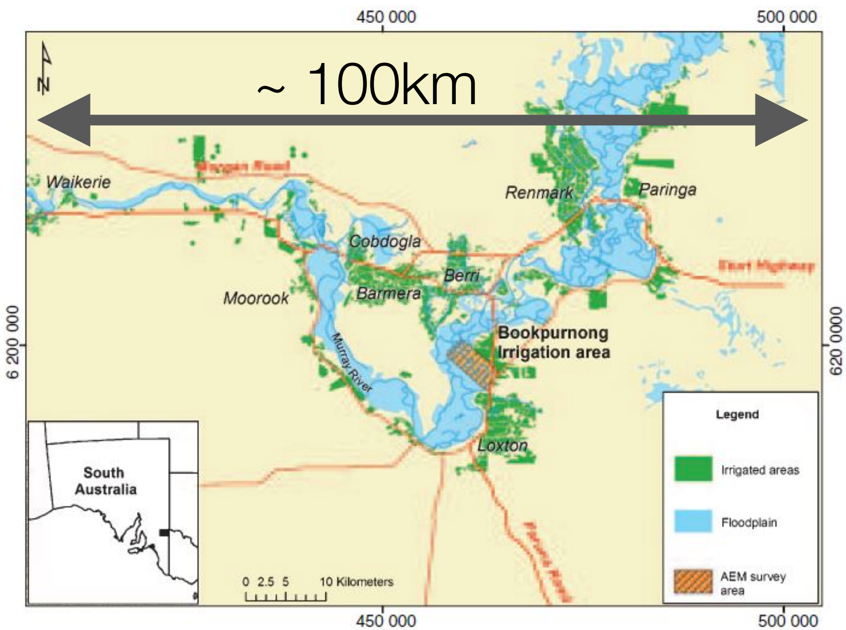
EM Fundamentals

DISC
2017
Electromagnetics
Fundamentals and Applications

Motivation: applications difficult for DC

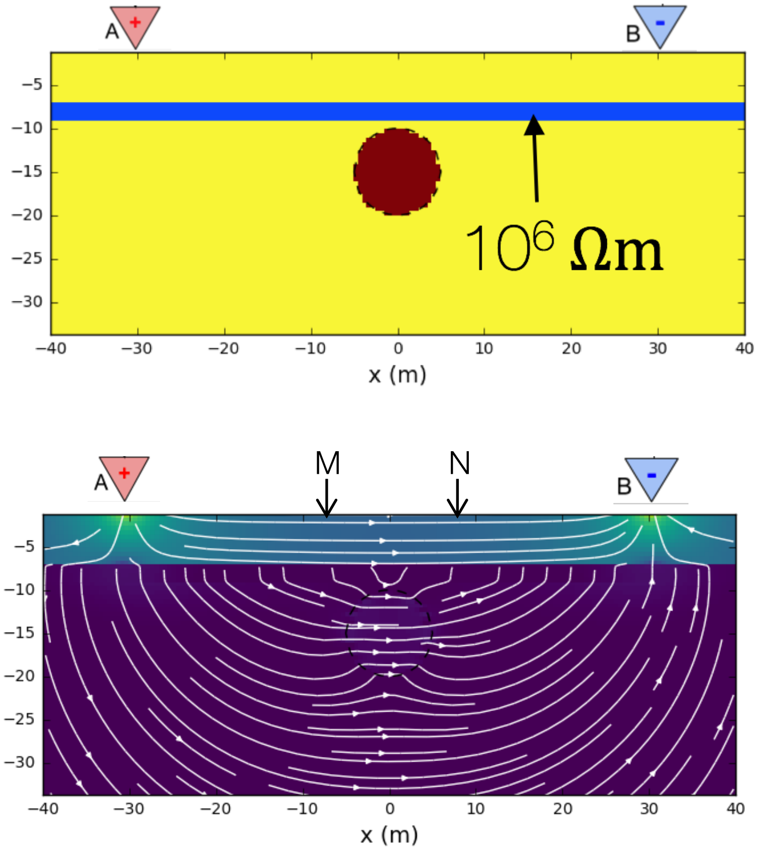
Large areas to be covered



Rugged terrain

Hard to inject

Resistive layer “shields” target

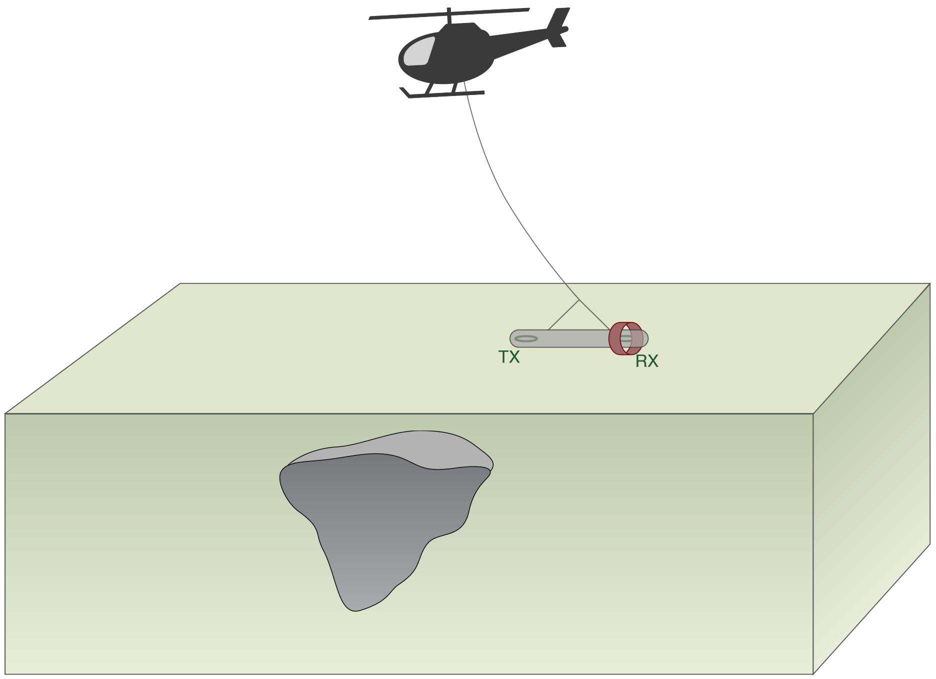


Outline

- Basic Survey
- Ampere's and Faraday's Laws (2-coil App)
- Circuit model for EM induction
- Frequency and time domain data
- Sphere in homogeneous earth
- Cyl code
- Energy losses in the ground

Basic Experiment

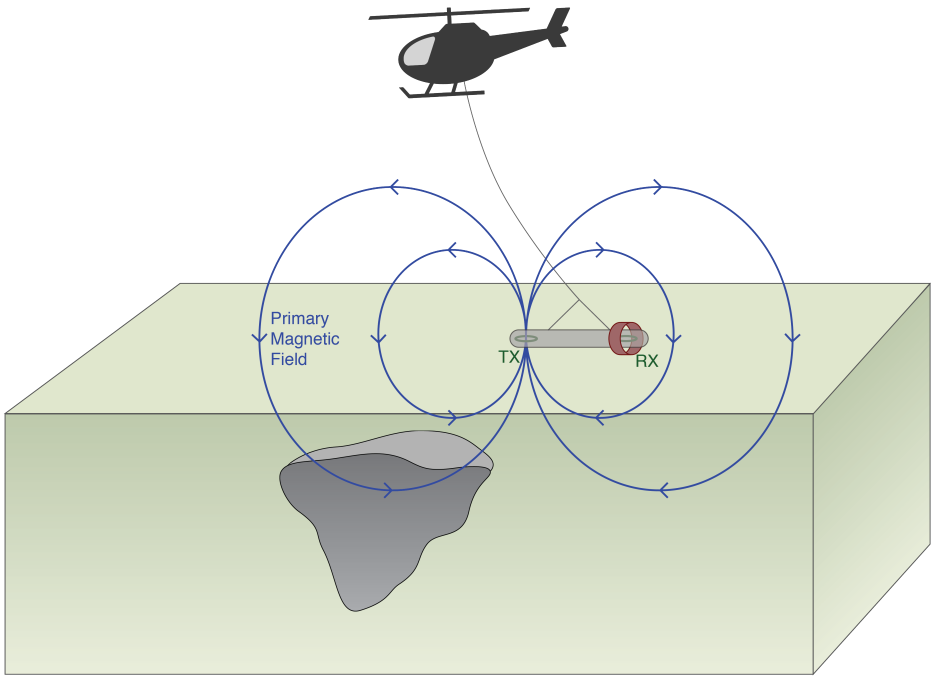
- **Setup:**
  - transmitter and receiver are in a towed bird



Basic Experiment

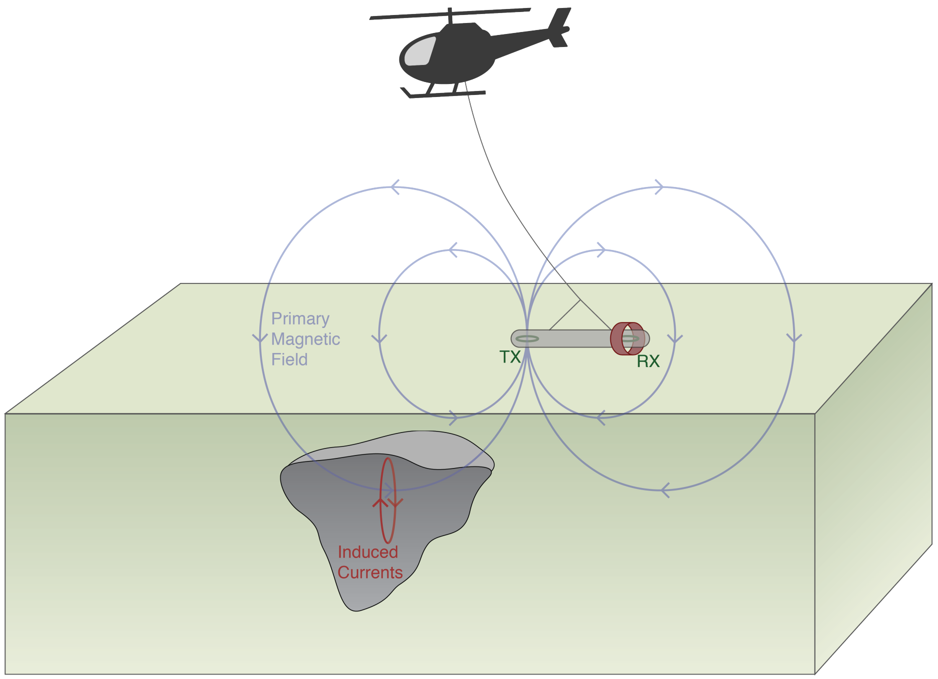
- **Setup:**
  - transmitter and receiver are in a towed bird

- **Primary:**
  - Transmitter produces a primary magnetic field



Basic Experiment

- **Setup:**
  - transmitter and receiver are in a towed bird
- **Primary:**
  - Transmitter produces a primary magnetic field
- **Induced Currents:**
  - Time varying magnetic fields generate electric fields everywhere and currents in conductors



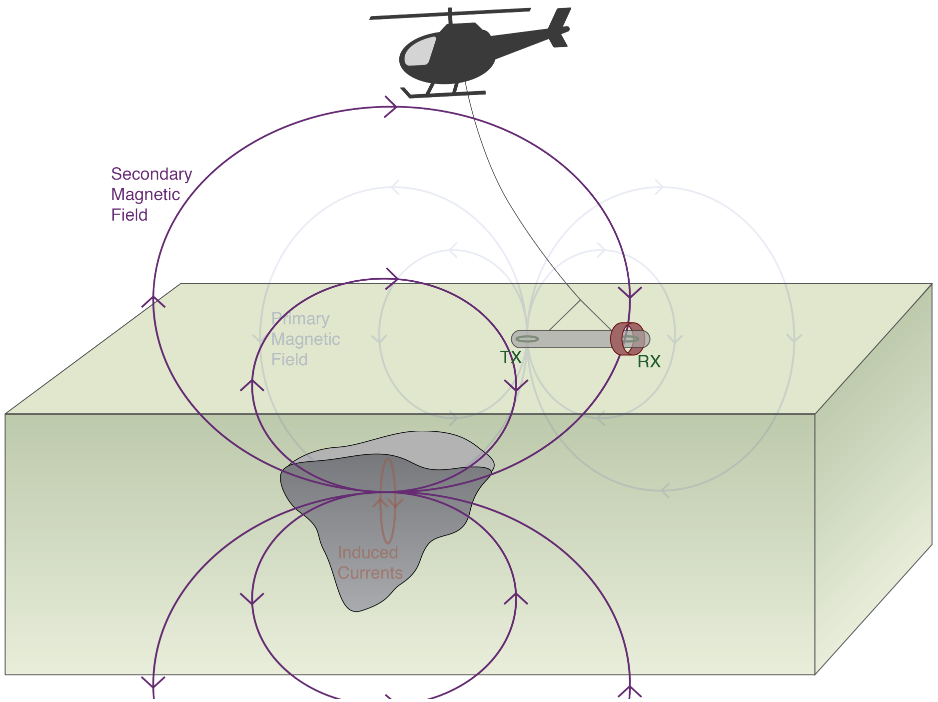
Basic Experiment

- **Setup:**
  - transmitter and receiver are in a towed bird

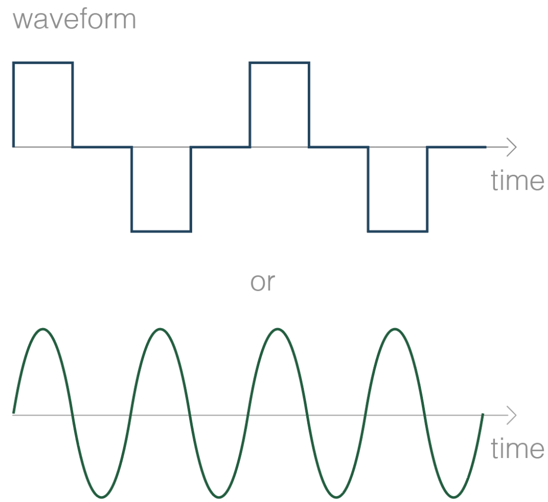
- **Primary:**
  - Transmitter produces a primary magnetic field

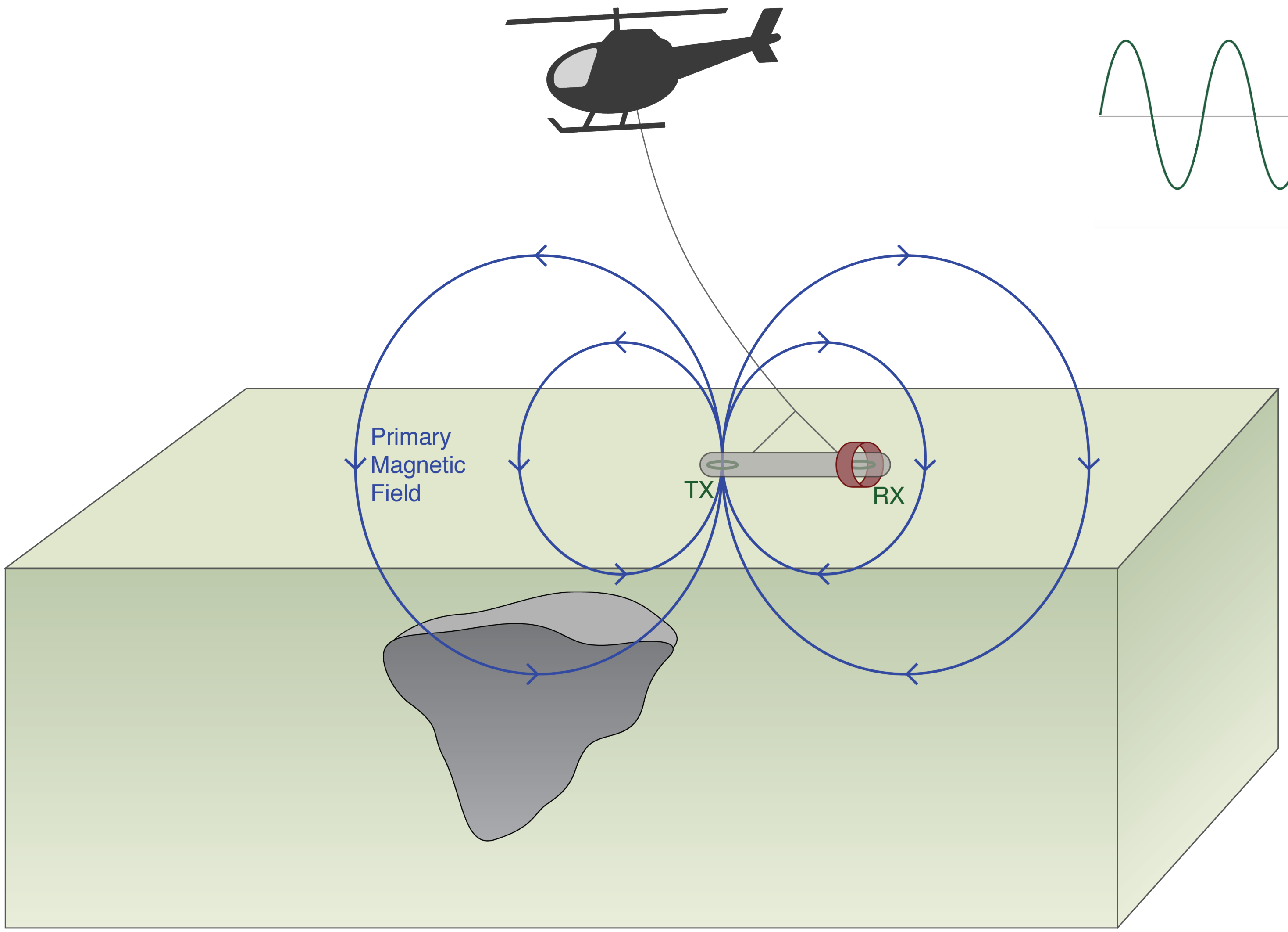
- **Induced Currents:**
  - Time varying magnetic fields generate electric fields everywhere and currents in conductors

- **Secondary Fields:**
  - The induced currents produce a secondary magnetic field.

