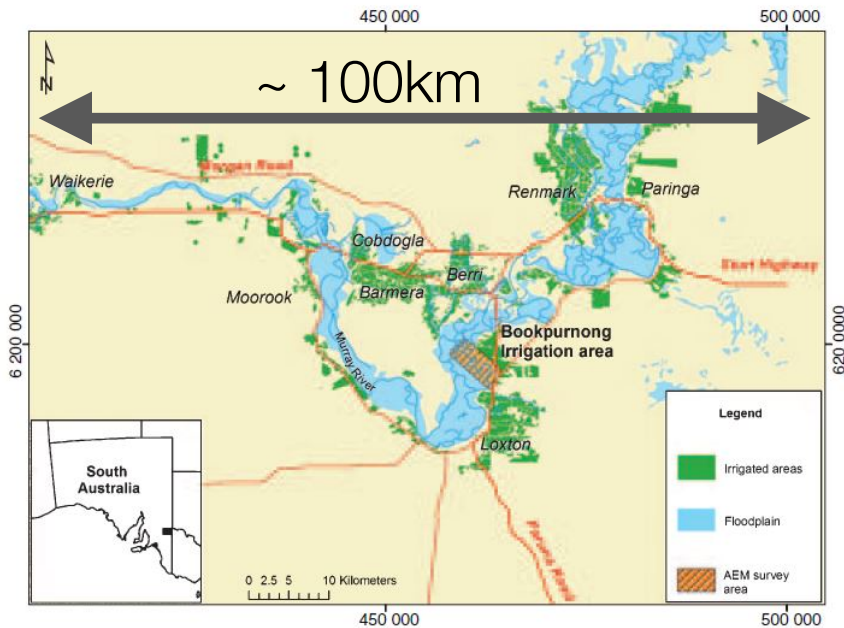


# EM Fundamentals

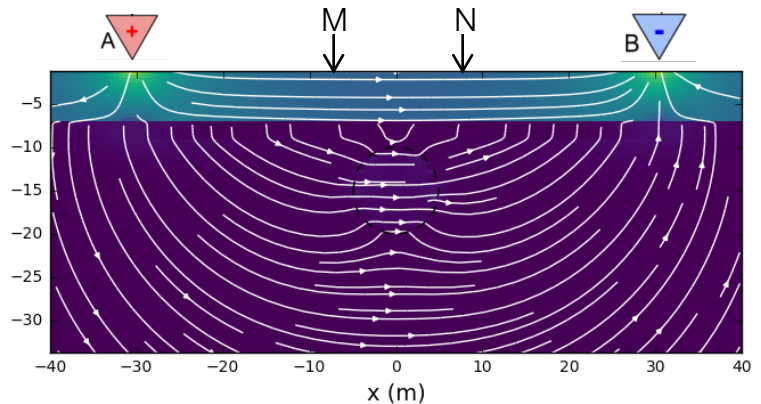
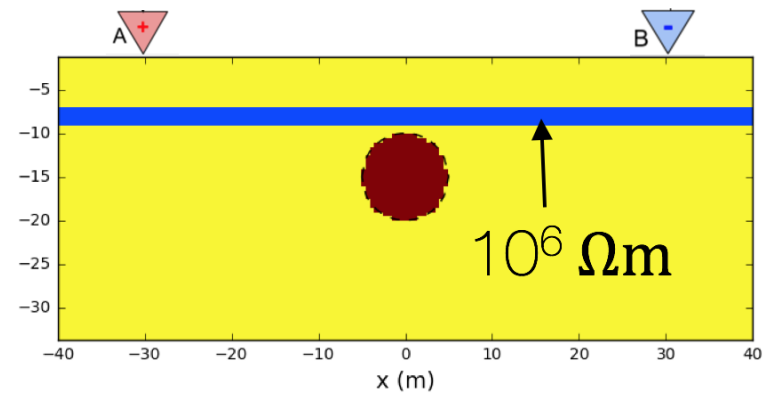


# Motivation: applications difficult for DC

Large areas to be covered



Resistive layer “shields” target



Rugged terrain



Hard to inject

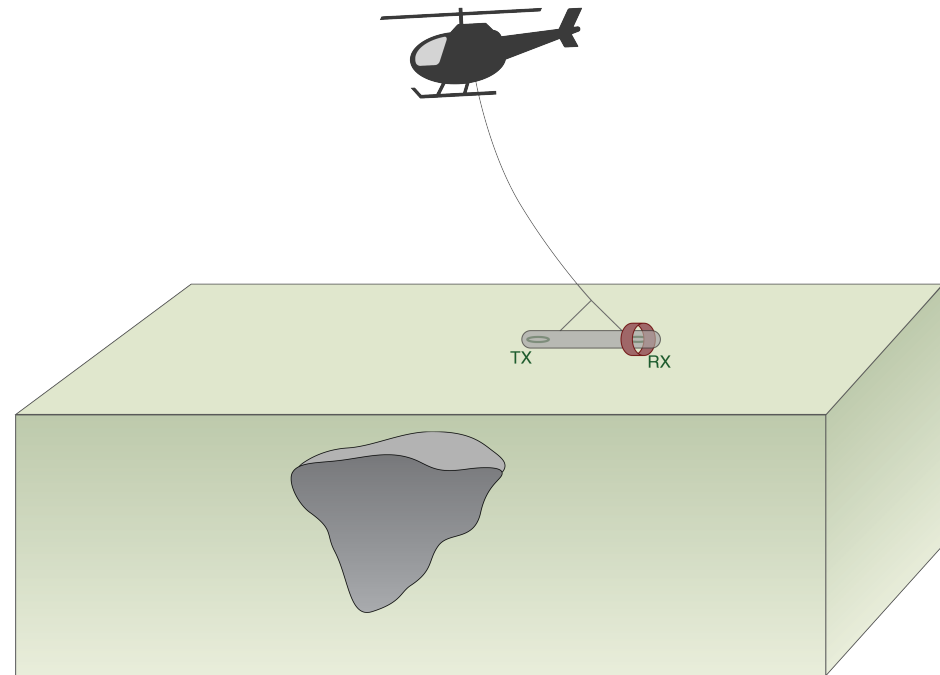


# Outline

- Basic Survey
- Ampere's and Faraday's Laws (2-coil App)
- Circuit model for EM induction
- Frequency and time domain data
- Sphere in homogeneous earth
- Cyl code
- Energy losses in the ground

# Basic Experiment

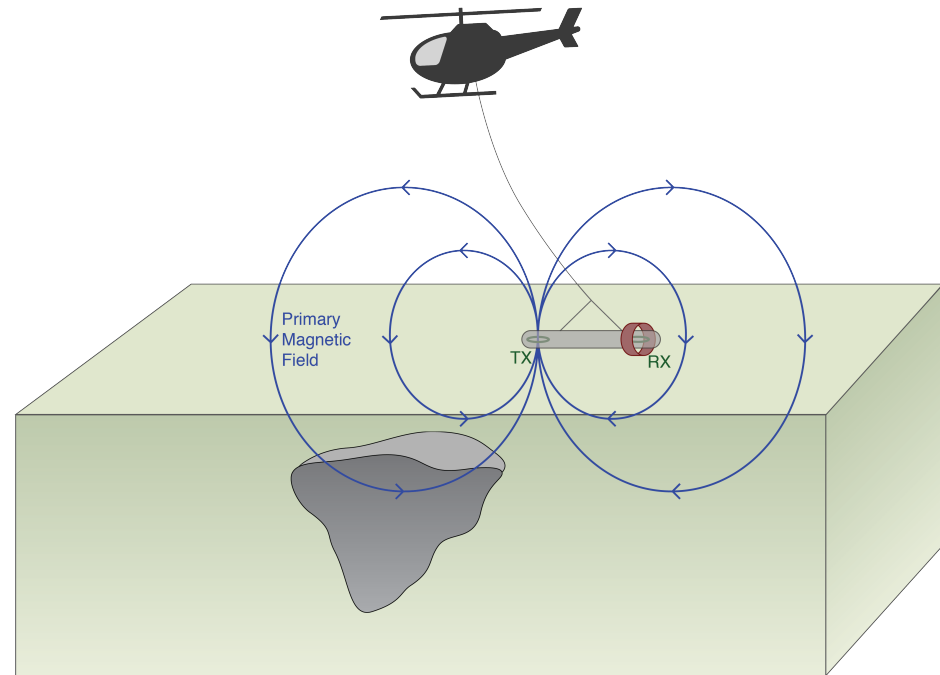
- **Setup:**
  - transmitter and receiver are in a towed bird





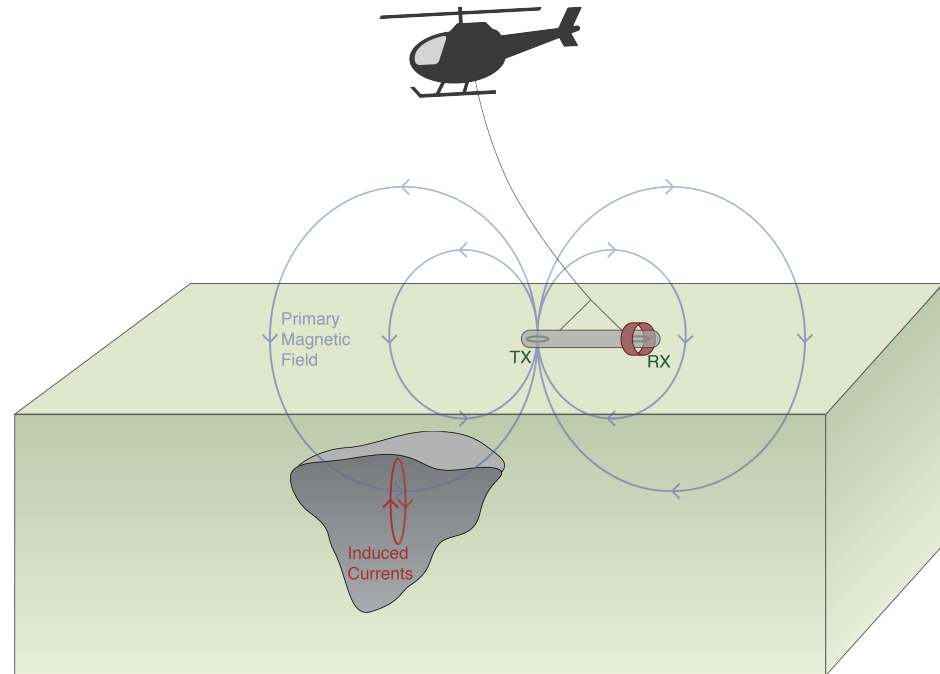
# Basic Experiment

- **Setup:**
  - transmitter and receiver are in a towed bird
- **Primary:**
  - Transmitter produces a primary magnetic field



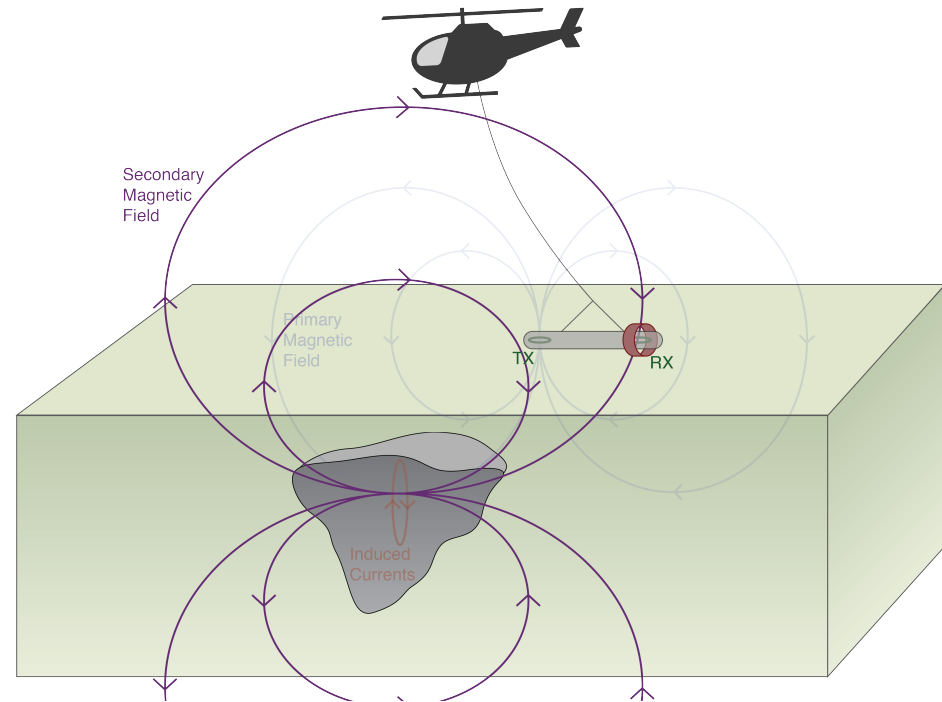
# Basic Experiment

- **Setup:**
  - transmitter and receiver are in a towed bird
- **Primary:**
  - Transmitter produces a primary magnetic field
- **Induced Currents:**
  - Time varying magnetic fields generate electric fields everywhere and currents in conductors



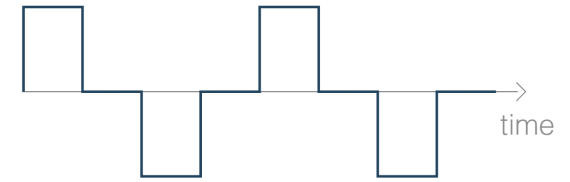
# Basic Experiment

- **Setup:**
  - transmitter and receiver are in a towed bird
- **Primary:**
  - Transmitter produces a primary magnetic field
- **Induced Currents:**
  - Time varying magnetic fields generate electric fields everywhere and currents in conductors
- **Secondary Fields:**
  - The induced currents produce a secondary magnetic field.

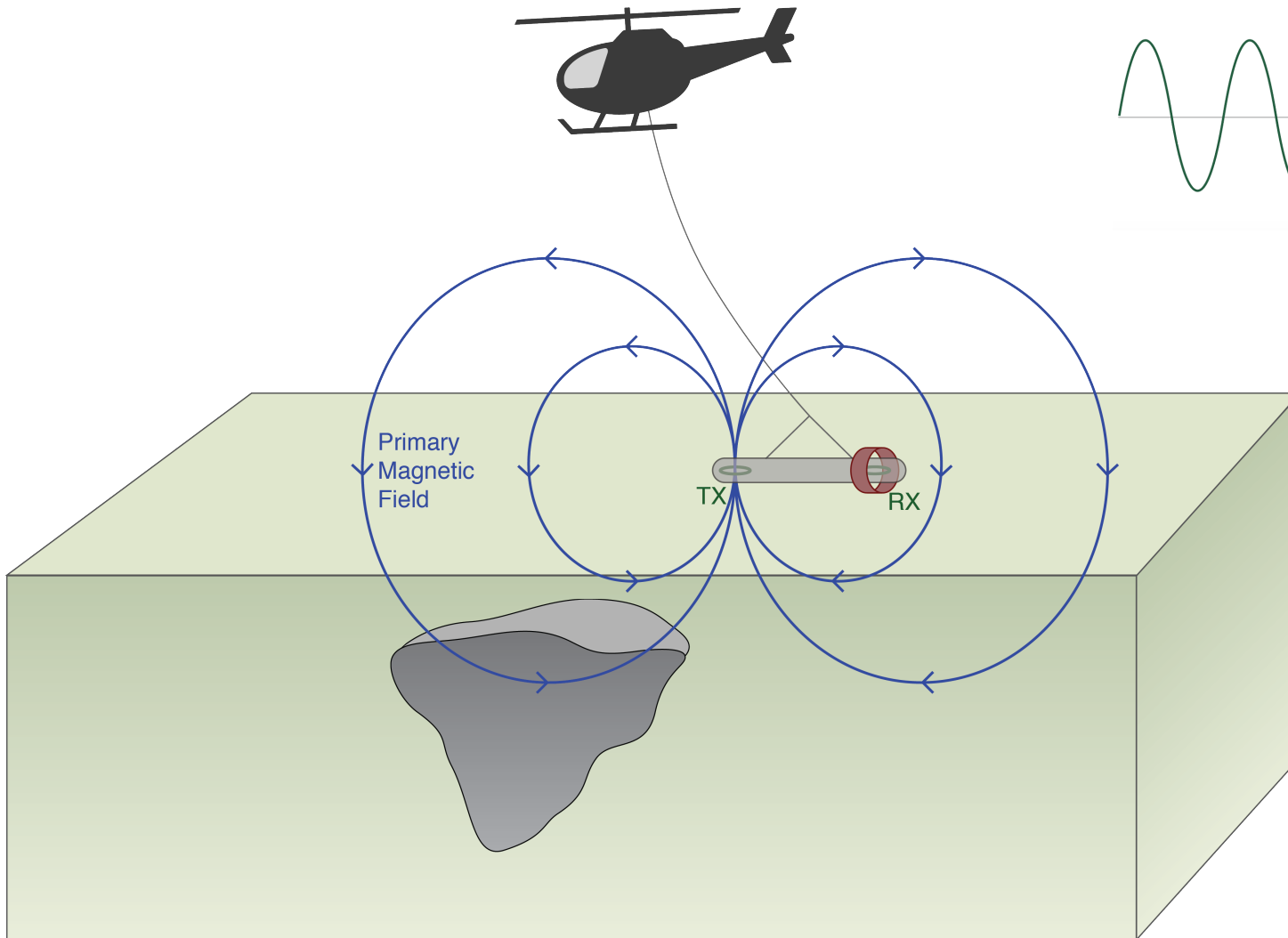
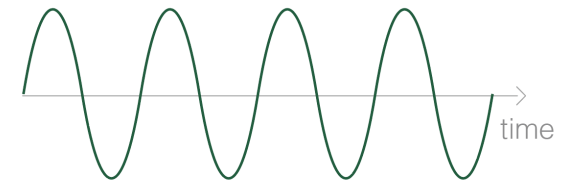


# Transmitter

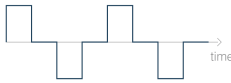
waveform



or



# Basic Equations: Quasi-static

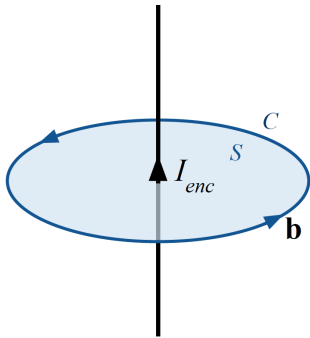
|                                             | Time               | Frequency         |
|---------------------------------------------|------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------|
| Faraday's Law                               | $\nabla \times \mathbf{e} = - \frac{\partial \mathbf{b}}{\partial t}$                                | $\nabla \times \mathbf{E} = - i\omega \mathbf{B}$                                                    |
| Ampere's Law                                | $\nabla \times \mathbf{h} = \mathbf{j} + \frac{\partial \mathbf{d}}{\partial t}$                     | $\nabla \times \mathbf{H} = \mathbf{J} + i\omega \mathbf{D}$                                         |
| No Magnetic Monopoles                       | $\nabla \cdot \mathbf{b} = 0$                                                                        | $\nabla \cdot \mathbf{B} = 0$                                                                        |
| Constitutive Relationships (non-dispersive) | $\mathbf{j} = \sigma \mathbf{e}$ $\mathbf{b} = \mu \mathbf{h}$ $\mathbf{d} = \varepsilon \mathbf{e}$ | $\mathbf{J} = \sigma \mathbf{E}$ $\mathbf{B} = \mu \mathbf{H}$ $\mathbf{D} = \varepsilon \mathbf{E}$ |

\* Solve with sources and boundary conditions

# Ampere's Law

$$\nabla \times \mathbf{H} = \mathbf{J}$$

Wire



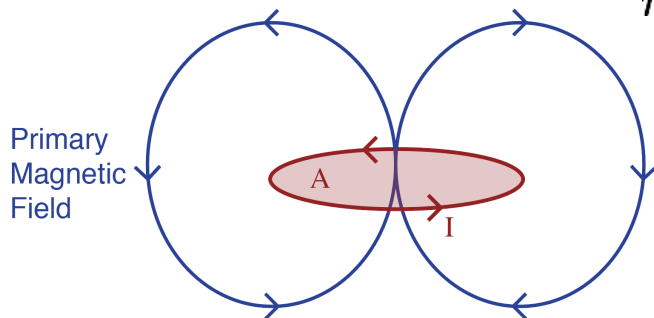
$$\mathbf{B} = \frac{\mu_0 I_{enc}}{2\pi r} \hat{\phi}$$

Right hand rule

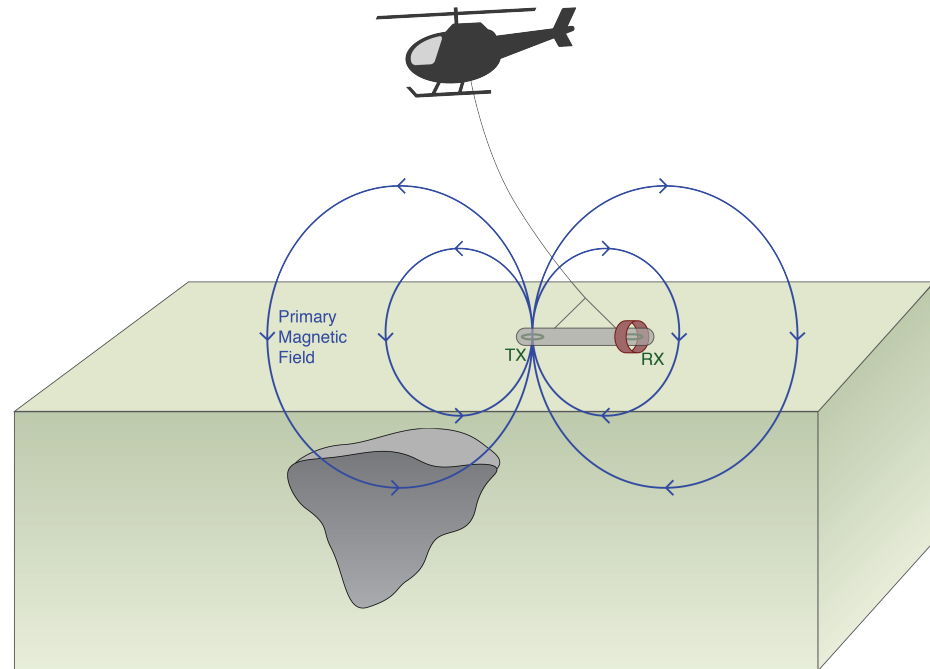
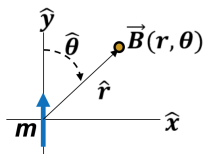
Current loop

$$\mathbf{B} = \frac{\mu_0 m}{4\pi r^3} (2 \cos \theta \hat{\mathbf{r}} + \sin \theta \hat{\boldsymbol{\theta}})$$

$$m = IA$$



Primary  
Magnetic  
Field



# Faraday's Law

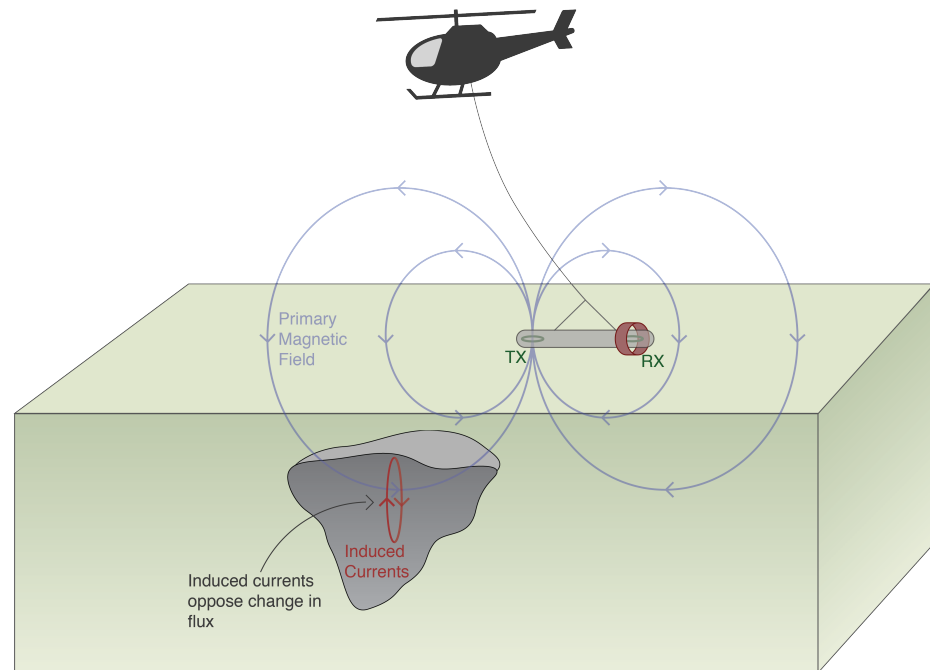
$$\nabla \times \mathbf{e} = - \frac{\partial \mathbf{b}}{\partial t}$$

