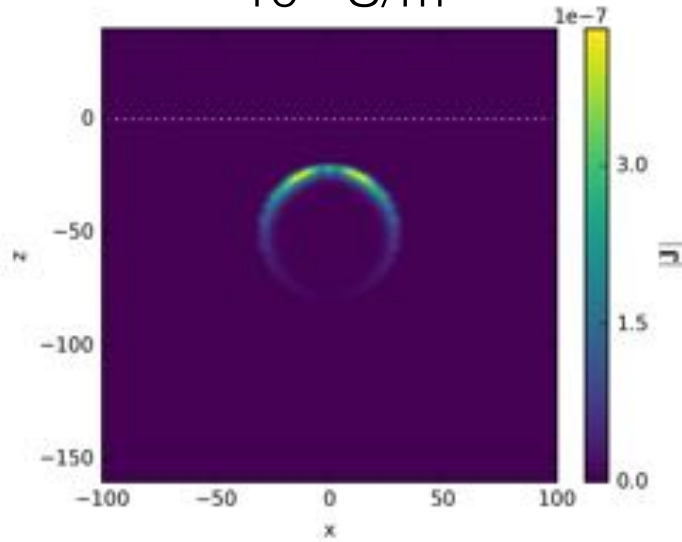
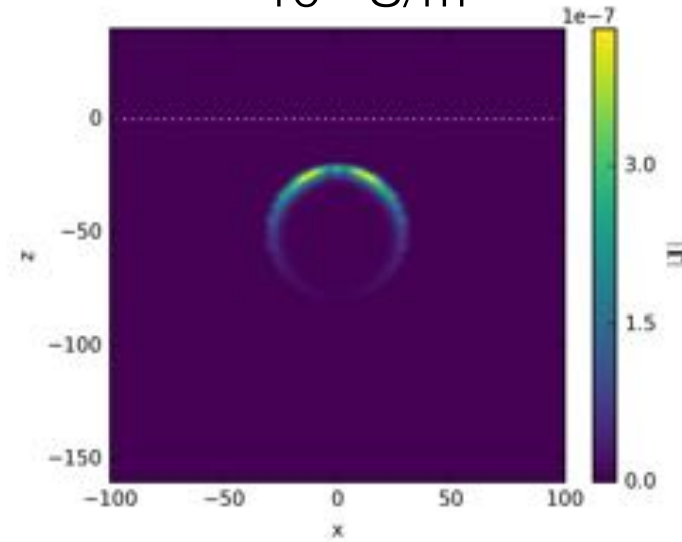
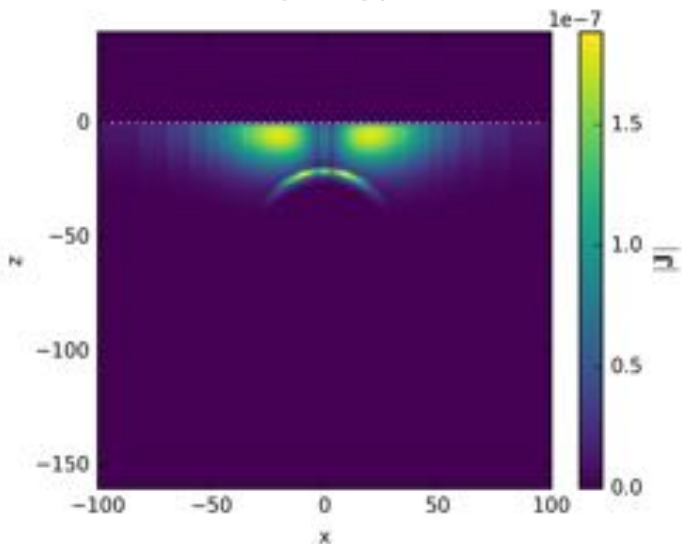
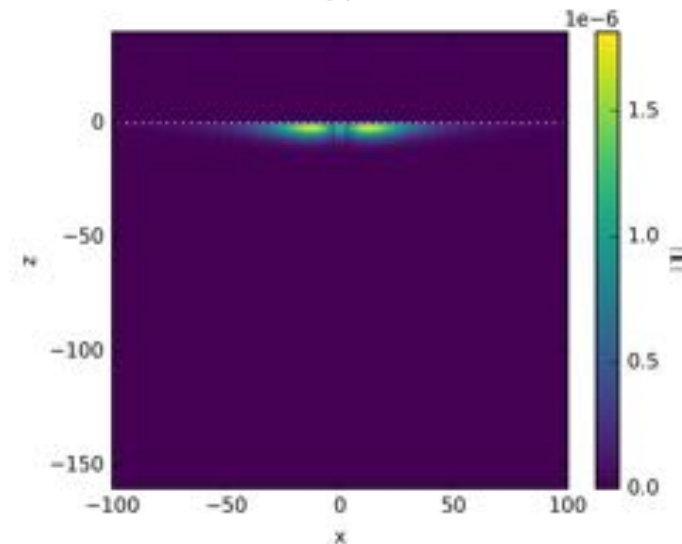
1 S/m background