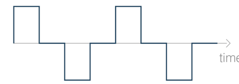
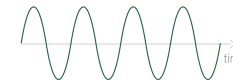


Transmitter





Basic Equations: Quasi-static

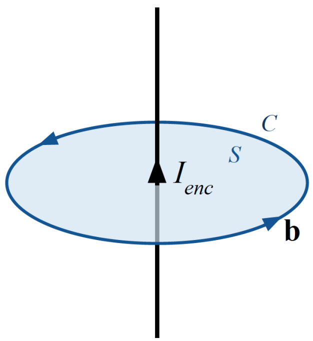
	Time 	Frequency 
Faraday's Law	$\nabla \times \mathbf{e} = -\dfrac{\partial \mathbf{b}}{\partial t}$	$\nabla \times \mathbf{E} = -i\omega \mathbf{B}$
Ampere's Law	$\nabla \times \mathbf{h} = \mathbf{j} + \dfrac{\partial \mathbf{d}}{\partial t}$	$\nabla \times \mathbf{H} = \mathbf{J} + i\omega \mathbf{D}$
No Magnetic Monopoles	$\nabla \cdot \mathbf{b} = 0$	$\nabla \cdot \mathbf{B} = 0$
Constitutive Relationships (non-dispersive)	$\mathbf{j} = \sigma \mathbf{e}$ $\mathbf{b} = \mu \mathbf{h}$ $\mathbf{d} = \varepsilon \mathbf{e}$	$\mathbf{J} = \sigma \mathbf{E}$ $\mathbf{B} = \mu \mathbf{H}$ $\mathbf{D} = \varepsilon \mathbf{E}$

* Solve with sources and boundary conditions

Ampere's Law

$$\nabla \times \mathbf{H} = \mathbf{J}$$

Wire



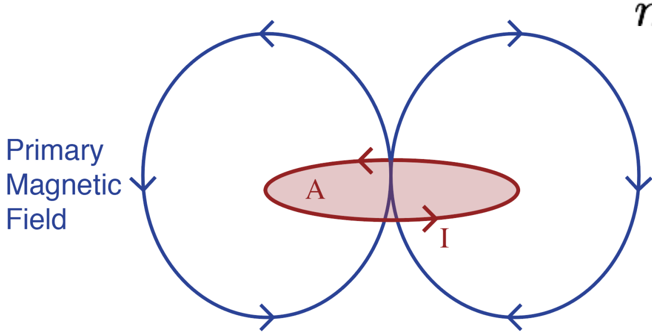
$$\mathbf{B} = \frac{\mu_0 I_{enc}}{2\pi r}\hat{\phi}$$

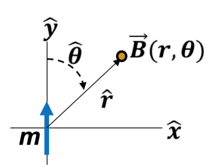
Right hand rule

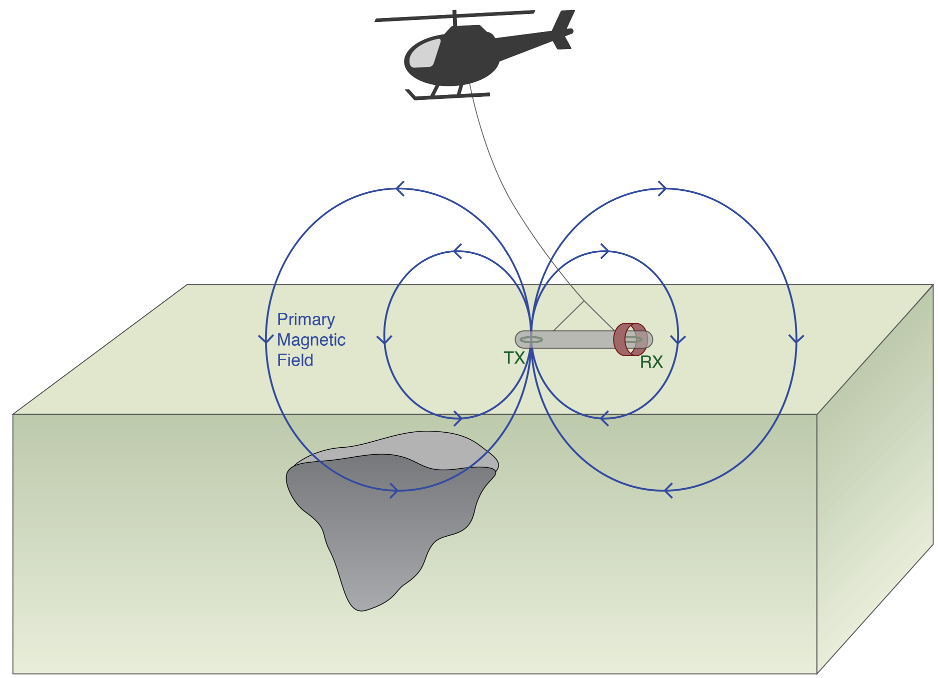
Current loop

$$\mathbf{B} = \frac{\mu_0 m}{4\pi r^3}(2\cos\theta\hat{\mathbf{r}} + \sin\theta\hat{\boldsymbol{\theta}})$$

$$m = IA$$







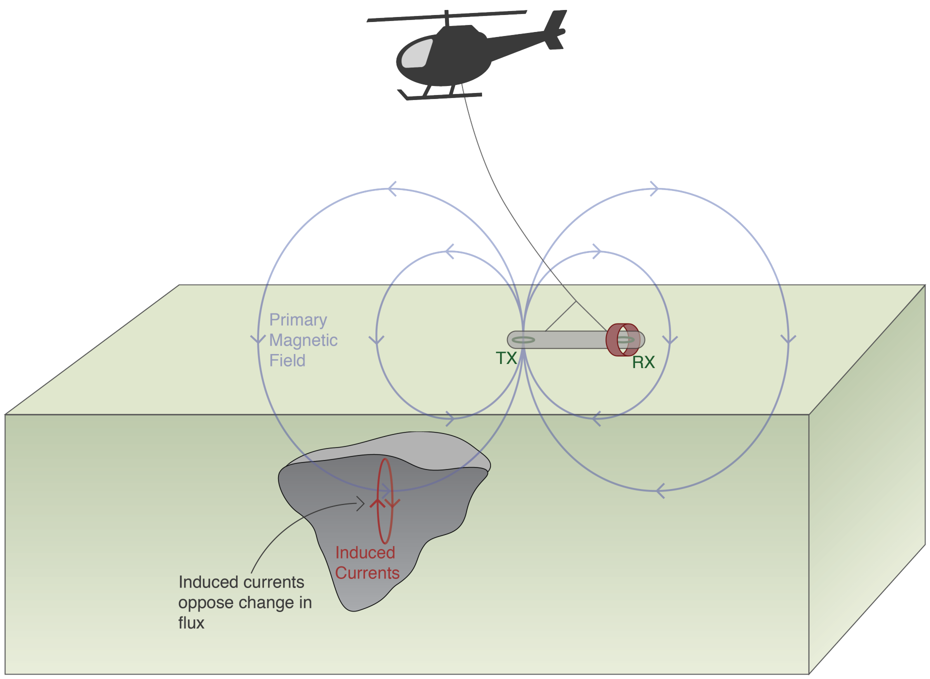
Faraday’s Law and Induced Currents

Lenz’ Law

$$\nabla \times \mathbf{e} = -\frac{\partial \mathbf{b}}{\partial t}$$

Ohm’s Law

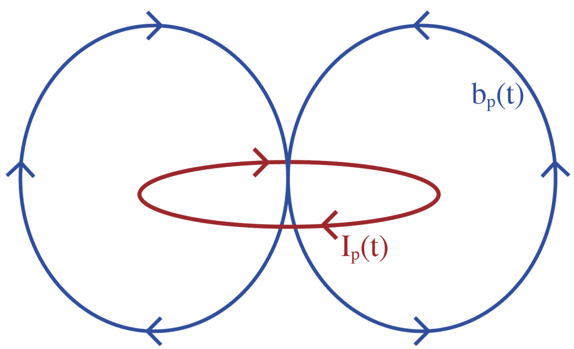
$$\mathbf{j} = \sigma \mathbf{e}$$



Two Coil Example: Harmonic

Source (red loop)

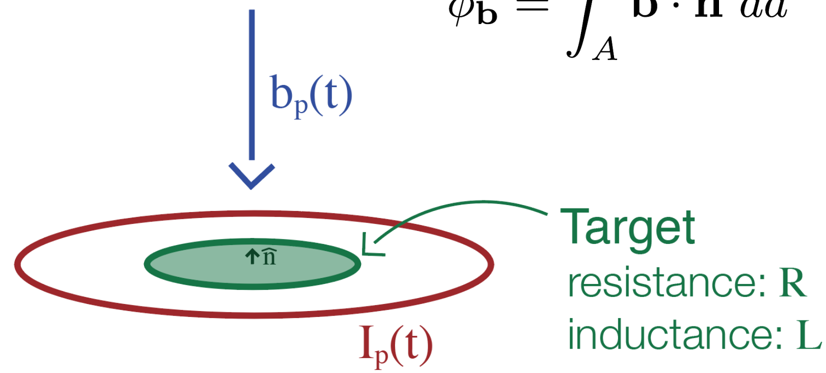
- Time varying current → Time varying magnetic flux



Target (green loop)

- Time varying magnetic flux

$$\phi_{\mathbf{b}} = \int_A \mathbf{b} \cdot \hat{\mathbf{n}} \, da$$



Faraday's Law

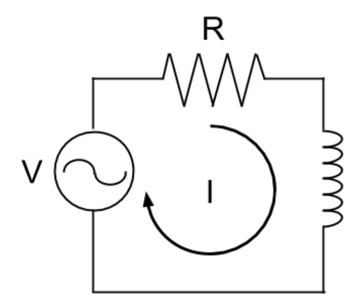
$$\nabla \times \mathbf{e} = -\frac{\partial \mathbf{b}}{\partial t}$$

Ohm's Law

$$\mathbf{j} = \sigma \mathbf{e}$$

EMF (voltage) is related to time rate of change in flux.

$$V = EMF = -\frac{d\phi_{\mathbf{b}}}{dt}$$

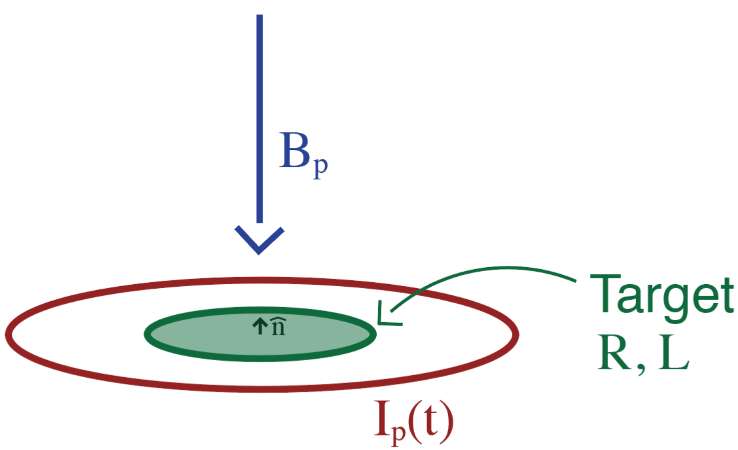


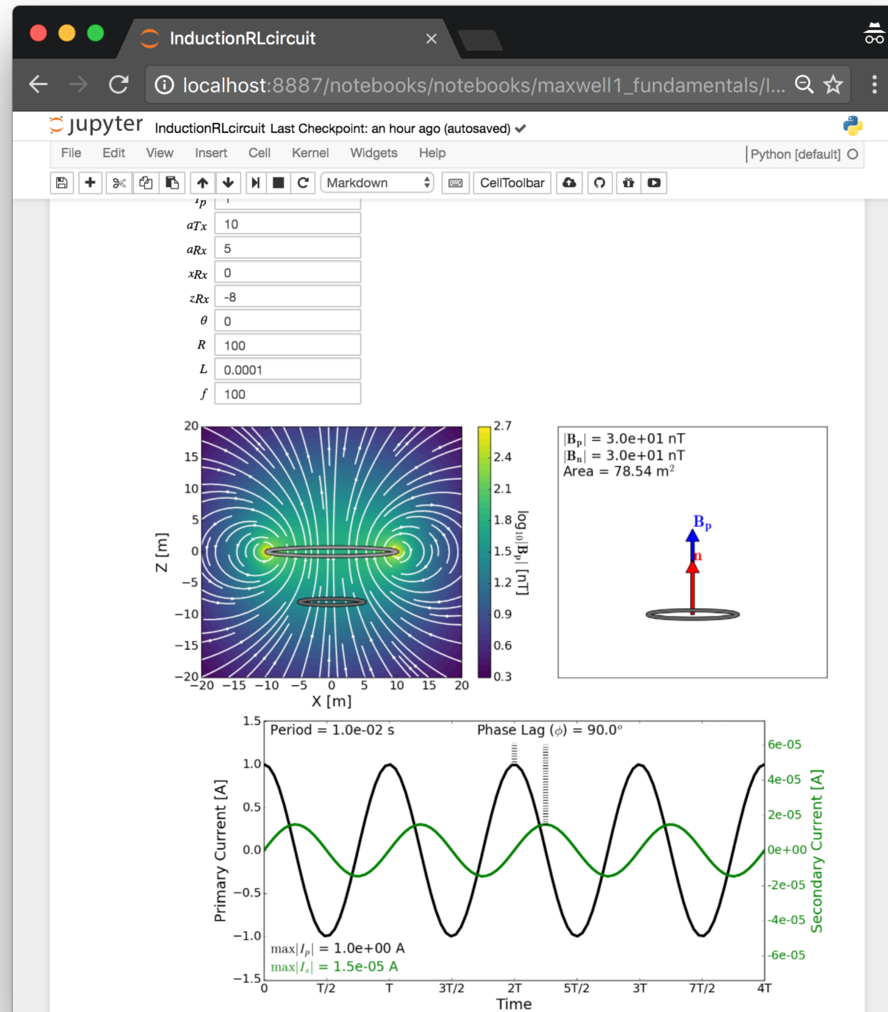
App for Faraday's Law

Lenz' Law

$$\nabla \times \mathbf{e} = -\frac{\partial \mathbf{b}}{\partial t}$$

$$\mathbf{j} = \sigma \mathbf{e}$$





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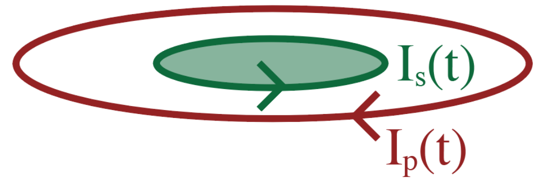


Induced Currents

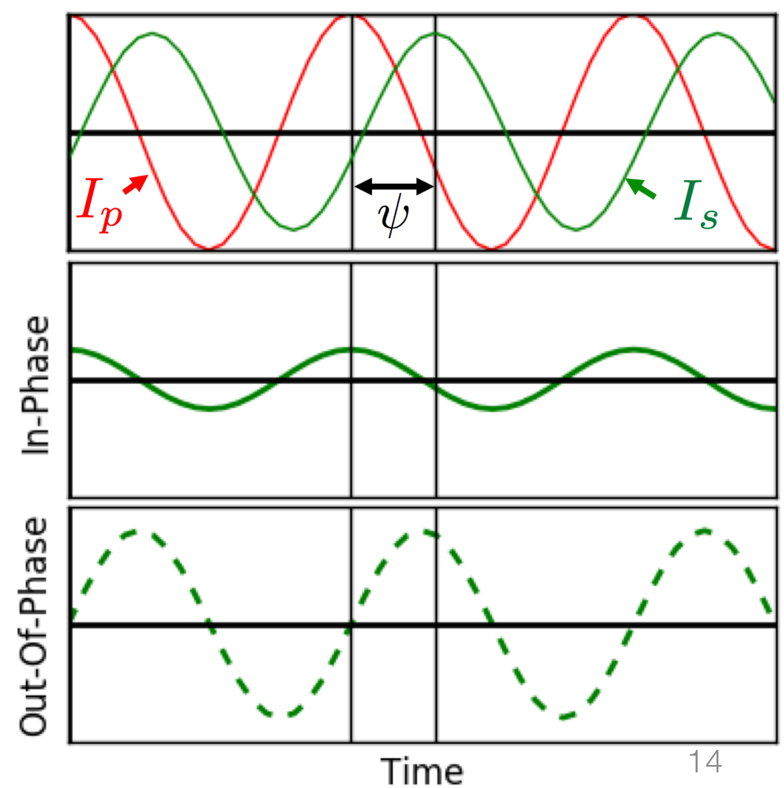
$$I_p(t) = I_p \cos \omega t$$

$$I_s(t) = I_s \cos(\omega t - \psi)$$

$$= \underbrace{I_s \cos \psi}_{\text{In-Phase, Real}} \cos \omega t + \underbrace{I_s \sin \psi}_{\text{Out-of-Phase, Quadrature, Imaginary}} \sin \omega t$$

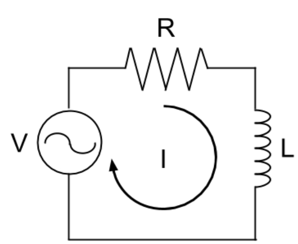
In-Phase
Real

Out-of-Phase
Quadrature
Imaginary

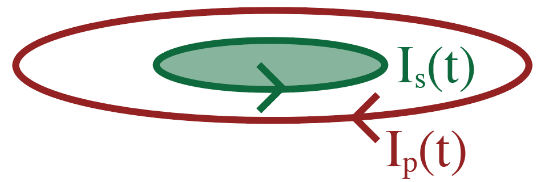


Phase Lag

$$\psi = \frac{\pi}{2} + \tan^{-1}\left(\frac{\omega L}{R}\right)$$



Two Coil Example: Harmonic



Induced Currents

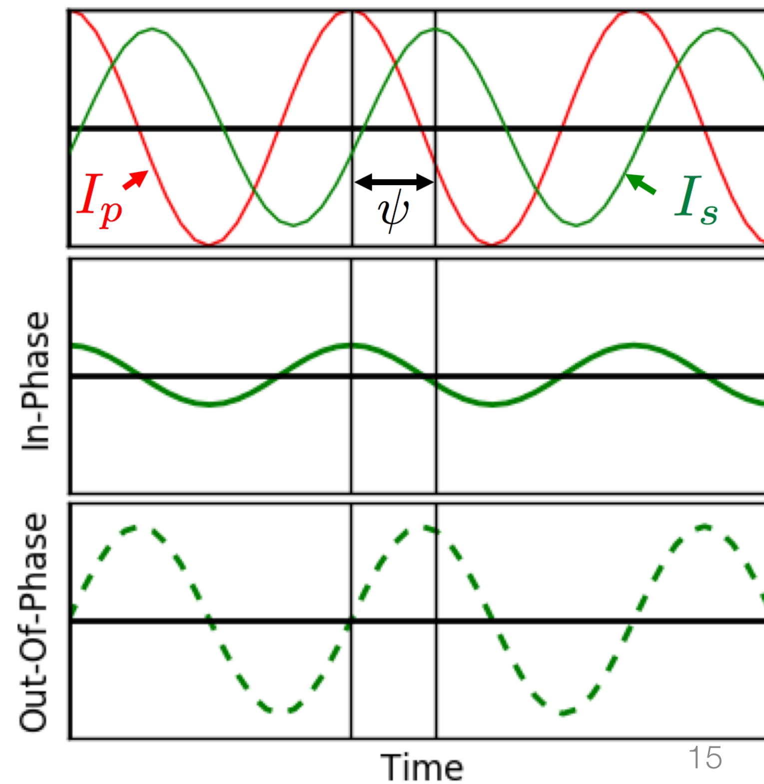
$$I_p(t) = I_p \cos \omega t$$

$$I_s(t) = I_s \cos(\omega t - \psi)$$

$$= \underbrace{I_s \cos \psi}_{\text{In-Phase Real}} \cos \omega t + \underbrace{I_s \sin \psi}_{\text{Out-of-Phase Quadrature Imaginary}} \sin \omega t$$

In-Phase
Real

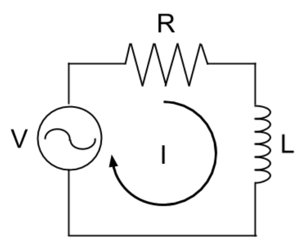
Out-of-Phase
Quadrature
Imaginary



Phase Lag

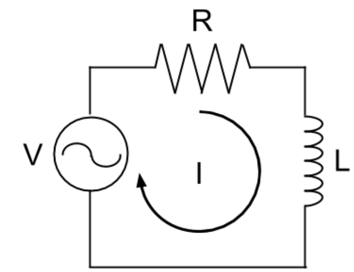
$$\psi = \frac{\pi}{2} + \tan^{-1} \underbrace{\left(\frac{\omega L}{R} \right)}_{\alpha}$$