Lenz'  
Law

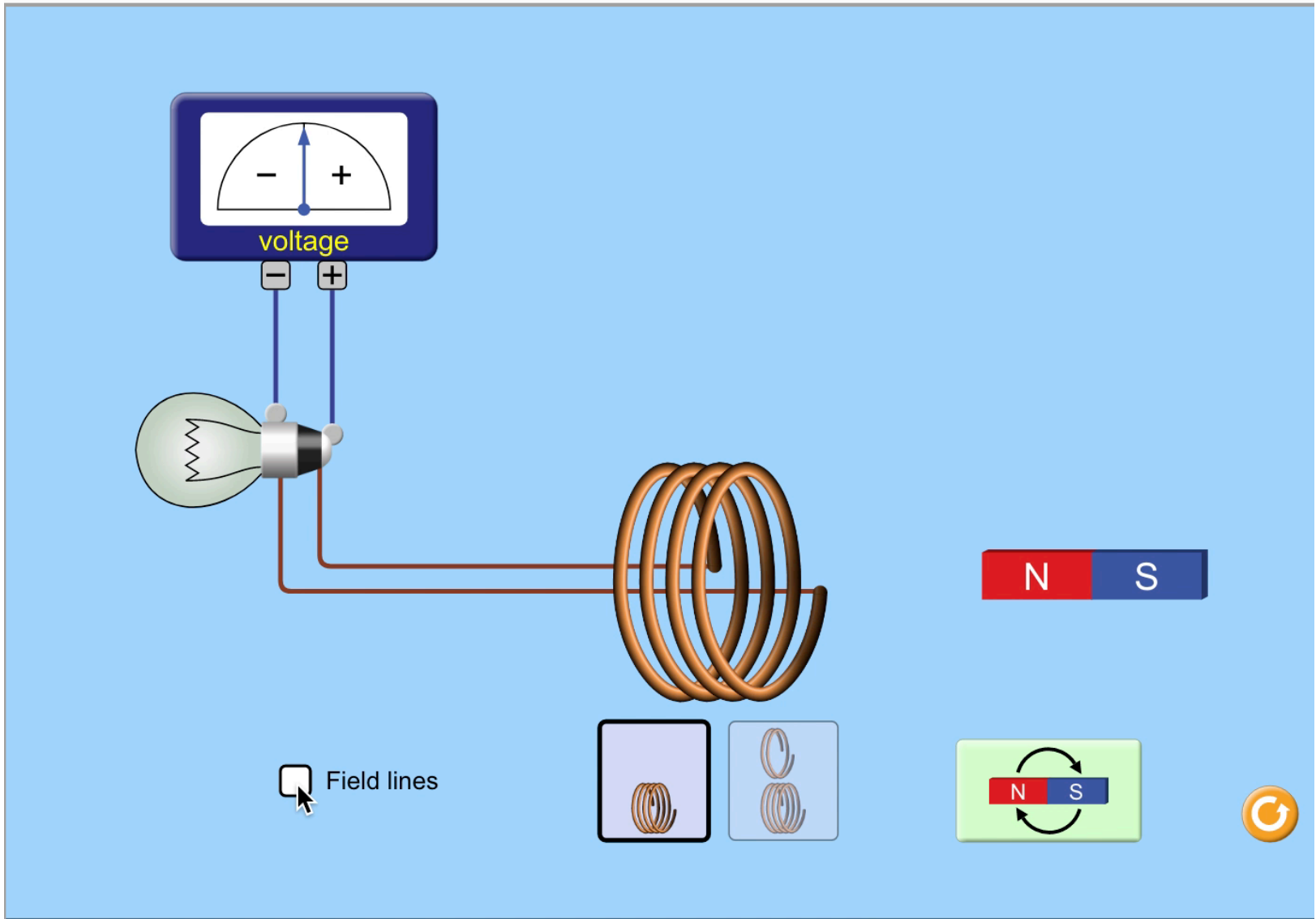


Ohm's Law

$$\mathbf{j} = \sigma \mathbf{e}$$



# Faraday's Law





# Faraday's Law

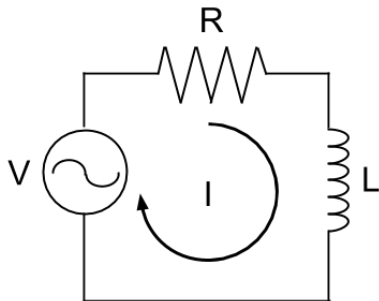
$$\nabla \times \mathbf{e} = - \frac{\partial \mathbf{b}}{\partial t}$$

Magnetic Flux

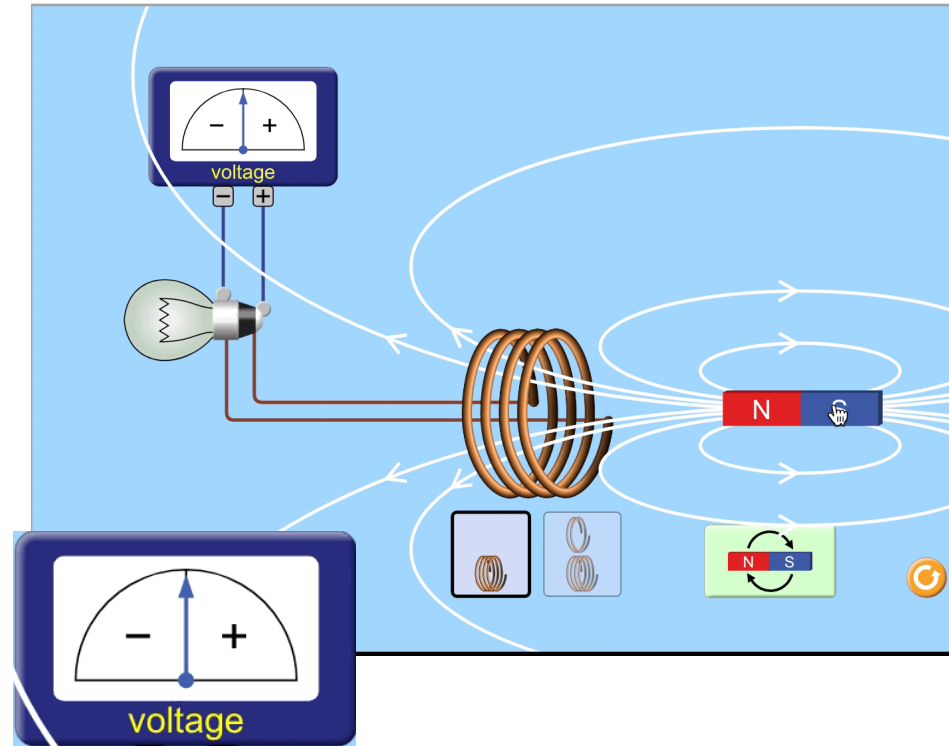
$$\phi_{\mathbf{b}} = \int_A \mathbf{b} \cdot \hat{\mathbf{n}} \, da$$

Induced EMF

$$V = EMF = - \frac{d\phi_{\mathbf{b}}}{dt} = \mathbf{0}$$



$\phi_b$  : constant



# Faraday's Law

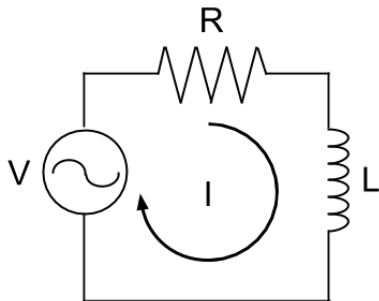
$$\nabla \times \mathbf{e} = - \frac{\partial \mathbf{b}}{\partial t}$$

Magnetic Flux

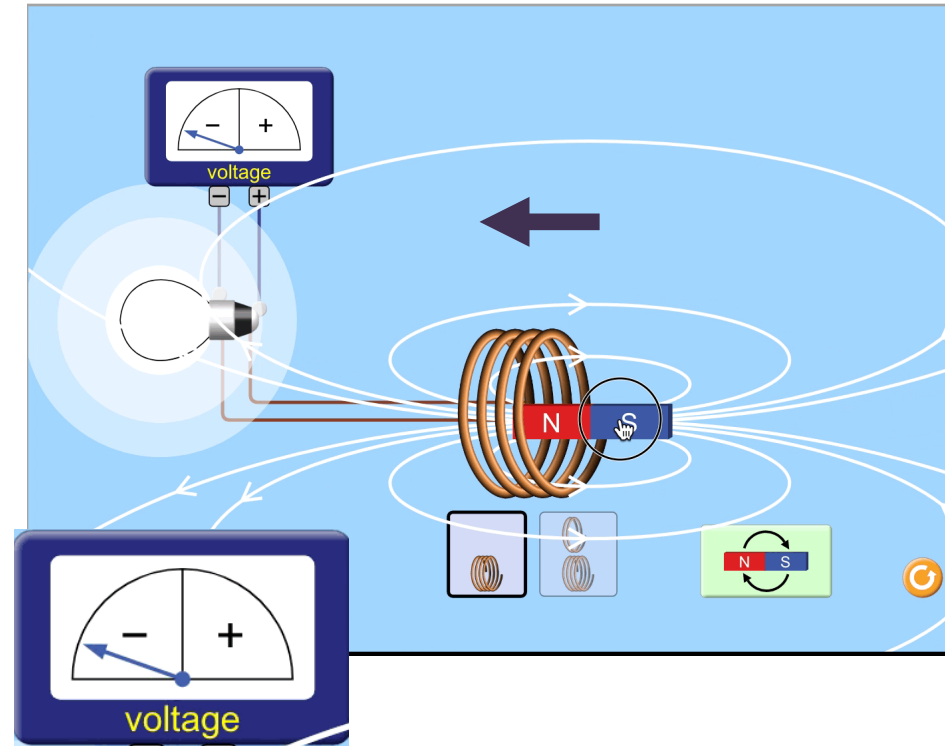
$$\phi_{\mathbf{b}} = \int_A \mathbf{b} \cdot \hat{\mathbf{n}} \, da$$

Induced EMF

$$V = EMF = - \frac{d\phi_{\mathbf{b}}}{dt} < 0$$



$\phi_b : \uparrow$



# Faraday's Law

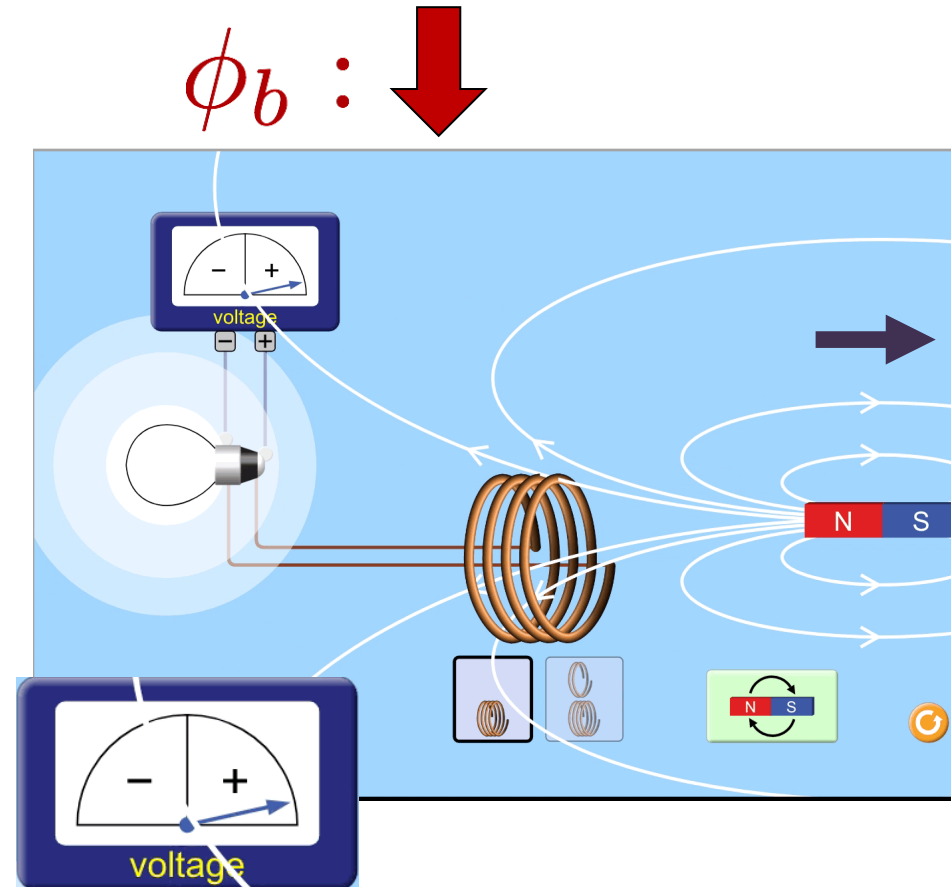
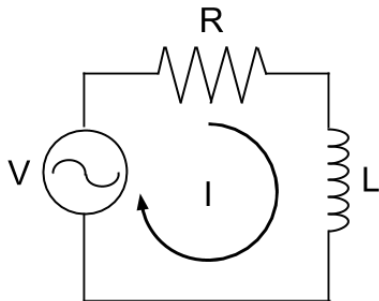
$$\nabla \times \mathbf{e} = - \frac{\partial \mathbf{b}}{\partial t}$$

Magnetic Flux

$$\phi_{\mathbf{b}} = \int_A \mathbf{b} \cdot \hat{\mathbf{n}} \, da$$

Induced EMF

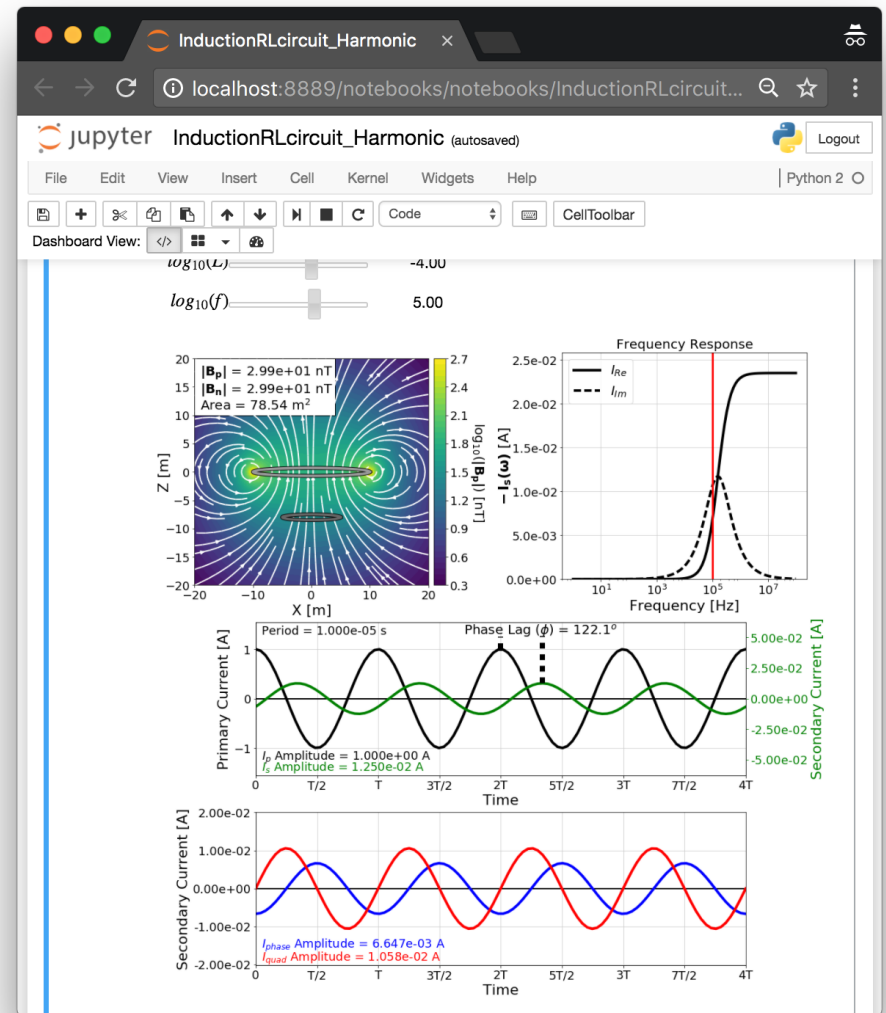
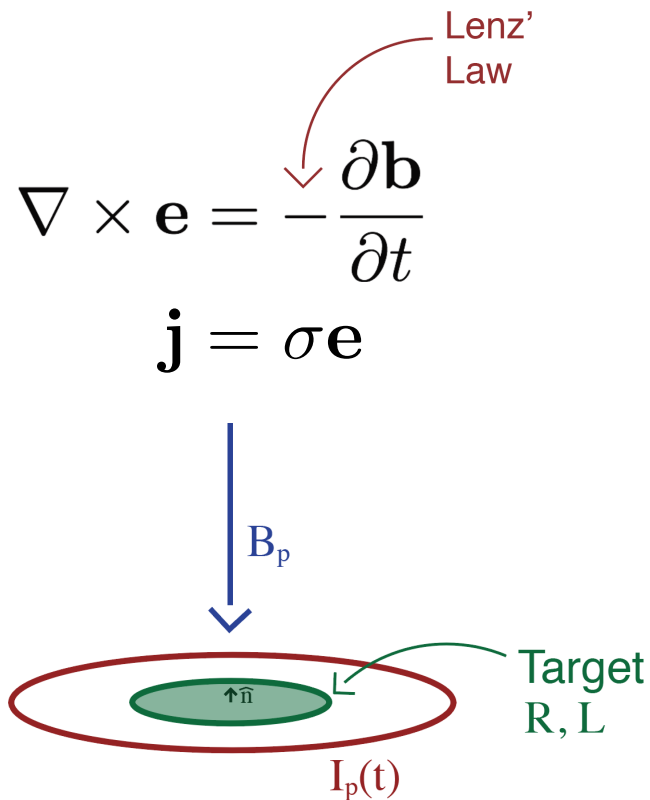
$$V = EMF = - \frac{d\phi_{\mathbf{b}}}{dt} > 0$$



# App for Faraday's Law

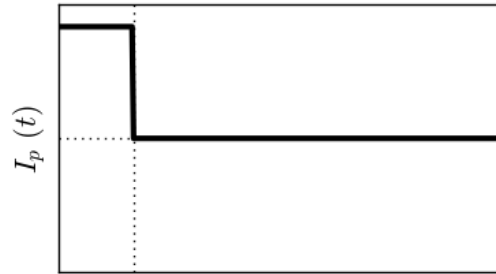
## 2 Apps:

- Harmonic
- Transient

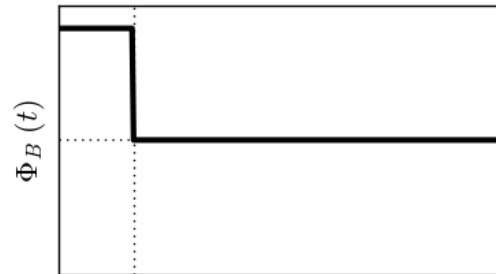


# Two Coil Example: Transient

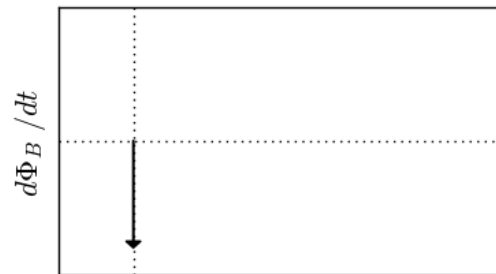
Primary currents



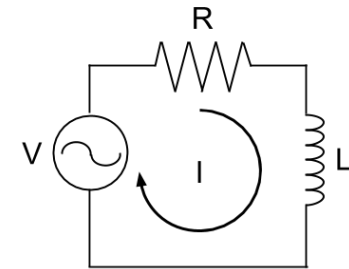
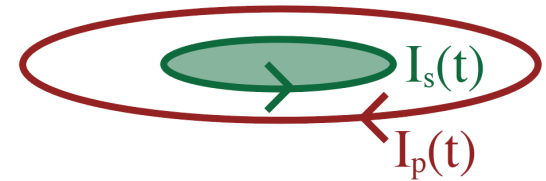
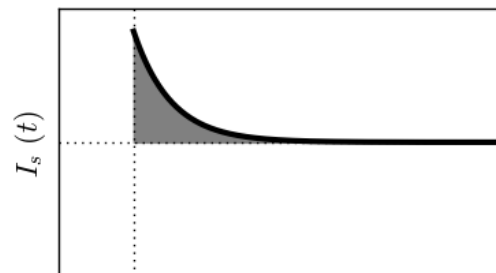
Magnetic flux