Current Density

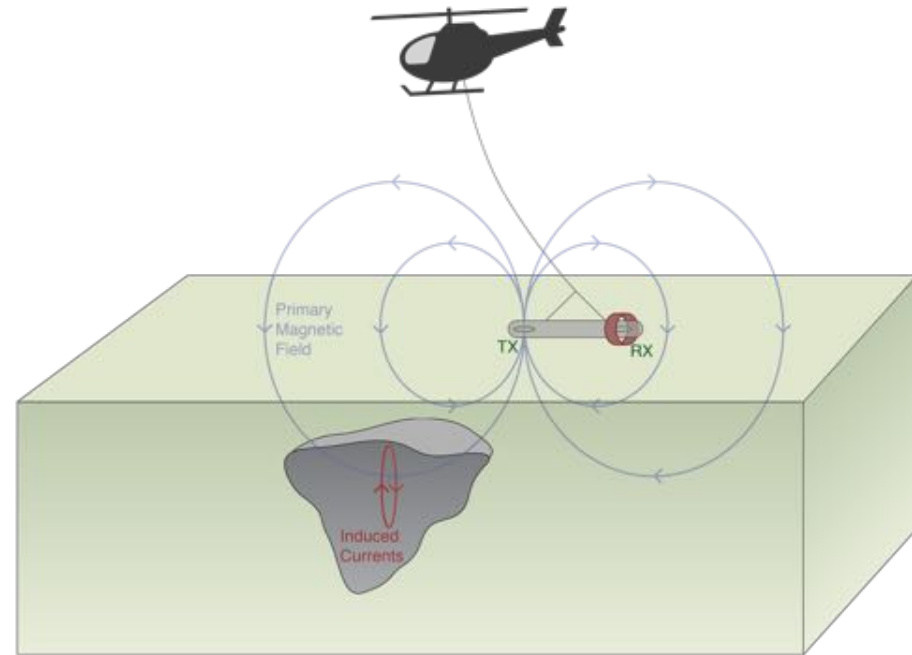


Effects of background resistivity: Time

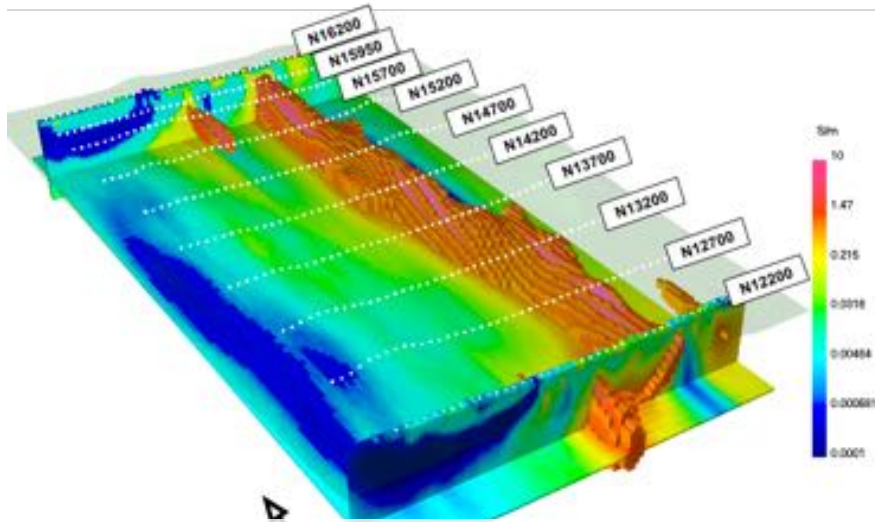
 10^{-8} S/m  10^{-2} S/m  10^{-1} S/m  1 S/m 

Recap: what have we learned?

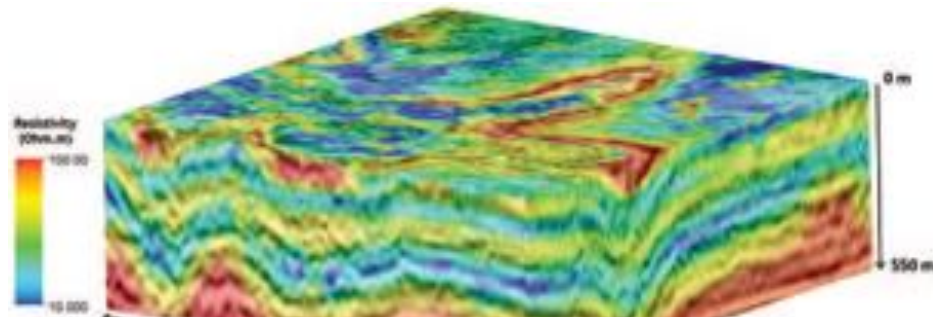
- Basics of EM induction
- Response functions
- Mutual coupling
- Data for frequency or time domain systems
- Circuit model is a good proxy
- Need to account for energy losses
- Ready to look at some field examples