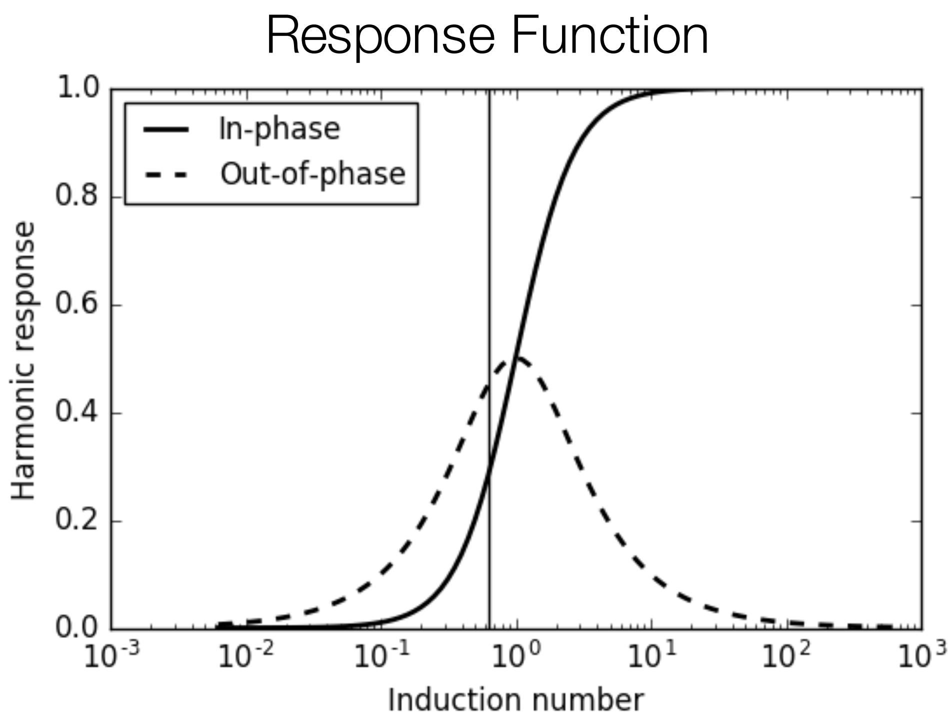
Induction number



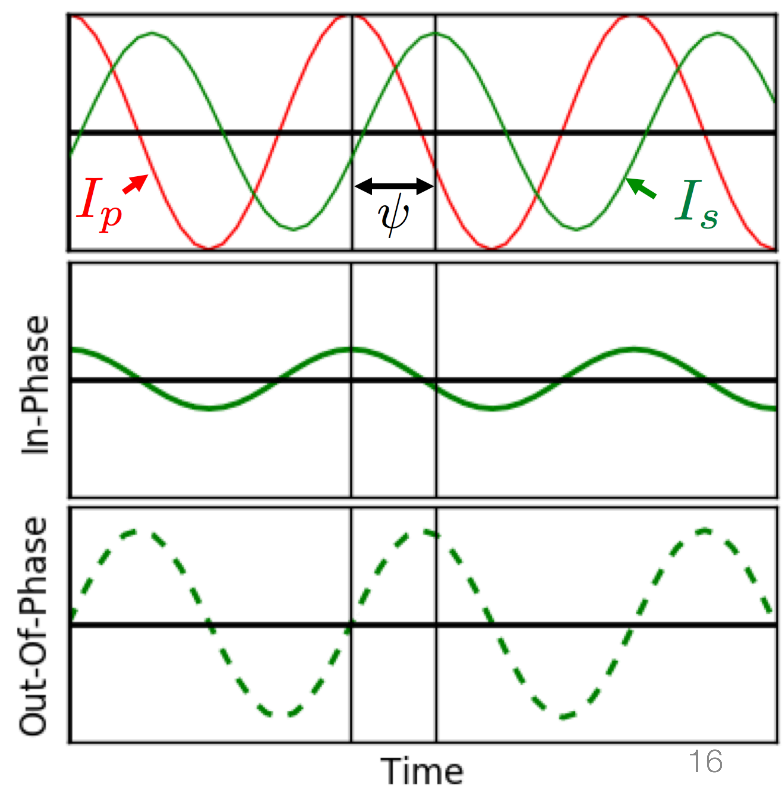
Response Function

- Quantifies how a target responds to a time varying magnetic field
- Partitions real and imaginary parts





$$\alpha = \frac{\omega L}{R}$$



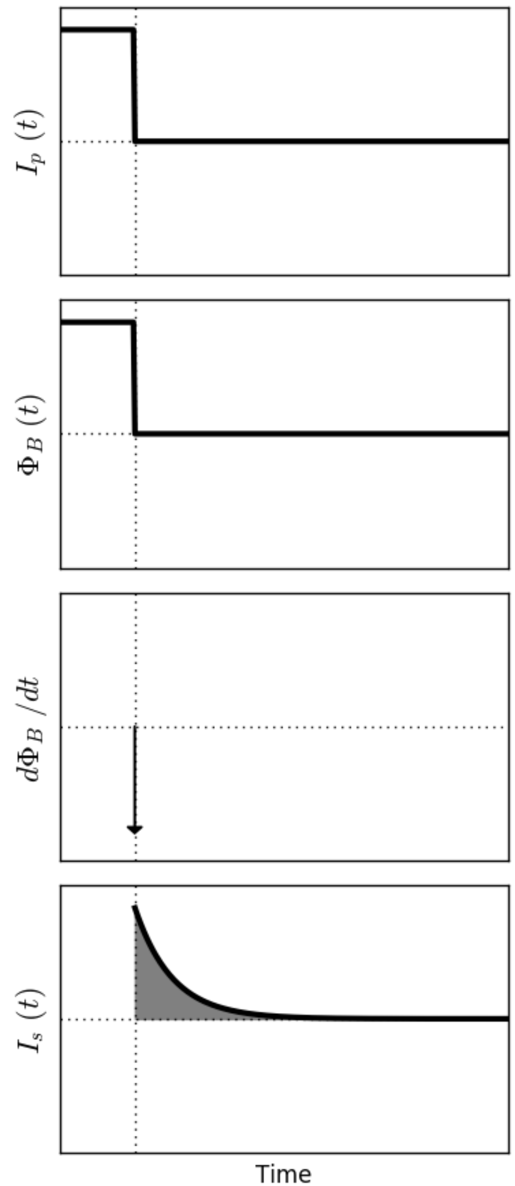
Two Coil Example: Transient

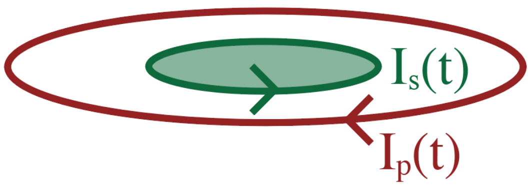
Primary currents

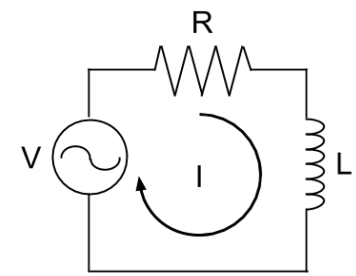
Magnetic flux

Time-variation of magnetic flux

Secondary currents







$$I_s(t) = I_s e^{-t/\tau}$$

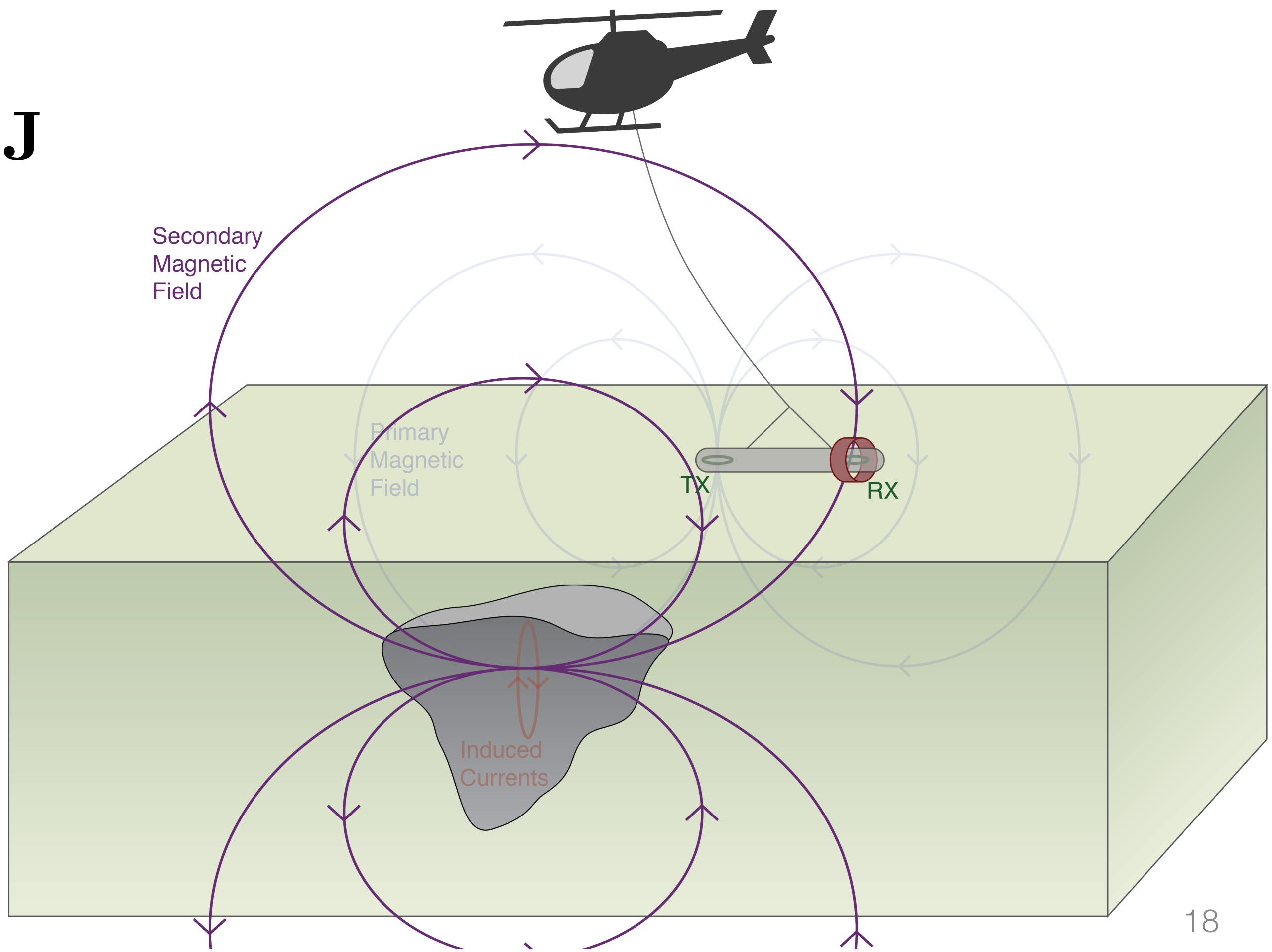
$$\tau = L/R$$

Secondary magnetic fields

Induced currents generate magnetic fields

- Ampere's Law

$$\nabla \times \mathbf{H} = \mathbf{J}$$

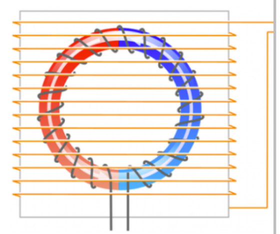


Receiver and Data

Magnetometer

- Measures:
  - Magnetic fields
  - 3 components
- eg. 3-component fluxgate

$$\mathbf{b}(t)$$

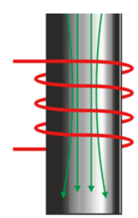


Fluxgate

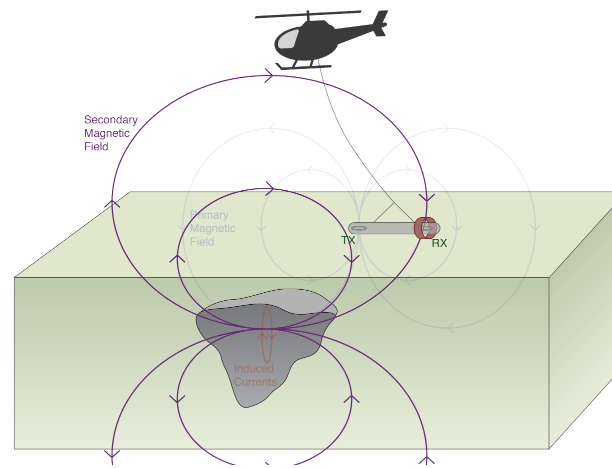
Coil

- Measures:
  - Voltage
  - Single component that depends on coil orientation
    - Coupling matters
- eg. airborne frequency domain
  - ratio of Hs/Hp is the same as Vs/Vp

$$\frac{\partial b}{\partial t}$$



Coil



Coupling

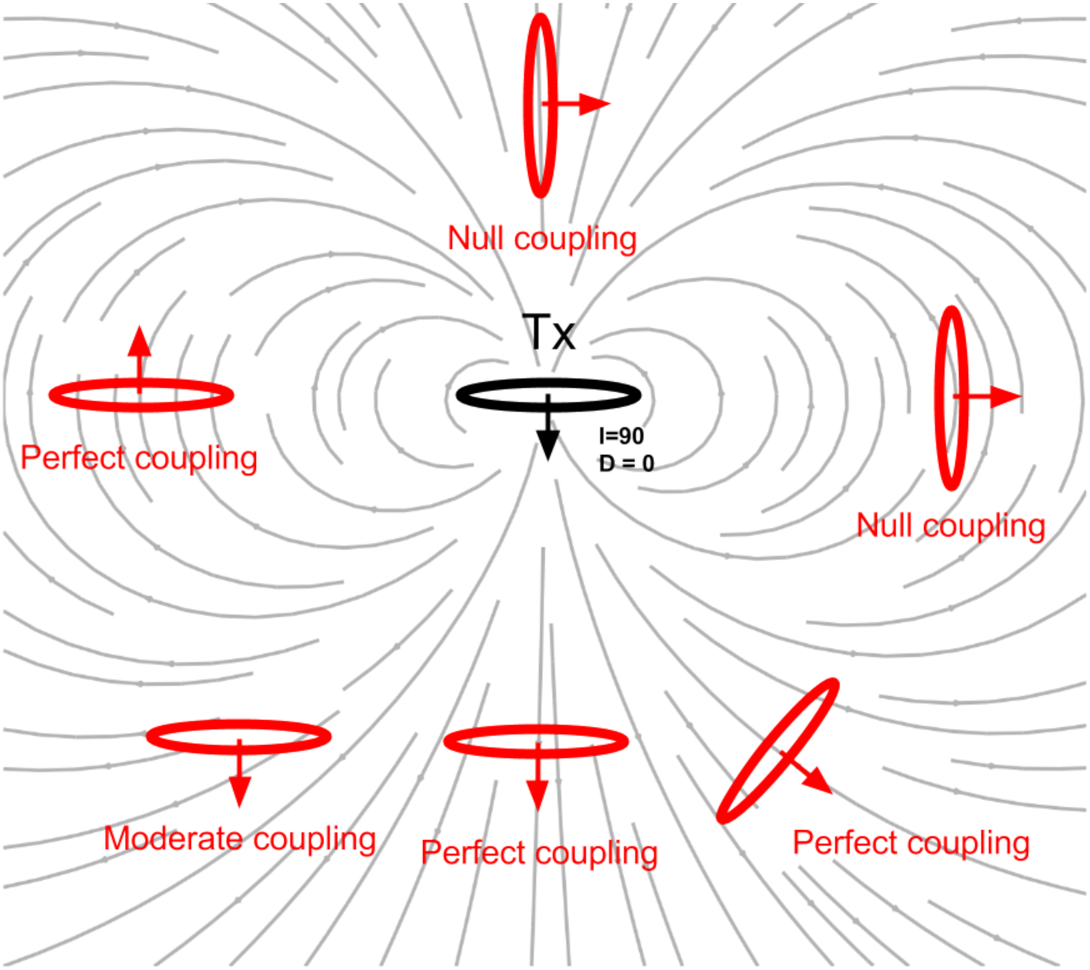
- Transmitter: Primary

$$I_p(t) = I_p \cos(\omega t)$$

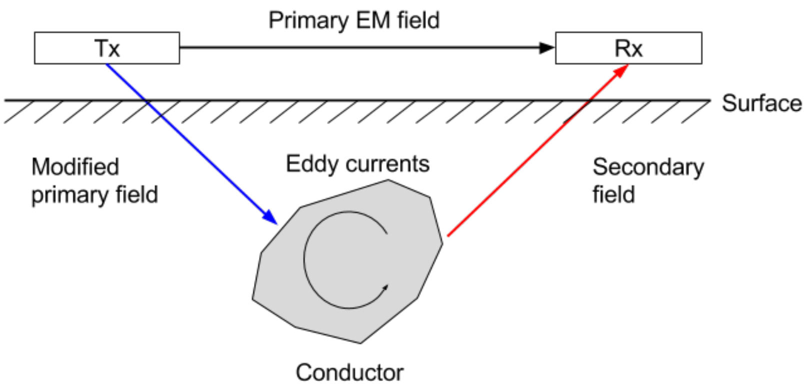
$$\mathbf{B}_p(t) \sim I_p cos(\omega t)$$

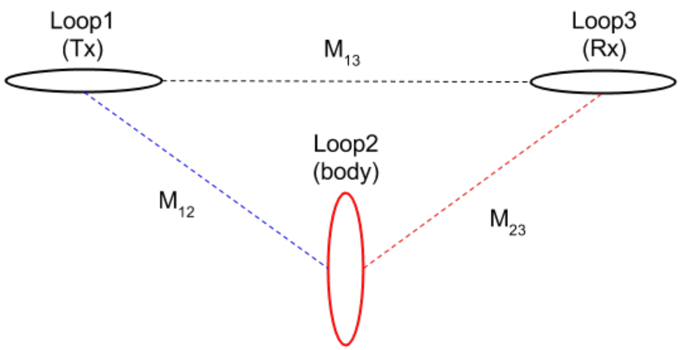
- Target: Secondary

$$EMF = -\frac{\partial \phi_{\mathbf{B}}}{\partial t}$$
$$= -\frac{\partial}{\partial t}\left(\mathbf{B}_p \cdot \hat{\mathbf{n}}\right) A$$



Circuit model of EM induction





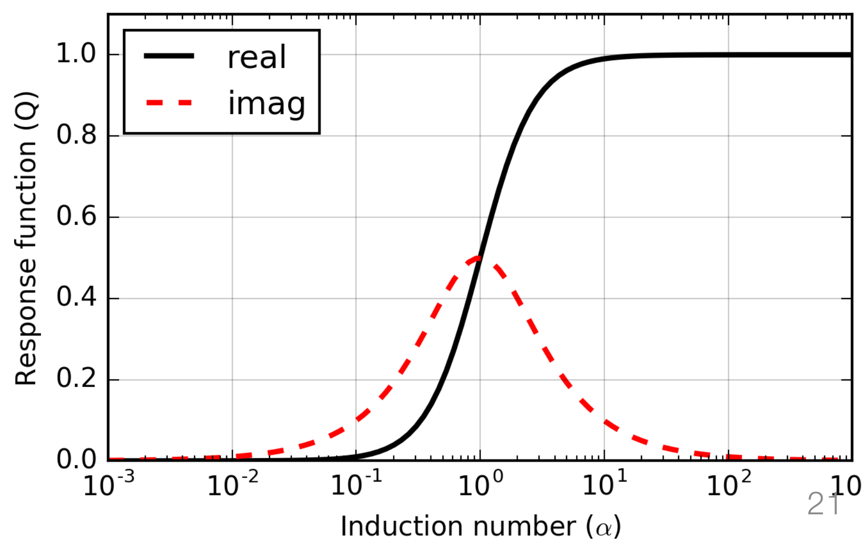
Magnetic field at the receiver

$$\frac{H^s}{H^p} = -\frac{M_{12}M_{23}}{M_{13}L}\underbrace{\left[\frac{\alpha^2 + \imath\alpha}{1+\alpha^2}\right]}_{Q}$$

Induction Number

- Depends on properties of target

$$\alpha = \frac{\omega L}{R}$$



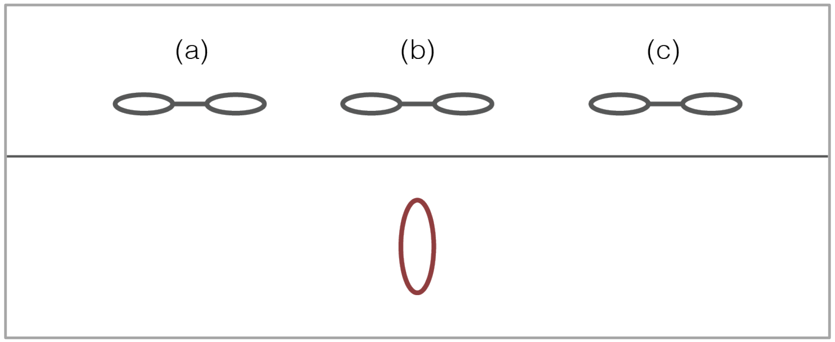
Coupling coefficient

- Depends on geometry

$$M_{12} = \frac{\mu_0}{4\pi}\oint\oint\frac{dl_1 \cdot dl_2}{|\mathbf{r}-\mathbf{r}'|^2}.$$