Time-variation of magnetic flux



Secondary currents

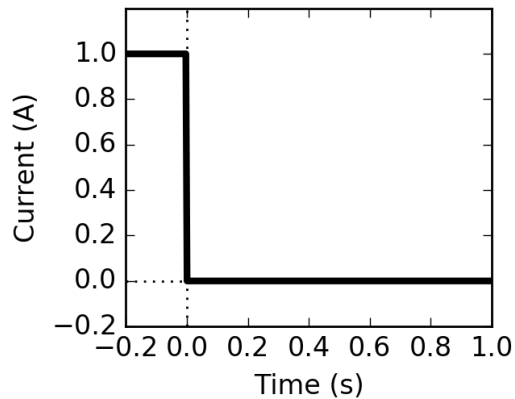


$$I_s(t) = I_s e^{-t/\tau}$$

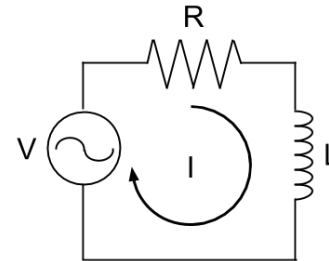
$$\tau = L/R$$

# Response Function: Transient

Step-off current in Tx

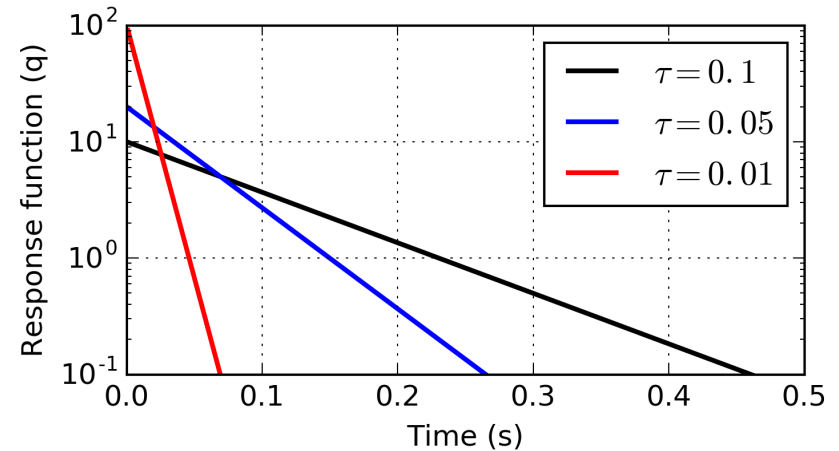
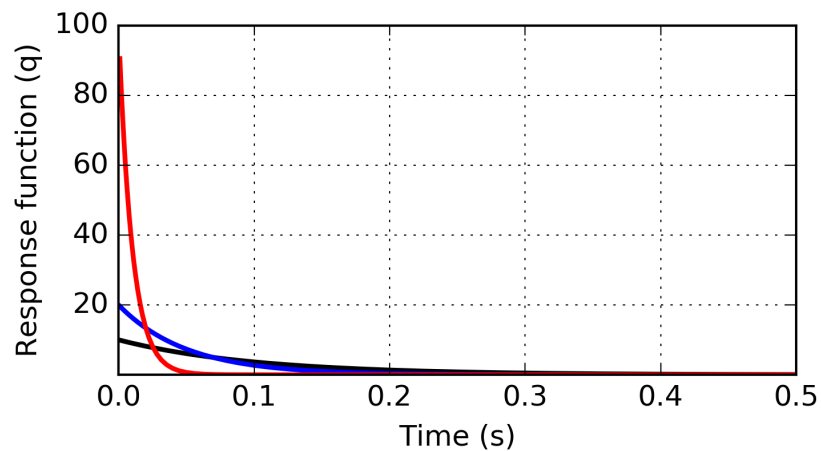


Time constant



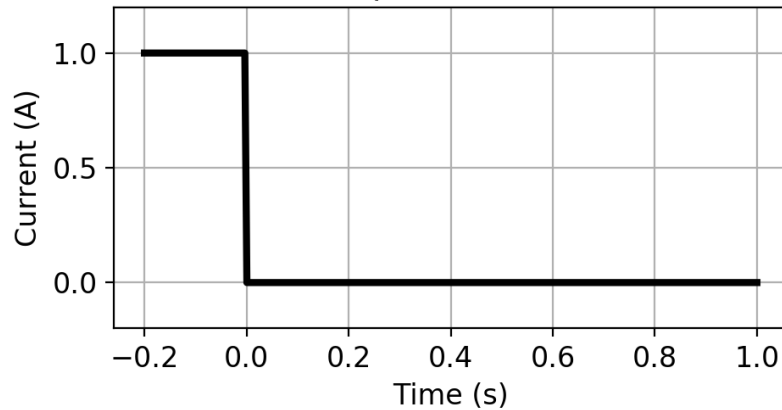
$$\tau = L/R$$

Response function:  $q(t) = e^{-t/\tau}$

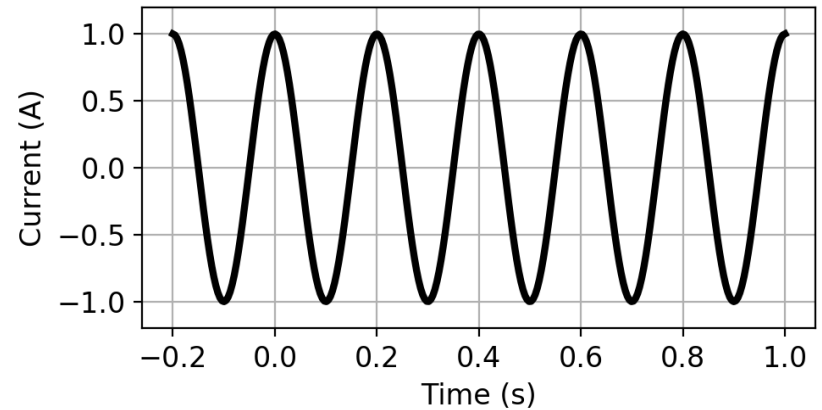


# Transient and Harmonic Signals

We have seen a transient pulse...



What happens when he have a harmonic?



# Two Coil Example: Harmonic

## Induced Currents

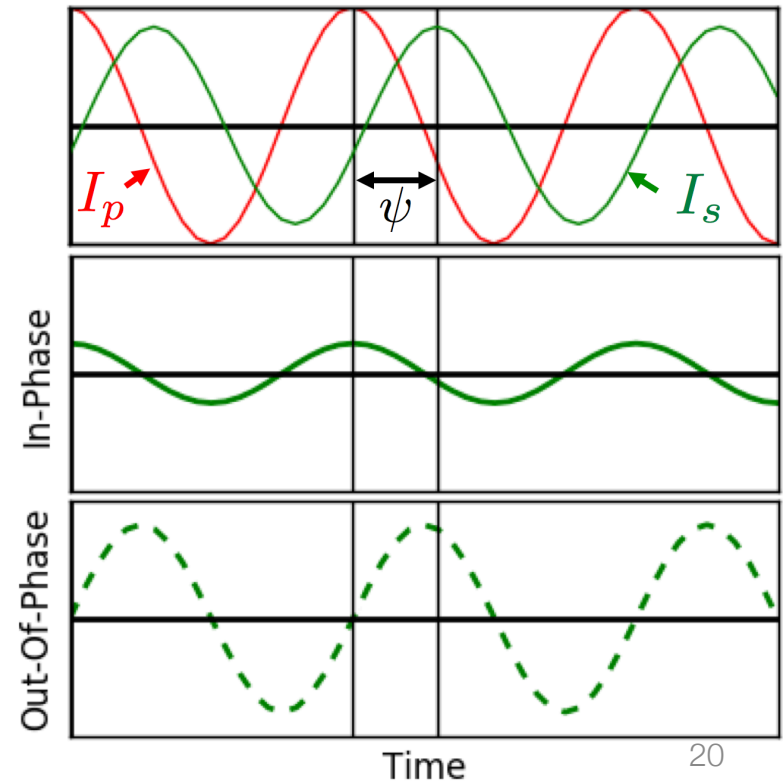
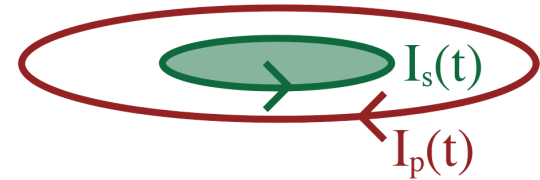
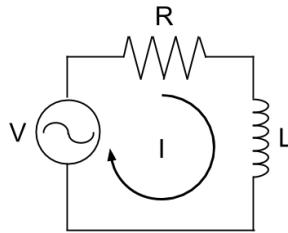
$$I_p(t) = I_p \cos \omega t$$

$$I_s(t) = I_s \cos(\omega t - \psi)$$

$$= \underbrace{I_s \cos \psi \cos \omega t}_{\substack{\text{In-Phase} \\ \text{Real}}} + \underbrace{I_s \sin \psi \sin \omega t}_{\substack{\text{Out-of-Phase} \\ \text{Quadrature} \\ \text{Imaginary}}}$$

## Phase Lag

$$\psi = \frac{\pi}{2} + \tan^{-1} \left( \frac{\omega L}{R} \right)$$





# Two Coil Example: Harmonic

## Induced Currents

$$I_p(t) = I_p \cos \omega t$$

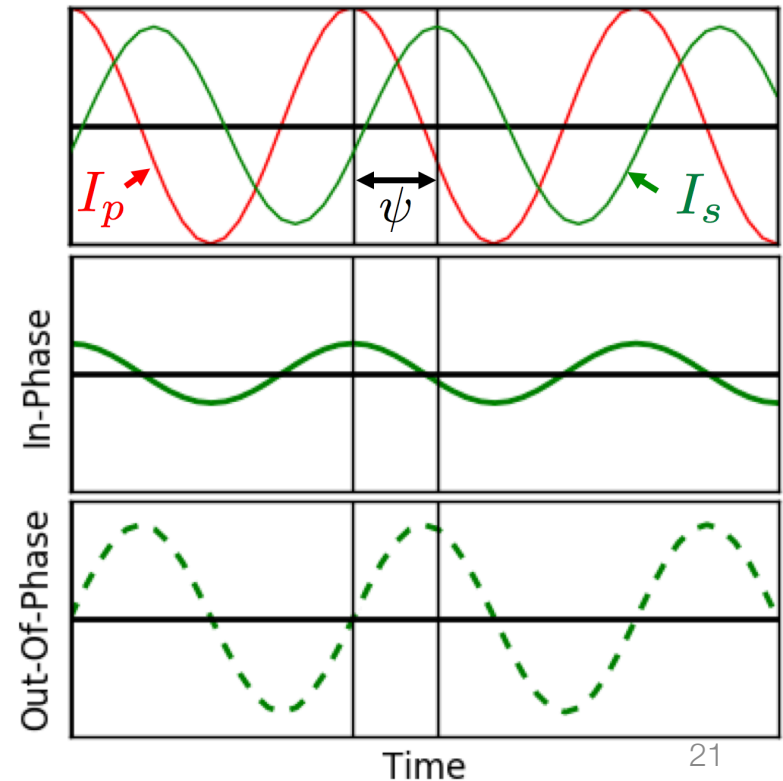
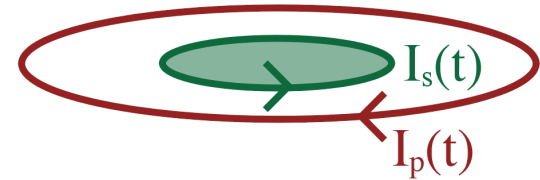
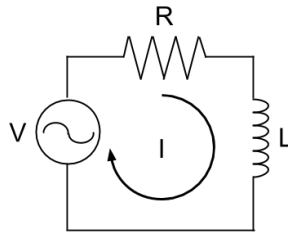
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## Phase Lag

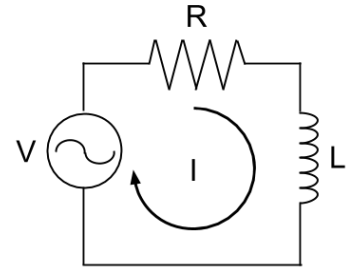
$$\psi = \frac{\pi}{2} + \underbrace{\tan^{-1} \left( \frac{\omega L}{R} \right)}_{\alpha}$$

Induction number

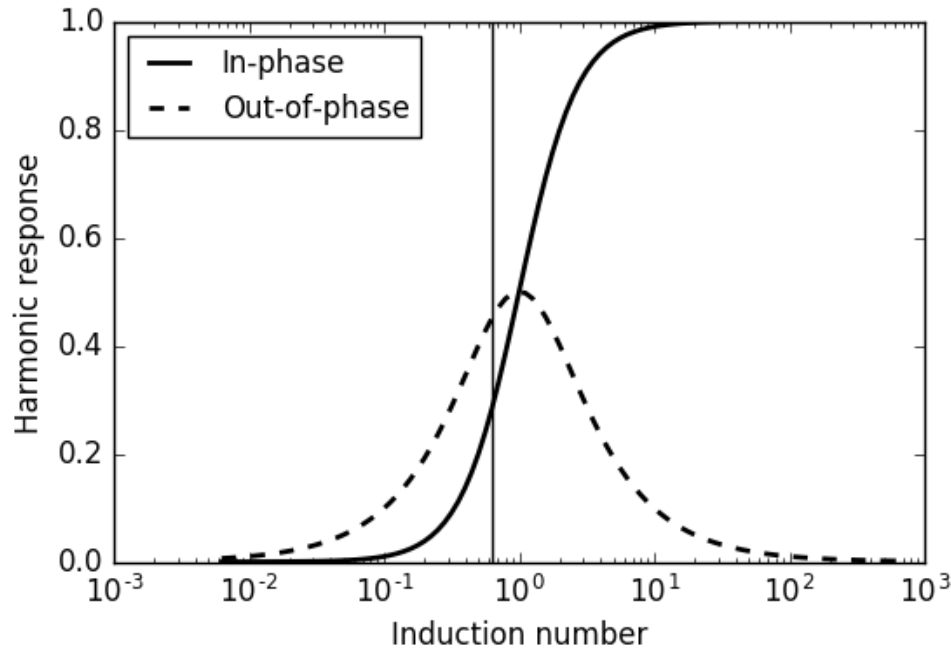


# Response Function

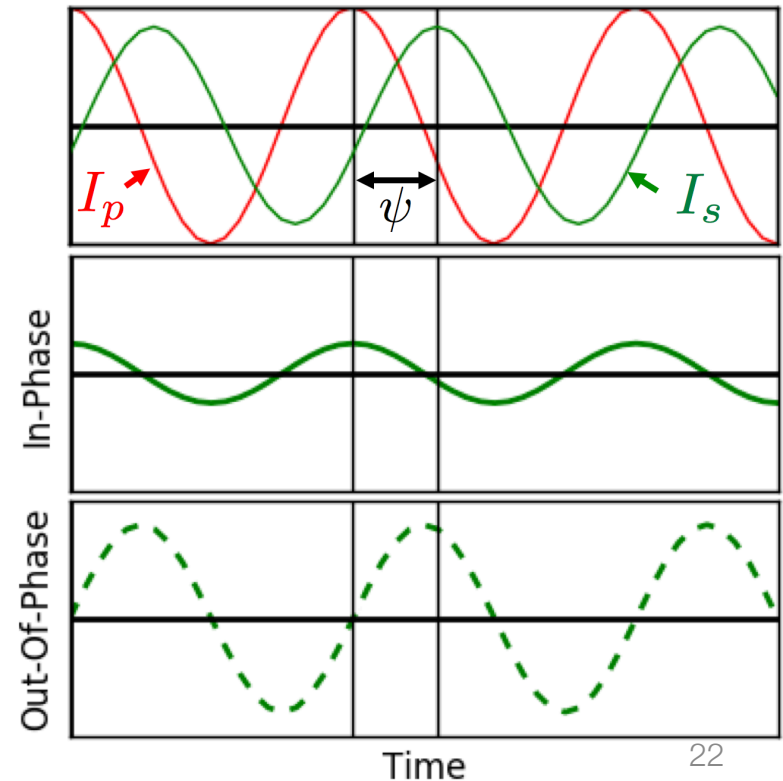
- Quantifies how a target responds to a time varying magnetic field
- Partitions real and imaginary parts



Response Function

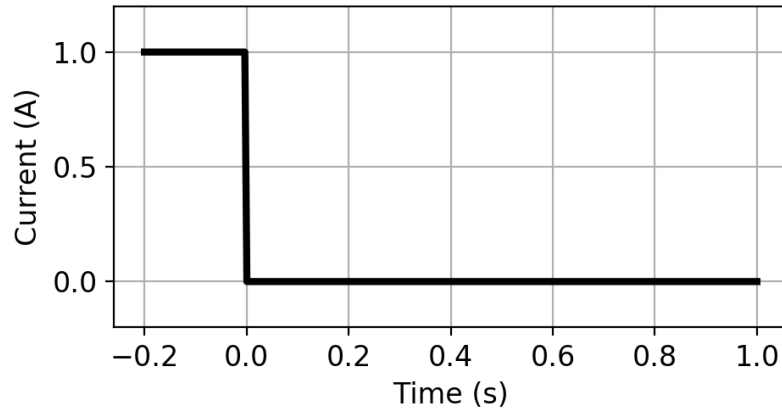


$$\alpha = \frac{\omega L}{R}$$

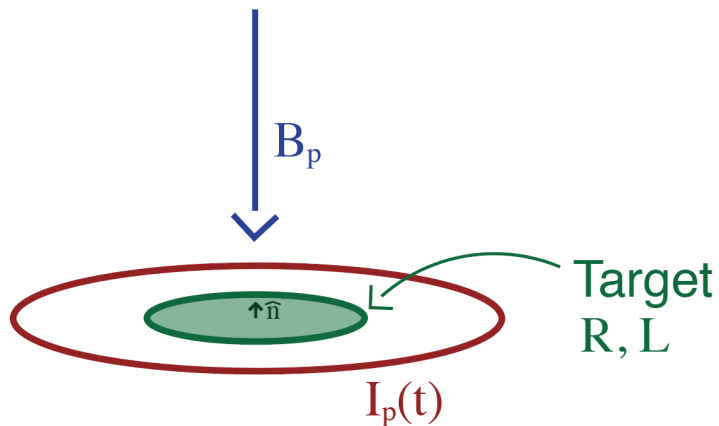
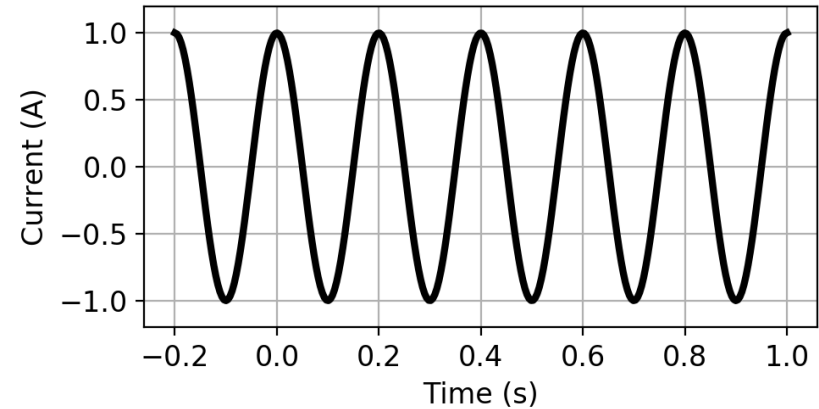


# Response Functions: Summary

Step-off



Harmonic



In both:

- Induce currents

$$\nabla \times \mathbf{e} = -\frac{\partial \mathbf{b}}{\partial t}$$

- Generate secondary magnetic fields

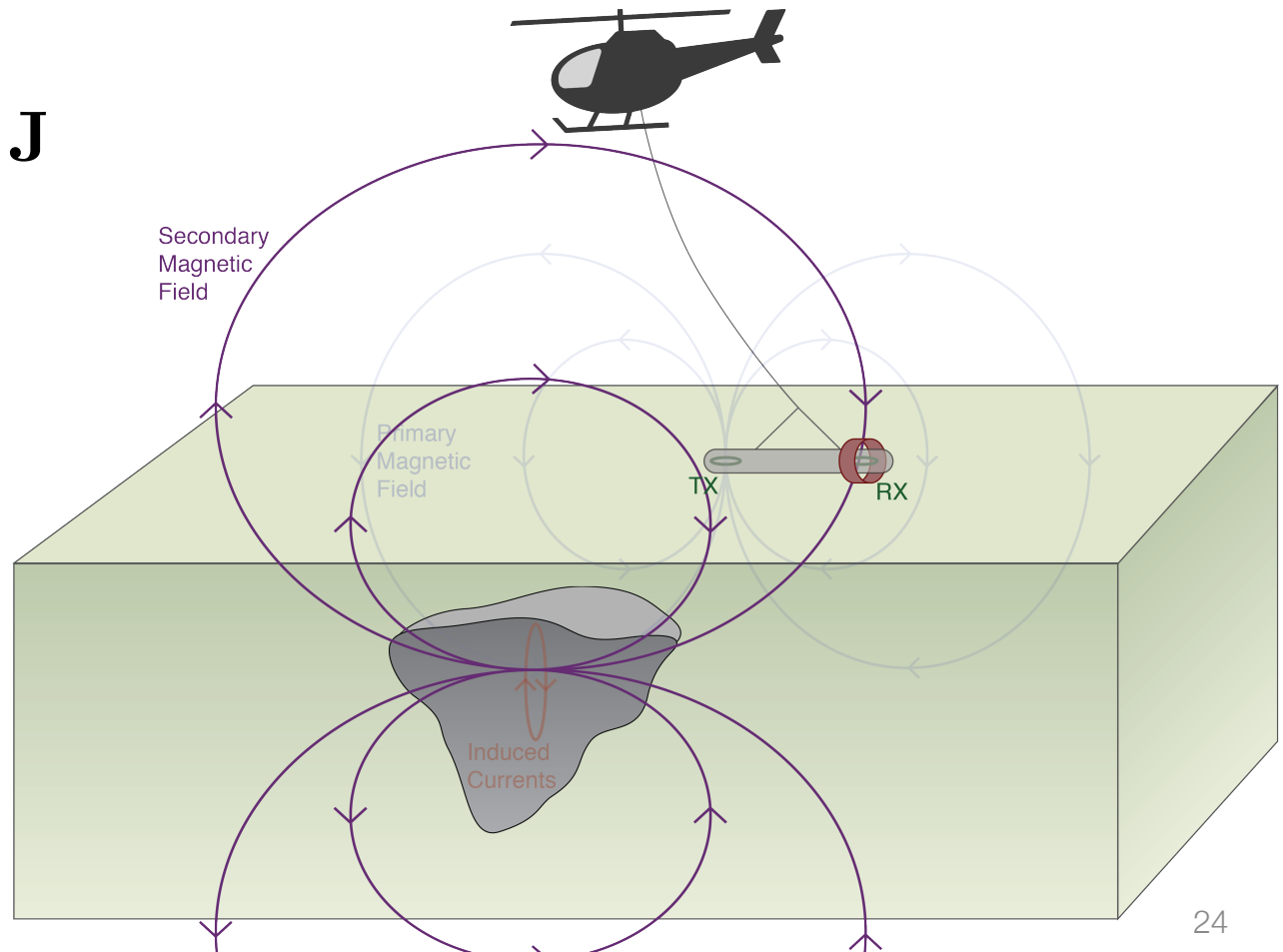
$$\nabla \times \mathbf{h} = \mathbf{j}$$

# Secondary magnetic fields

Induced currents generate magnetic fields

- Ampere's Law

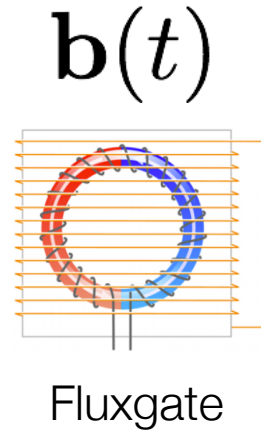
$$\nabla \times \mathbf{H} = \mathbf{J}$$



