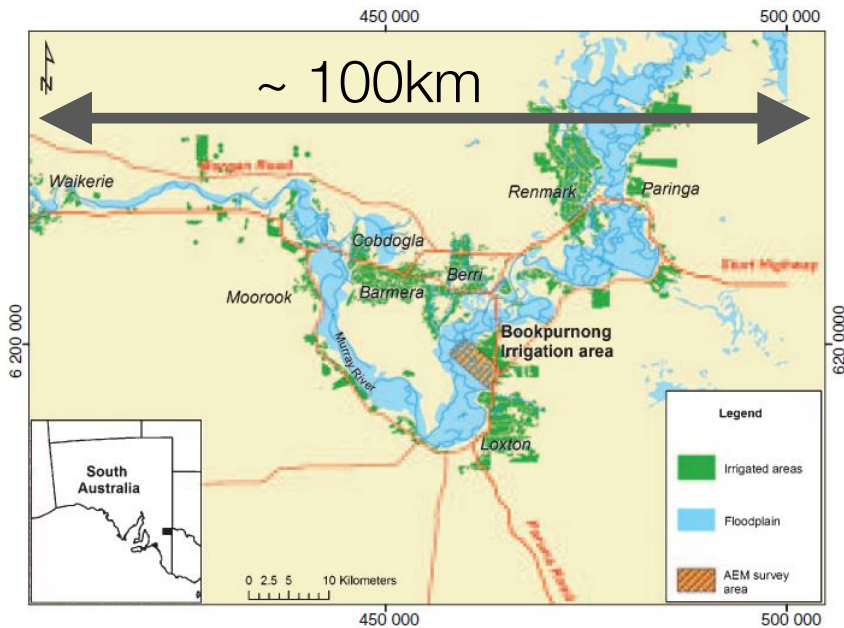


EM Fundamentals

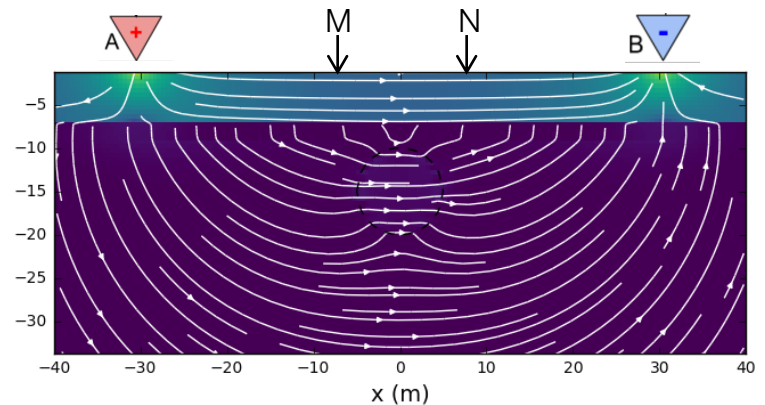
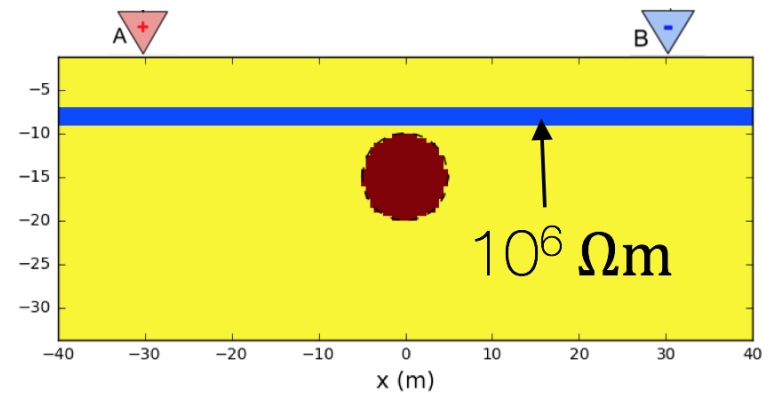


Motivation: applications difficult for DC

Large areas to be covered



Resistive layer “shields” target



Rugged terrain



Hard to inject

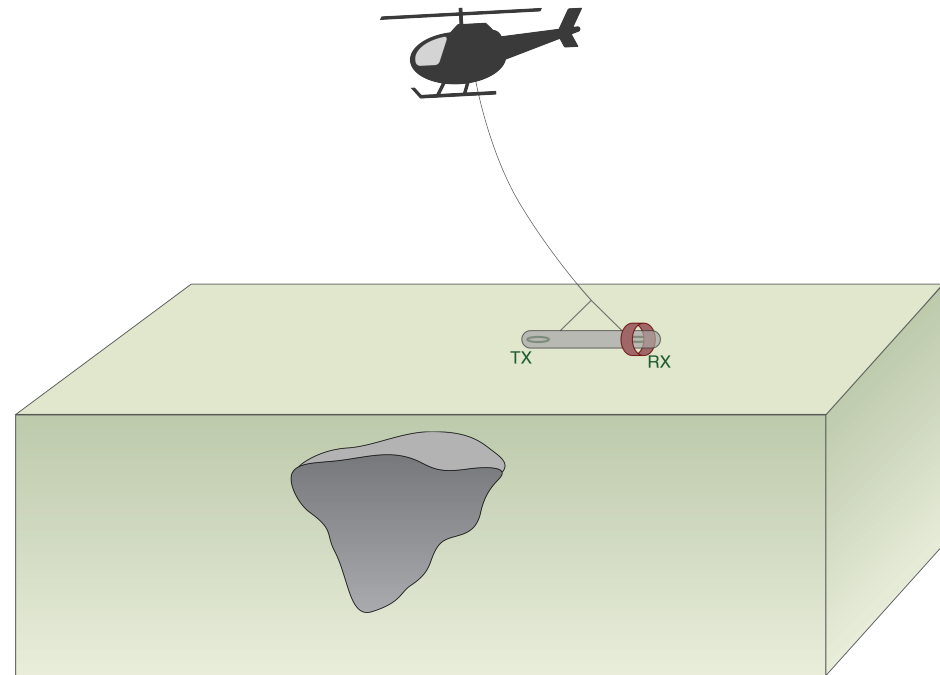


Outline

- Basic Survey
- Ampere's and Faraday's Laws (2-coil App)
- Circuit model for EM induction
- Frequency and time domain data
- Sphere in homogeneous earth
- Cyl code
- Energy losses in the ground

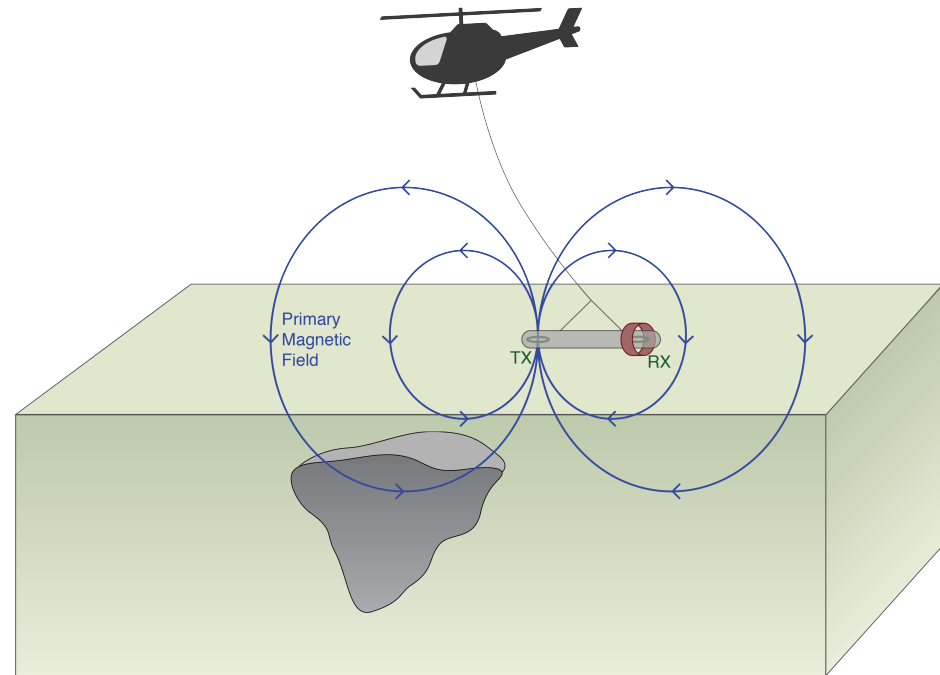
Basic Experiment

- **Setup:**
 - transmitter and receiver are in a towed bird



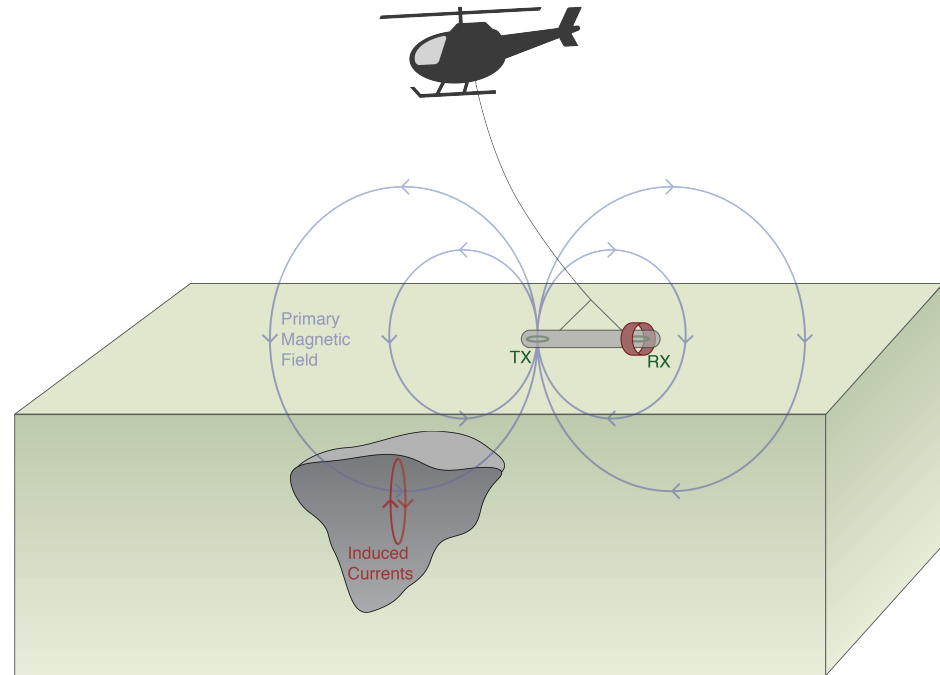
Basic Experiment

- **Setup:**
 - transmitter and receiver are in a towed bird
- **Primary:**
 - Transmitter produces a primary magnetic field