Conductor in a resistive earth: Frequency

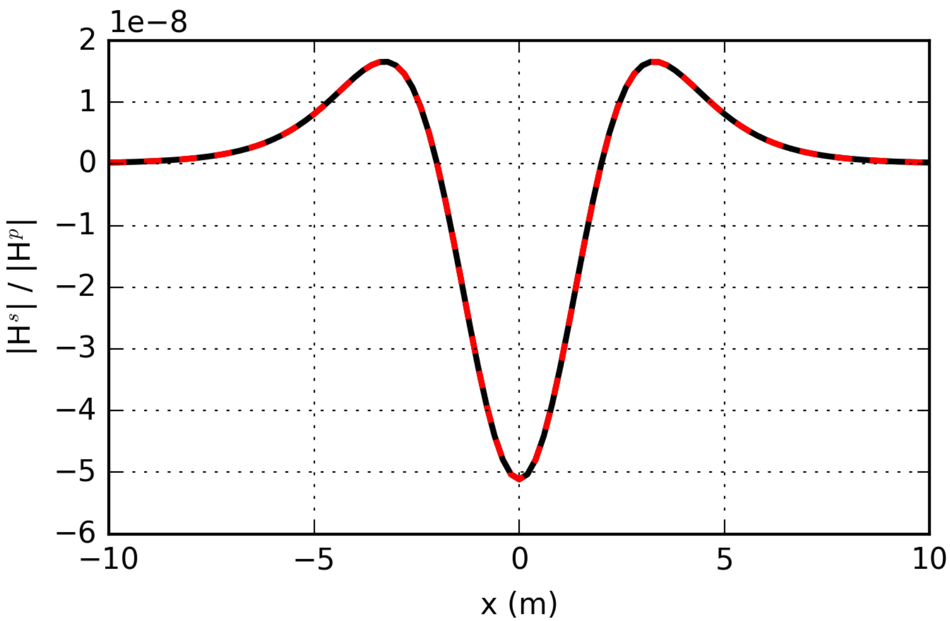
Profile over the loop

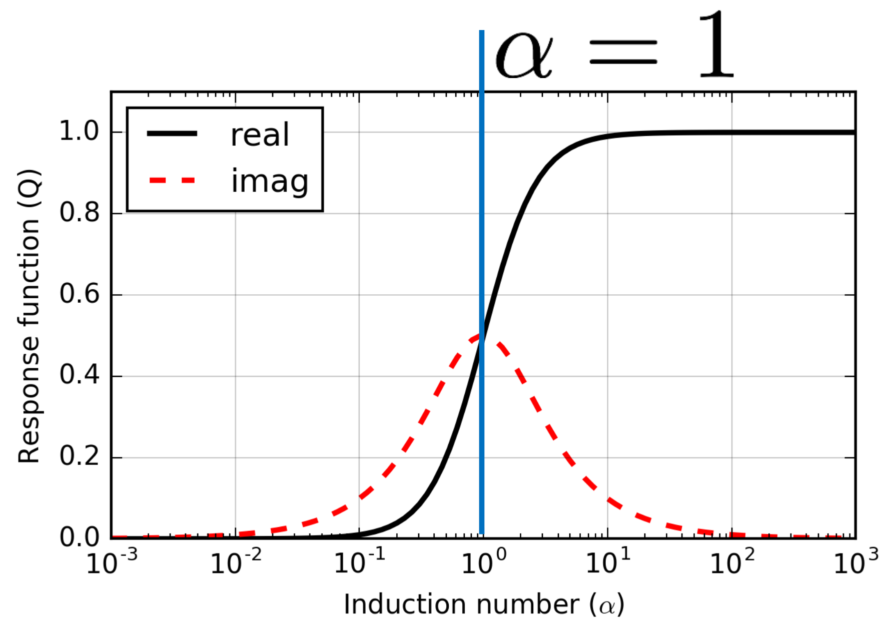


- Induction number

$$\alpha = \frac{\omega L}{R}$$

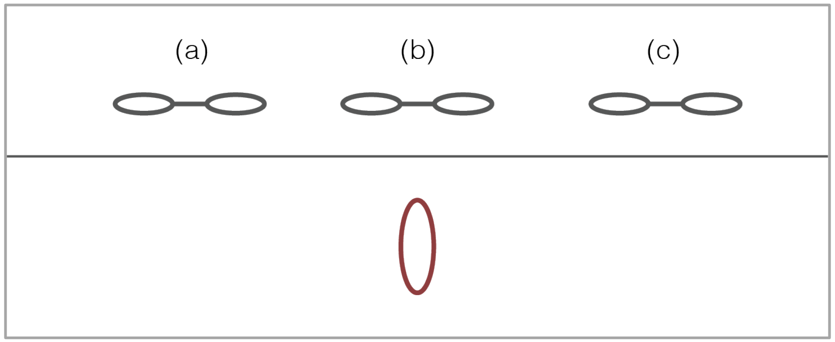
- When $\alpha = 1$
  - Real = Imag

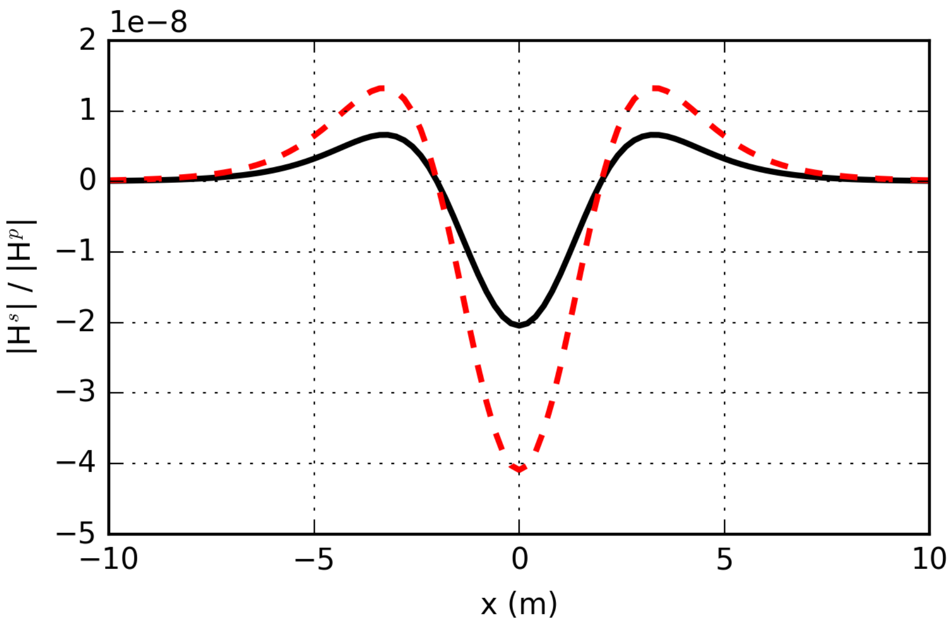




Conductor in a resistive earth: Frequency

Profile over the loop

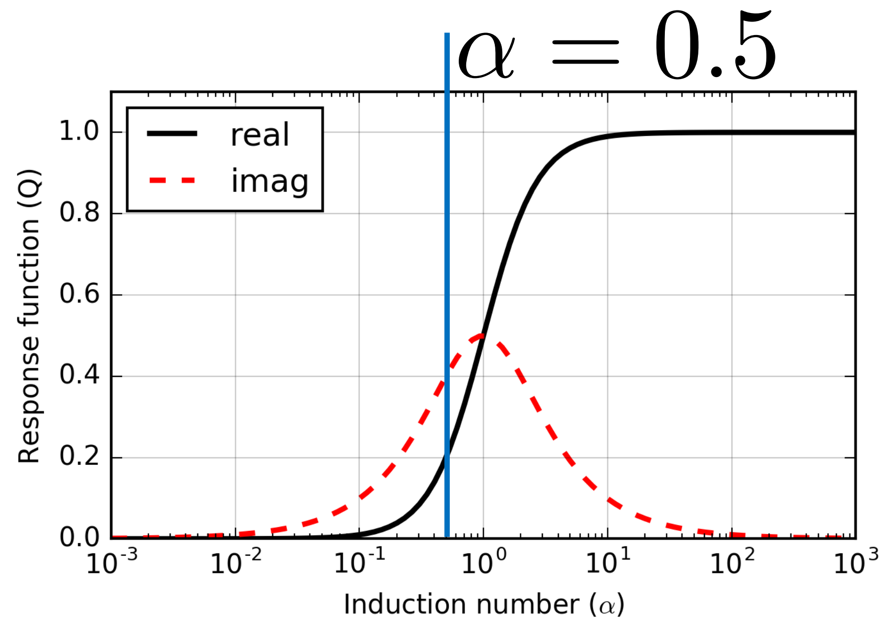




- Induction number

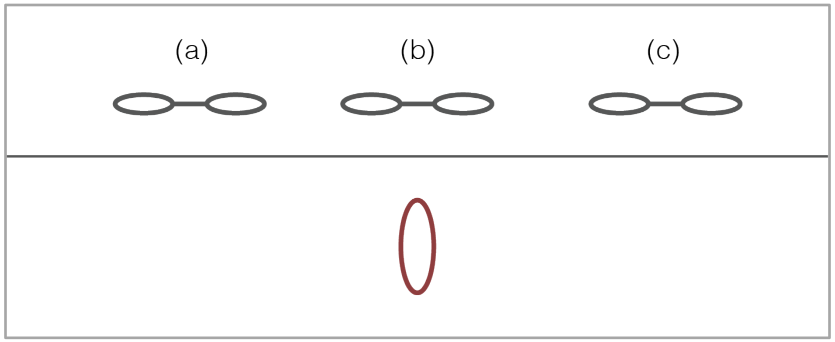
$$\alpha = \frac{\omega L}{R}$$

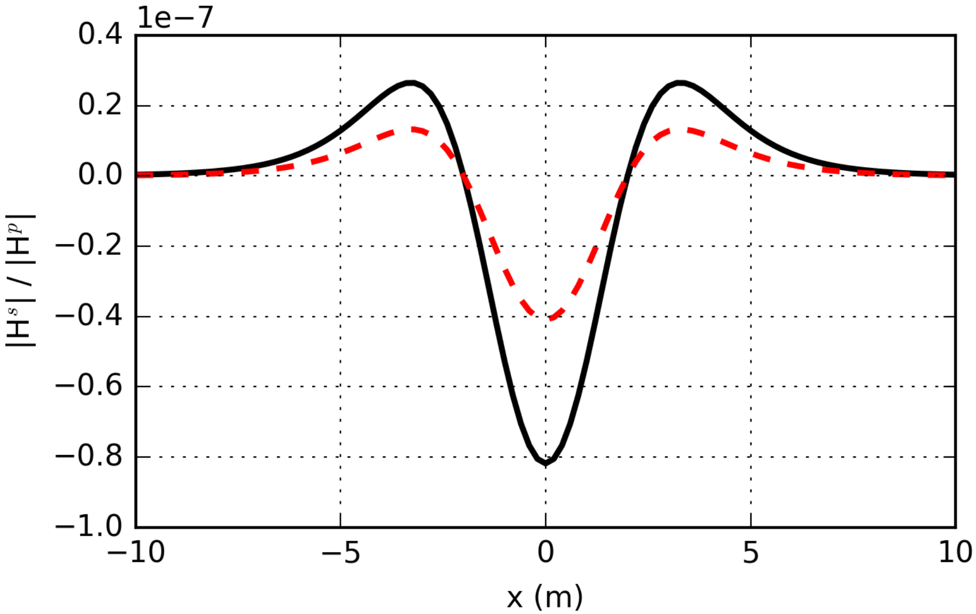
- When $\alpha < 1$
  - Real < Imag



Conductor in a resistive earth: Frequency

Profile over the loop

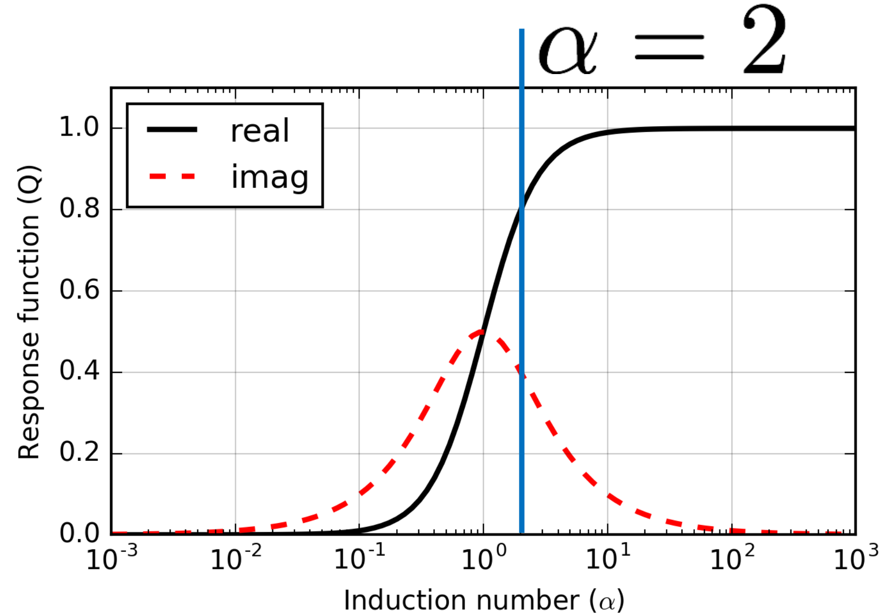




- Induction number

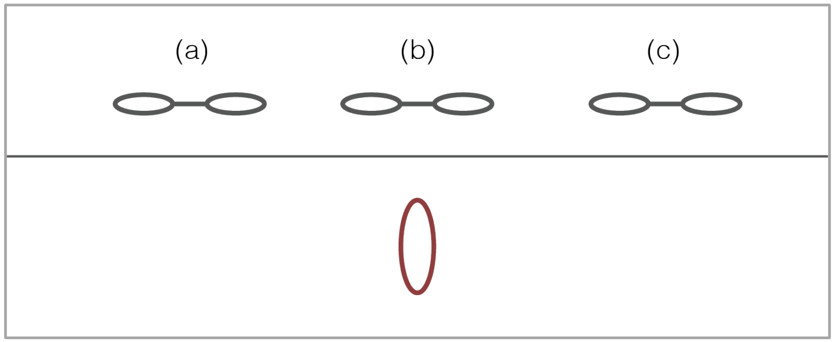
$$\alpha = \frac{\omega L}{R}$$

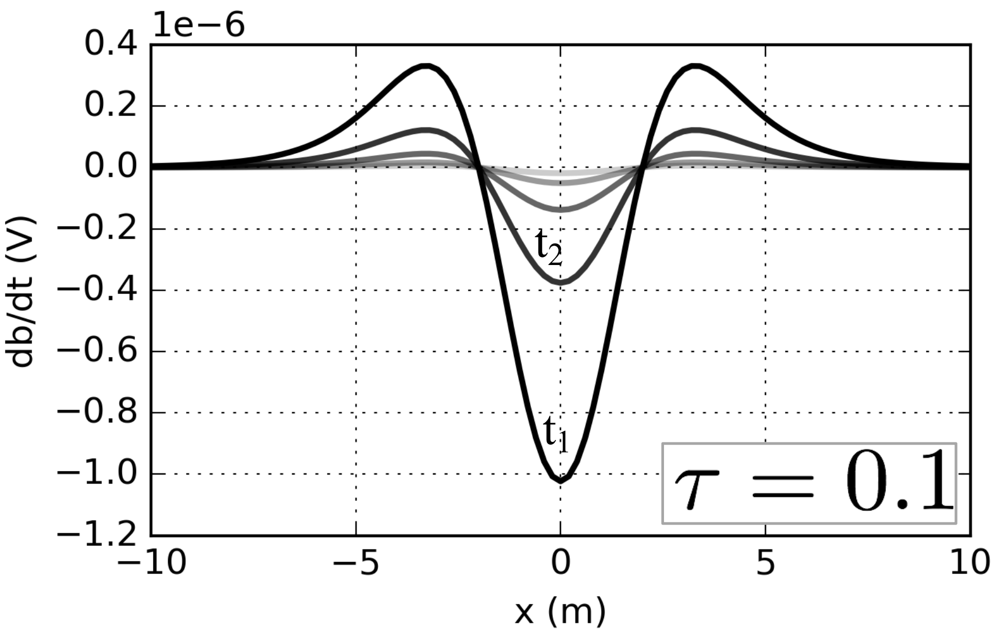
- When $\alpha > 1$
  - Real > Imag



Conductor in a resistive earth: Transient

Profile over the loop

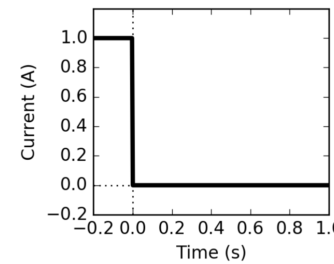




- Time constant

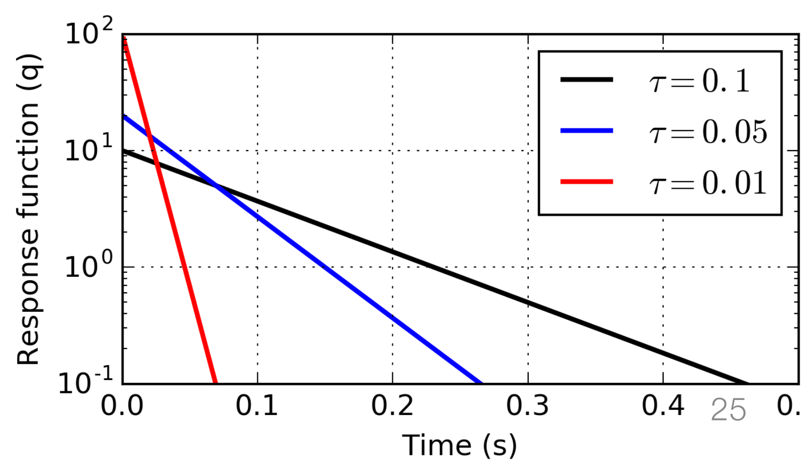
$$\tau = L/R$$

- Step-off current in Tx



- Response function depends on time, τ

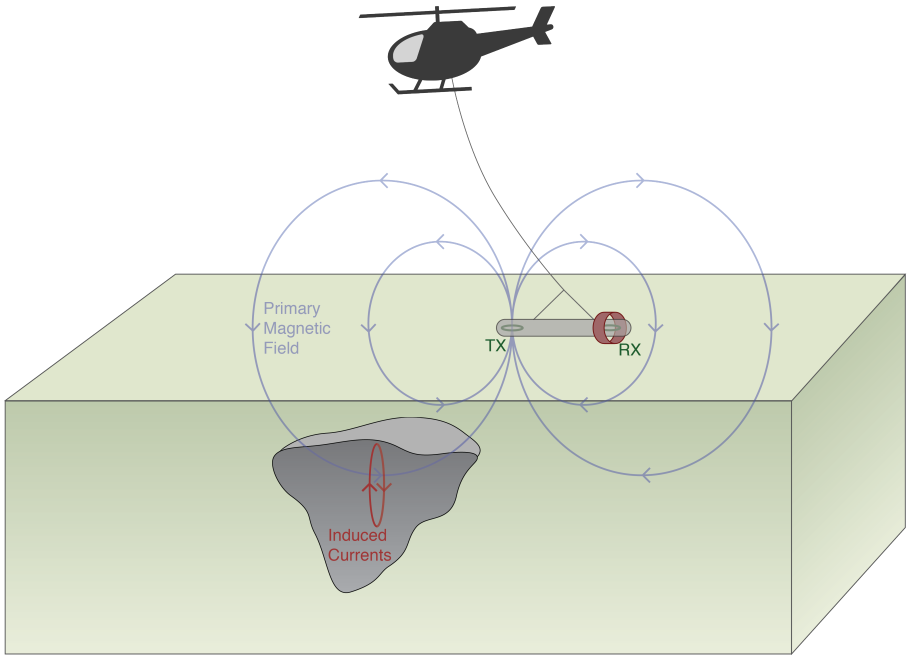
$$q(t) = e^{-t/\tau}$$



Recap: what have we learned?

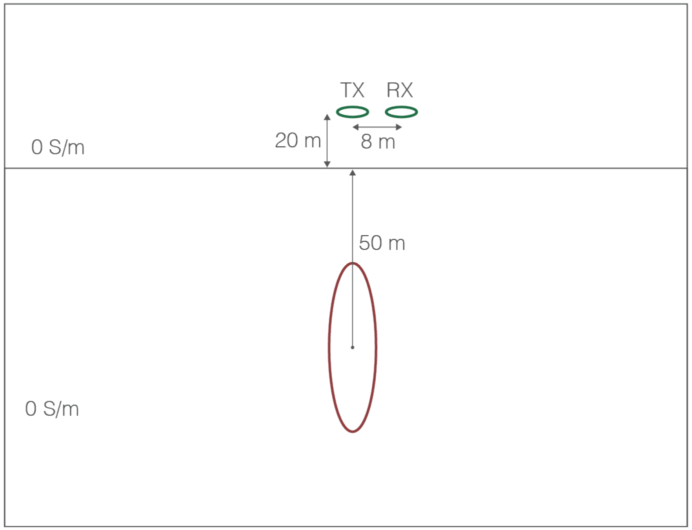
- Basics of EM induction
- Response functions
- Mutual coupling
- Data for frequency or time domain systems

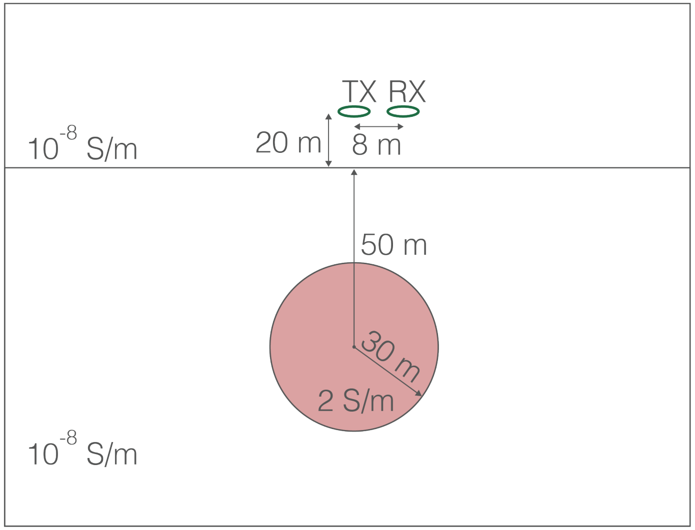
- Circuit model provides representative results
  - Applicable to geologic targets?