# Receiver and Data

## Magnetometer

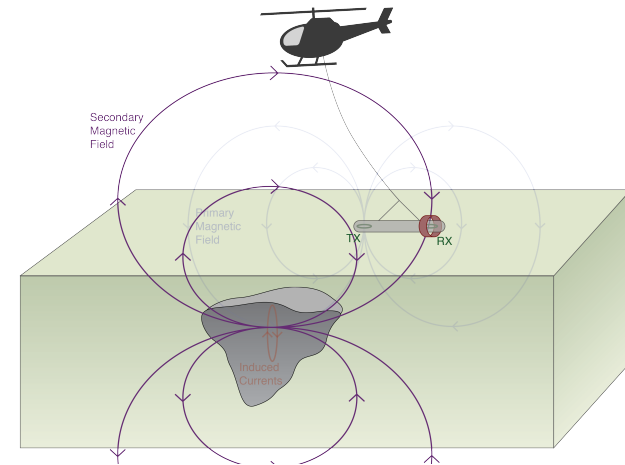
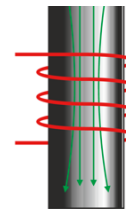
- Measures:
  - Magnetic fields
  - 3 components
- eg. 3-component fluxgate



## Coil

- Measures:
  - Voltage
  - Single component that depends on coil orientation
    - Coupling matters
- eg. airborne frequency domain
  - ratio of  $H_s/H_p$  is the same as  $V_s/V_p$

$$\frac{\partial b}{\partial t}$$



# Coupling

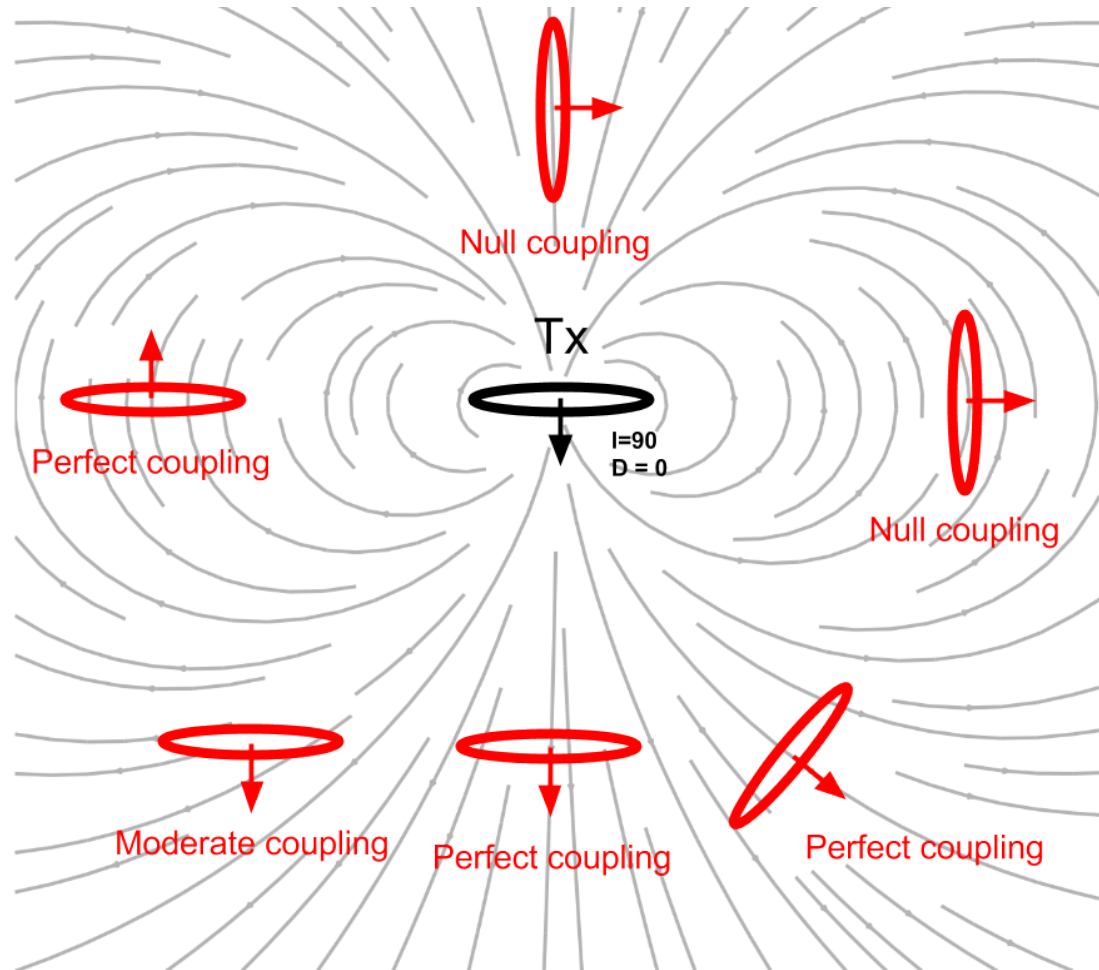
- Transmitter: Primary

$$I_p(t) = I_p \cos(\omega t)$$

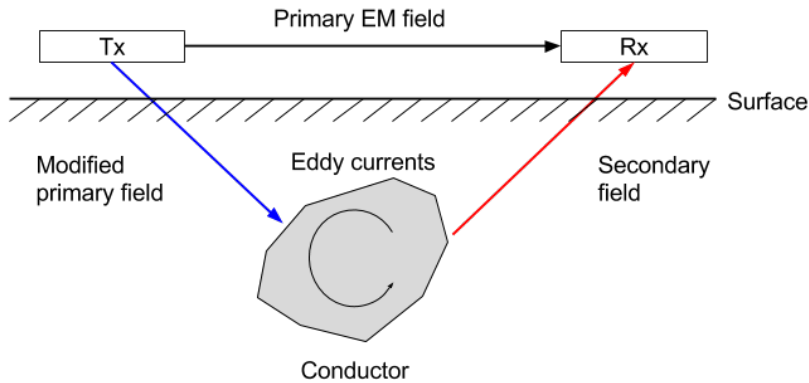
$$\mathbf{B}_p(t) \sim I_p \cos(\omega t)$$

- Target: Secondary

$$\begin{aligned} EMF &= -\frac{\partial \phi_{\mathbf{B}}}{\partial t} \\ &= -\frac{\partial}{\partial t} (\mathbf{B}_p \cdot \hat{\mathbf{n}}) A \end{aligned}$$



# Circuit model of EM induction

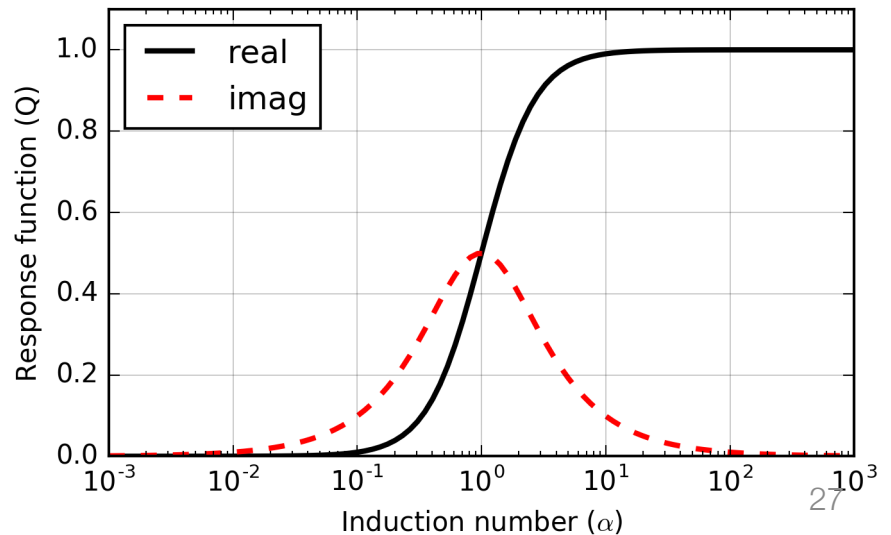


Magnetic field at the receiver

$$\frac{H^s}{H^p} = - \frac{M_{12}M_{23}}{M_{13}L} \underbrace{\left[ \frac{\alpha^2 + i\alpha}{1 + \alpha^2} \right]}_Q$$

Induction Number

- Depends on properties of target  $\alpha = \frac{\omega L}{R}$



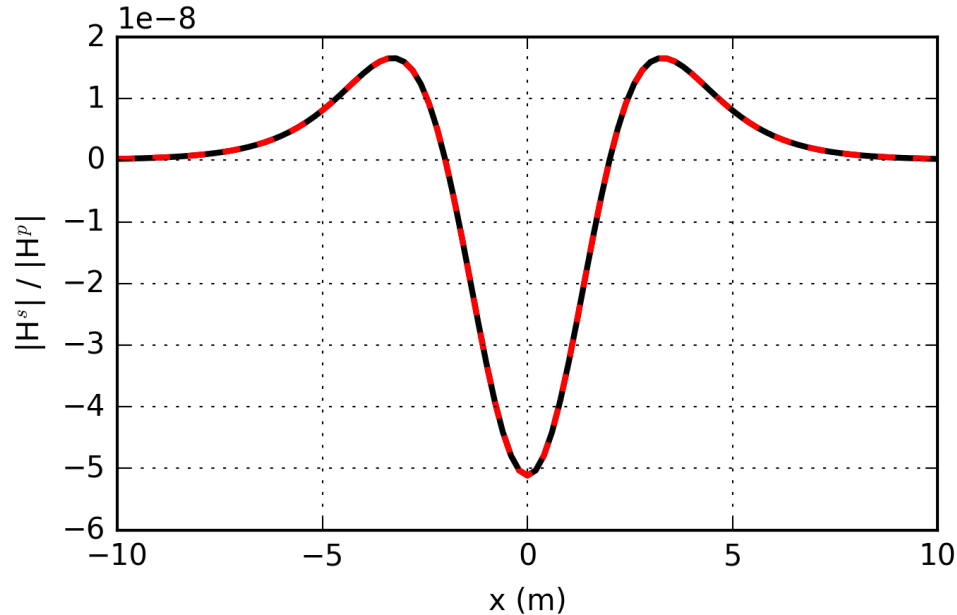
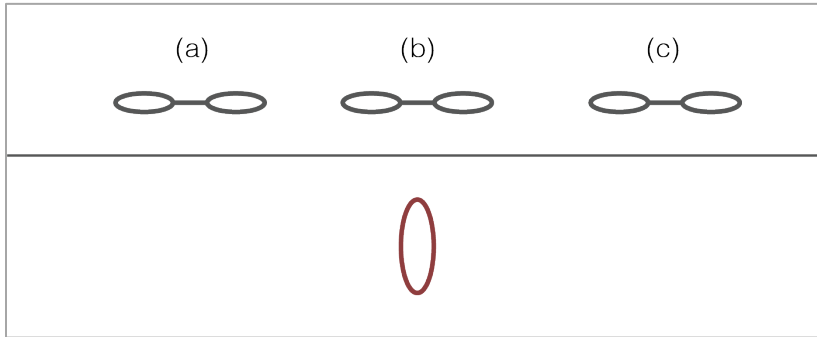
Coupling coefficient

- Depends on geometry

$$M_{12} = \frac{\mu_0}{4\pi} \oint \oint \frac{dl_1 \cdot dl_2}{|\mathbf{r} - \mathbf{r}'|^2}$$

# Conductor in a resistive earth: Frequency

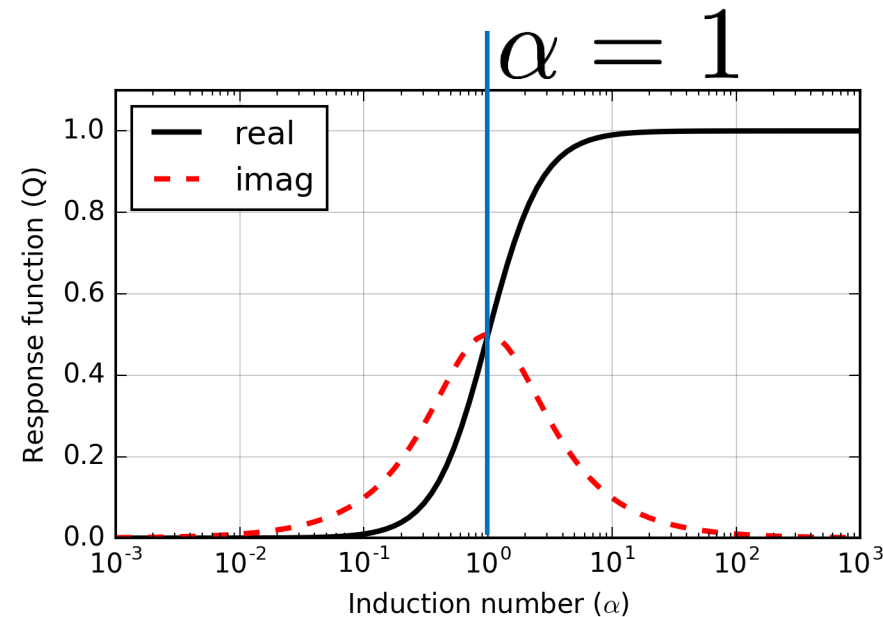
Profile over the loop



- Induction number

$$\alpha = \frac{\omega L}{R}$$

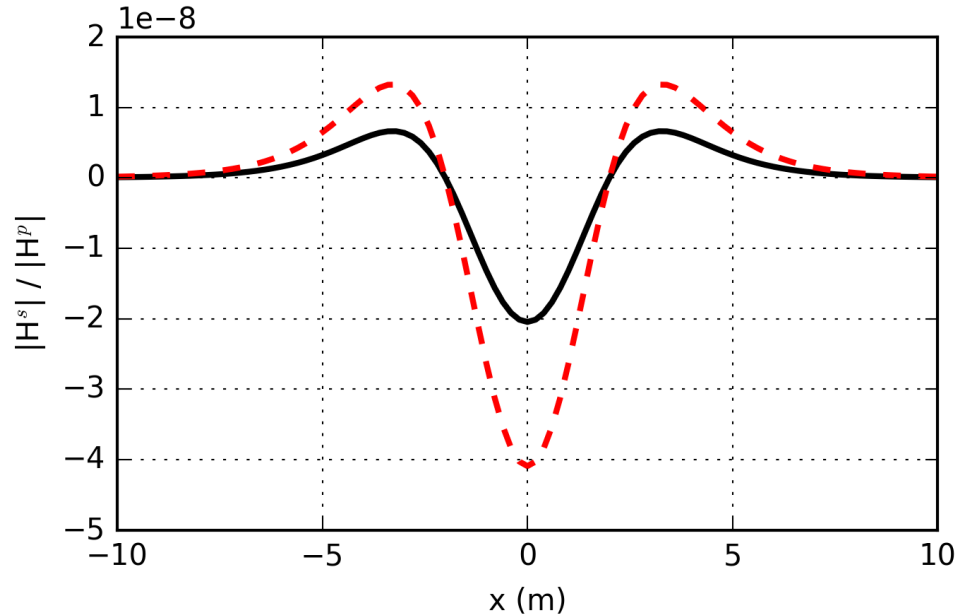
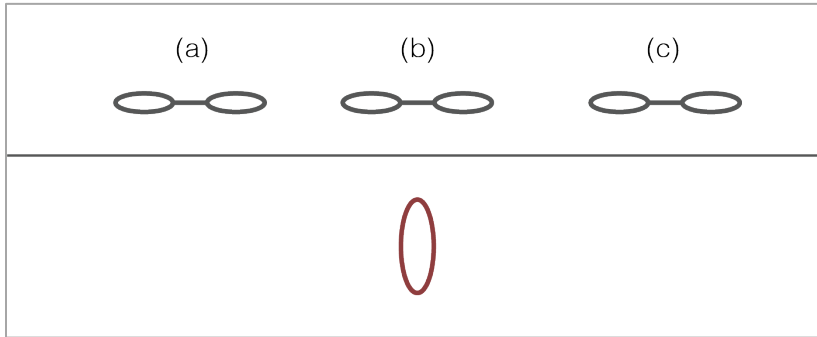
- When  $\alpha = 1$ 
  - Real = Imag





# Conductor in a resistive earth: Frequency

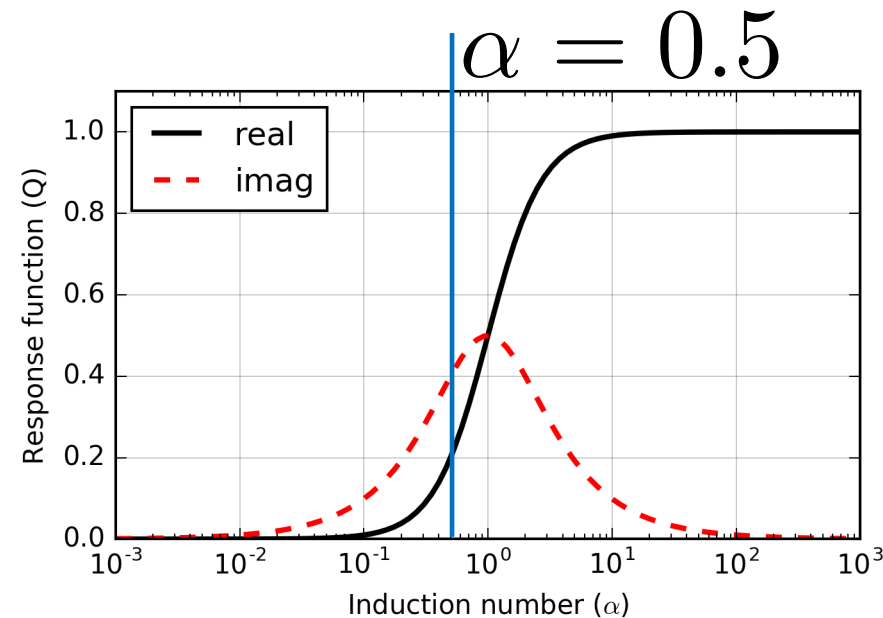
Profile over the loop



- Induction number

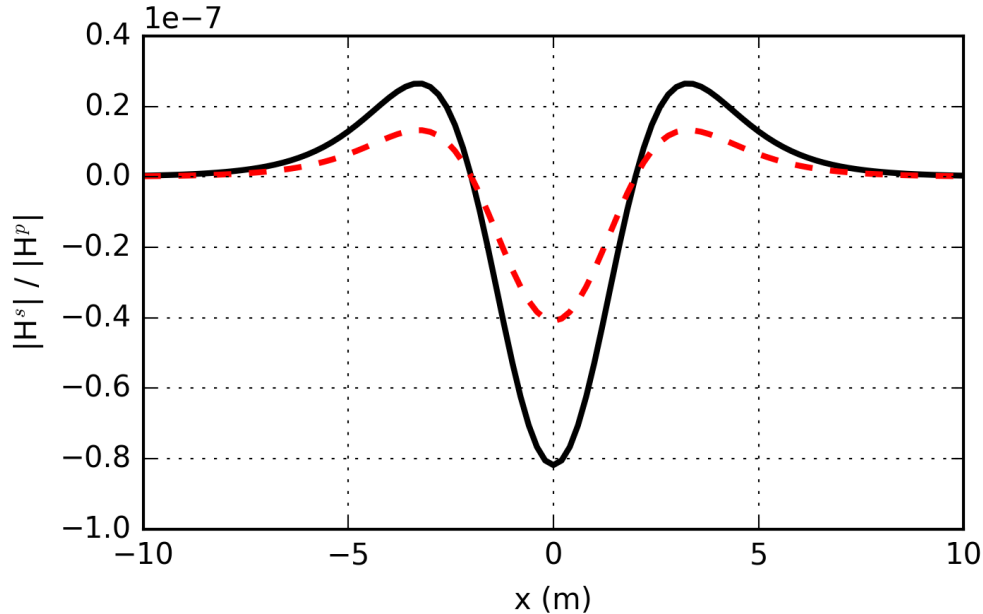
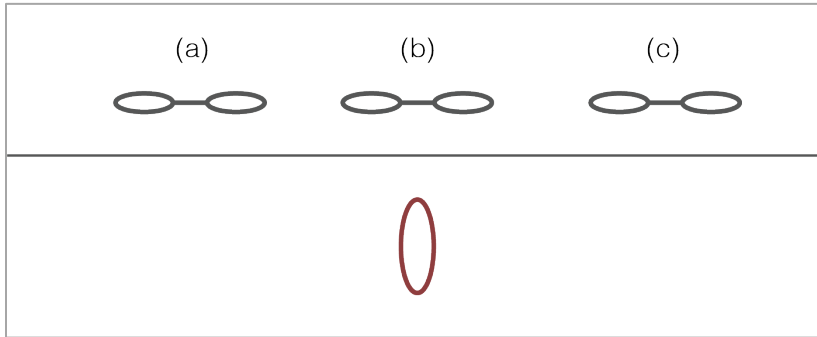
$$\alpha = \frac{\omega L}{R}$$