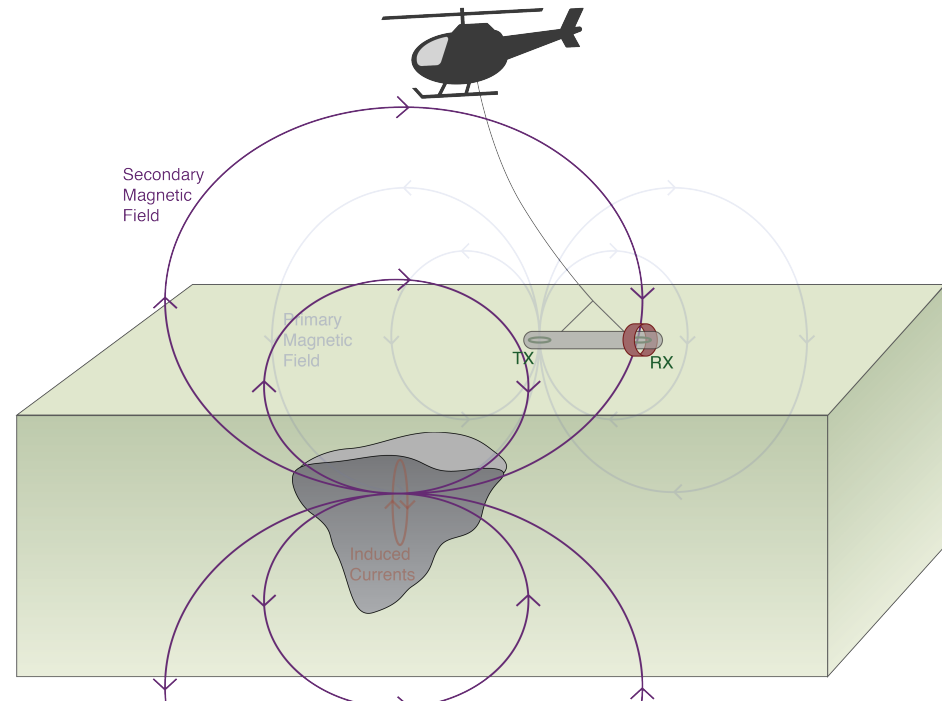
Basic Experiment

- **Setup:**
 - transmitter and receiver are in a towed bird
- **Primary:**
 - Transmitter produces a primary magnetic field
- **Induced Currents:**
 - Time varying magnetic fields generate electric fields everywhere and currents in conductors



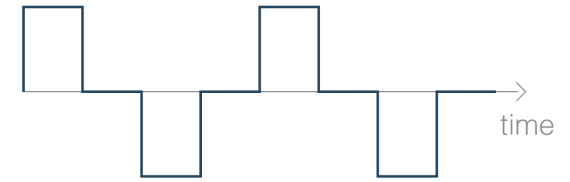
Basic Experiment

- **Setup:**
 - transmitter and receiver are in a towed bird
- **Primary:**
 - Transmitter produces a primary magnetic field
- **Induced Currents:**
 - Time varying magnetic fields generate electric fields everywhere and currents in conductors
- **Secondary Fields:**
 - The induced currents produce a secondary magnetic field.

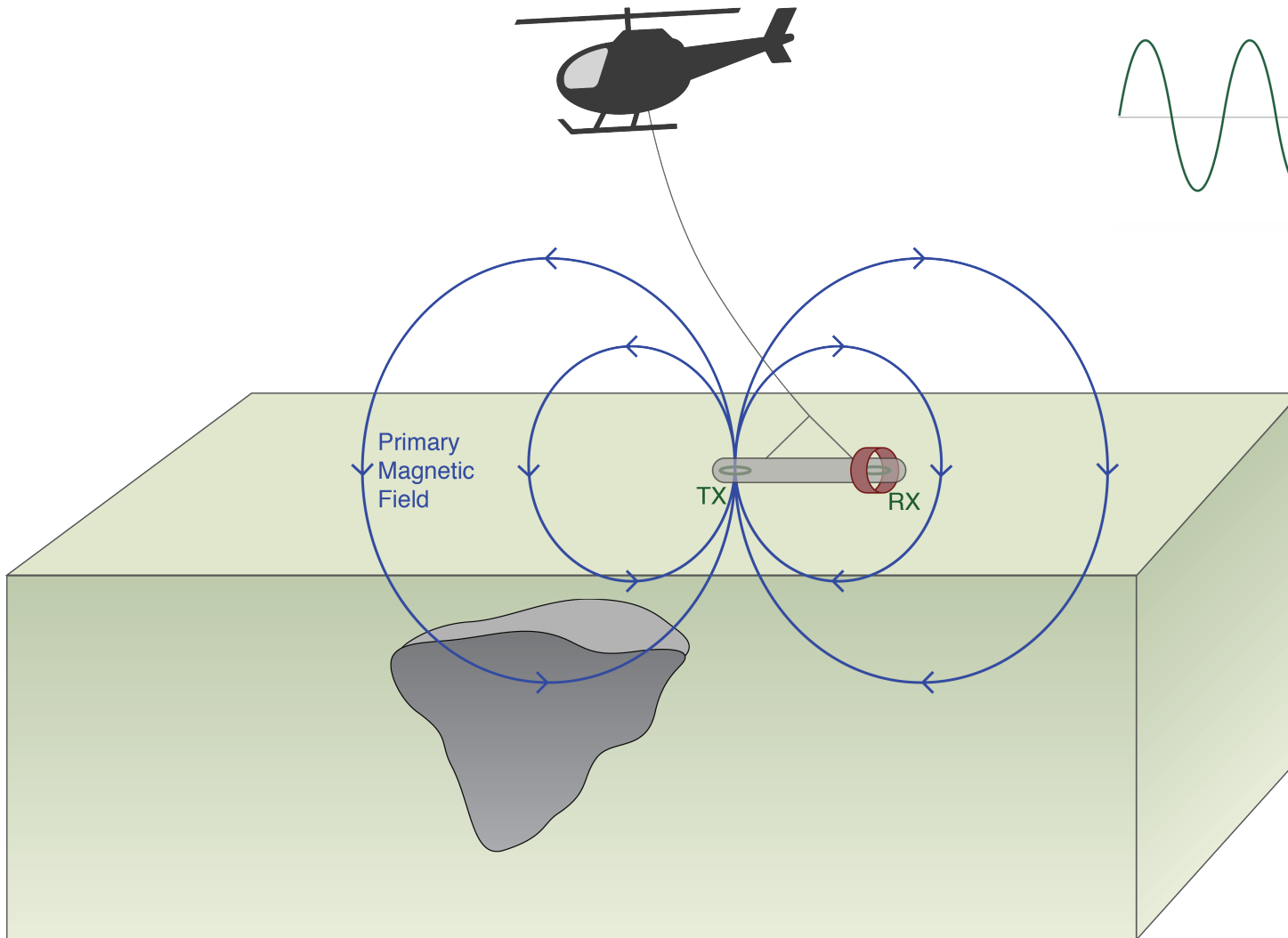
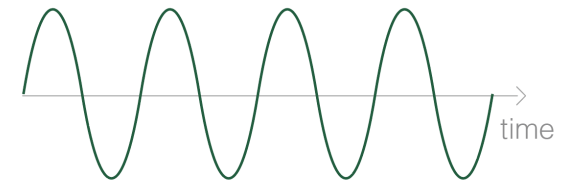