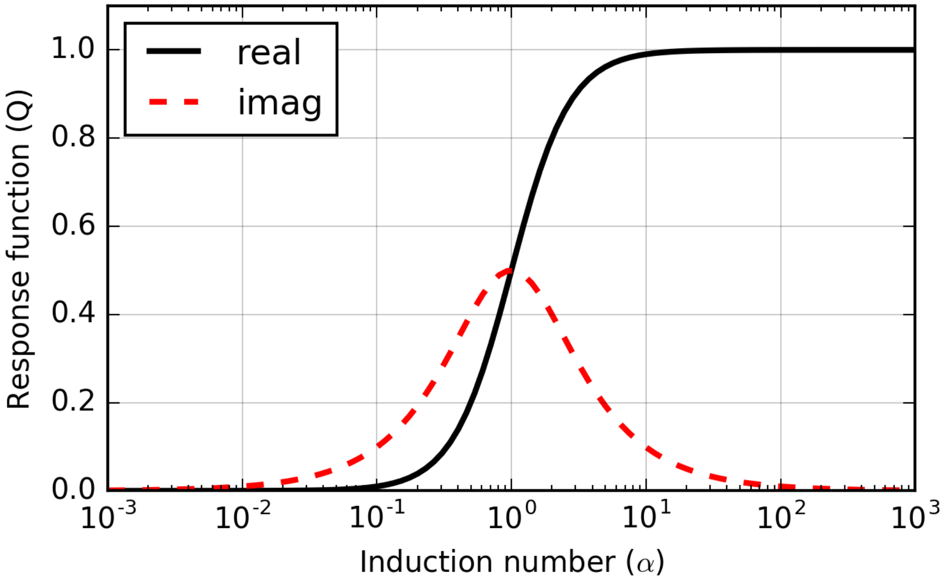


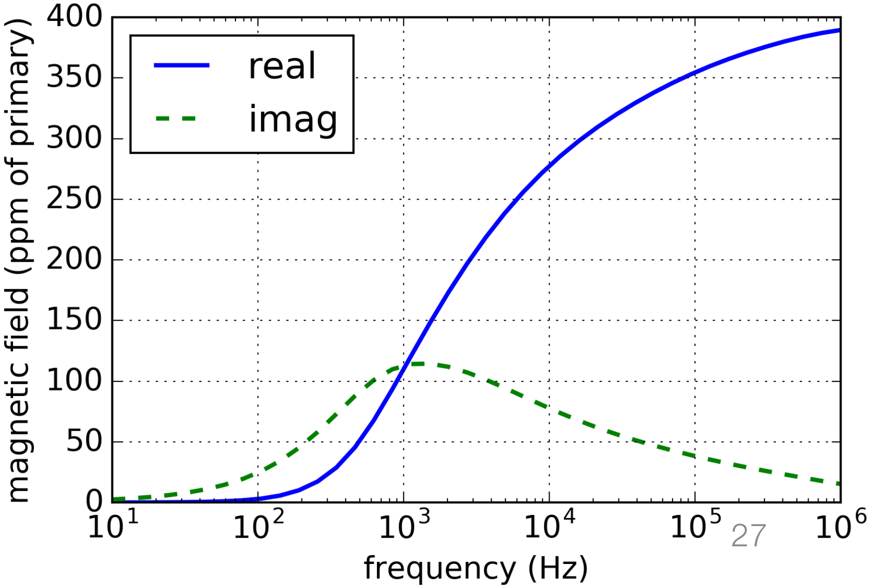
Sphere in a resistive background

How representative is a circuit model?



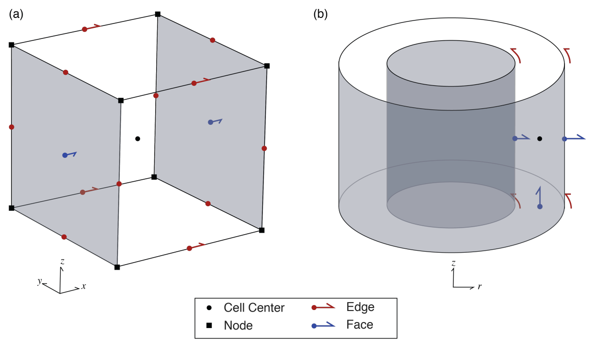






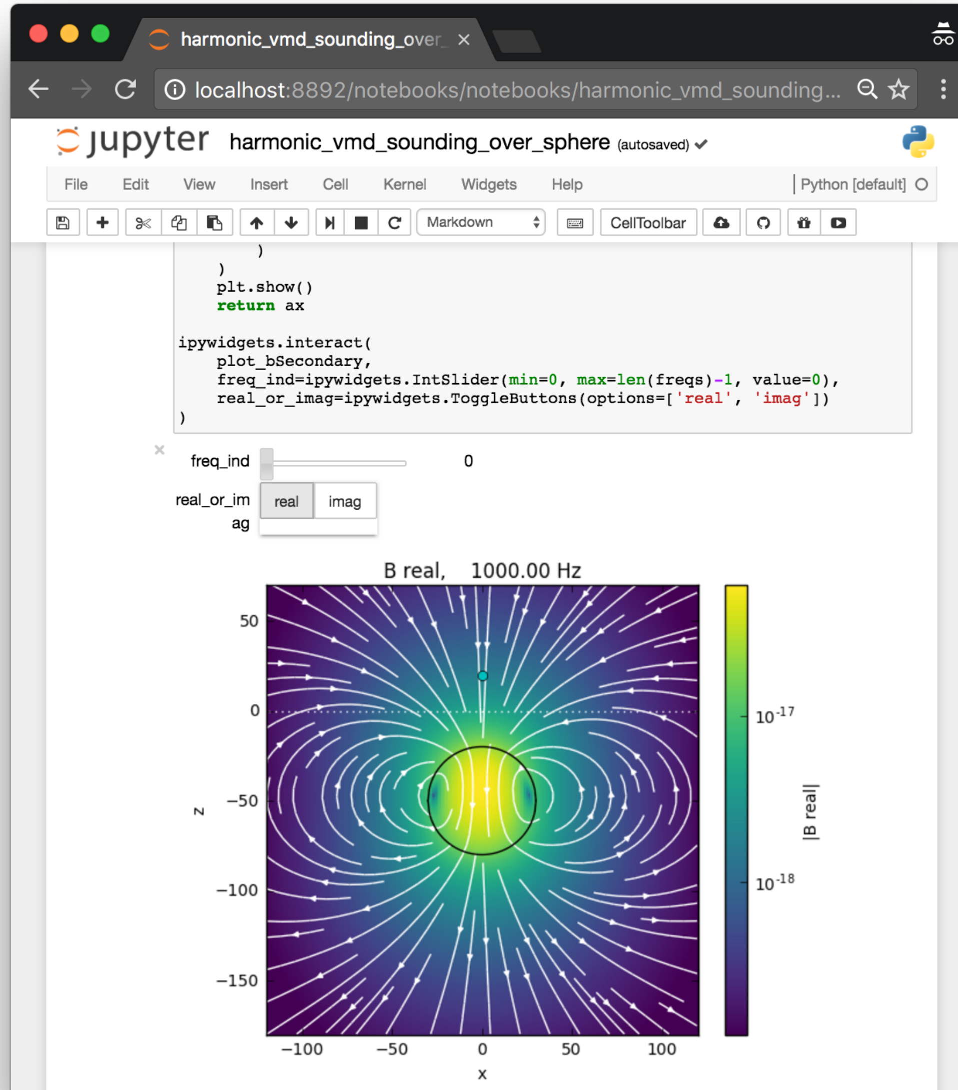
Cyl Code

- Finite Volume EM
  - Frequency and Time



- Built on SimPEG
- Open source, available at:

http://em.geosci.xyz/apps.html

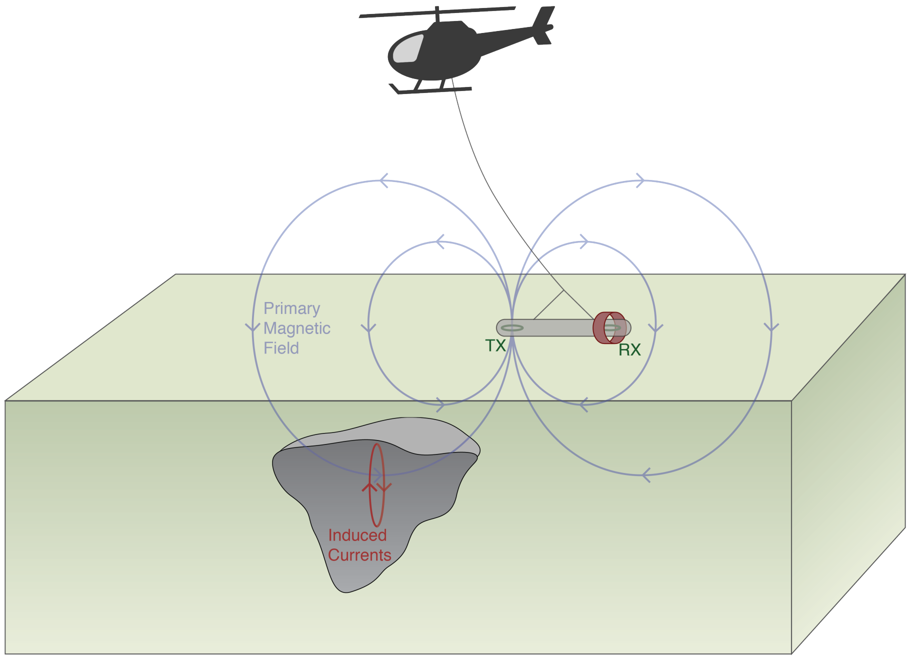


Recap: what have we learned?

- Basics of EM induction
- Response functions
- Mutual coupling
- Data for frequency or time domain systems
- Circuit model is a good proxy

Major item not yet accounted for…

- Propagation of energy from
  - Transmitter to target
  - Target to receiver



How do EM fields and fluxes behave in a conductive background?

Revisit Maxwell's equations

First order equations

$$\nabla \times \mathbf{e} = -\frac{\partial \mathbf{b}}{\partial t}$$

$$\nabla \times \mathbf{h} = \mathbf{j} + \frac{\partial \mathbf{d}}{\partial t}$$

$$\mathbf{j} = \sigma \mathbf{e}$$

$$\mathbf{b} = \mu \mathbf{h}$$

$$\mathbf{d} = \varepsilon \mathbf{e}$$

Second order equations

$$\nabla^2 \mathbf{h} - \underbrace{\mu\sigma \frac{\partial \mathbf{h}}{\partial t}}_{\text{diffusion}} - \underbrace{\mu\epsilon \frac{\partial^2 \mathbf{h}}{\partial t^2}}_{\text{wave propagation}} = 0$$

In frequency

$$\nabla^2 \mathbf{H} + k^2 \mathbf{H} = 0$$

$$k^2 = \omega^2 \mu\varepsilon - i\omega\mu\sigma$$

* Same equation holds for E

Plane waves in a homogeneous media

In frequency

$$\nabla^2 \mathbf{H} + k^2 \mathbf{H} = 0$$

$$k^2 = \omega^2 \mu \varepsilon - i\omega\mu\sigma$$

Quasi-static

$$\frac{\omega\varepsilon}{\sigma} \ll 1$$

even if…

$$\sigma = 10^{-4} S/m$$

$$f = 10^4 Hz$$

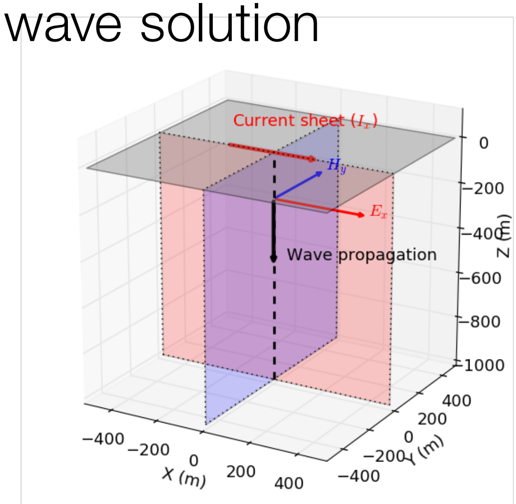
then

$$\frac{\omega\varepsilon}{\sigma} \sim 0.005$$

$$k = \sqrt{-i\omega\mu\sigma} = (1-i)\sqrt{\frac{\omega\mu\sigma}{2}}$$

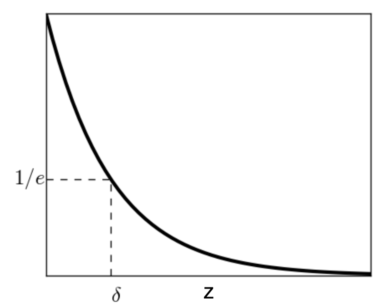
$$\equiv \alpha - i\beta$$

Plane wave solution



$$\mathbf{H} = \mathbf{H_0} \underbrace{e^{-\alpha z}}_{\text{attenuation}} \underbrace{e^{-i(\beta z - \omega t)}}_{\text{phase}}$$

Skin depth



δ : skin depth

$$\delta = \sqrt{\frac{2}{\omega\mu\sigma}} = 503\sqrt{\frac{1}{\sigma f}}$$

Plane waves in a homogeneous media

In time

$$\nabla^2 \mathbf{h} - \mu\epsilon \frac{\partial^2 \mathbf{h}}{\partial t^2} - \mu\sigma \frac{\partial \mathbf{h}}{\partial t} = 0$$

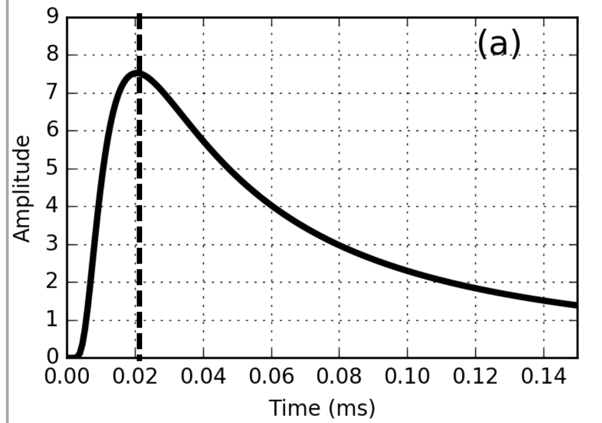
$$\mathbf{h}(t=0) = \mathbf{h}_0 \delta(t)$$

Solution for quasi-static

$$\mathbf{h}(t) = -\frac{(\mu\sigma)^{1/2} z}{2\pi^{1/2} t^{3/2}} e^{-\mu\sigma z^2/(4t)}$$

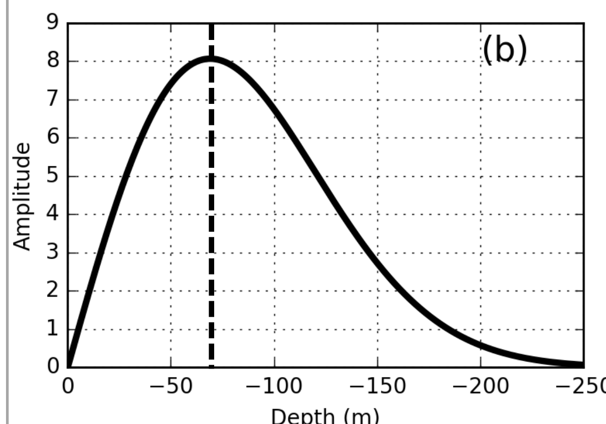
z: depth (m)

Peak time:



$$t_{max} = \frac{\mu\sigma z^2}{6}$$

Diffusion distance

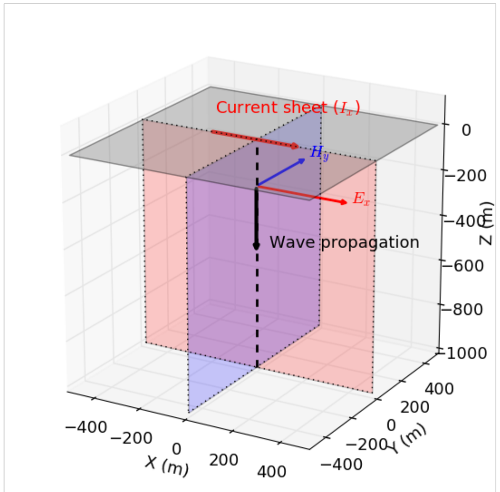


$$d = \sqrt{\frac{2t}{\mu\sigma}}$$

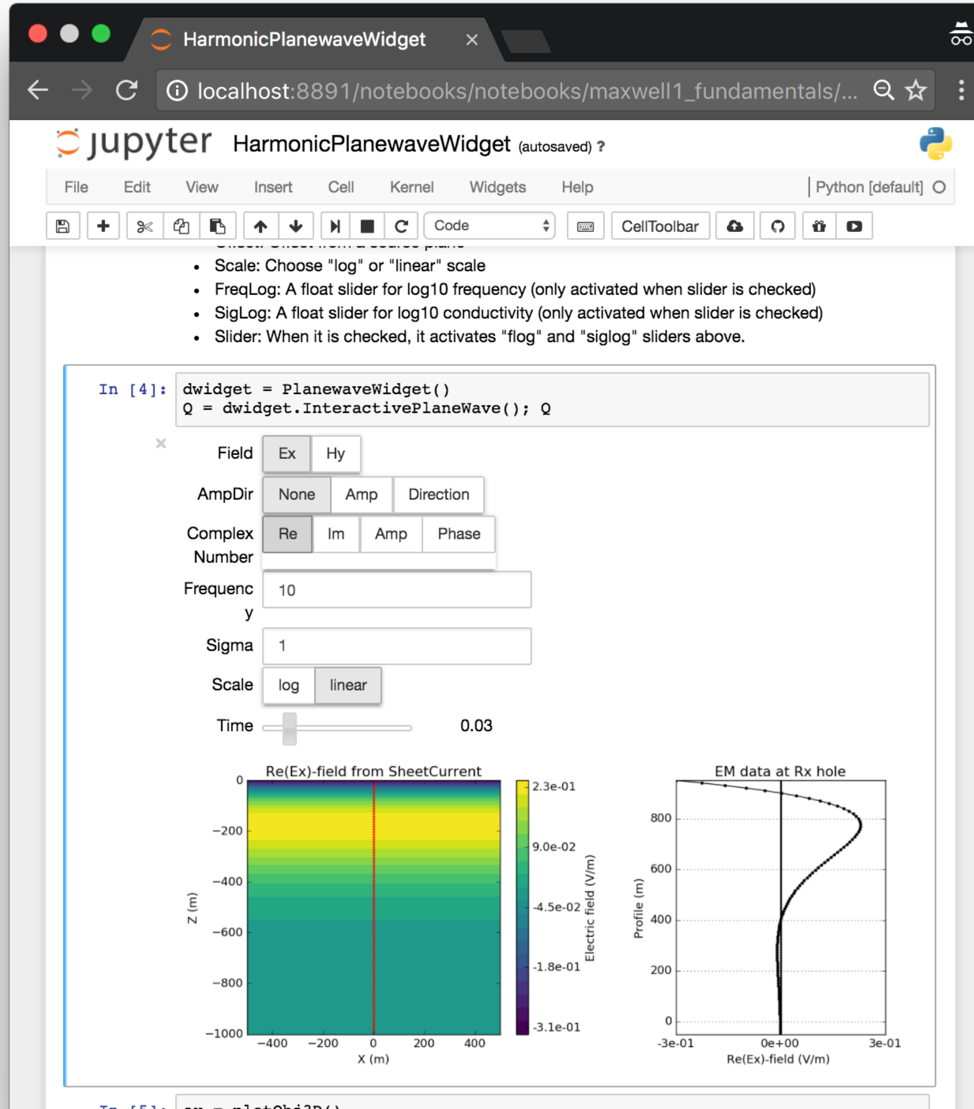
$$\approx 1260\sqrt{\frac{t}{\sigma}}$$

Frequency Domain App: Plane waves

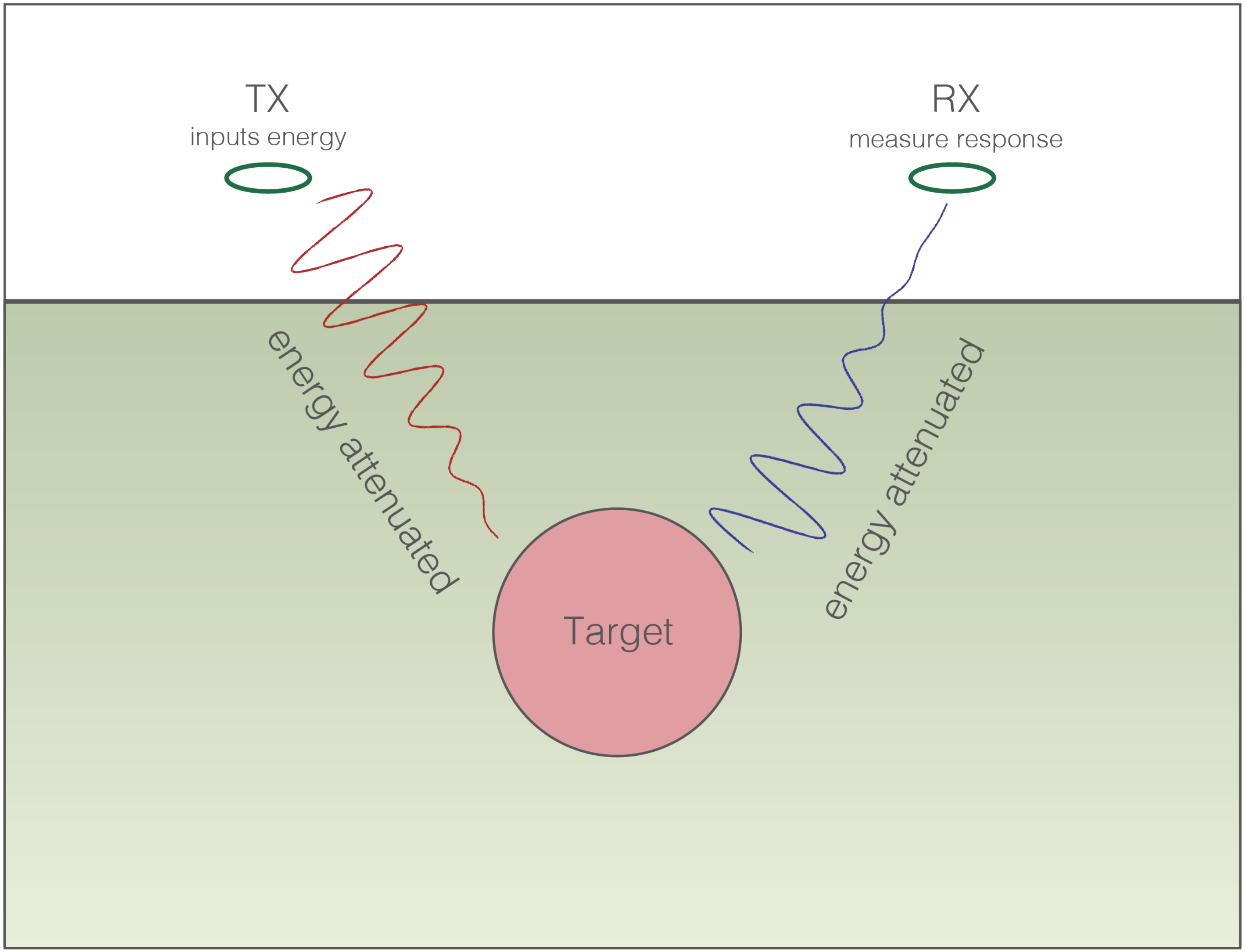
- Plane wave



$$\mathbf{H} = \mathbf{H_0}\underbrace{e^{-\alpha z}}_{\text{attenuation}}\underbrace{e^{-i(\beta z - \omega t)}}_{\text{phase}}$$

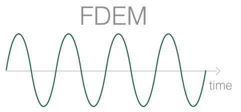


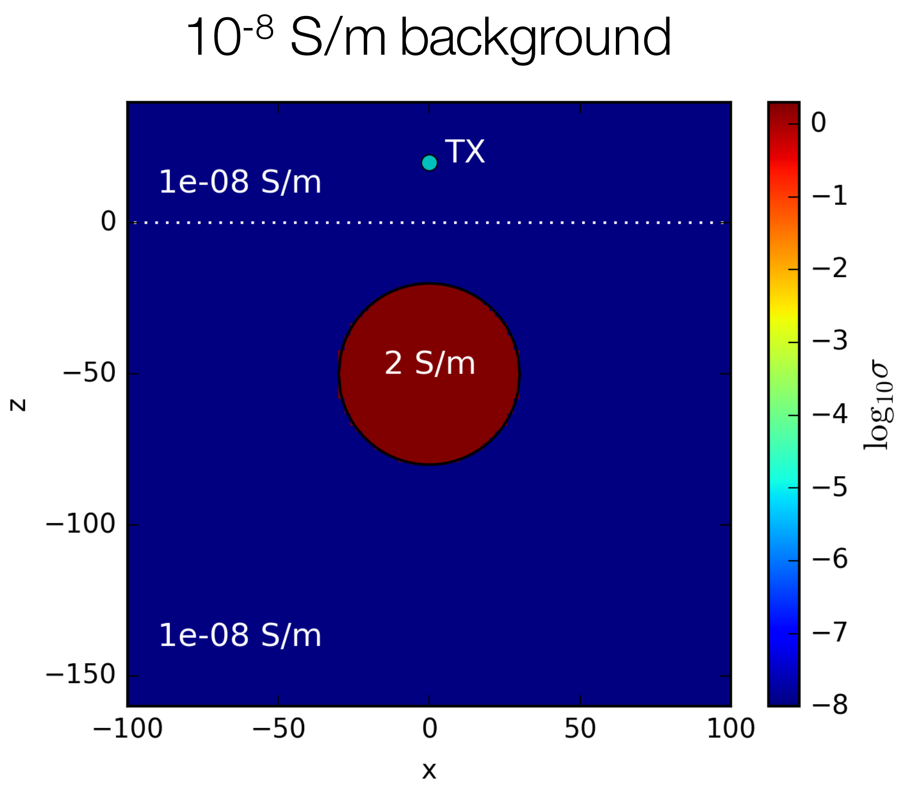
Effects of background resistivity

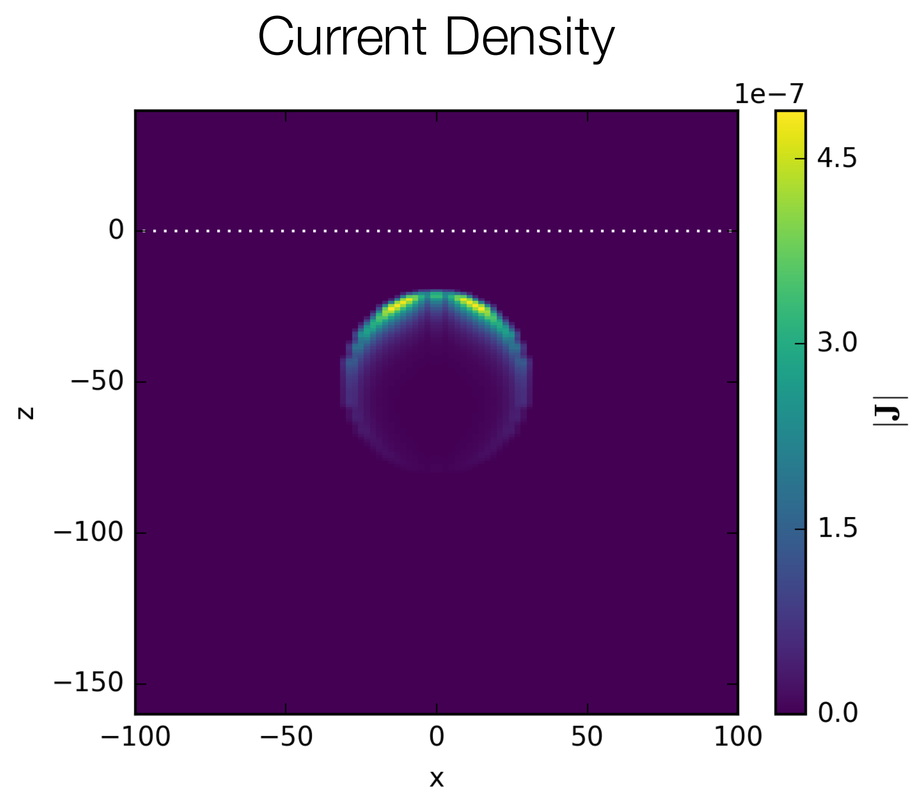


Effects of background resistivity: Frequency

- Buried, conductive sphere
- Vary background conductivity
- Frequency: 10^4 Hz







Effects of background resistivity: Frequency

- Buried, conductive sphere
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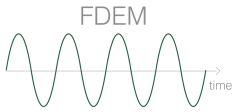
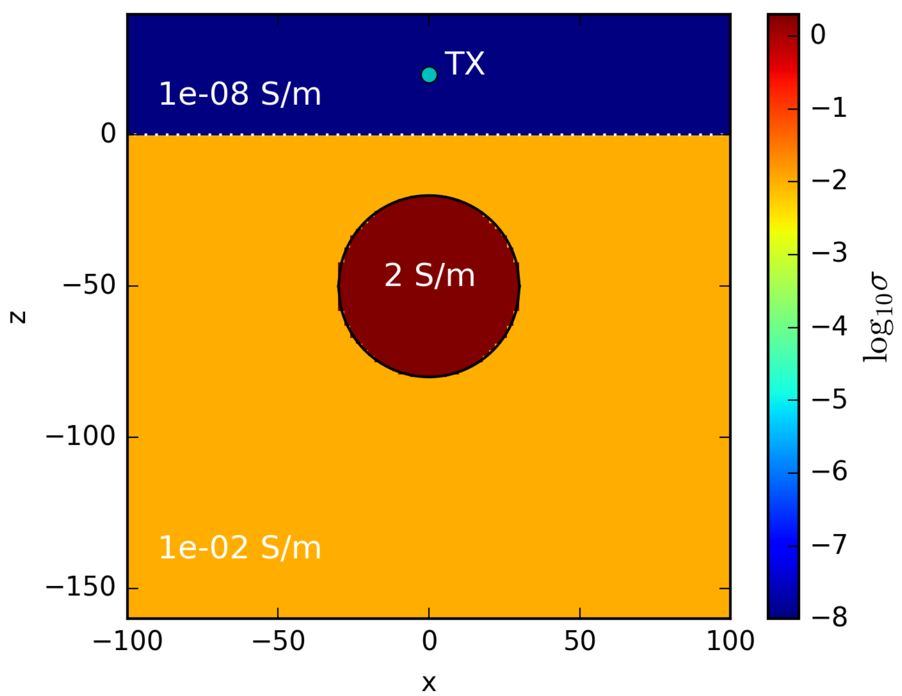
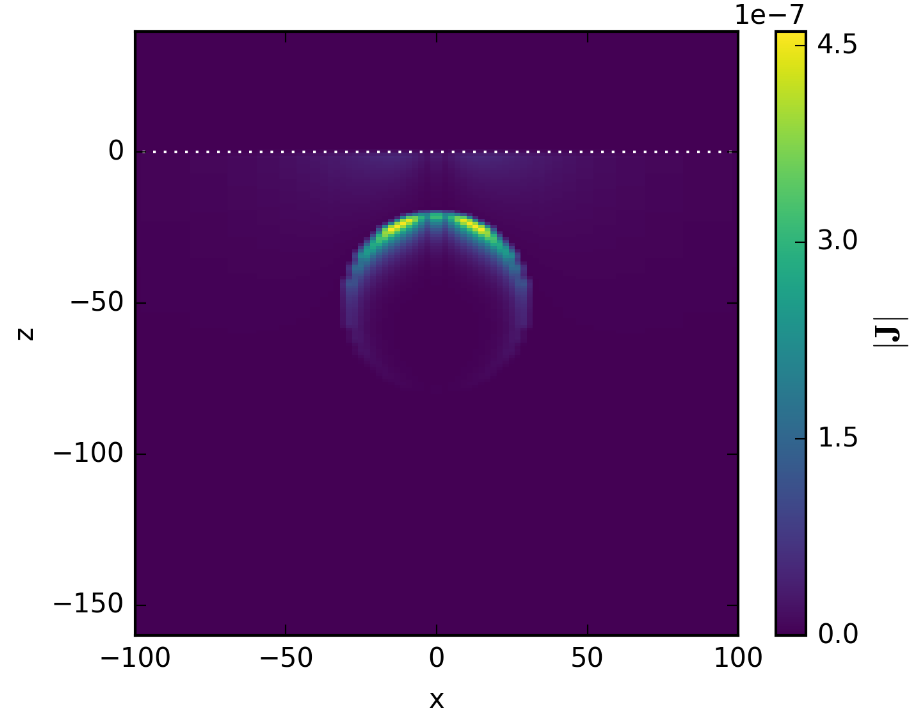


10^{-2} S/m background

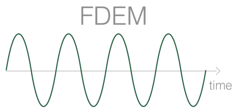


Current Density

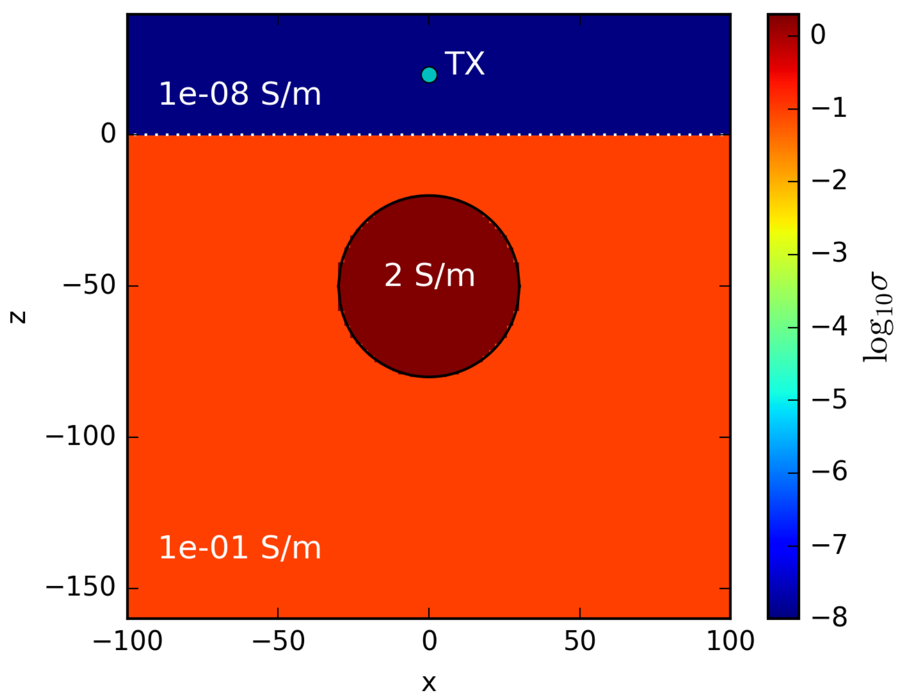


Effects of background resistivity: Frequency

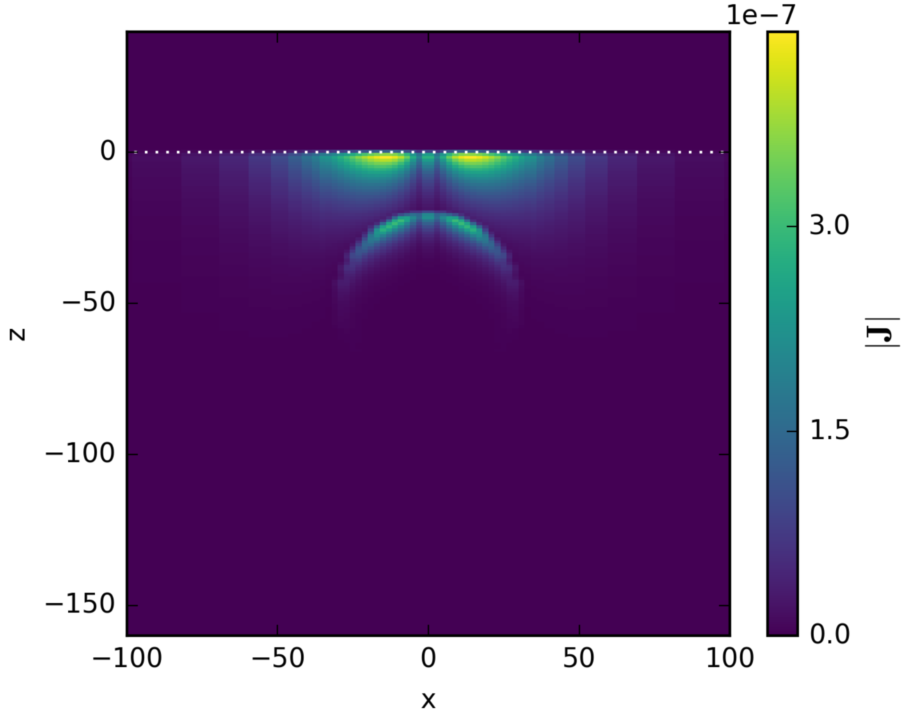
- Buried, conductive sphere
- Vary background conductivity
- Frequency: 10^4 Hz