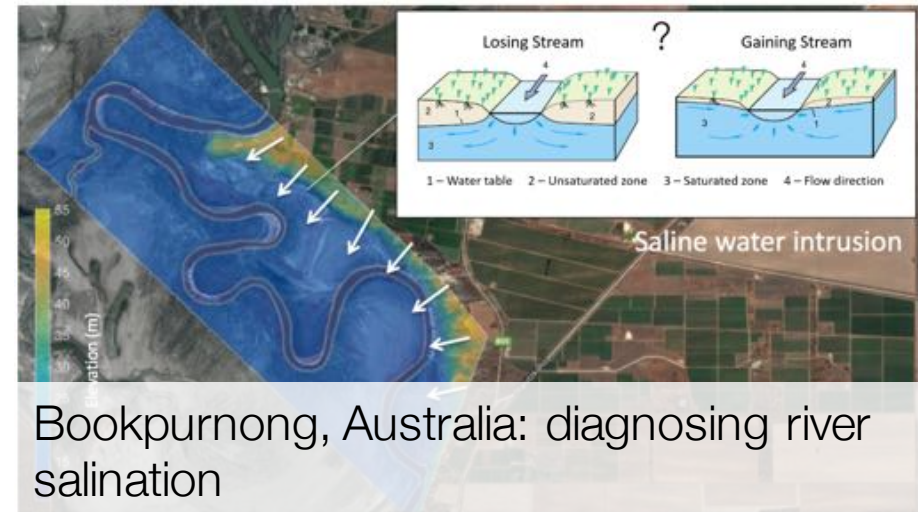
Today's Case Histories



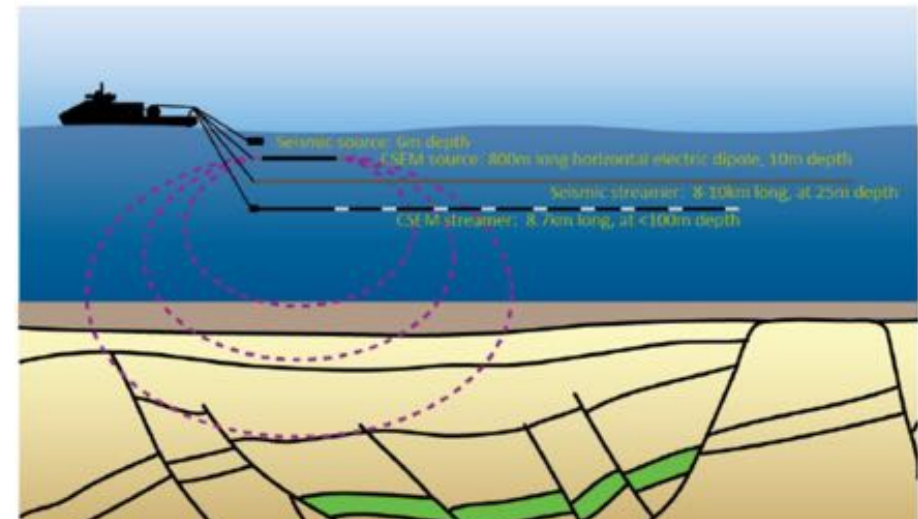
Mt. Isa, Australia: Mineral Exploration



Wadi Sahba, Saudi Arabia: Improving seismic imaging using airborne EM

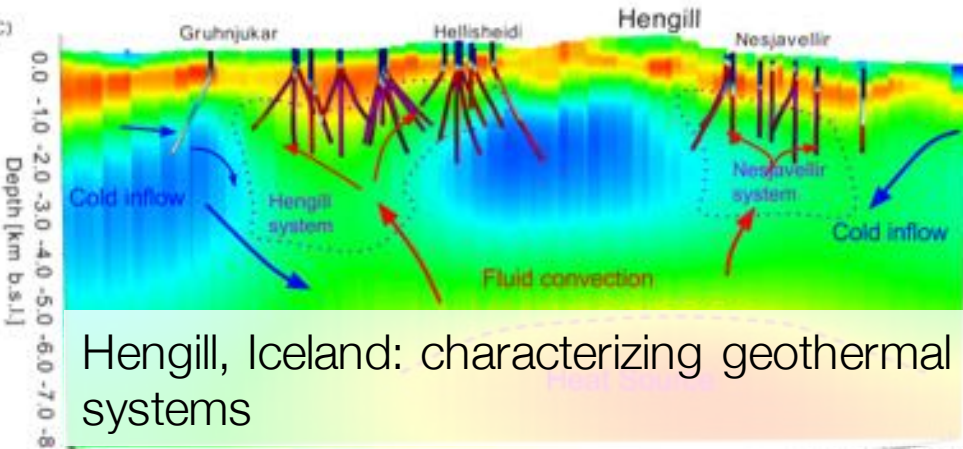