Transmitter

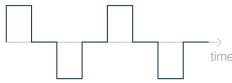

waveform



or



Basic Equations: Quasi-static

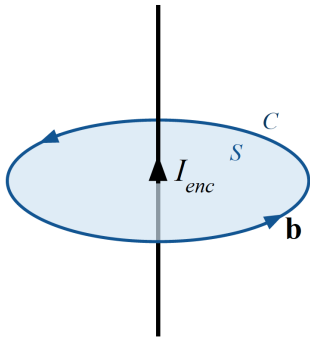
	Time 	Frequency 
Faraday's Law	$\nabla \times \mathbf{e} = - \frac{\partial \mathbf{b}}{\partial t}$	$\nabla \times \mathbf{E} = - i\omega \mathbf{B}$
Ampere's Law	$\nabla \times \mathbf{h} = \mathbf{j} + \frac{\partial \mathbf{d}}{\partial t}$	$\nabla \times \mathbf{H} = \mathbf{J} + i\omega \mathbf{D}$
No Magnetic Monopoles	$\nabla \cdot \mathbf{b} = 0$	$\nabla \cdot \mathbf{B} = 0$
Constitutive Relationships (non-dispersive)	$\mathbf{j} = \sigma \mathbf{e}$ $\mathbf{b} = \mu \mathbf{h}$ $\mathbf{d} = \varepsilon \mathbf{e}$	$\mathbf{J} = \sigma \mathbf{E}$ $\mathbf{B} = \mu \mathbf{H}$ $\mathbf{D} = \varepsilon \mathbf{E}$

* Solve with sources and boundary conditions

Ampere's Law

$$\nabla \times \mathbf{H} = \mathbf{J}$$

Wire



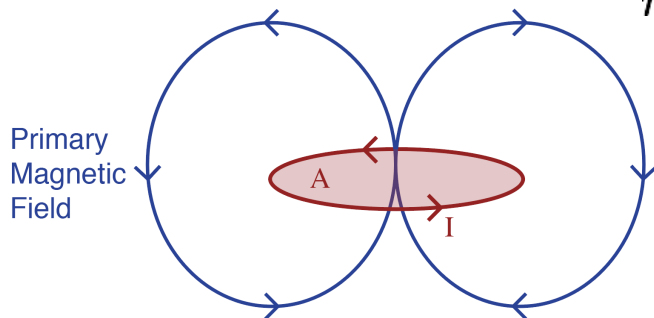
$$\mathbf{B} = \frac{\mu_0 I_{enc}}{2\pi r} \hat{\phi}$$

Right hand rule

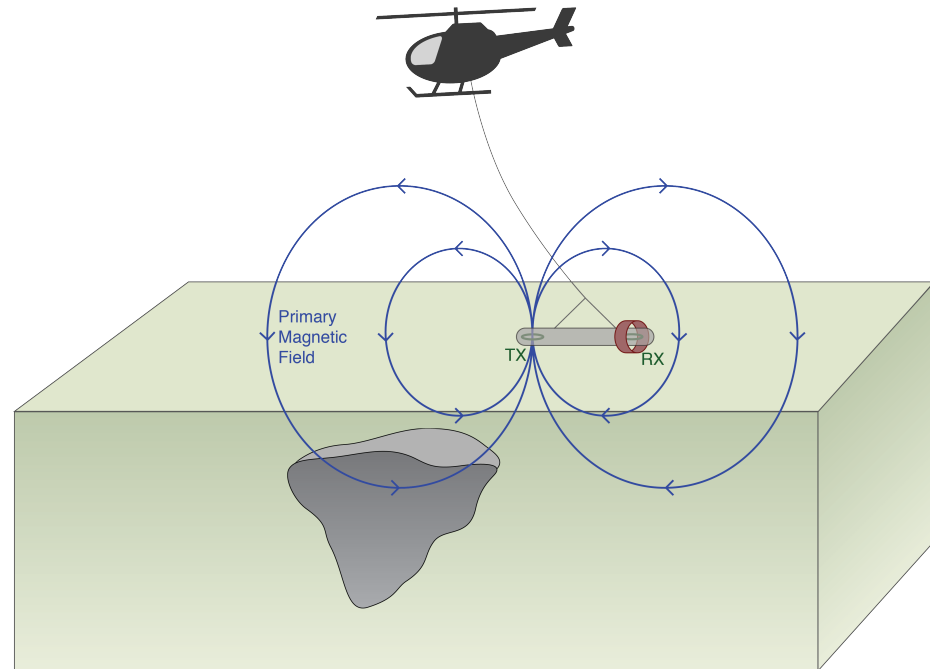
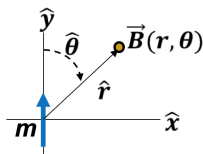
Current loop

$$\mathbf{B} = \frac{\mu_0 m}{4\pi r^3} (2 \cos \theta \hat{\mathbf{r}} + \sin \theta \hat{\boldsymbol{\theta}})$$

$$m = IA$$



Primary
Magnetic
Field



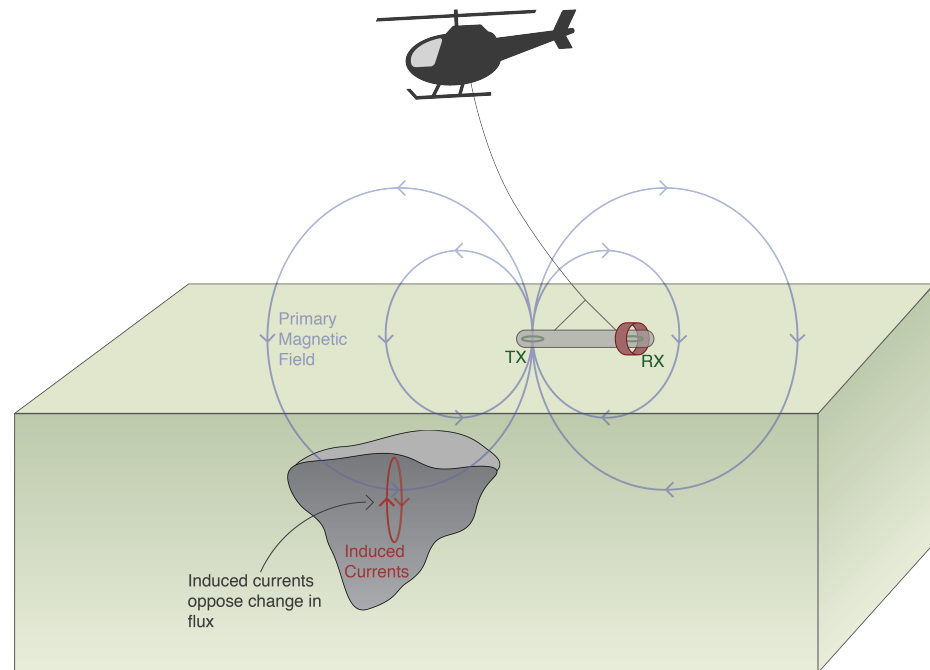
Faraday's Law

$$\nabla \times \mathbf{e} = - \frac{\partial \mathbf{b}}{\partial t}$$

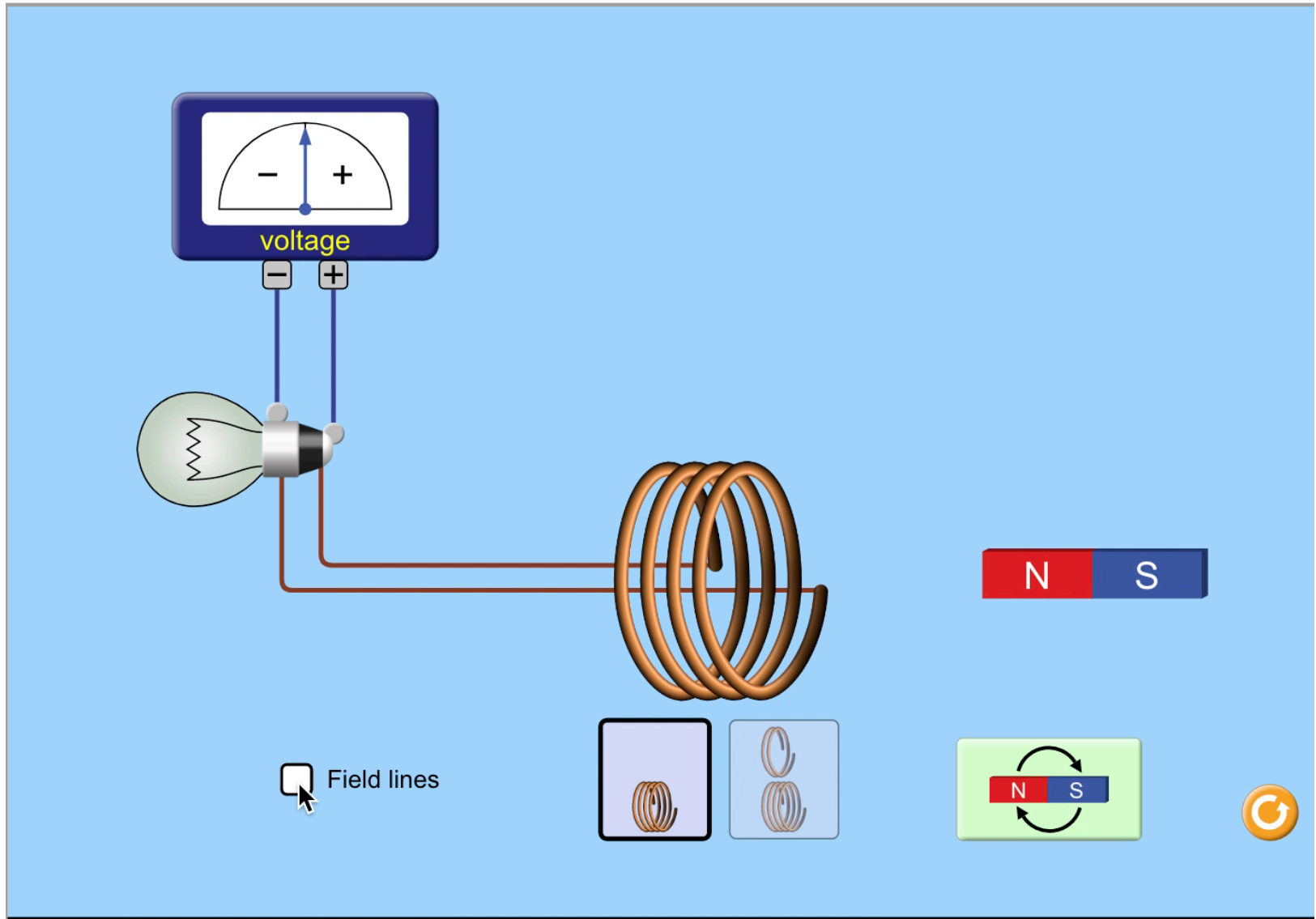
Lenz'
Law

Ohm's Law

$$\mathbf{j} = \sigma \mathbf{e}$$



Faraday's Law



Faraday's Law

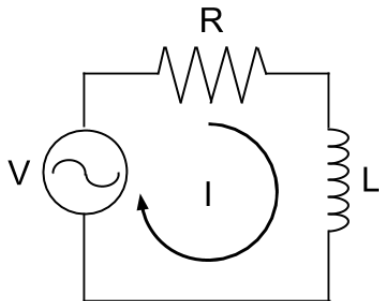
$$\nabla \times \mathbf{e} = - \frac{\partial \mathbf{b}}{\partial t}$$