10^{-1} S/m background

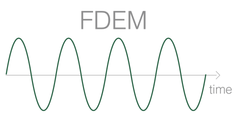


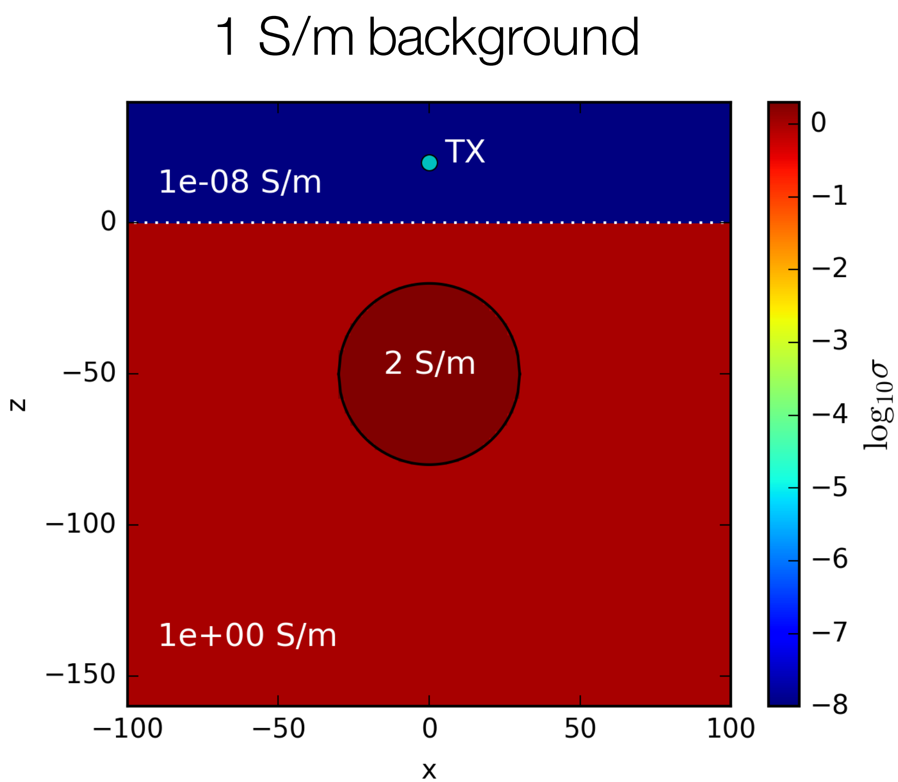
Current Density

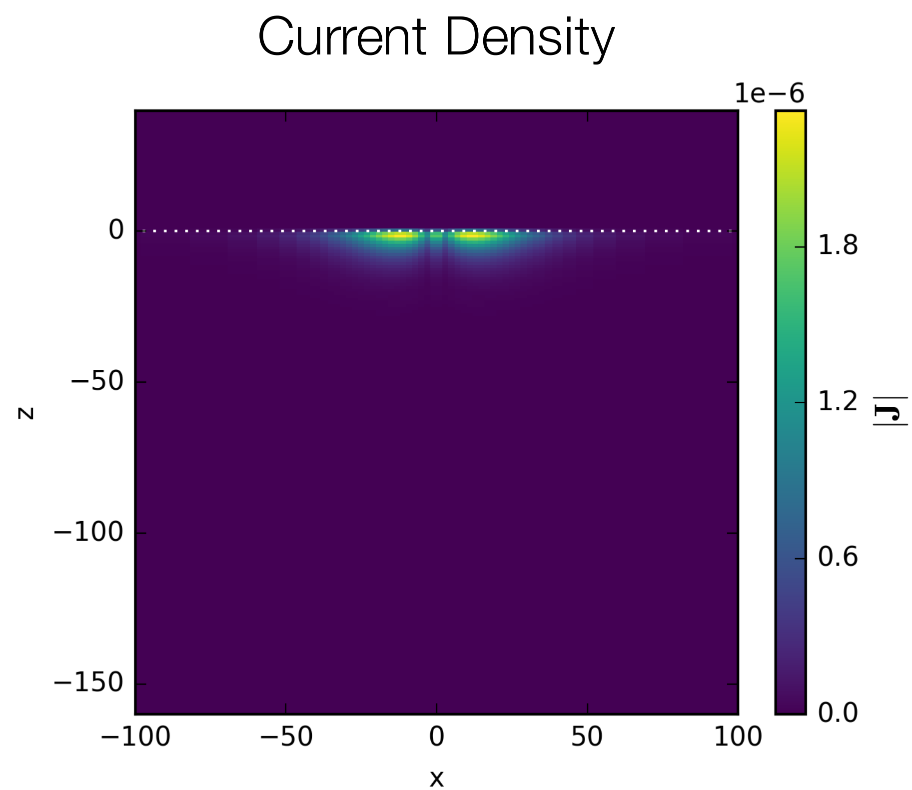


Effects of background resistivity: Frequency

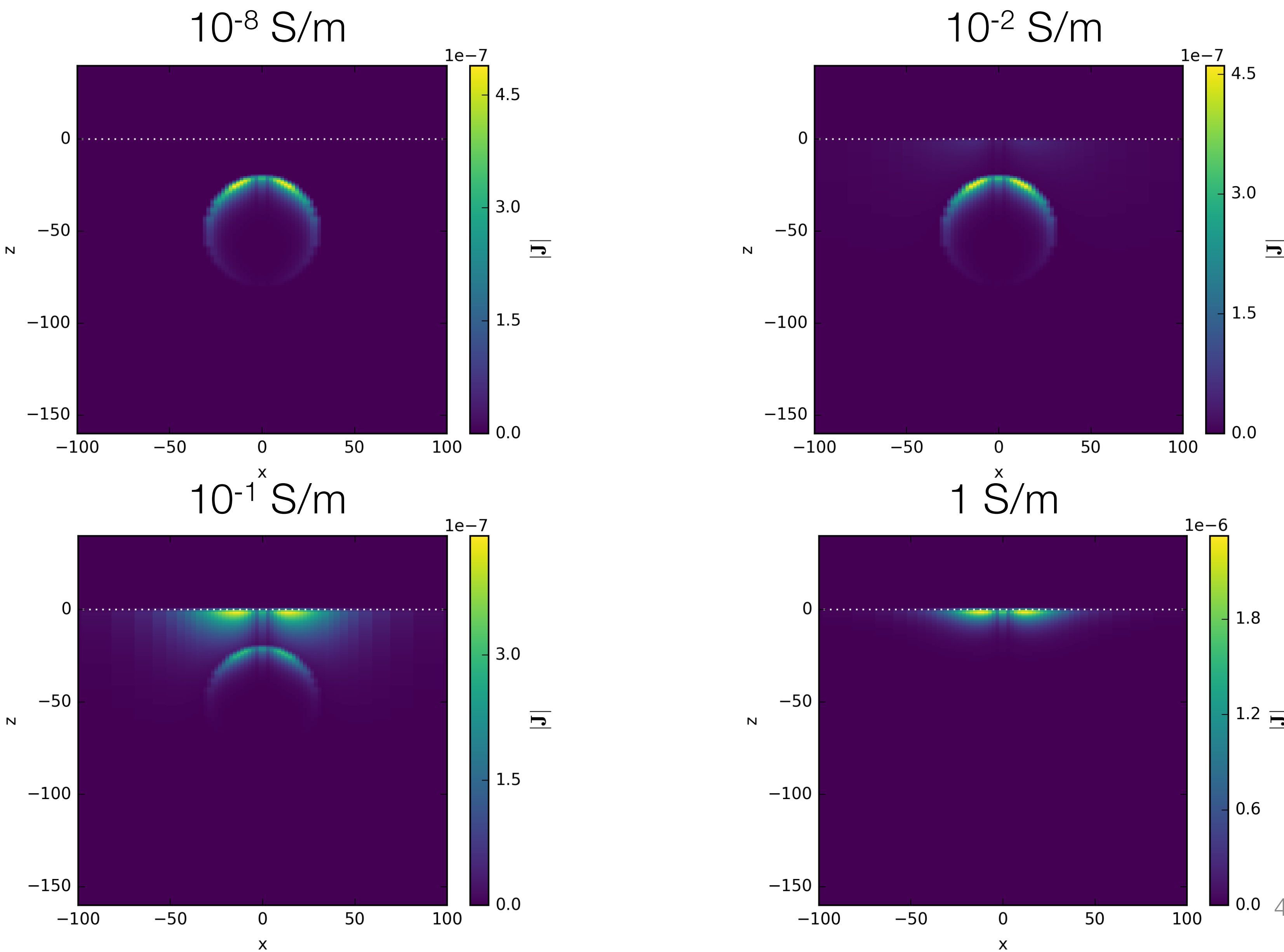
- Buried, conductive sphere
- Vary background conductivity
- Frequency: 10^4 Hz





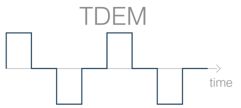


Effects of background resistivity: Frequency



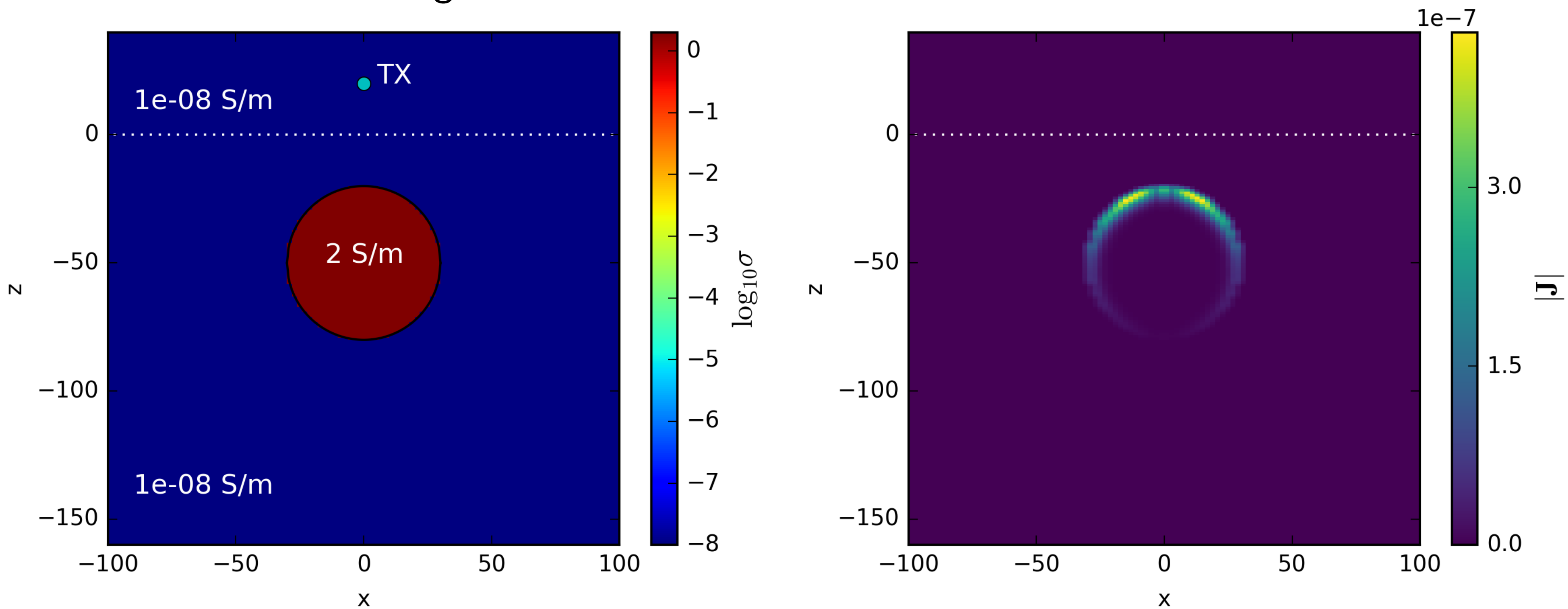
Effects of background resistivity: Time

- Buried, conductive sphere
- Vary background conductivity
- Time: 10^{-5} s



10^{-8} S/m background

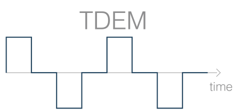
Current Density



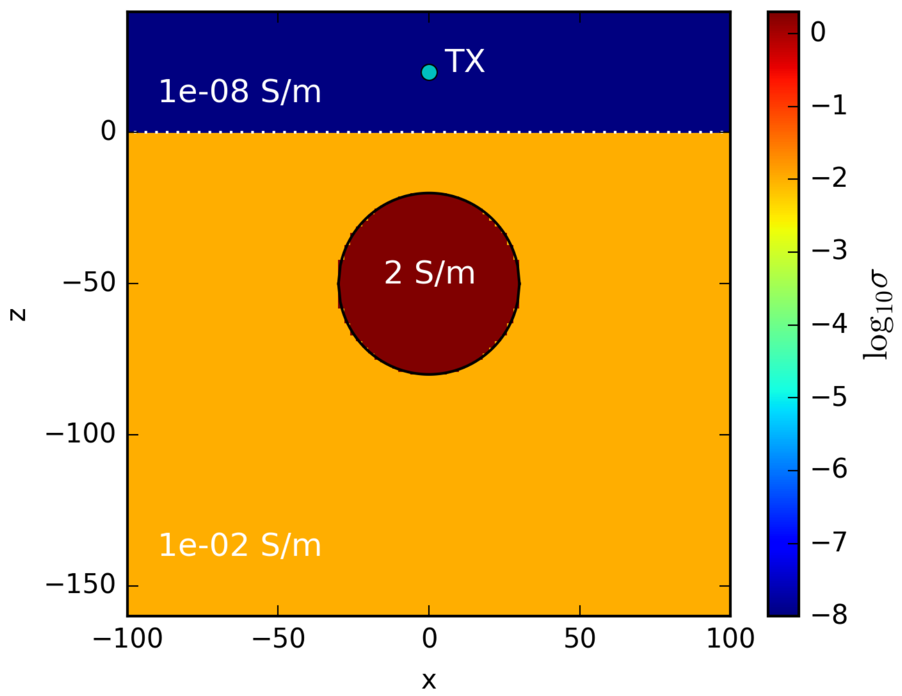
Effects of background resistivity: Time

- Buried, conductive sphere
- Vary background conductivity

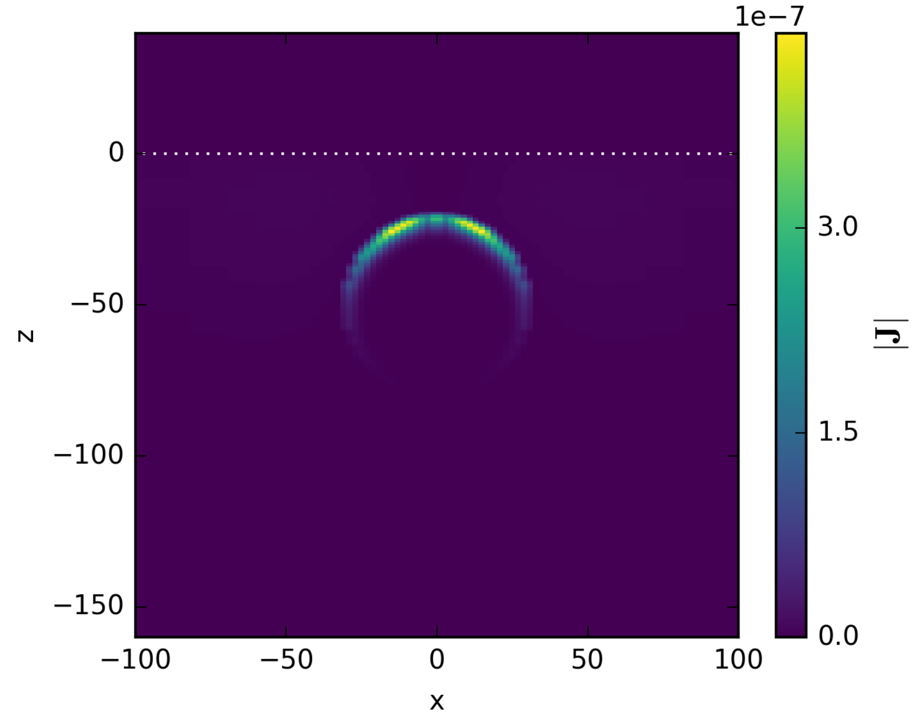
- Time: 10^{-5} s



10^{-2} S/m background

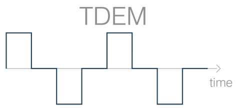


Current Density

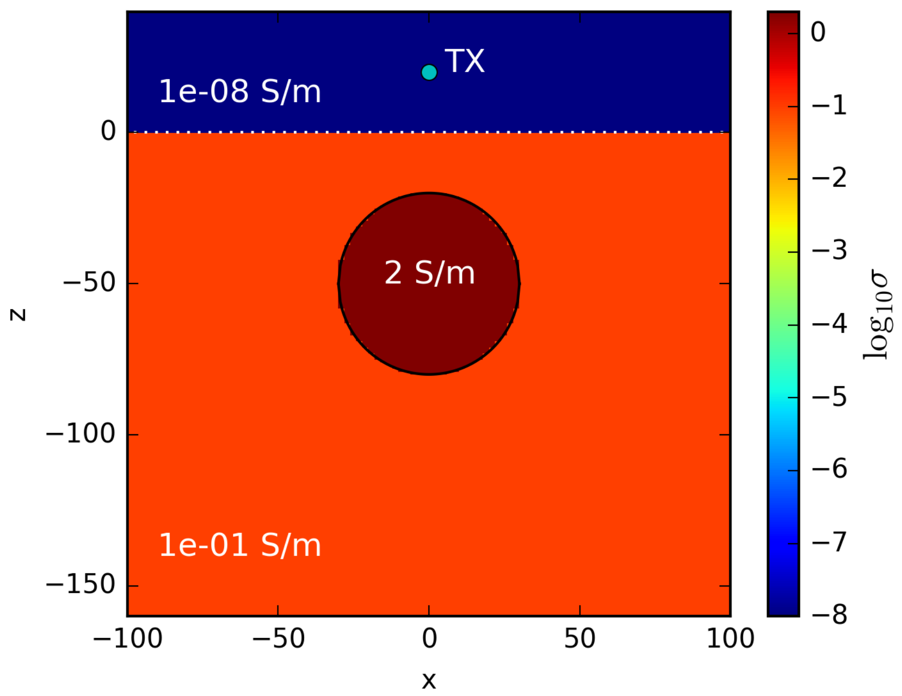


Effects of background resistivity: Time

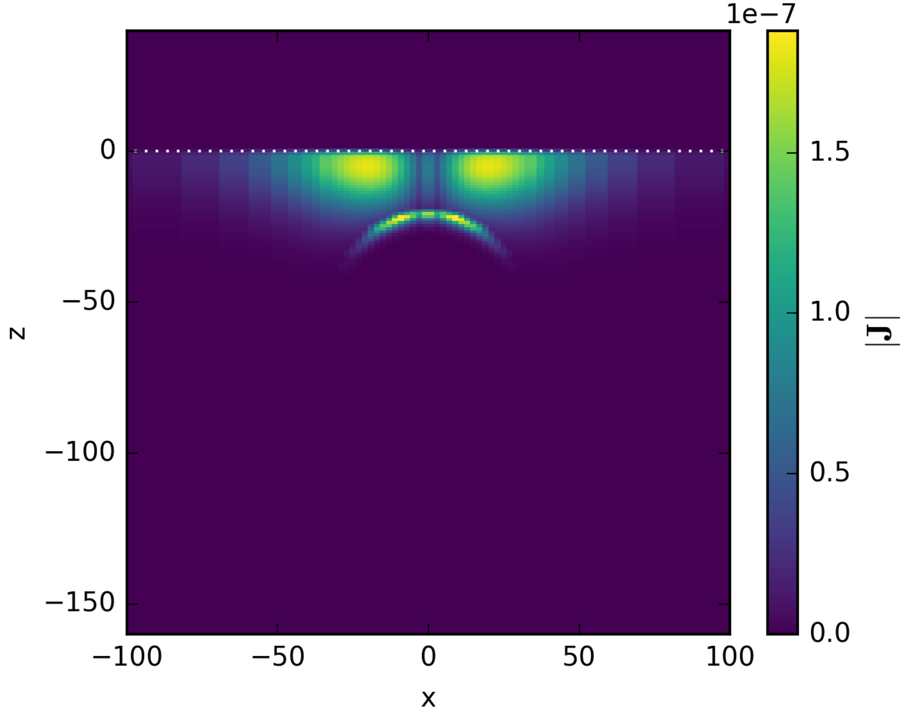
- Buried, conductive sphere
- Vary background conductivity
- Time: 10^{-5} s