- When  $\alpha < 1$ 
  - Real < Imag



# Conductor in a resistive earth: Frequency

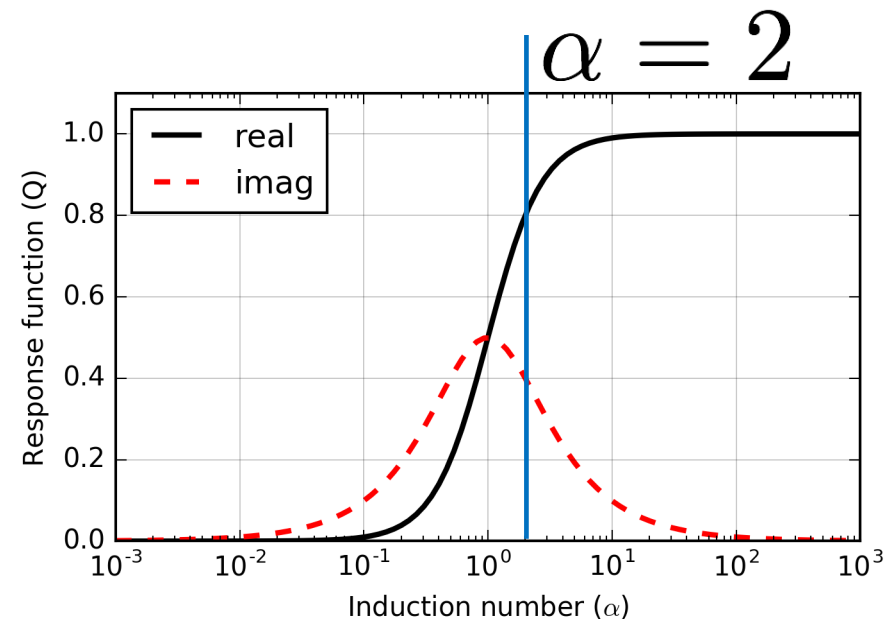
Profile over the loop



- Induction number

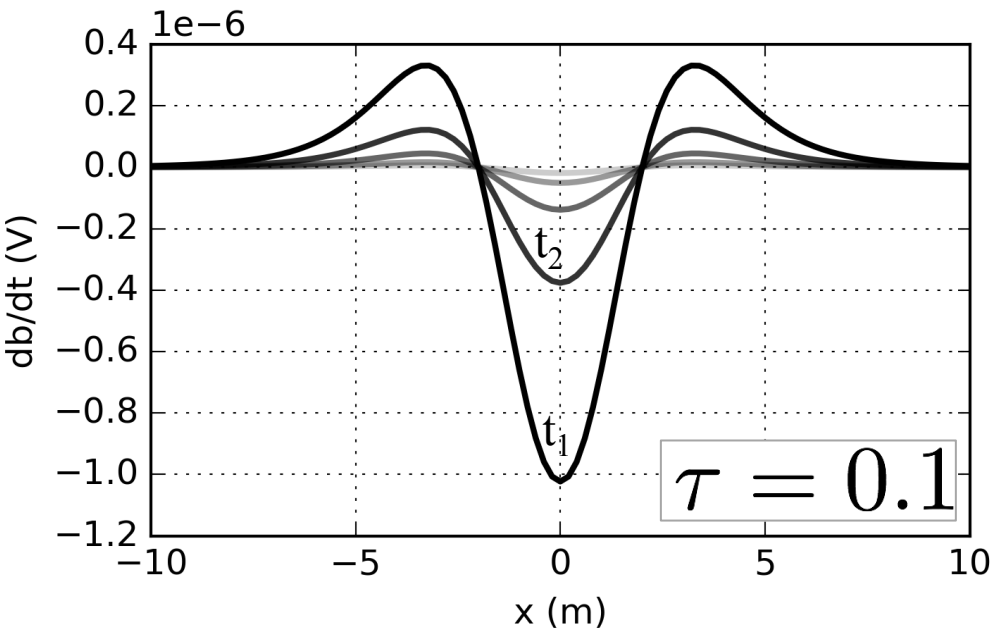
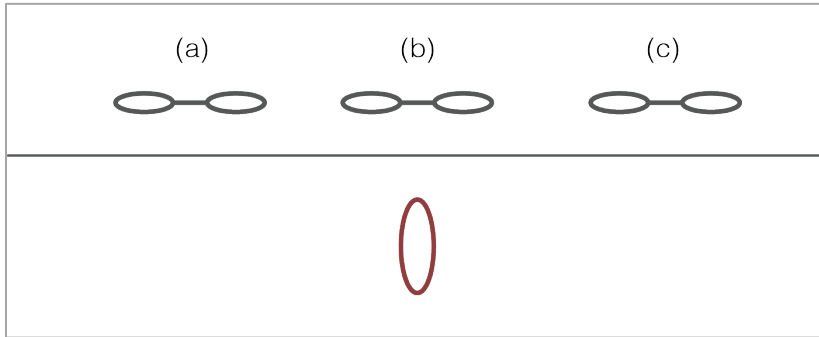
$$\alpha = \frac{\omega L}{R}$$

- When  $\alpha > 1$ 
  - Real > Imag



# Conductor in a resistive earth: Transient

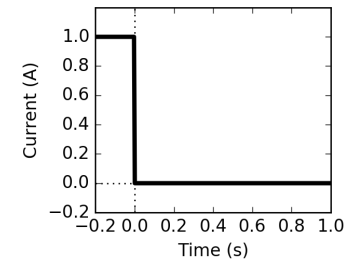
Profile over the loop



- Time constant

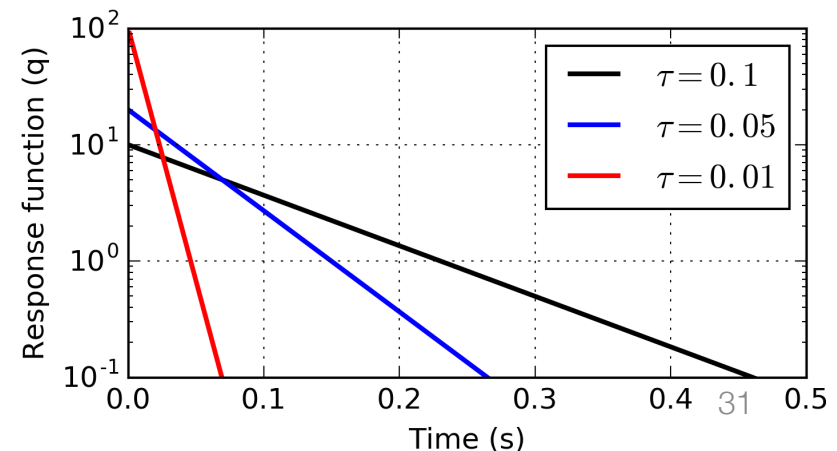
$$\tau = L/R$$

- Step-off current in Tx



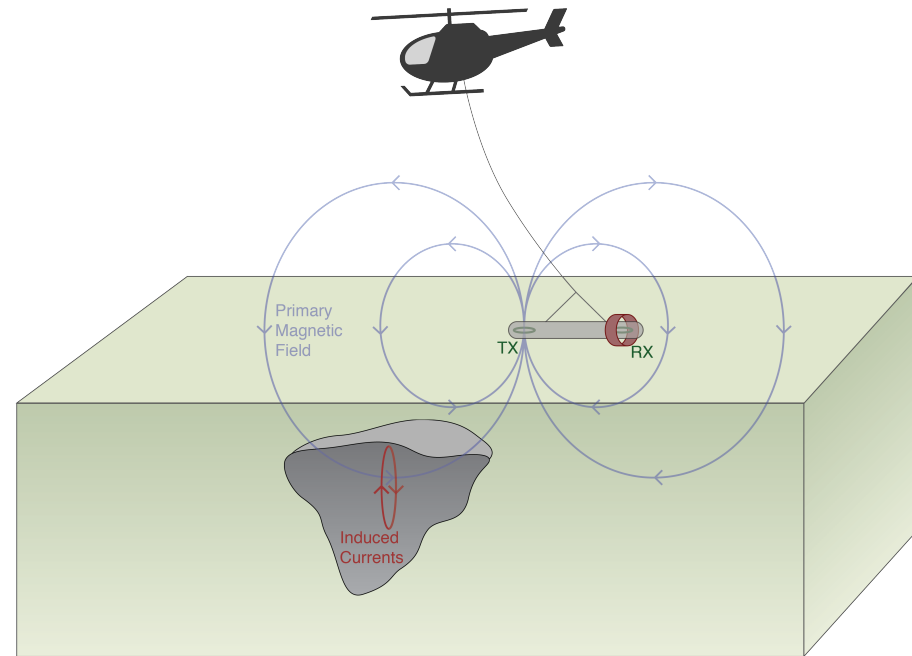
- Response function depends on time,  $\tau$

$$q(t) = e^{-t/\tau}$$



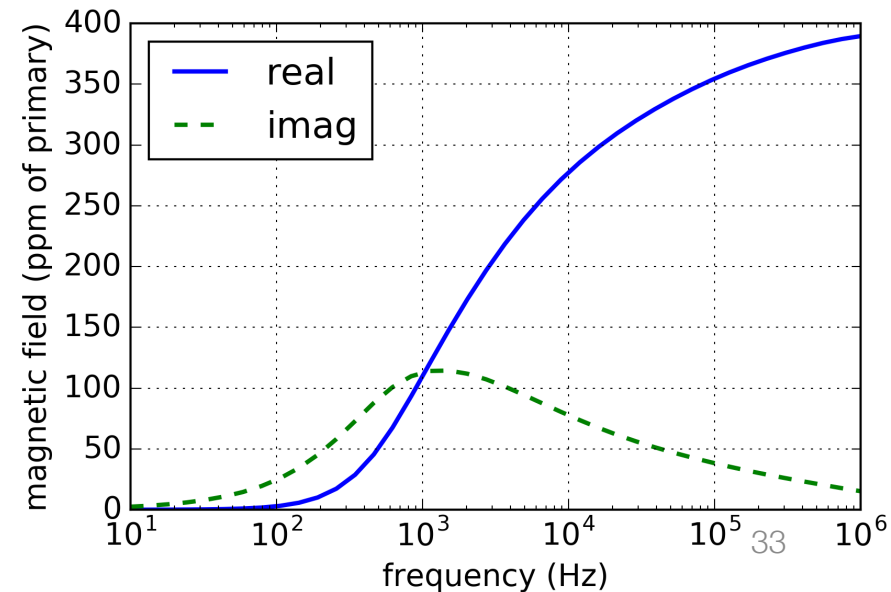
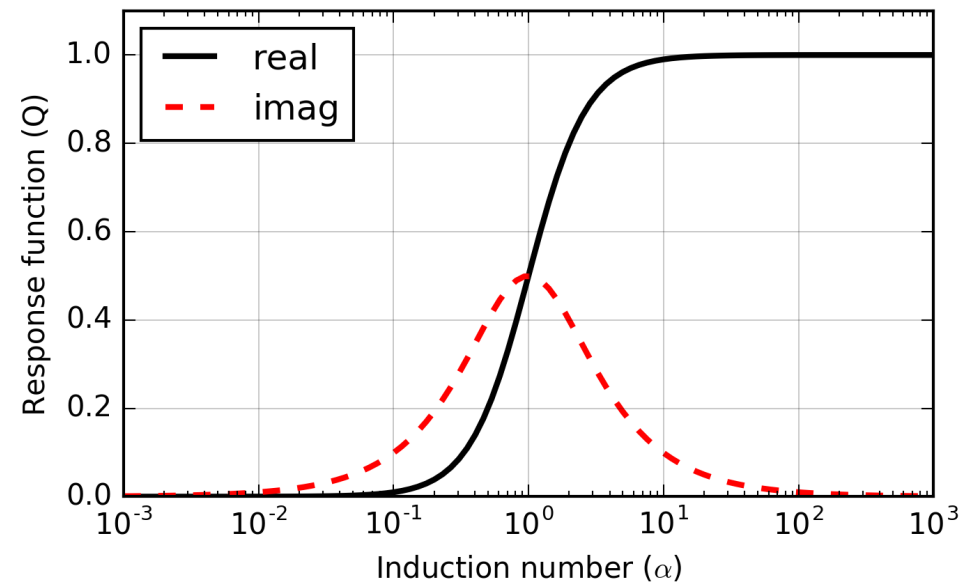
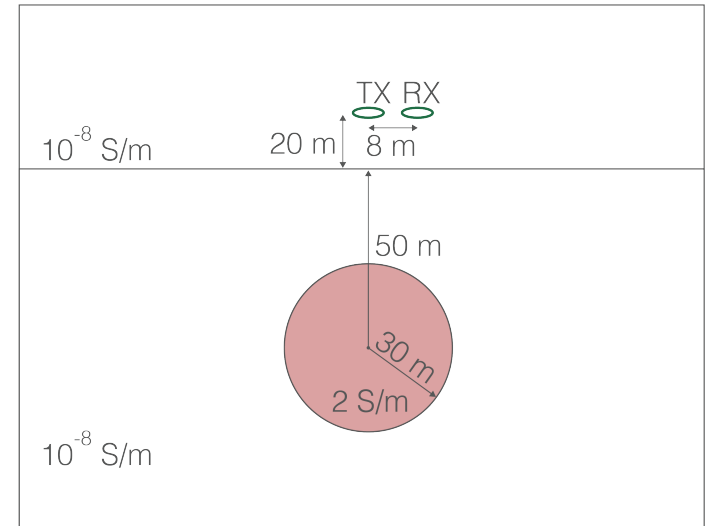
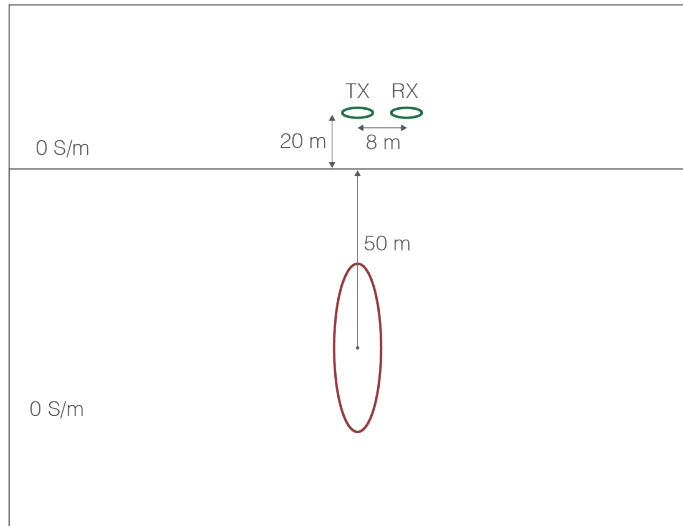
# Recap: what have we learned?

- Basics of EM induction
- Response functions
- Mutual coupling
- Data for frequency or time domain systems
- Circuit model provides representative results
  - Applicable to geologic targets?



# Sphere in a resistive background

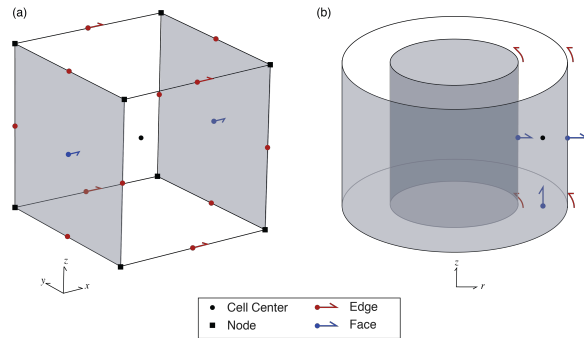
How representative is a circuit model?



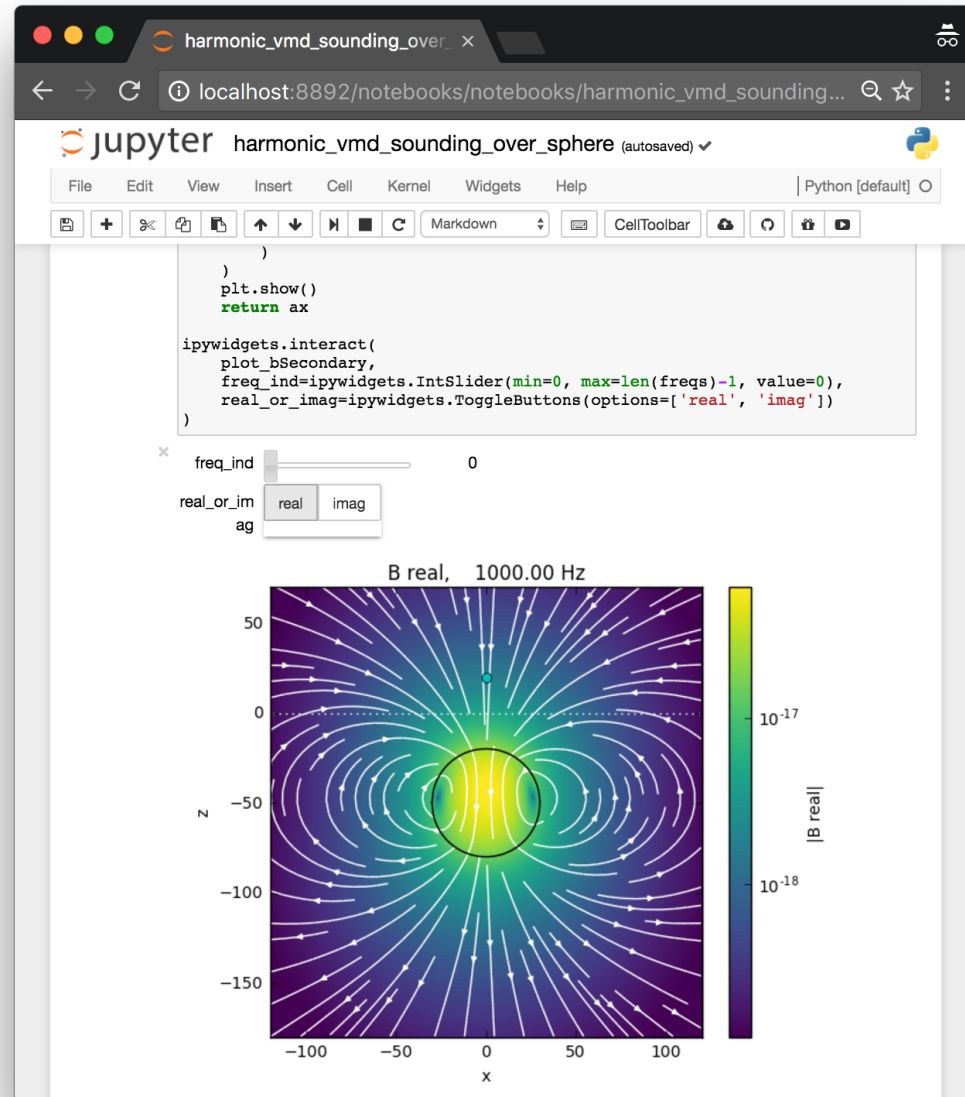
# Cyl Code



- Finite Volume EM
  - Frequency and Time



- Built on SimPEG
- Open source, available at:  
<http://em.geosci.xyz/apps.html>
- Papers
  - [Cockett et al, 2015](#)
  - [Heagy et al, 2017](#)



# Recap: what have we learned?

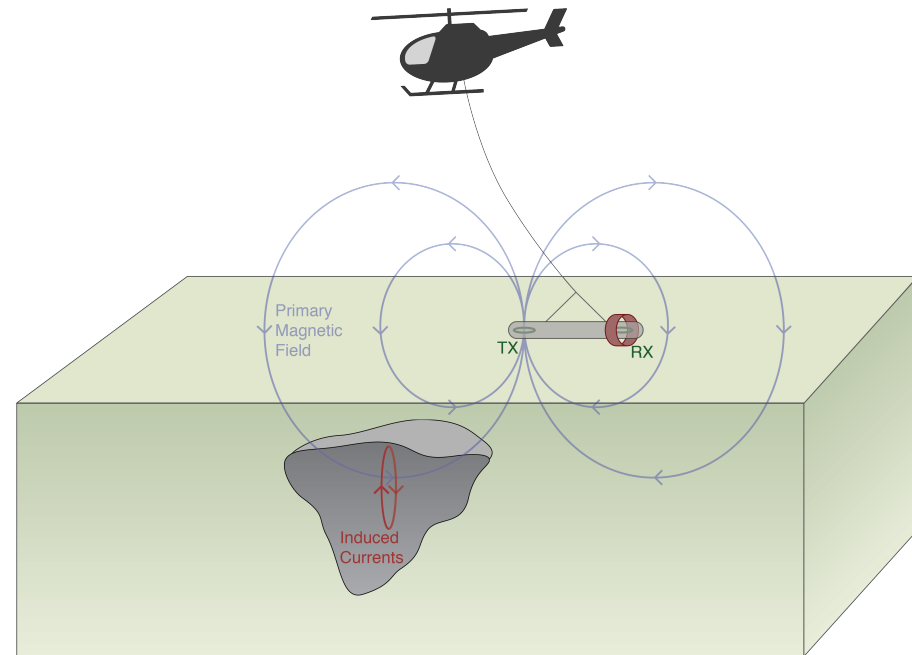
- Basics of EM induction
- Response functions
- Mutual coupling
- Data for frequency or time domain systems
- Circuit model is a good proxy

## 2-Coil Apps

- Frequency domain
- Time domain

Major item not yet accounted for...

- Propagation of energy from
  - Transmitter to target
  - Target to receiver



How do EM fields and fluxes behave in a  
conductive background?



# Revisit Maxwell's equations

First order equations

$$\begin{aligned}\nabla \times \mathbf{e} &= -\frac{\partial \mathbf{b}}{\partial t} & \mathbf{j} &= \sigma \mathbf{e} \\ \nabla \times \mathbf{h} &= \mathbf{j} + \frac{\partial \mathbf{d}}{\partial t} & \mathbf{b} &= \mu \mathbf{h} \\ & & \mathbf{d} &= \epsilon \mathbf{e}\end{aligned}$$

Second order equations

$$\nabla^2 \mathbf{h} - \underbrace{\mu\sigma \frac{\partial \mathbf{h}}{\partial t}}_{\text{diffusion}} - \underbrace{\mu\epsilon \frac{\partial^2 \mathbf{h}}{\partial t^2}}_{\text{wave propagation}} = 0$$

In frequency

$$\begin{aligned}\nabla^2 \mathbf{H} + k^2 \mathbf{H} &= 0 \\ k^2 &= \omega^2 \mu \epsilon - i\omega \mu \sigma\end{aligned}$$

# Plane waves in a homogeneous media

In frequency

$$\nabla^2 \mathbf{H} + k^2 \mathbf{H} = 0$$

$$k^2 = \omega^2 \mu \epsilon - i \omega \mu \sigma$$

Quasi-static

$$\frac{\omega \epsilon}{\sigma} \ll 1$$

even if...

$$\sigma = 10^{-4} \text{ S/m}$$

$$f = 10^4 \text{ Hz}$$

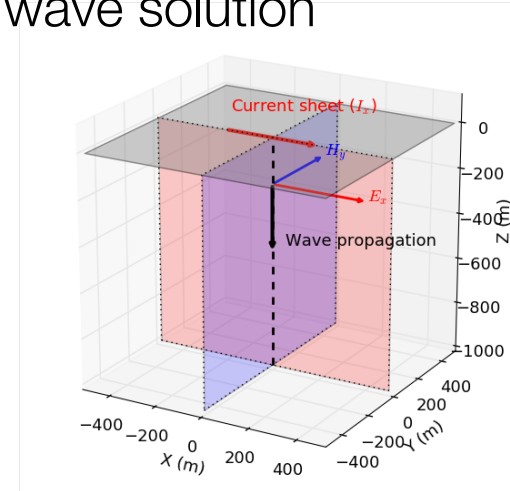
then

$$\frac{\omega \epsilon}{\sigma} \sim 0.005$$

$$k = \sqrt{-i \omega \mu \sigma} = (1 - i) \sqrt{\frac{\omega \mu \sigma}{2}}$$

$$\equiv \alpha - i \beta$$

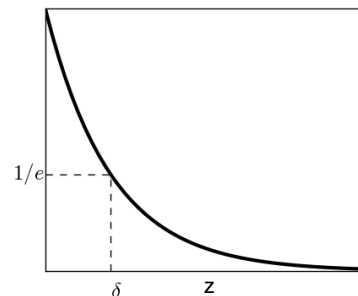
Plane wave solution



$$\mathbf{H} = \underbrace{\mathbf{H}_0 e^{-\alpha z}}_{\text{attenuation}} \underbrace{e^{-i(\beta z - \omega t)}}_{\text{phase}}$$

Skin depth

$\delta$  : skin depth



$$\delta = \sqrt{\frac{2}{\omega \mu \sigma}} = 503 \sqrt{\frac{1}{\sigma f}}$$

# Plane waves in a homogeneous media

In time

$$\nabla^2 \mathbf{h} - \mu\epsilon \frac{\partial^2 \mathbf{h}}{\partial t^2} - \mu\sigma \frac{\partial \mathbf{h}}{\partial t} = 0$$