Magnetic Flux

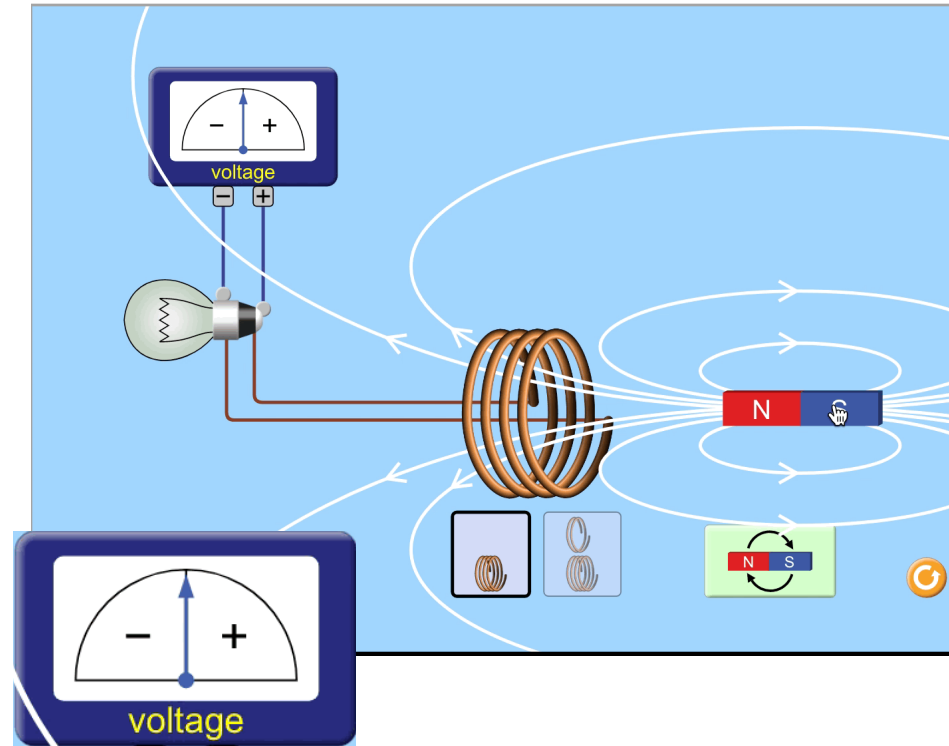
$$\phi_{\mathbf{b}} = \int_A \mathbf{b} \cdot \hat{\mathbf{n}} \, da$$

Induced EMF

$$V = EMF = - \frac{d\phi_{\mathbf{b}}}{dt} = \mathbf{0}$$



ϕ_b : constant



Faraday's Law

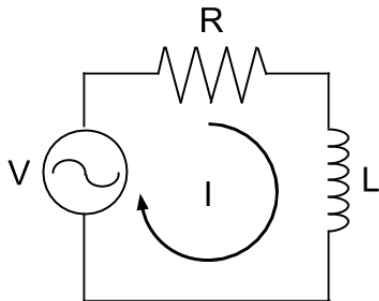
$$\nabla \times \mathbf{e} = - \frac{\partial \mathbf{b}}{\partial t}$$

Magnetic Flux

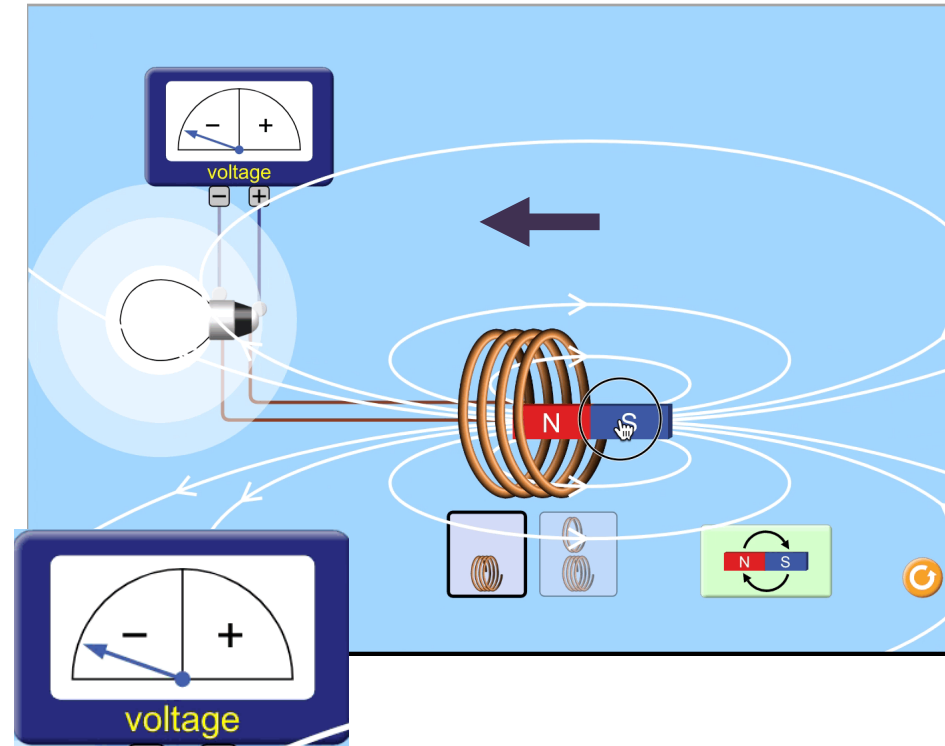
$$\phi_{\mathbf{b}} = \int_A \mathbf{b} \cdot \hat{\mathbf{n}} \, da$$

Induced EMF

$$V = EMF = - \frac{d\phi_{\mathbf{b}}}{dt} < 0$$



$\phi_b : \uparrow$



Faraday's Law

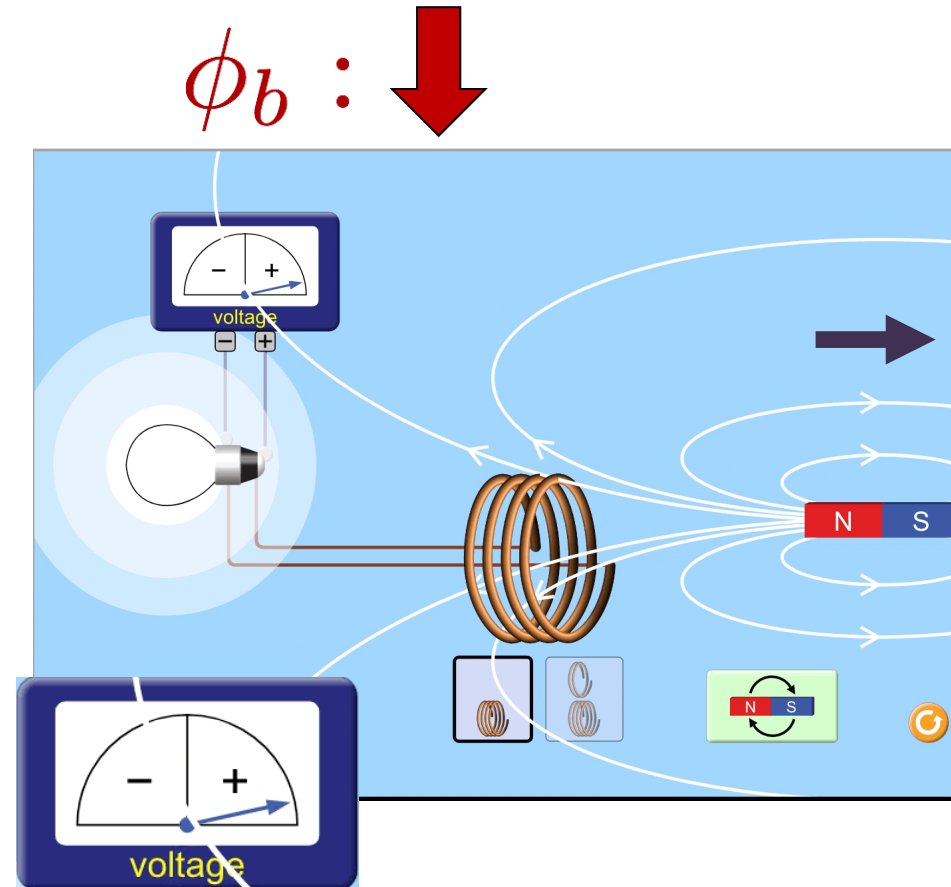
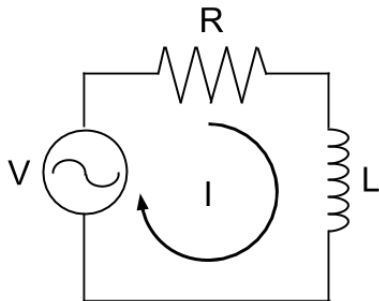
$$\nabla \times \mathbf{e} = - \frac{\partial \mathbf{b}}{\partial t}$$