10^{-1} S/m background



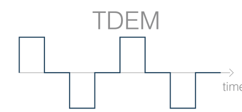
Current Density



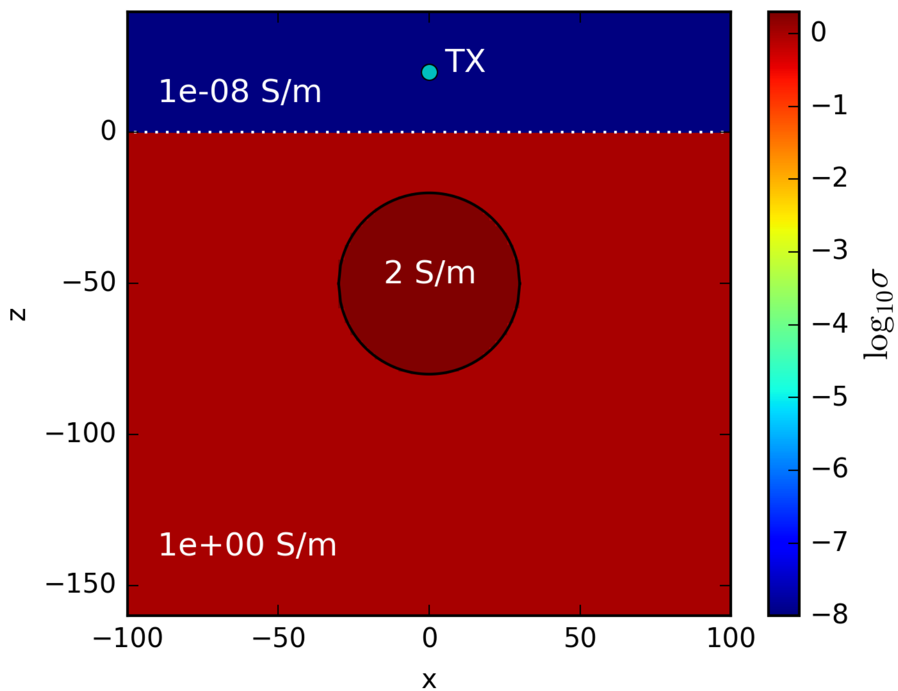
Effects of background resistivity: Time

- Buried, conductive sphere
- Vary background conductivity

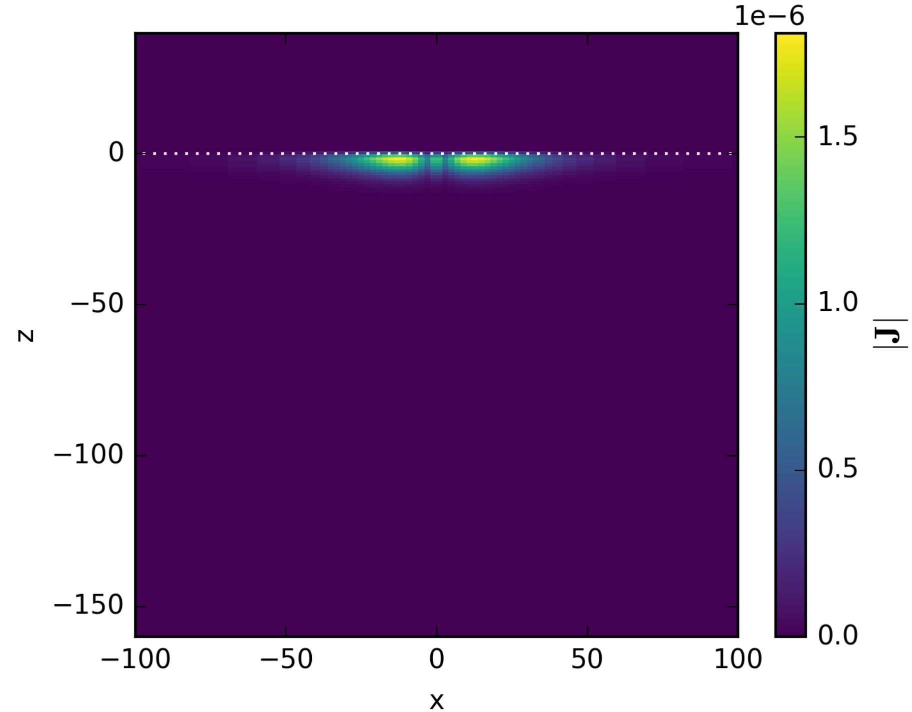
- Time: 10^{-5} s



1 S/m background



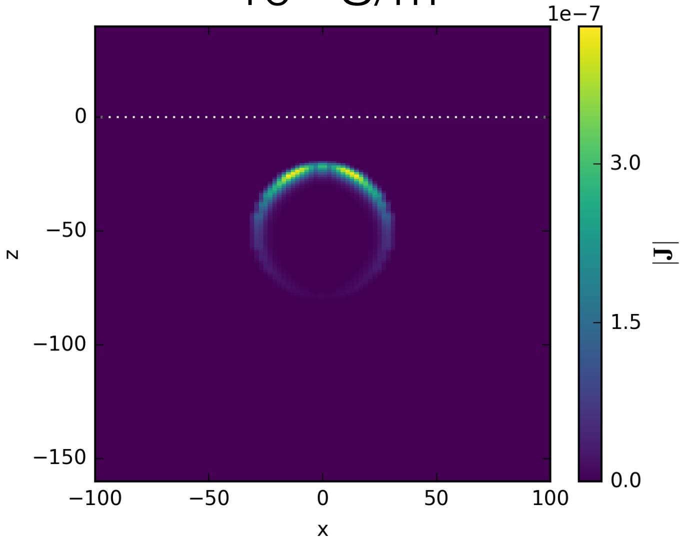
Current Density



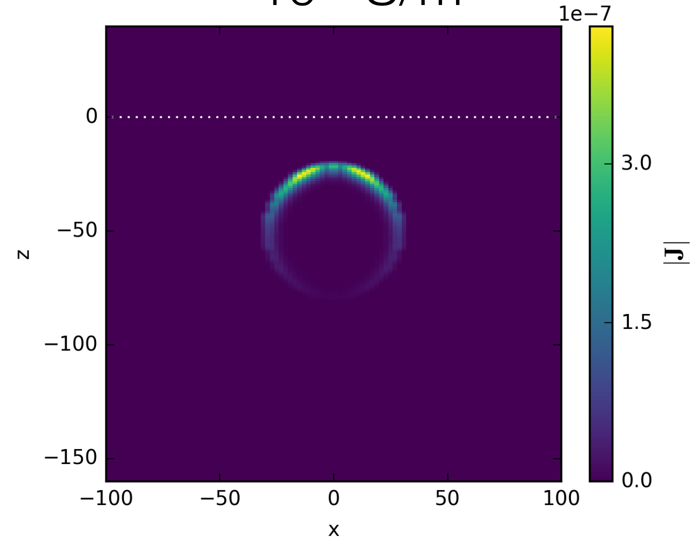
10^{-5} s

Effects of background resistivity: Time

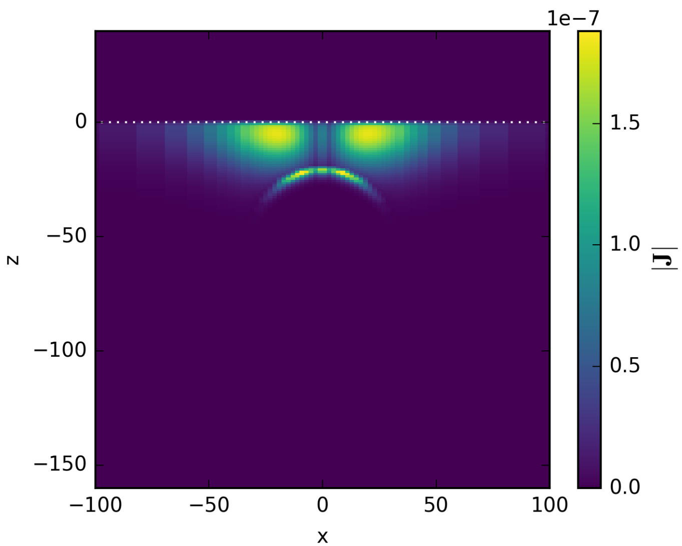
10^{-8} S/m



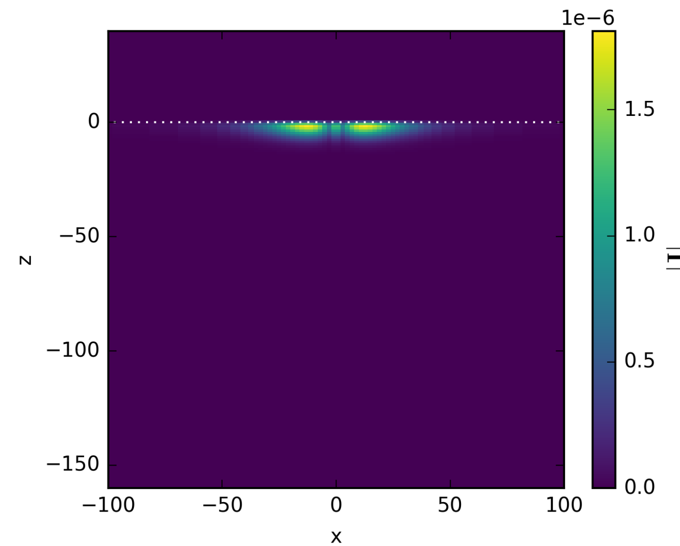
10^{-2} S/m



10^{-1} S/m



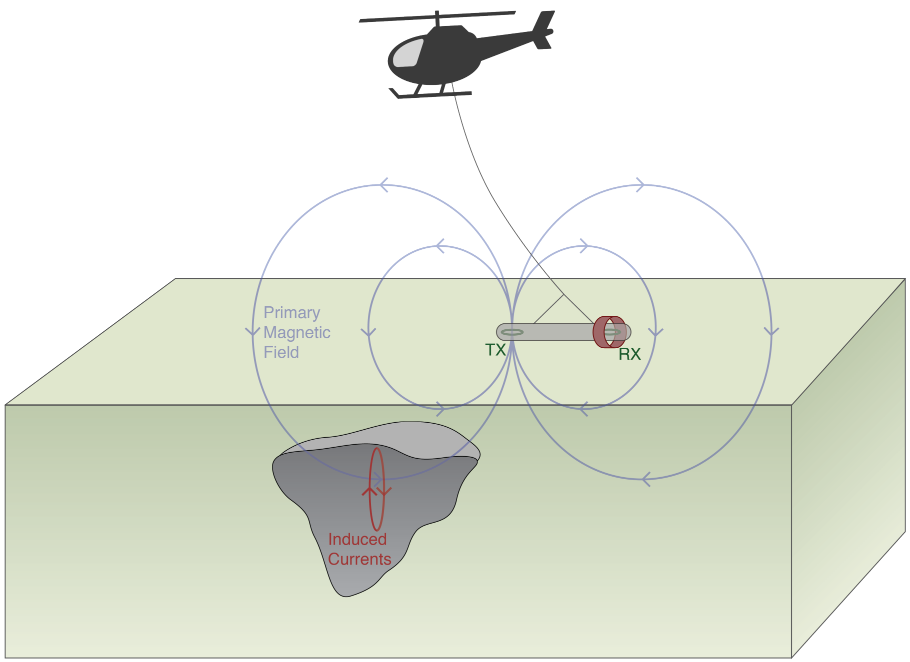
1 S/m



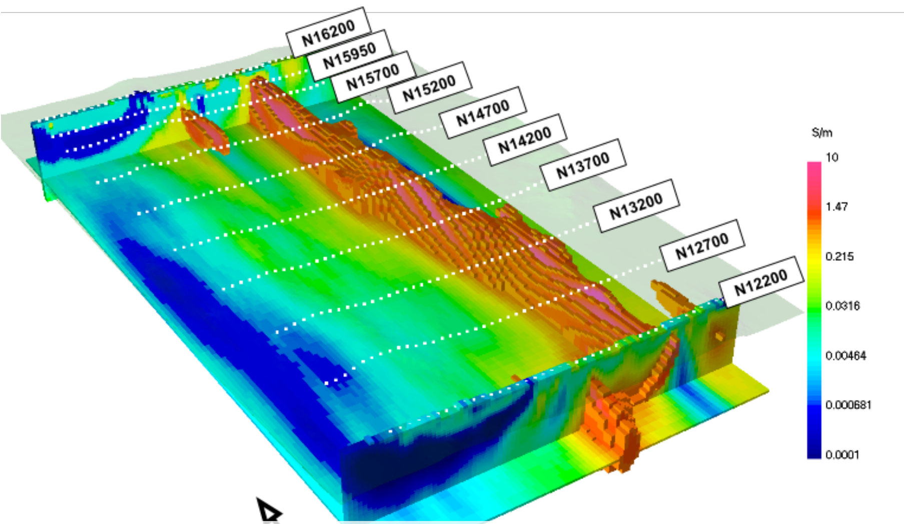
Recap: what have we learned?

- Basics of EM induction
- Response functions
- Mutual coupling
- Data for frequency or time domain systems
- Circuit model is a good proxy
- Need to account for energy losses

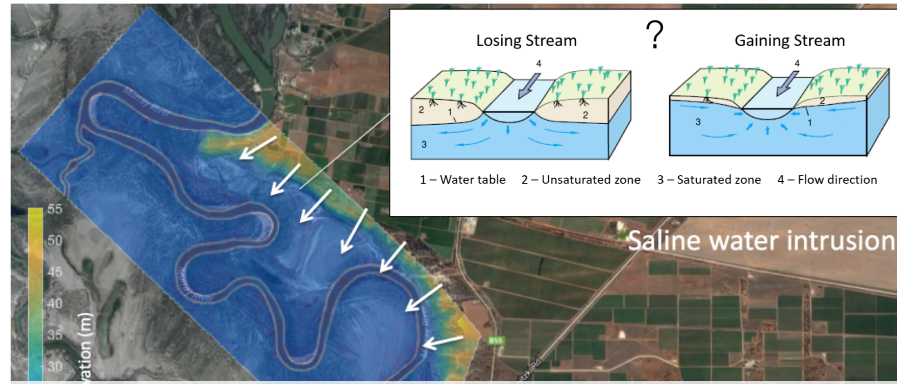
- Ready to look at some field examples



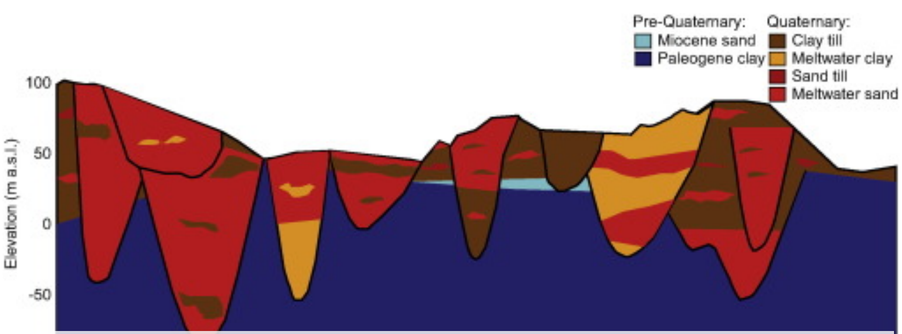
Today's Case Histories



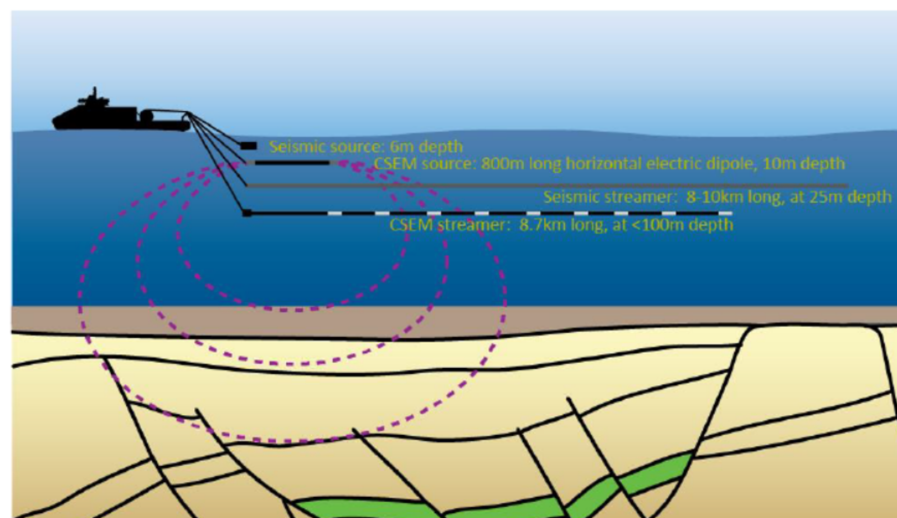
Mt. Isa, Australia: Mineral Exploration



Bookpurnong, Australia: diagnosing river salination

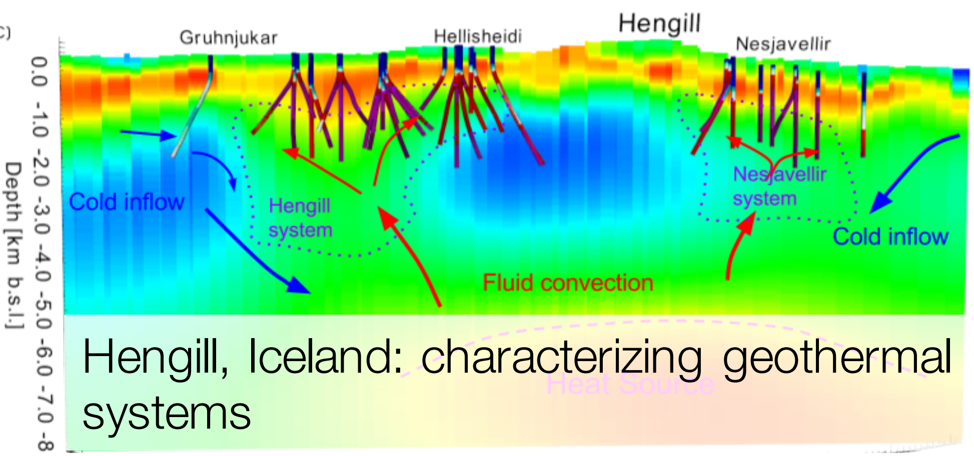


Kasted, Denmark: mapping paleo channel for hydrology

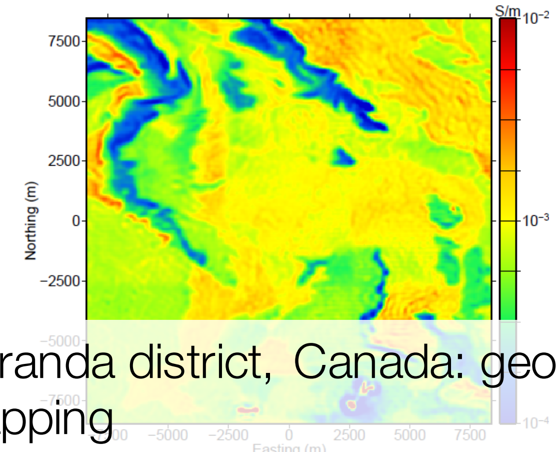


Barents Sea, Norway: Hydrocarbon de-risking

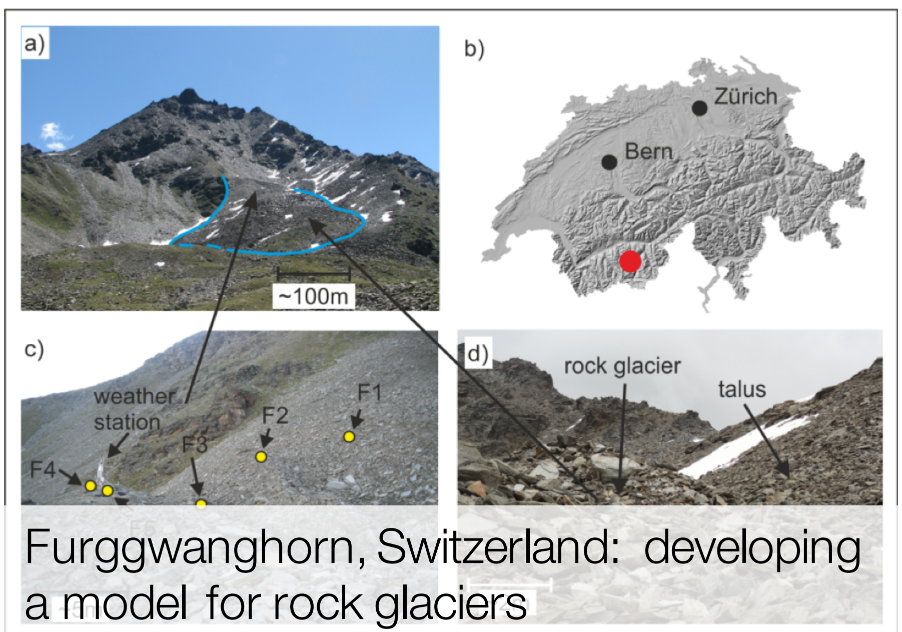
Today's Case Histories



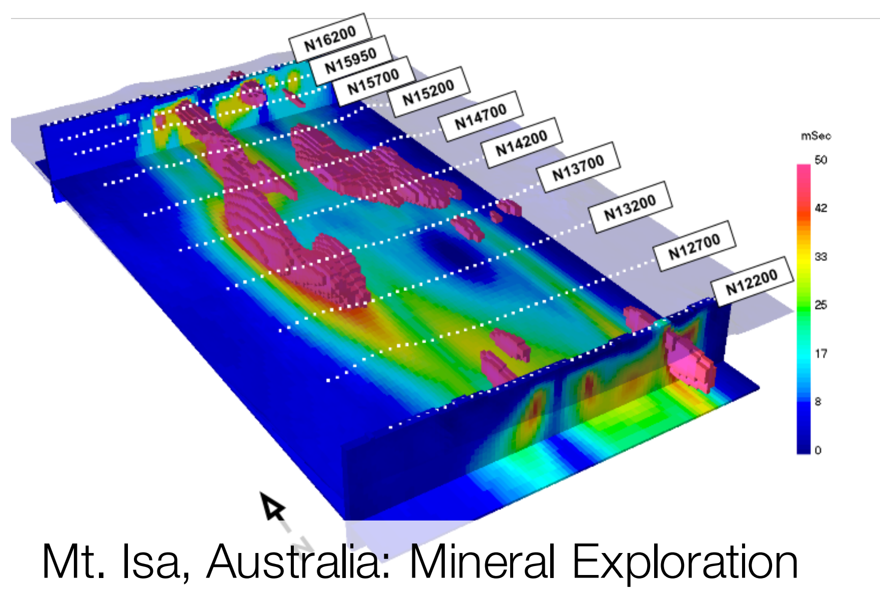
Hengill, Iceland: characterizing geothermal systems



Noranda district, Canada: geologic mapping



Furggwanghorn, Switzerland: developing a model for rock glaciers



Mt. Isa, Australia: Mineral Exploration

End of EM Fundamentals