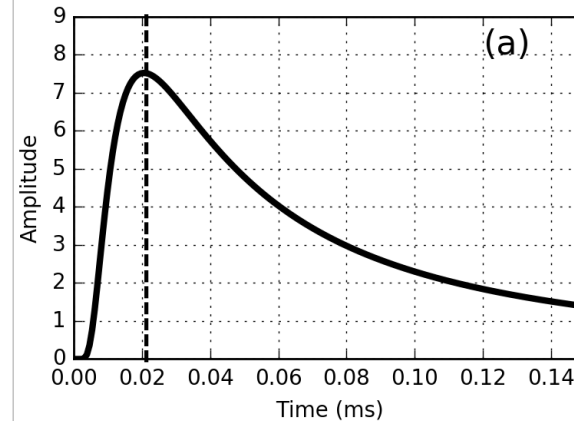
$$\mathbf{h}(t = 0) = \mathbf{h}_0 \delta(t)$$

Solution for quasi-static

$$\mathbf{h}(t) = -\frac{(\mu\sigma)^{1/2} z}{2\pi^{1/2} t^{3/2}} e^{-\mu\sigma z^2/(4t)}$$

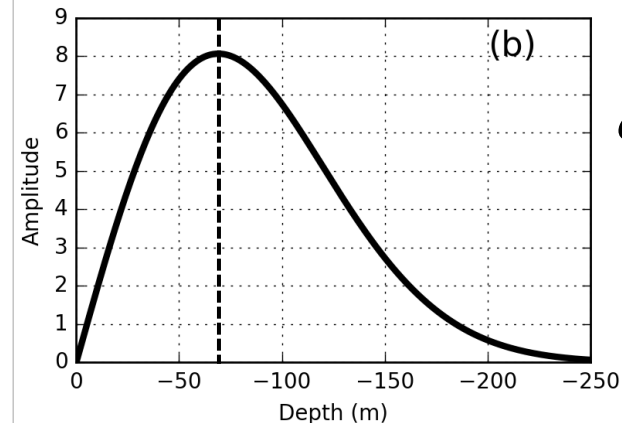
$z$ : depth (m)

Peak time:



$$t_{max} = \frac{\mu\sigma z^2}{6}$$

Diffusion distance



$$d = \sqrt{\frac{2t}{\mu\sigma}}$$

$$\approx 1260 \sqrt{\frac{t}{\sigma}}$$

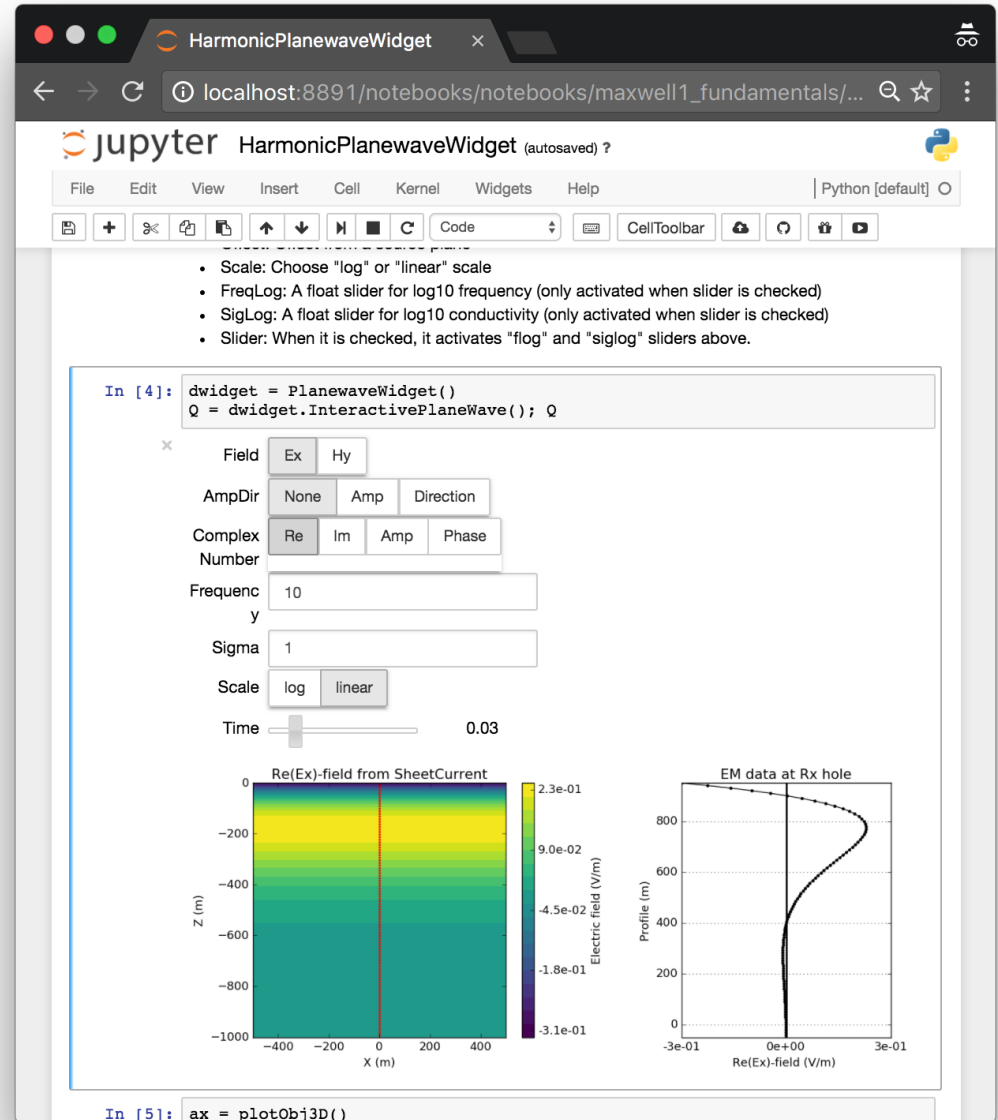
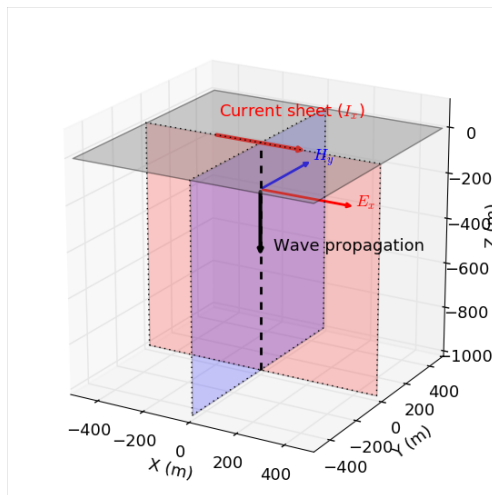
# Plane Wave apps

- 2 apps:
  - Transient

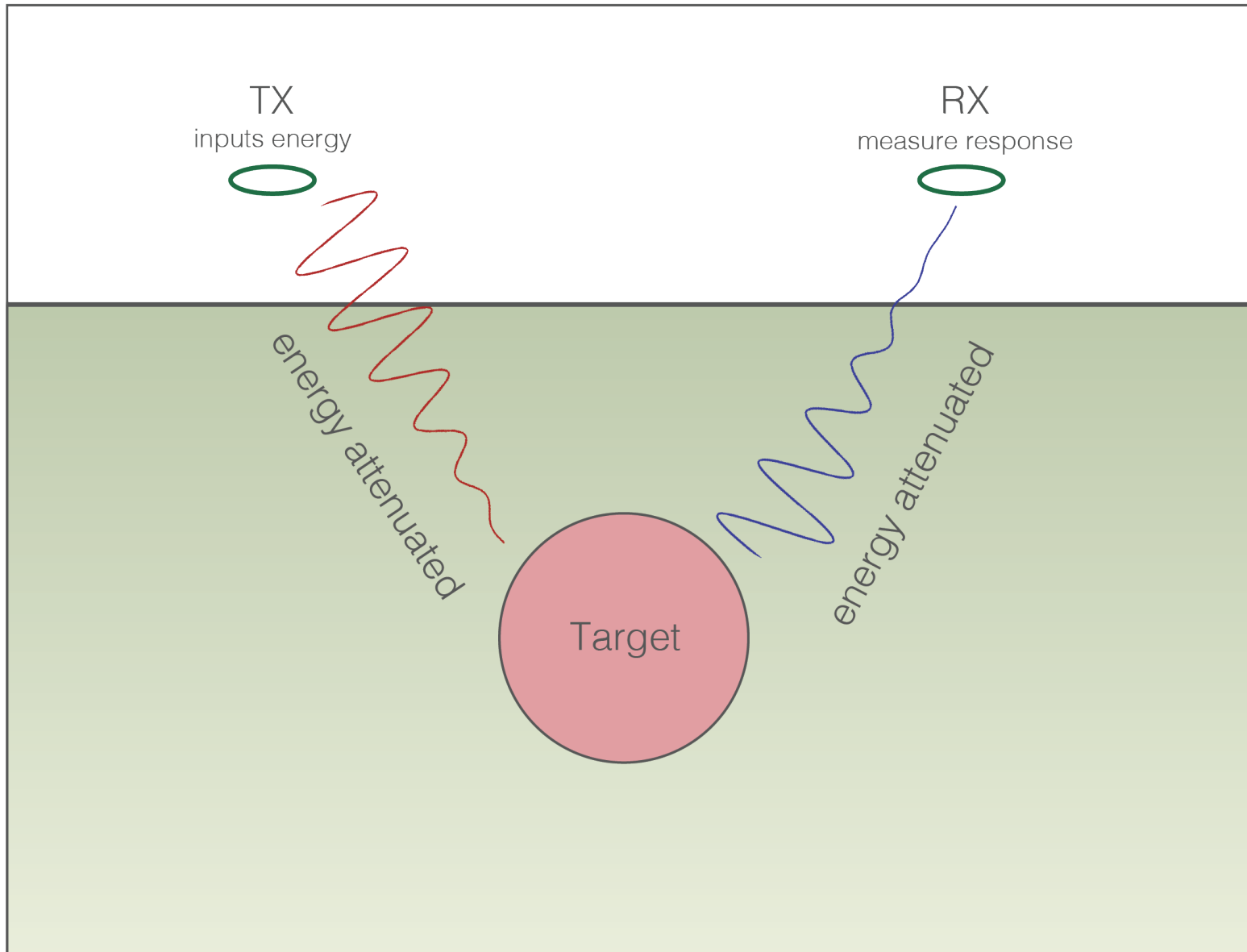
$$\mathbf{h}(t) = -\frac{(\mu\sigma)^{1/2}z}{2\pi^{1/2}t^{3/2}}e^{-\mu\sigma z^2/(4t)}$$

- Harmonic

$$\mathbf{H} = \mathbf{H}_0 \underbrace{e^{-\alpha z}}_{\text{attenuation}} \underbrace{e^{-i(\beta z - \omega t)}}_{\text{phase}}$$

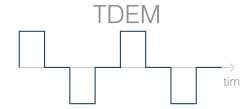


# Effects of background resistivity

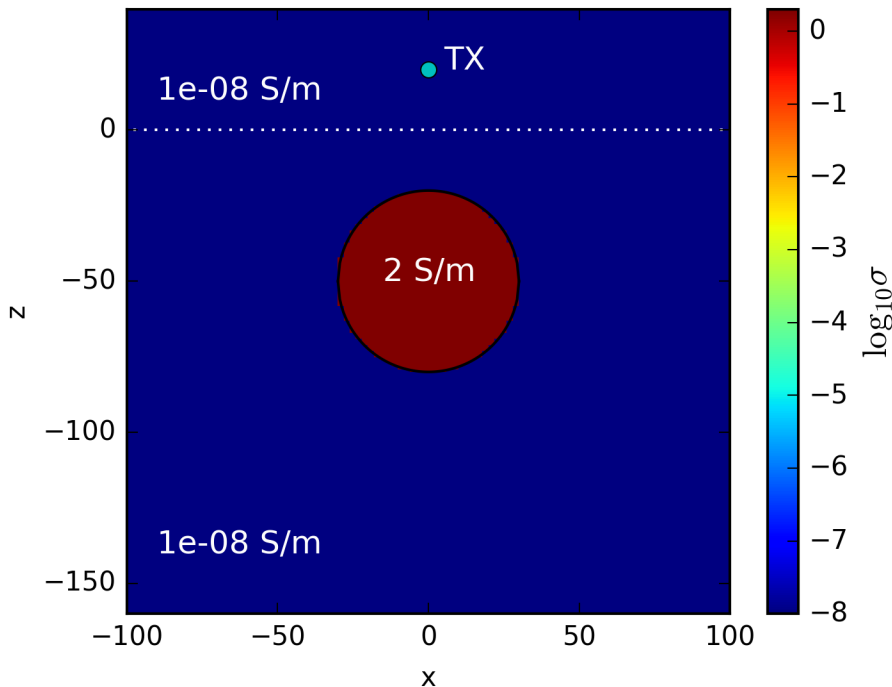


# Effects of background resistivity: Time

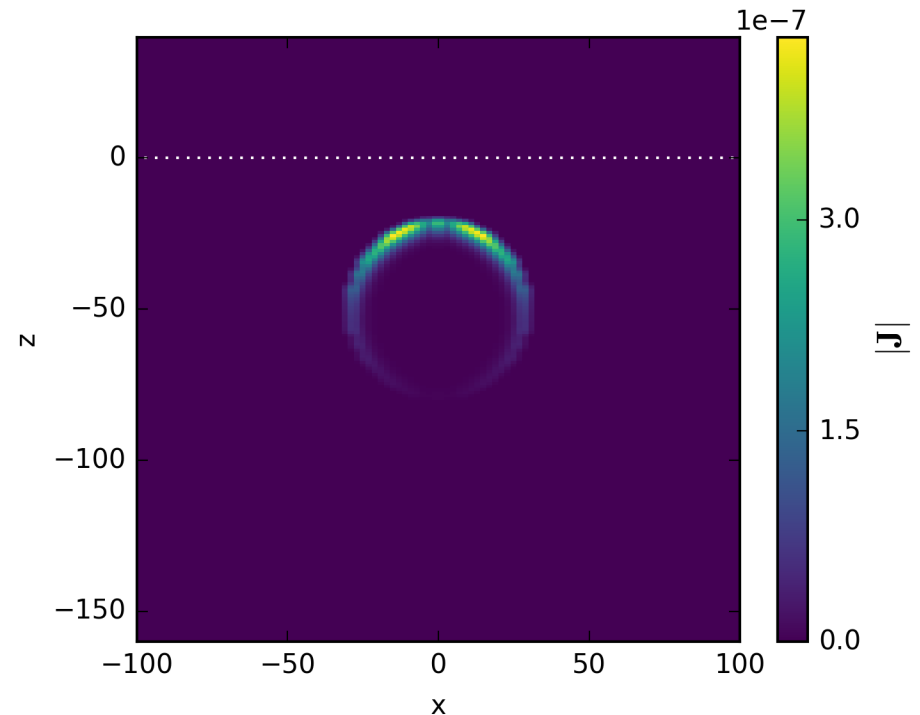
- Buried, conductive sphere
- Vary background conductivity
- Time:  $10^{-5}$  s



$10^{-8}$  S/m background

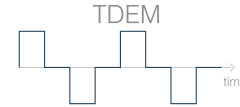


Current Density

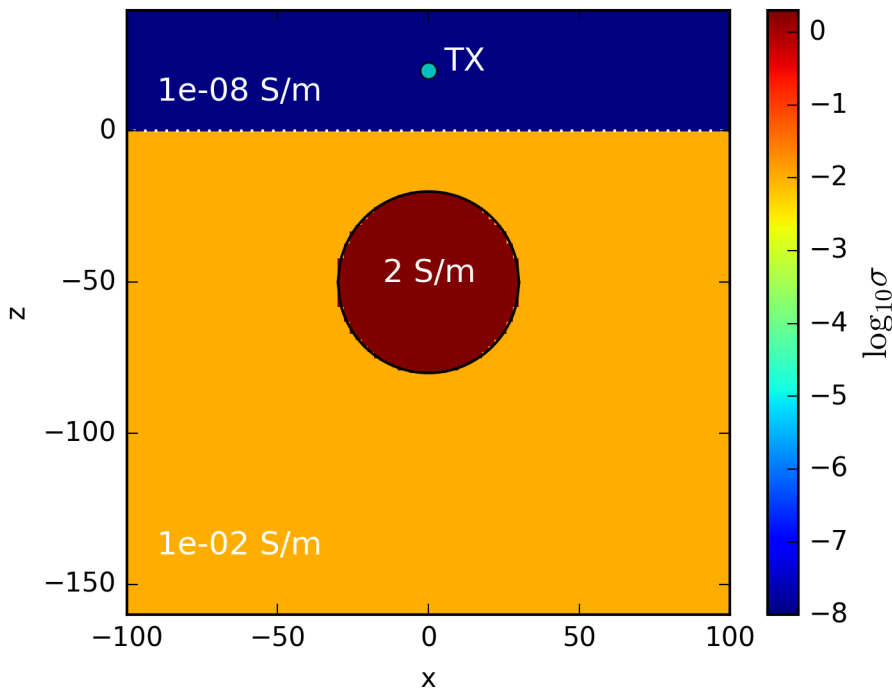


# Effects of background resistivity: Time

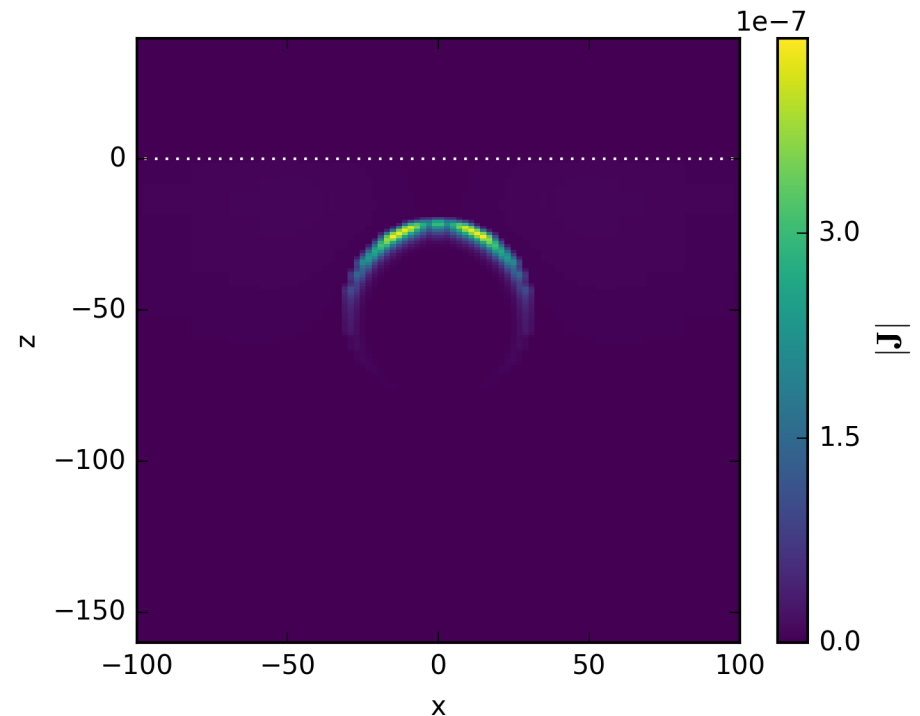
- Buried, conductive sphere
- Vary background conductivity
- Time:  $10^{-5}$  s



$10^{-2}$  S/m background

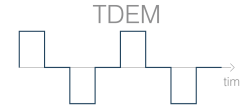


Current Density

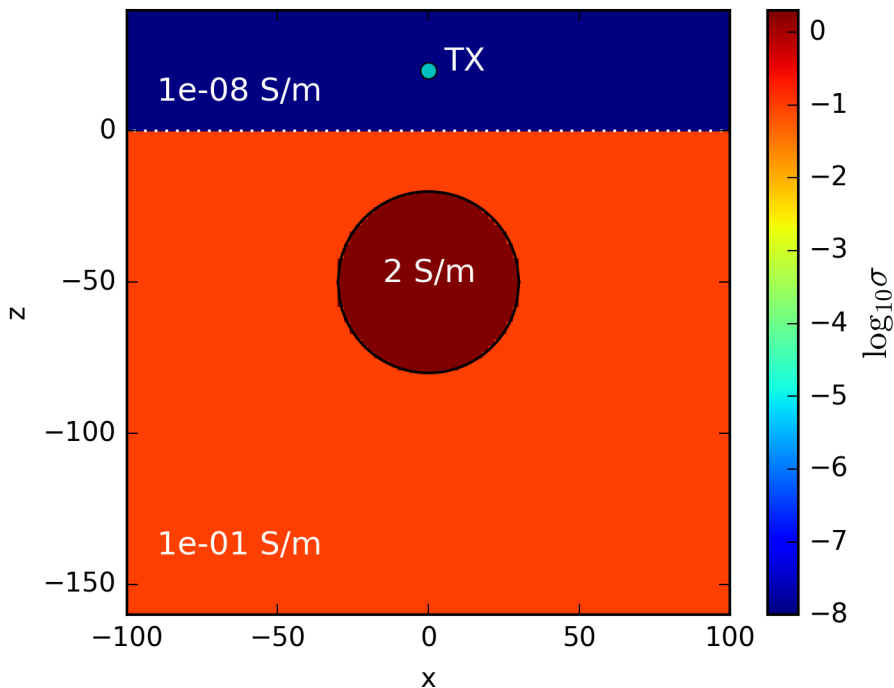


# Effects of background resistivity: Time

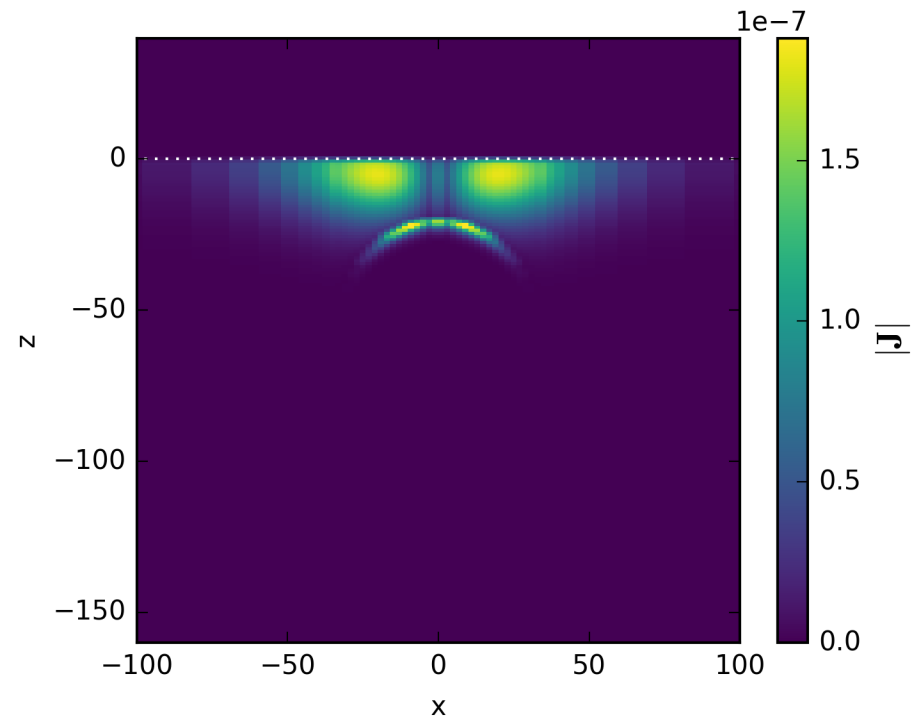
- Buried, conductive sphere
- Vary background conductivity
- Time:  $10^{-5}$  s