Magnetic Flux

$$\phi_{\mathbf{b}} = \int_A \mathbf{b} \cdot \hat{\mathbf{n}} \, da$$

Induced EMF

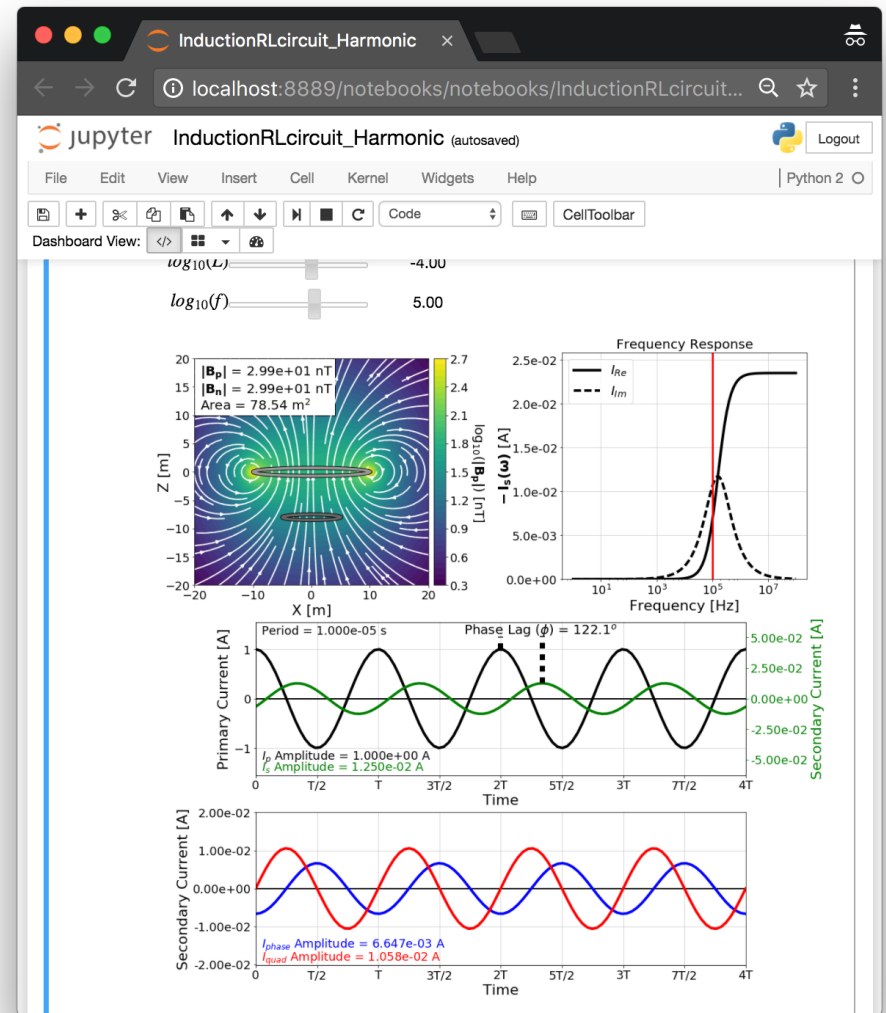
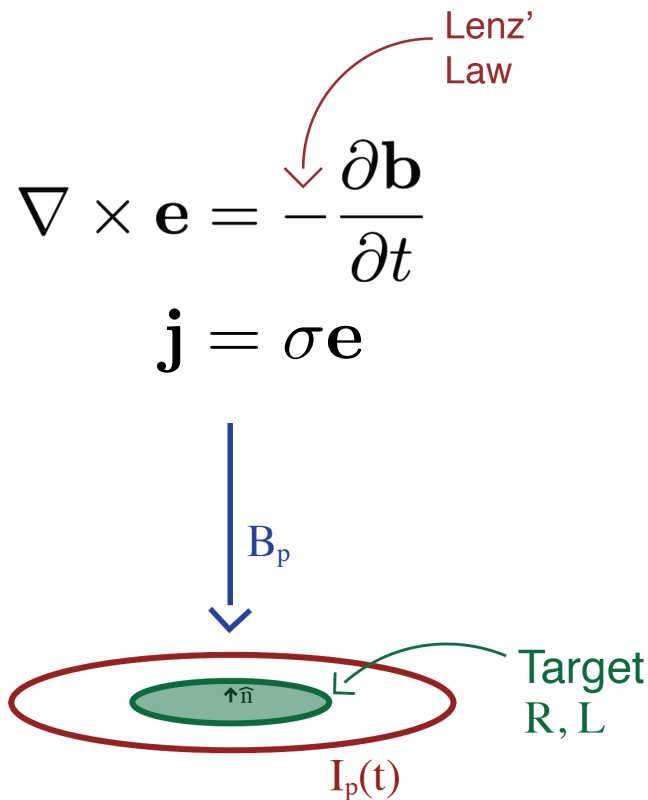
$$V = EMF = - \frac{d\phi_{\mathbf{b}}}{dt} > 0$$



App for Faraday's Law

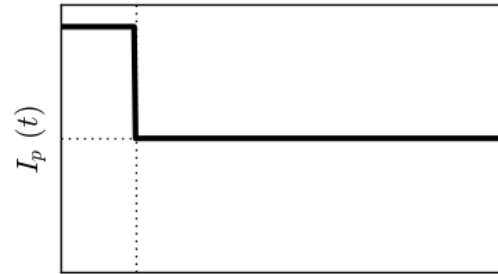
2 Apps:

- Harmonic
- Transient

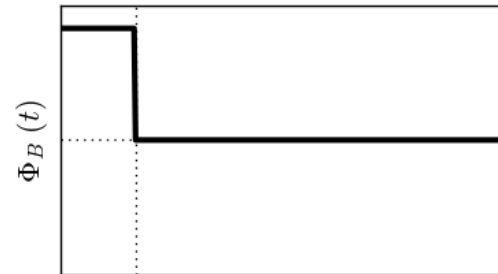


Two Coil Example: Transient

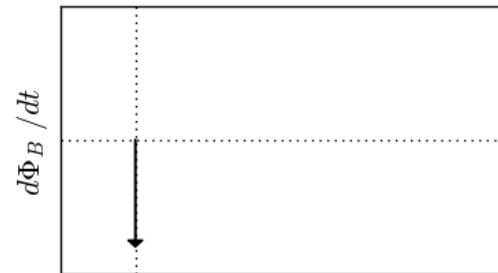
Primary currents



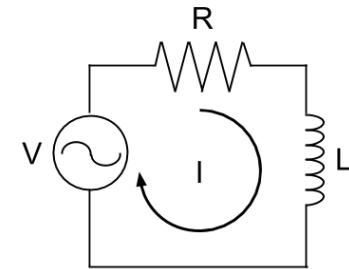
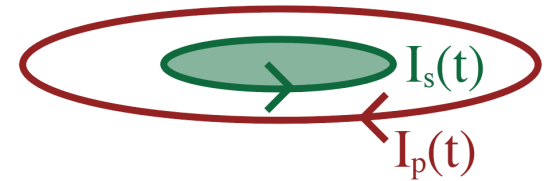
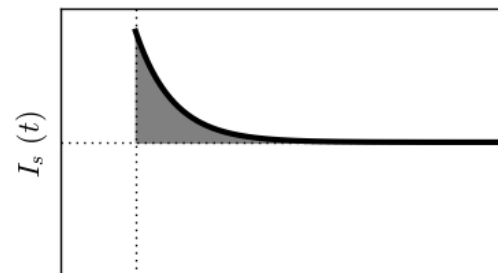
Magnetic flux