Bookpurnong, Australia: diagnosing river salination

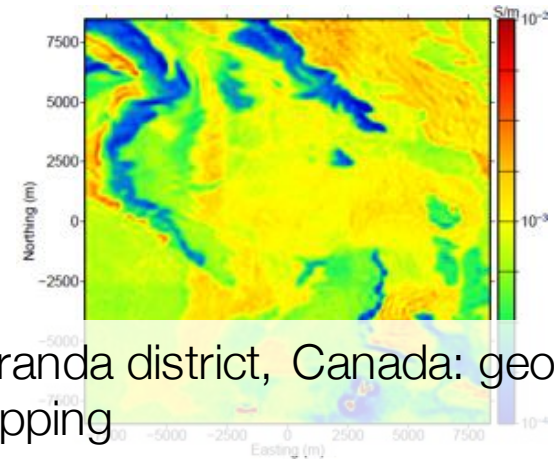


Barents Sea, Norway: Hydrocarbon de-risking

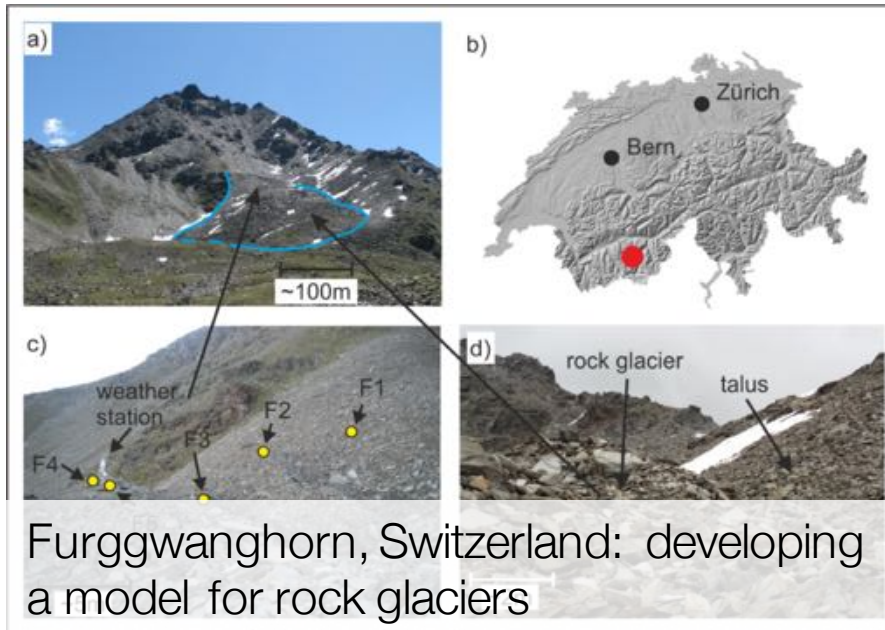
Today's Case Histories



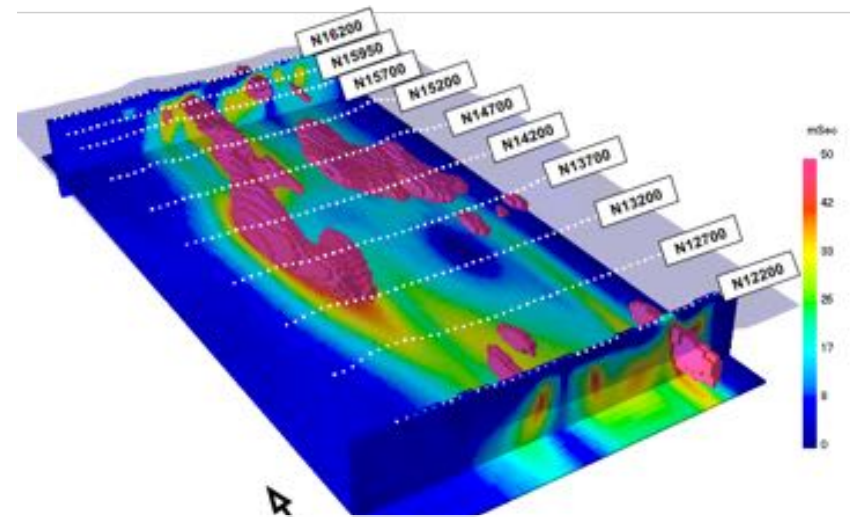
Hengill, Iceland: characterizing geothermal systems



Noranda district, Canada: geologic mapping



Furggwanghorn, Switzerland: developing a model for rock glaciers



Mt. Isa, Australia: Mineral Exploration

End of EM Fundamentals