$10^{-1}$  S/m background



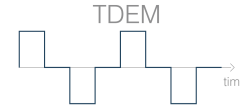
Current Density



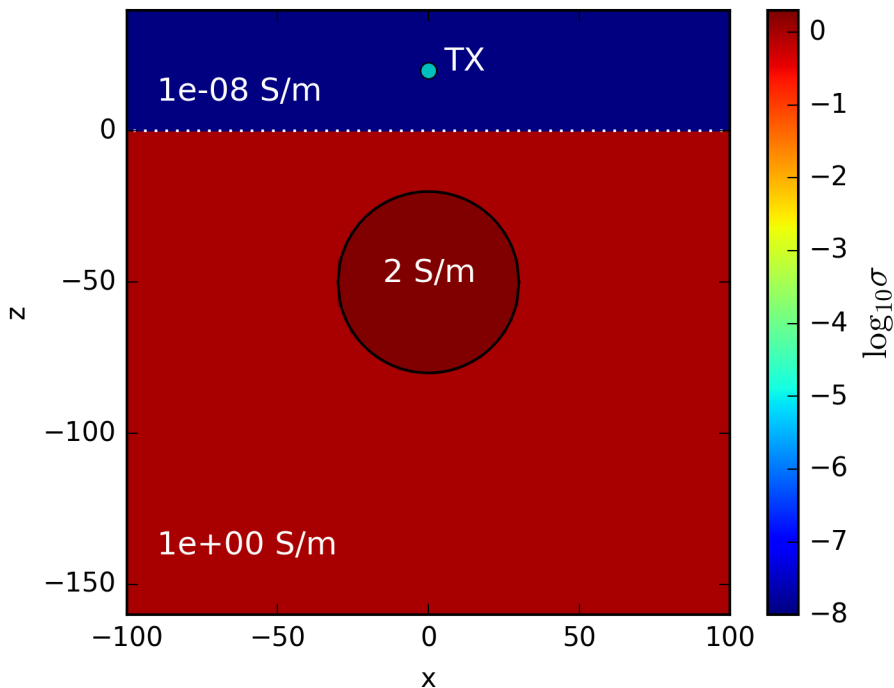


# Effects of background resistivity: Time

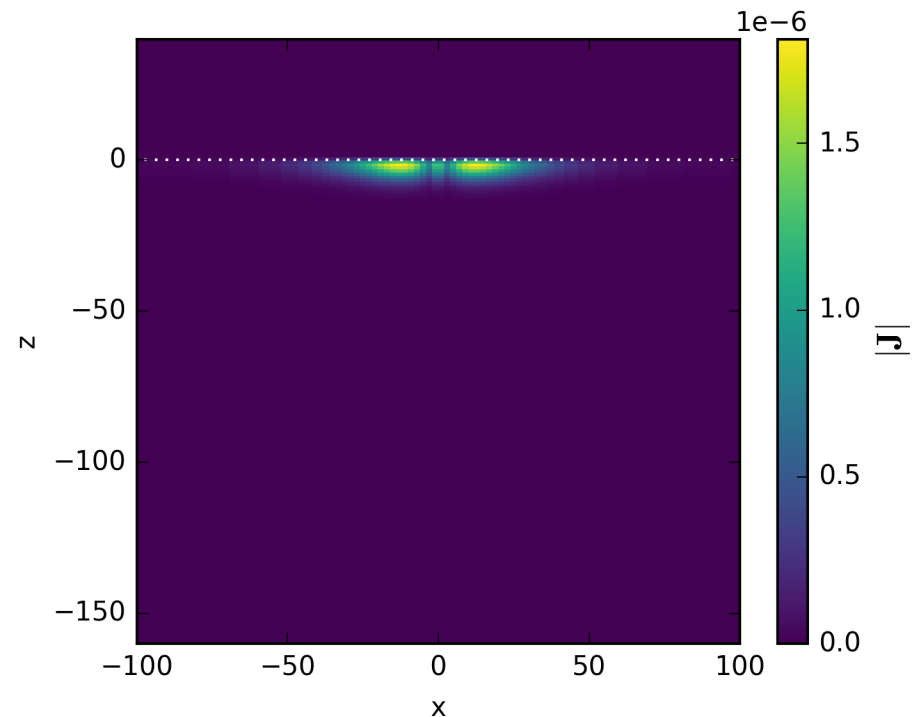
- Buried, conductive sphere
- Vary background conductivity
- Time:  $10^{-5}$  s



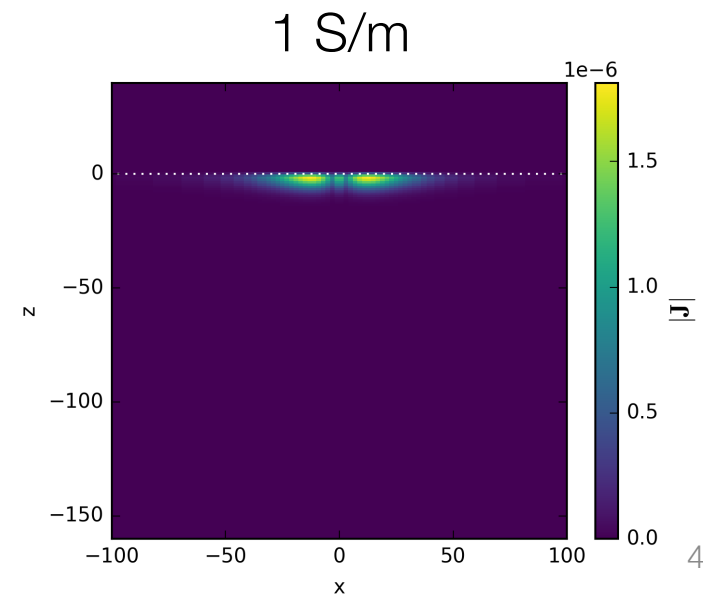
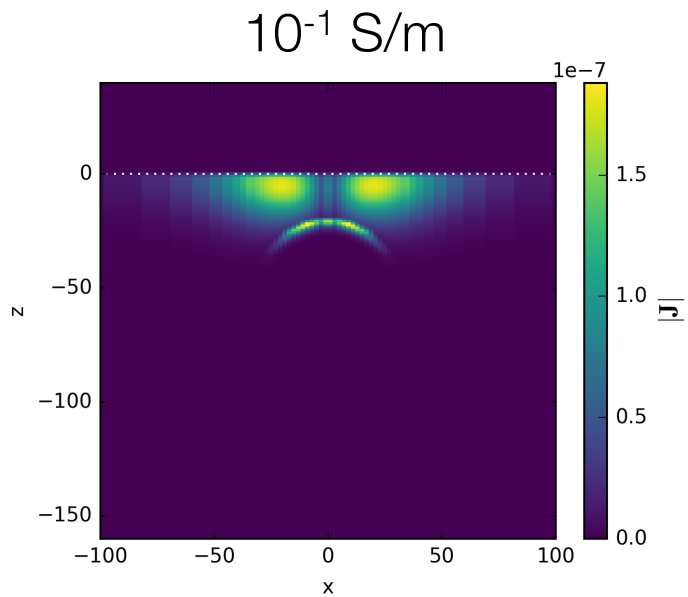
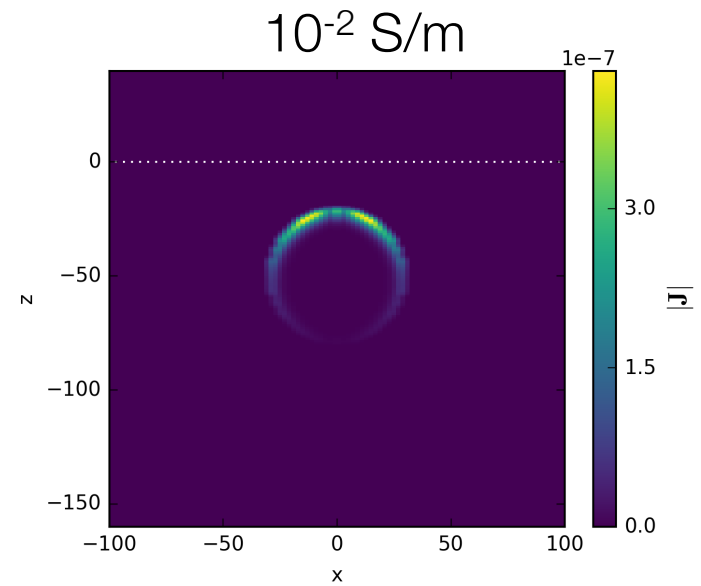
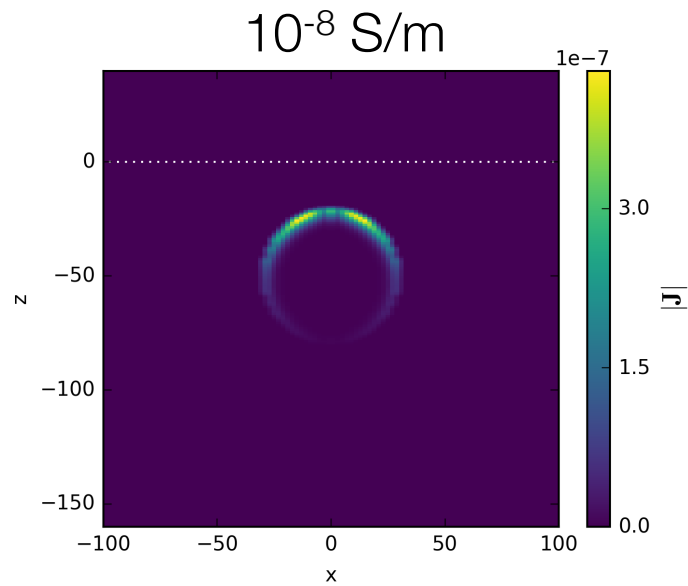
1 S/m background



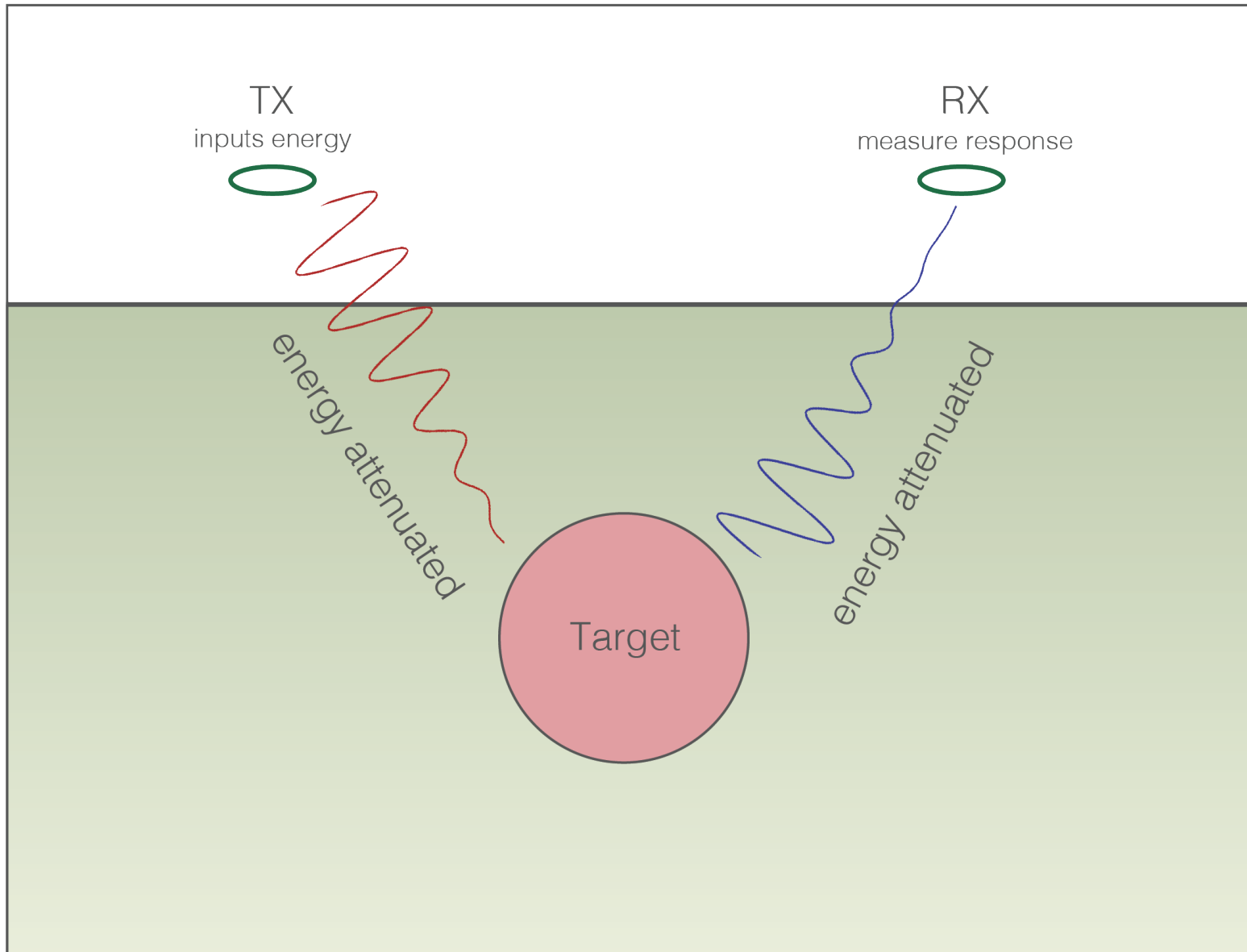
Current Density



# Effects of background resistivity: Time

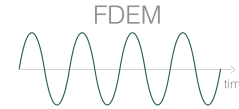


# Effects of background resistivity

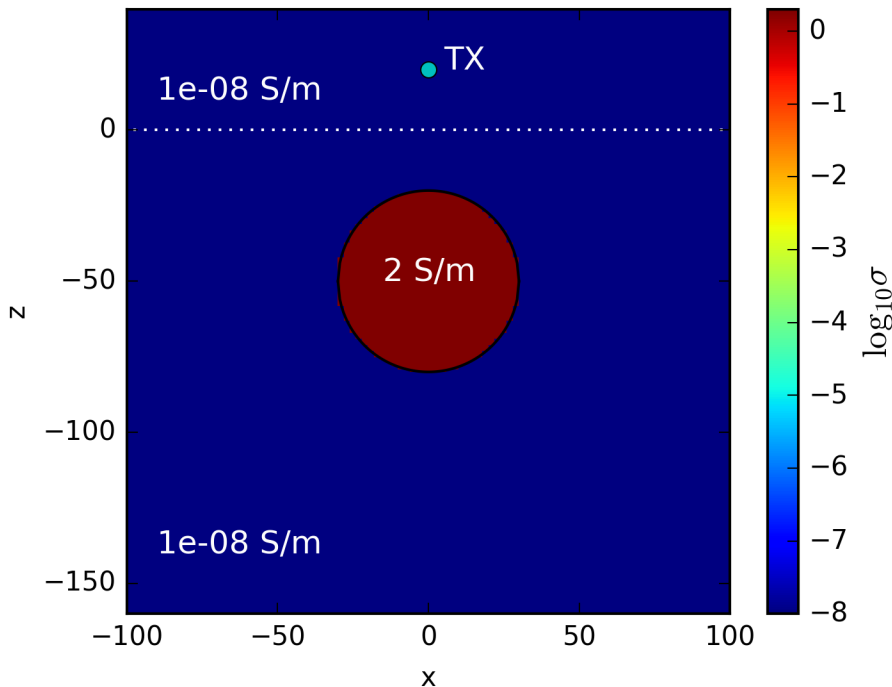


# Effects of background resistivity: Frequency

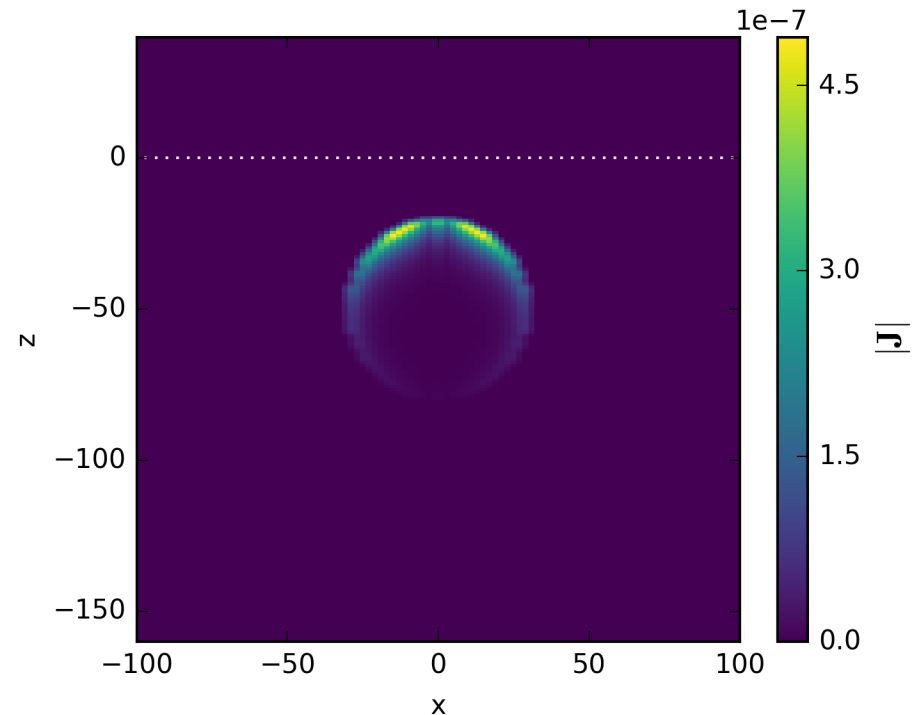
- Buried, conductive sphere
- Vary background conductivity
- Frequency:  $10^4$  Hz



$10^{-8}$  S/m background



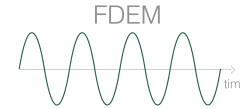
Current Density



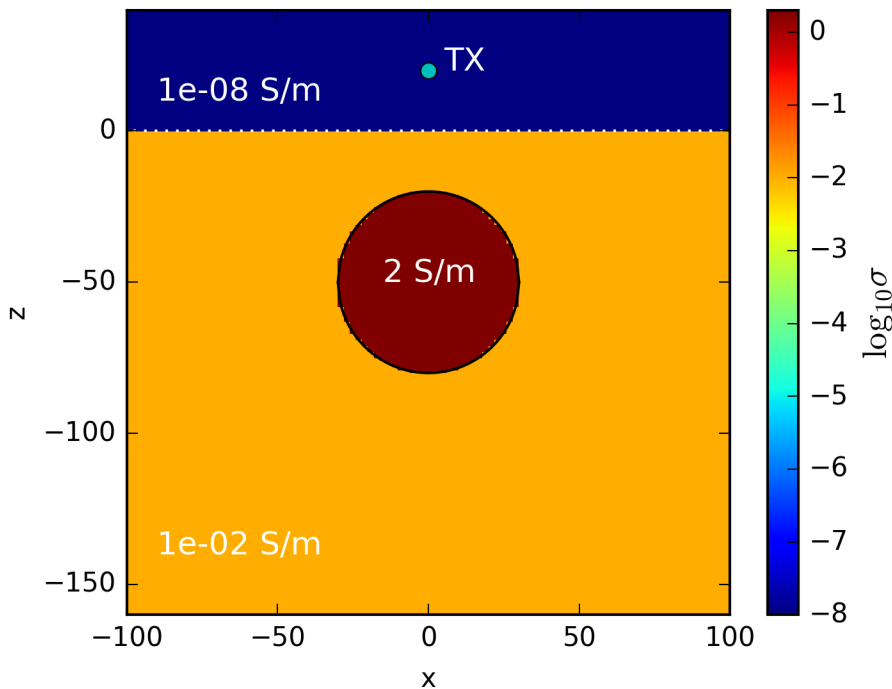
# Effects of background resistivity: Frequency