Time-variation of magnetic flux



Secondary currents

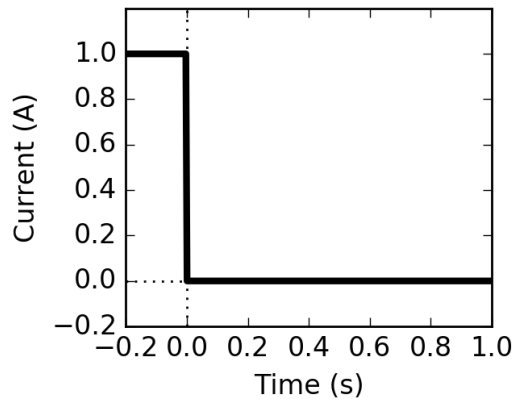


$$I_s(t) = I_s e^{-t/\tau}$$

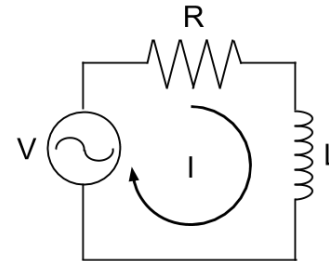
$$\tau = L/R$$

Response Function: Transient

Step-off current in Tx

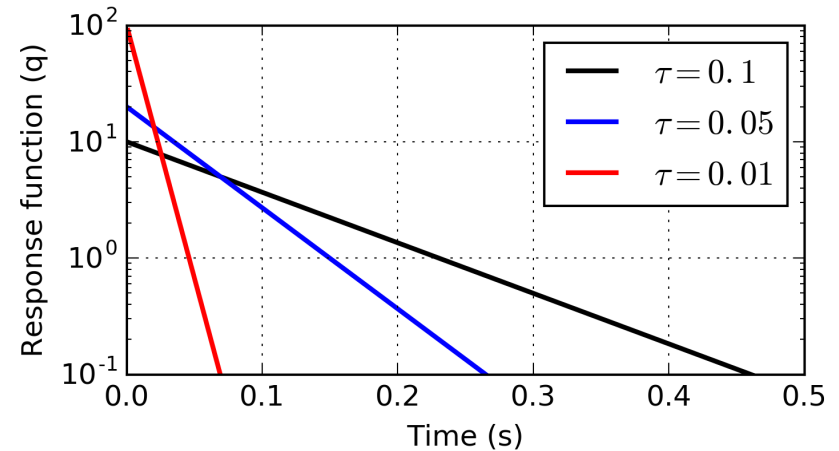
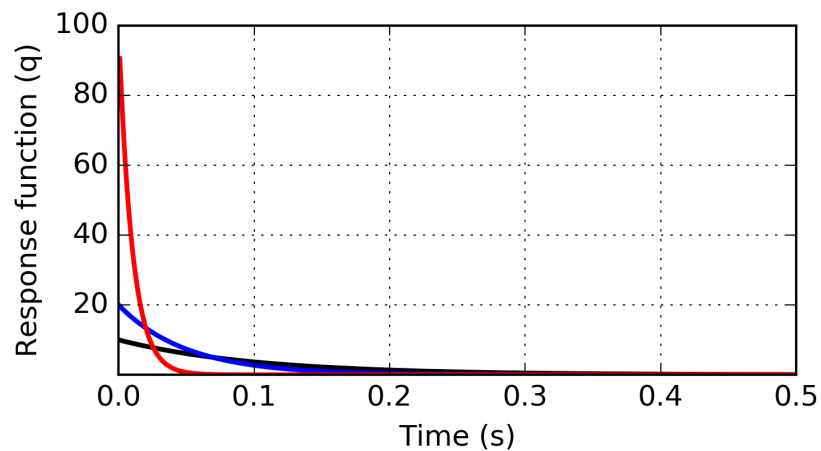


Time constant



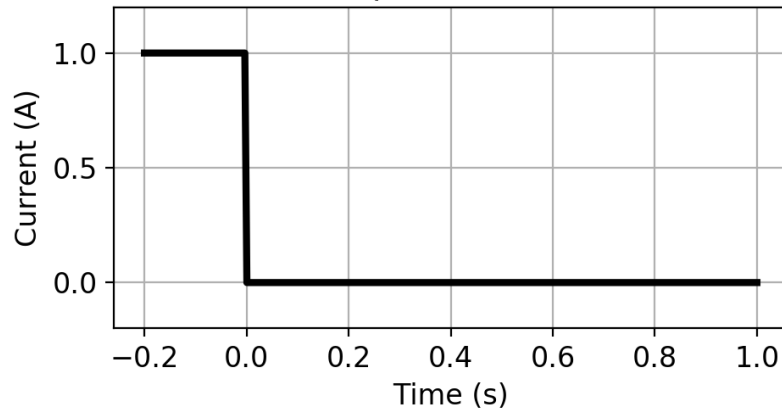
$$\tau = L/R$$

Response function: $q(t) = e^{-t/\tau}$

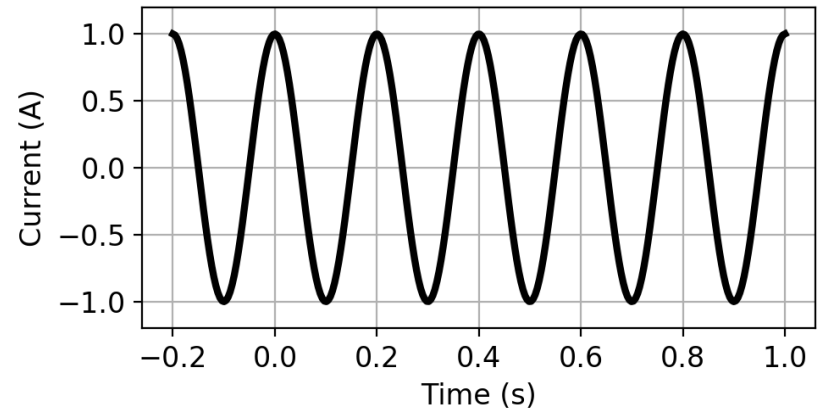


Transient and Harmonic Signals

We have seen a transient pulse...



What happens when he have a harmonic?



Two Coil Example: Harmonic

Induced Currents

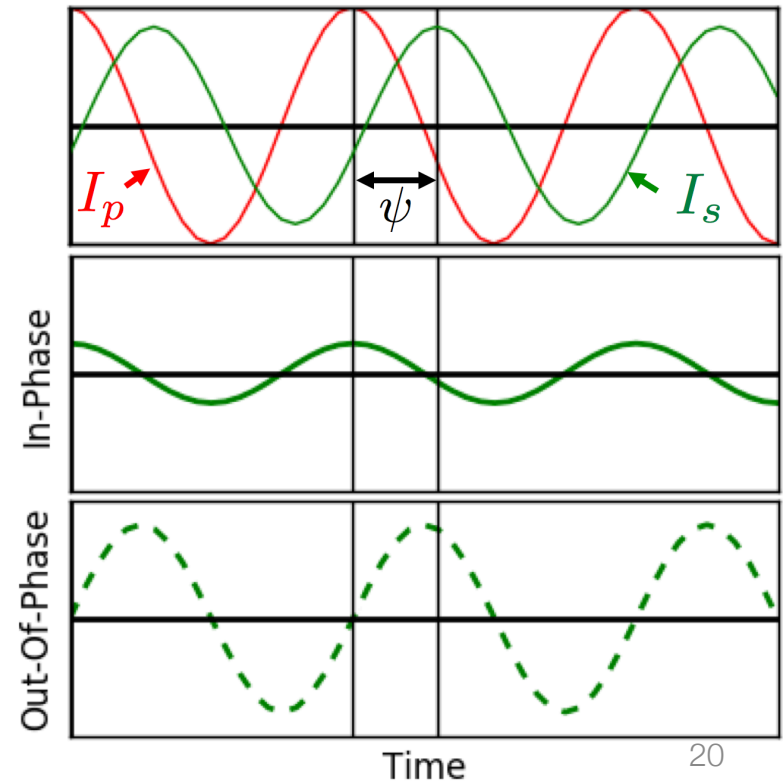
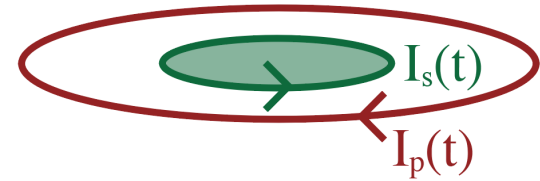
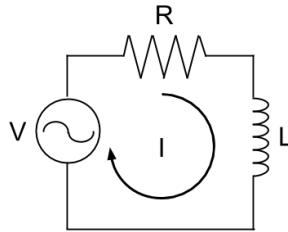
$$I_p(t) = I_p \cos \omega t$$

$$I_s(t) = I_s \cos(\omega t - \psi)$$

$$= \underbrace{I_s \cos \psi \cos \omega t}_{\substack{\text{In-Phase} \\ \text{Real}}} + \underbrace{I_s \sin \psi \sin \omega t}_{\substack{\text{Out-of-Phase} \\ \text{Quadrature} \\ \text{Imaginary}}}$$

Phase Lag

$$\psi = \frac{\pi}{2} + \tan^{-1} \left(\frac{\omega L}{R} \right)$$



Two Coil Example: Harmonic

Induced Currents

$$I_p(t) = I_p \cos \omega t$$

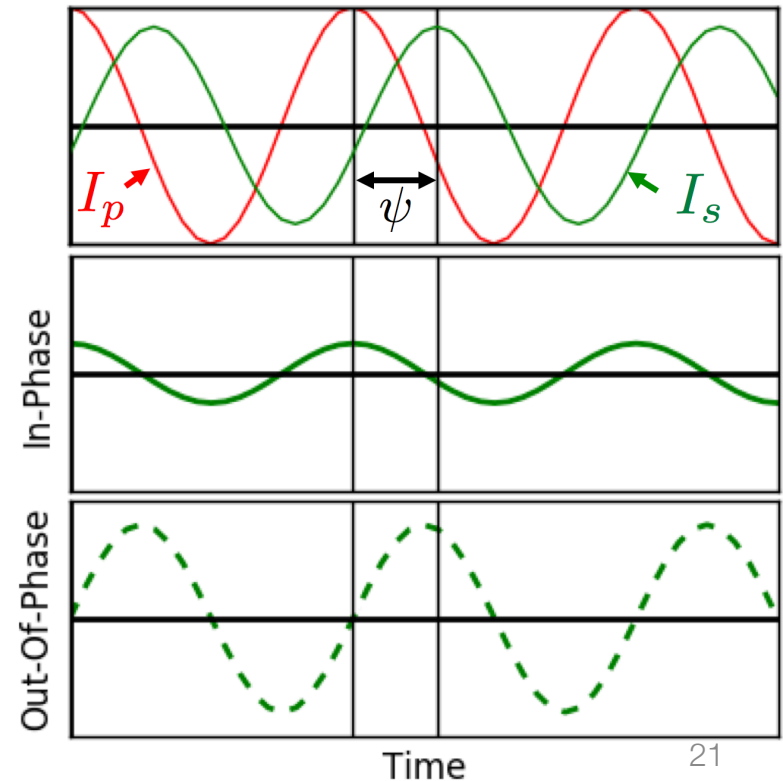
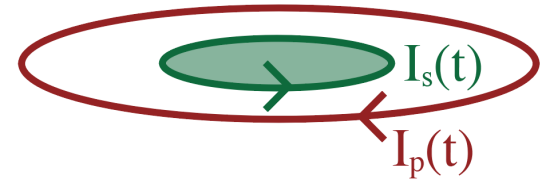
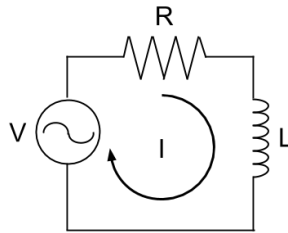
$$I_s(t) = I_s \cos(\omega t - \psi)$$