- Buried, conductive sphere
- Vary background conductivity

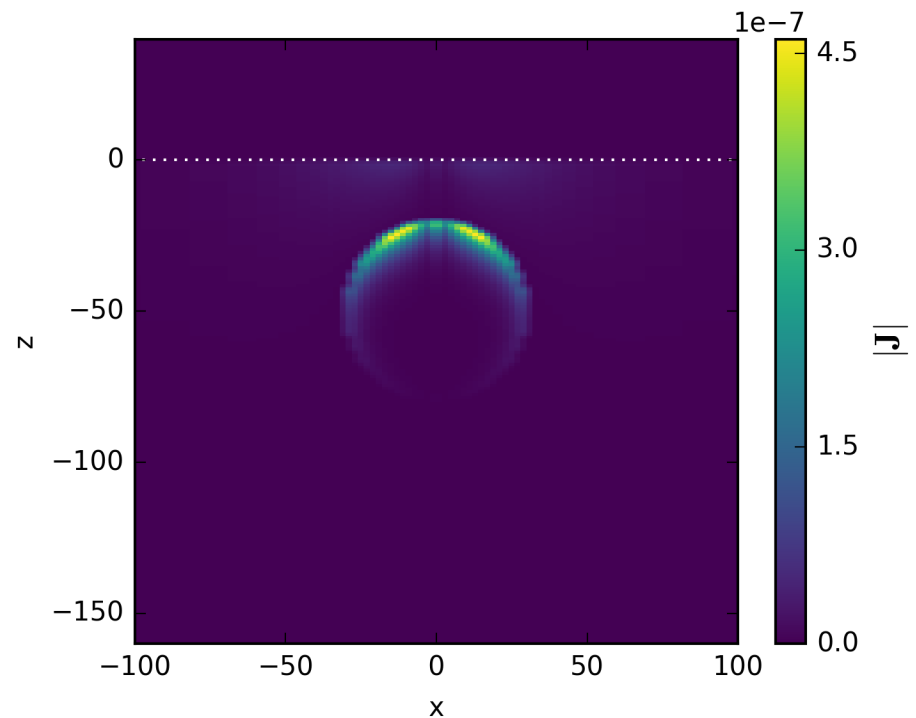
- Frequency:  $10^4$  Hz



$10^{-2}$  S/m background

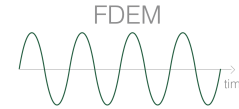


Current Density

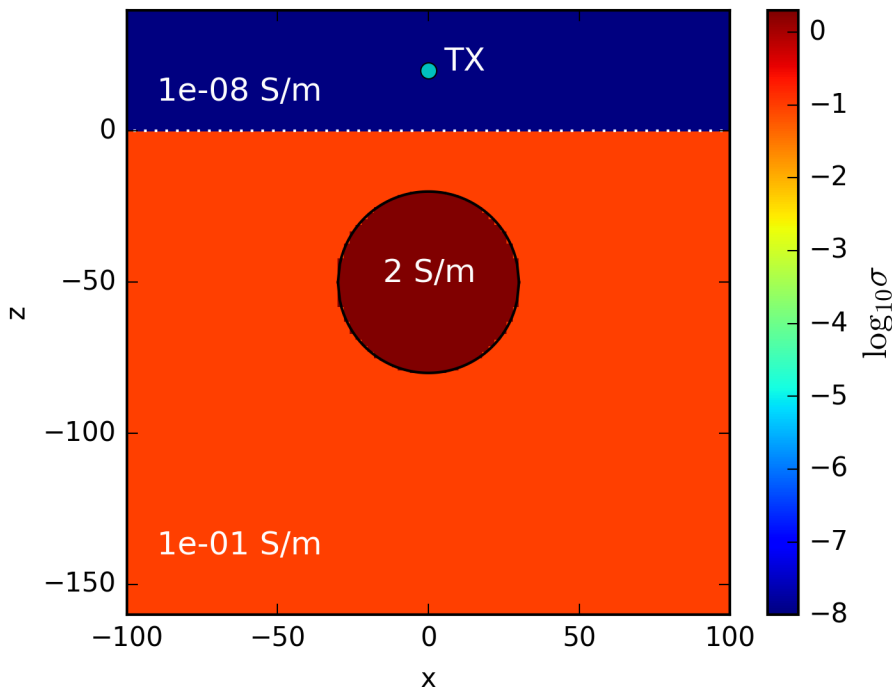


# Effects of background resistivity: Frequency

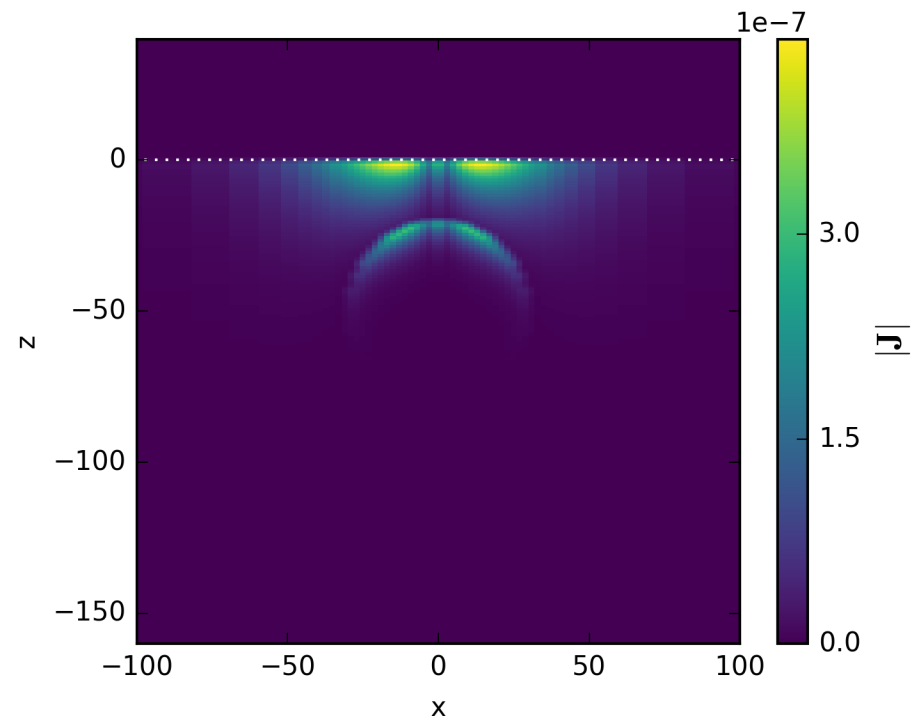
- Buried, conductive sphere
- Vary background conductivity
- Frequency:  $10^4$  Hz



$10^{-1}$  S/m background

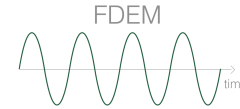


Current Density

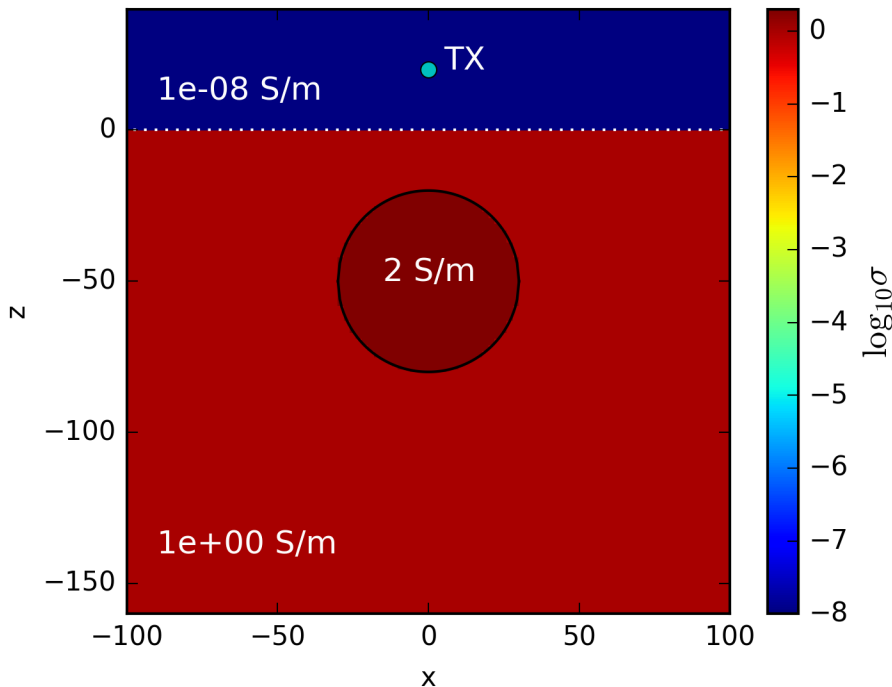


# Effects of background resistivity: Frequency

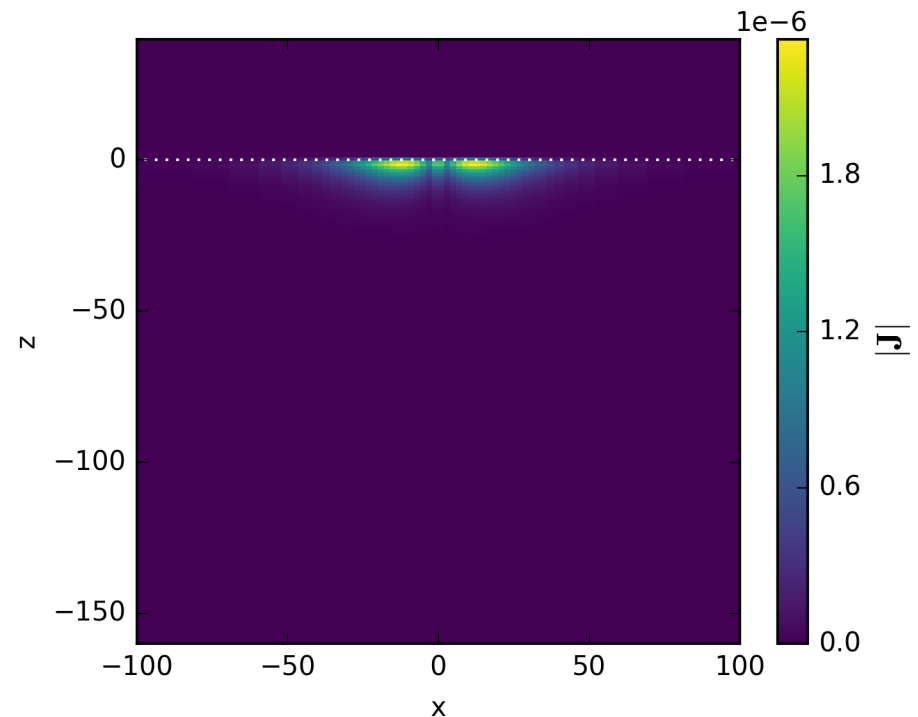
- Buried, conductive sphere
- Vary background conductivity
- Frequency:  $10^4$  Hz



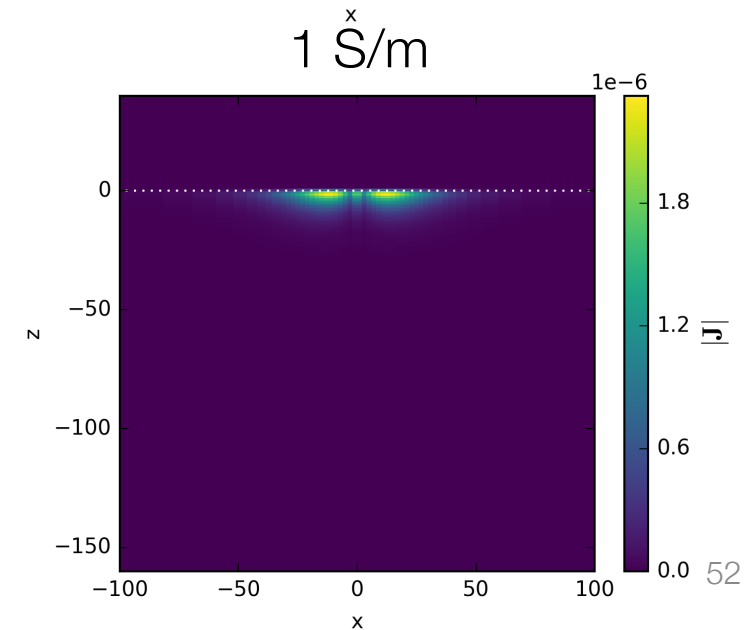
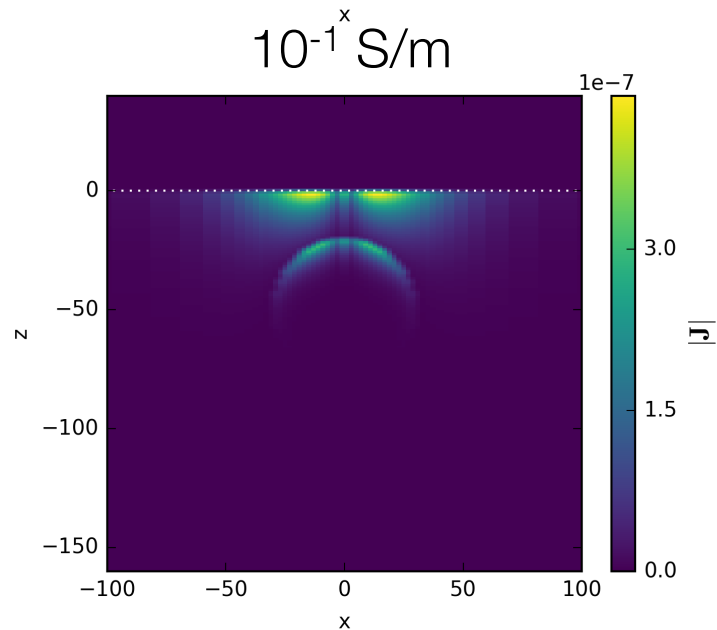
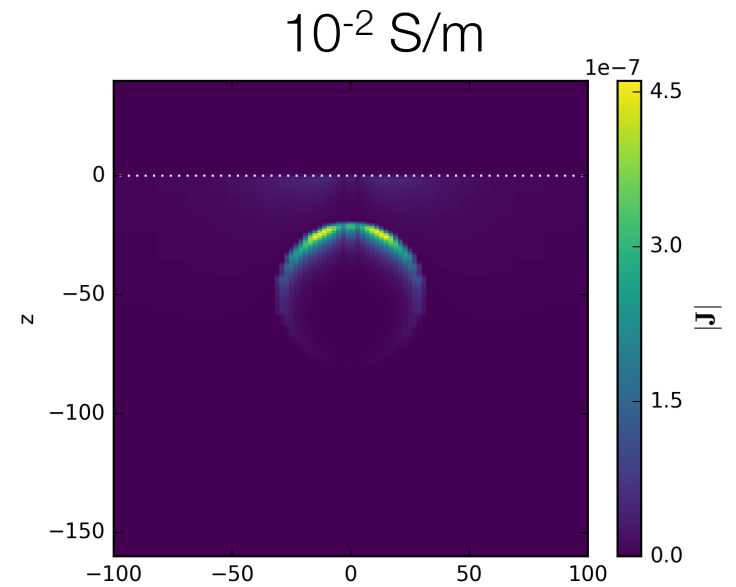
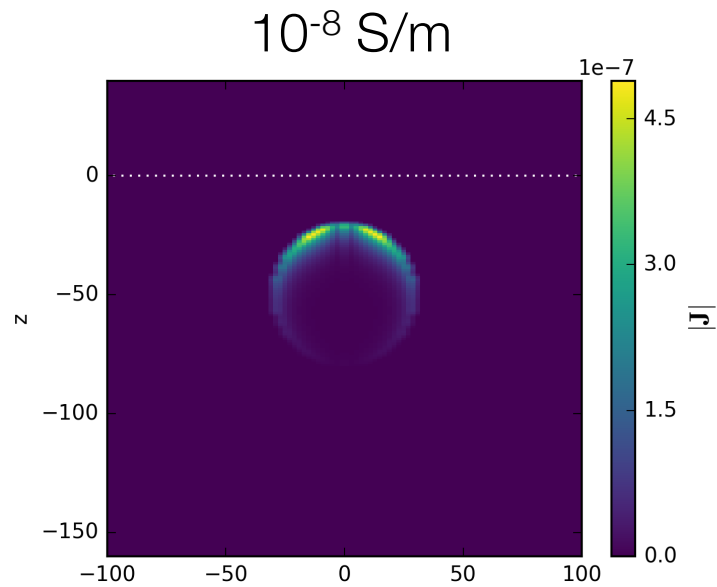
1 S/m background



Current Density



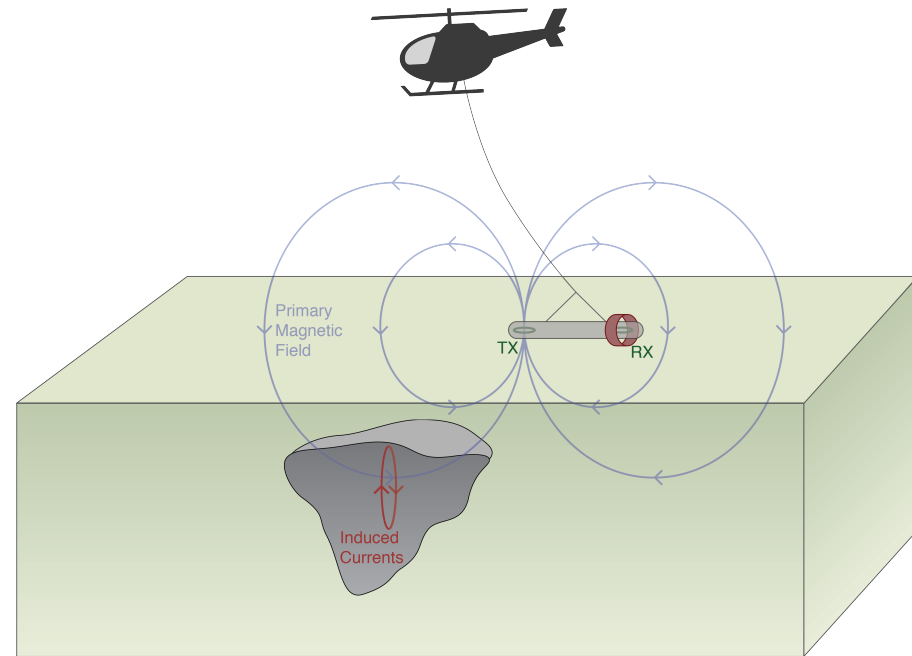
# Effects of background resistivity: Frequency



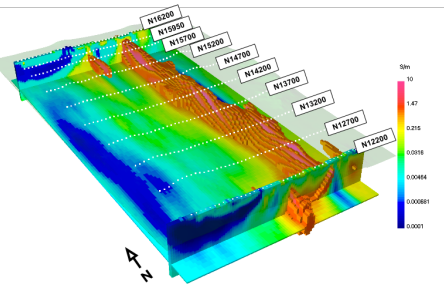


# Recap: what have we learned?

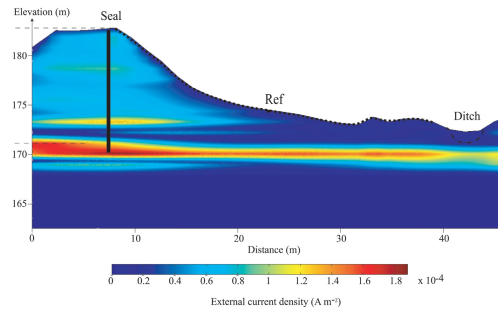
- Basics of EM induction
- Response functions
- Mutual coupling
- Data for frequency or time domain systems
- Circuit model is a good proxy
- Need to account for energy losses
- Ready to look at some field examples



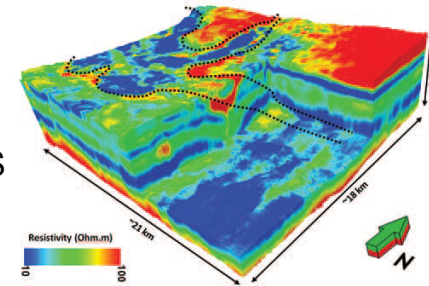
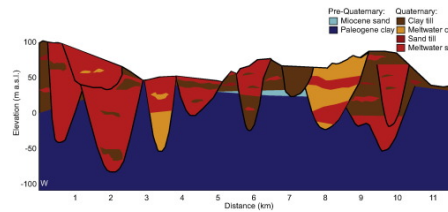
# Today's Case Histories



Mt. Isa, Australia:  
Mineral Exploration

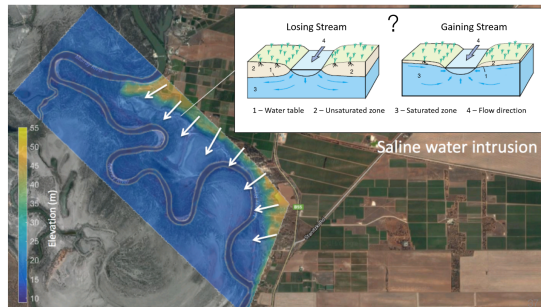


Kasted, Denmark:  
mapping paleochannels

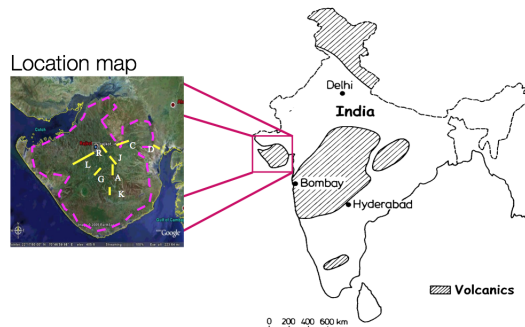


France: Dam Monitoring

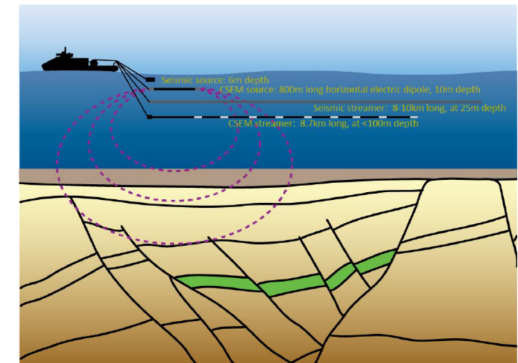
Saudi Arabia: Improving  
a seismic velocity model



Bookpurnong, Australia:  
diagnosing river  
salinization

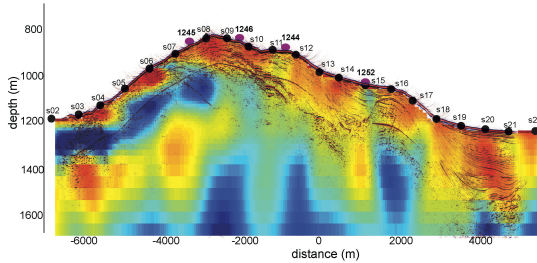


Deccan Traps, India:  
mapping sediment beneath  
basalt

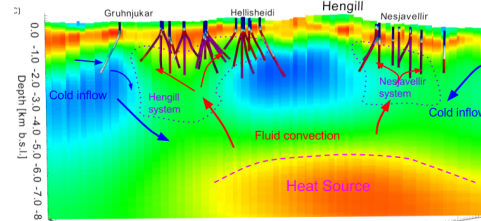


Barents Sea, Norway:  
Hydrocarbon de-risking

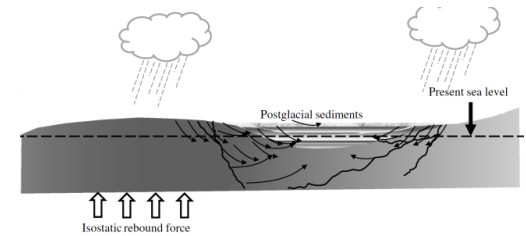
# Today's Case Histories



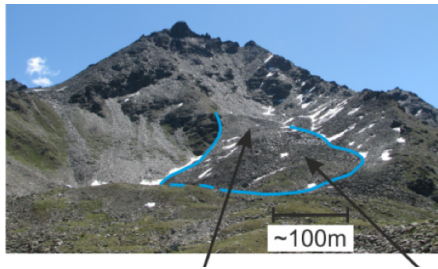
Oregon, USA: methane hydrate



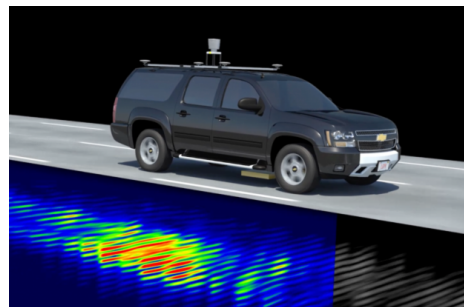
Iceland: characterizing geothermal systems



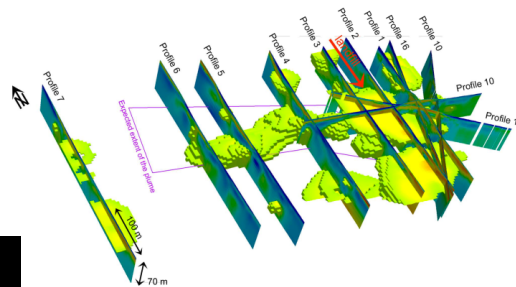
Sweden: Landslides, quick clays



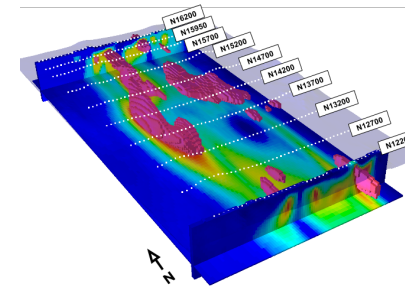
Switzerland: Rock Glaciers



USA: Self-driving vehicles



Denmark: Landfill



Mt. Isa, Australia: Mineral Exploration

# End of EM Fundamentals