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Phase Lag

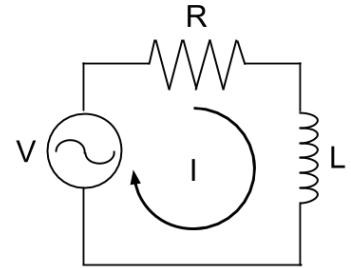
$$\psi = \frac{\pi}{2} + \underbrace{\tan^{-1} \left(\frac{\omega L}{R} \right)}_{\alpha}$$

Induction number

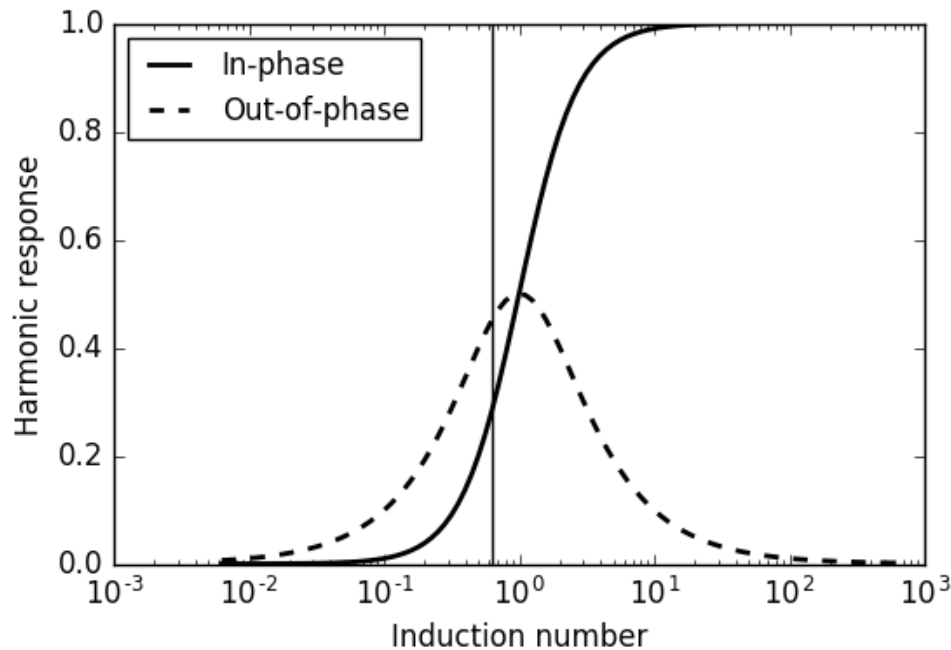


Response Function

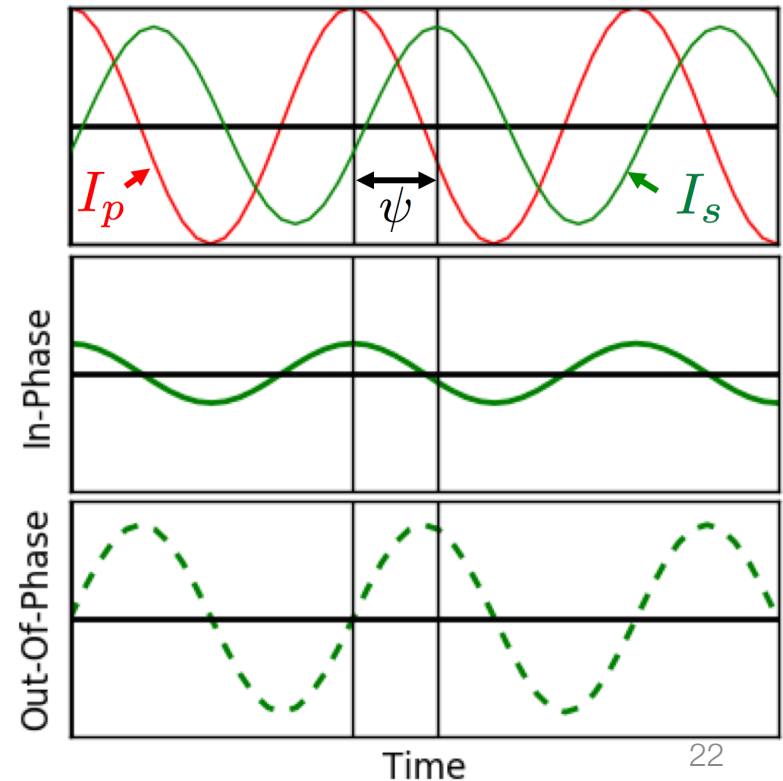
- Quantifies how a target responds to a time varying magnetic field
- Partitions real and imaginary parts



Response Function

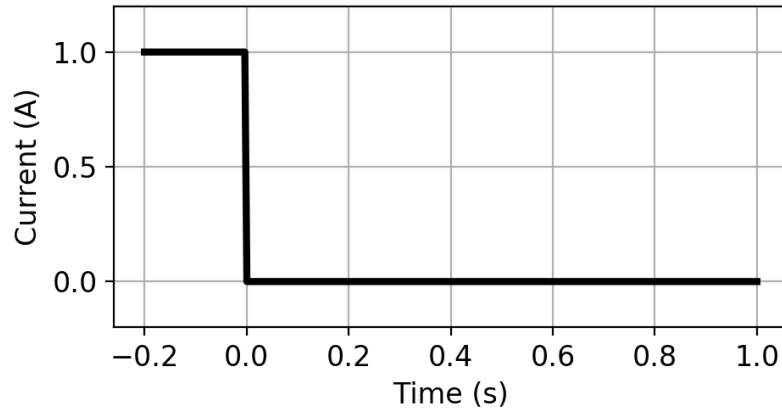


$$\alpha = \frac{\omega L}{R}$$

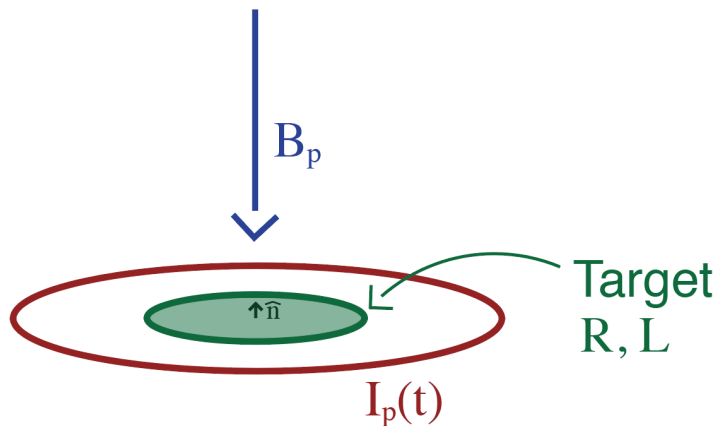
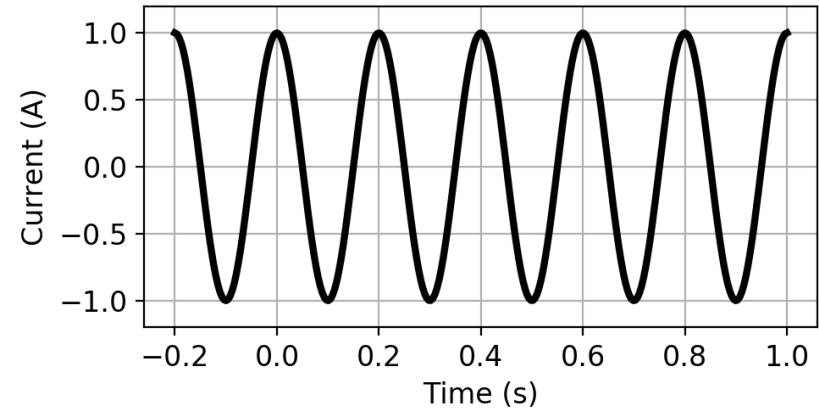


Response Functions: Summary

Step-off



Harmonic



In both:

- Induce currents

$$\nabla \times \mathbf{e} = -\frac{\partial \mathbf{b}}{\partial t}$$

- Generate secondary magnetic fields

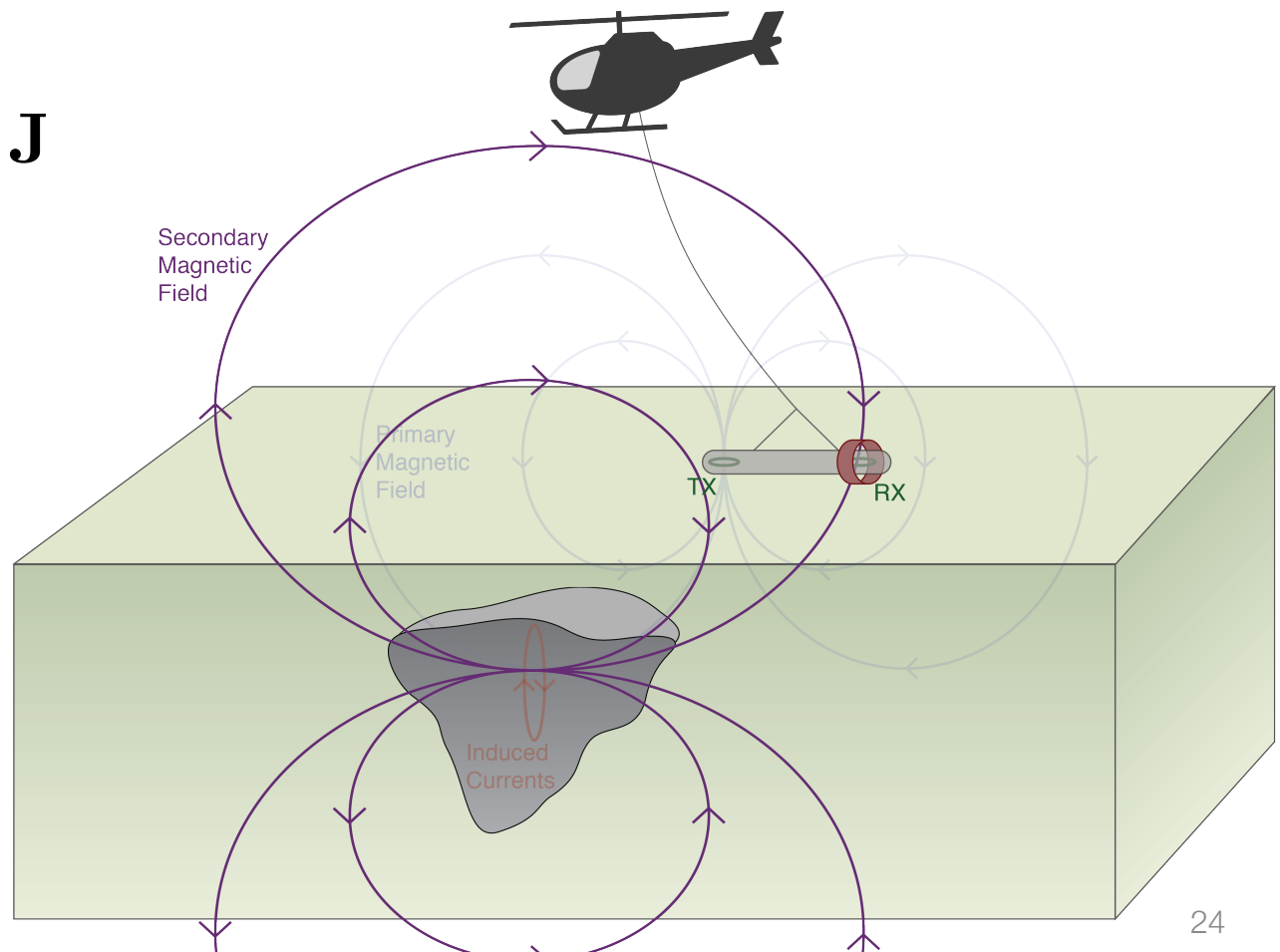
$$\nabla \times \mathbf{h} = \mathbf{j}$$

Secondary magnetic fields

Induced currents generate magnetic fields

- Ampere's Law

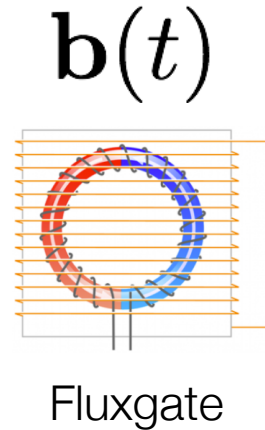
$$\nabla \times \mathbf{H} = \mathbf{J}$$



Receiver and Data

Magnetometer

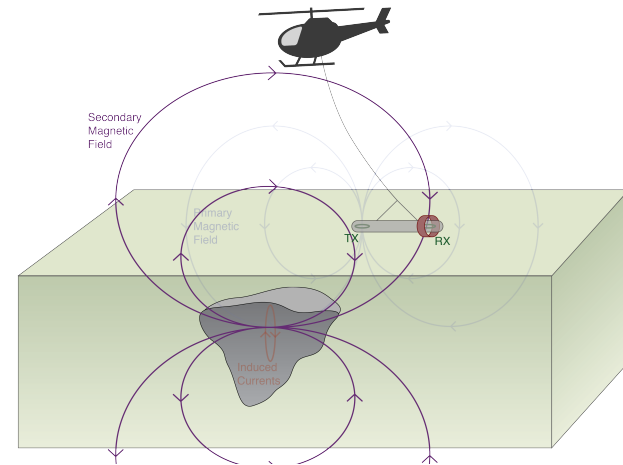
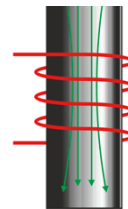
- Measures:
 - Magnetic fields
 - 3 components
- eg. 3-component fluxgate



Coil

- Measures:
 - Voltage
 - Single component that depends on coil orientation
 - Coupling matters
- eg. airborne frequency domain
 - ratio of H_s/H_p is the same as V_s/V_p

$$\frac{\partial b}{\partial t}$$



Coupling

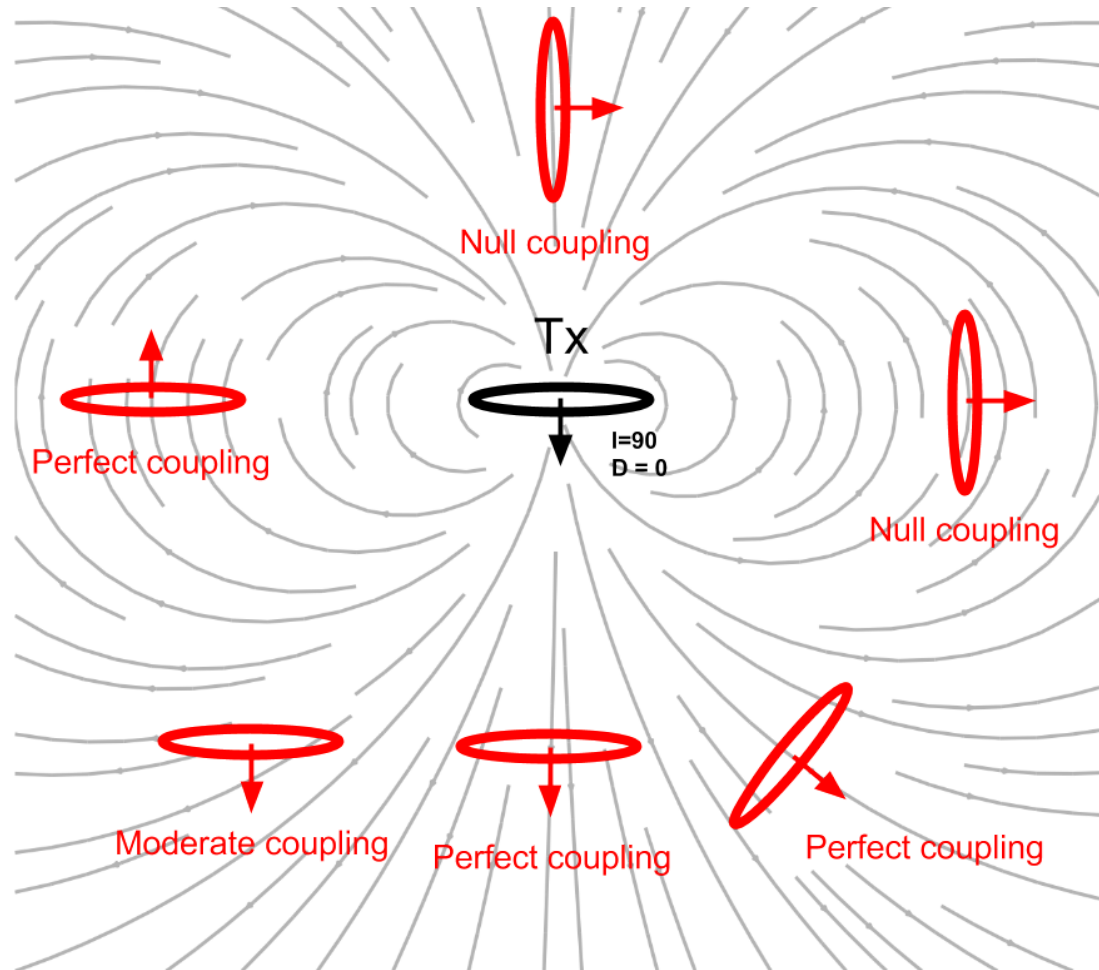
- Transmitter: Primary

$$I_p(t) = I_p \cos(\omega t)$$

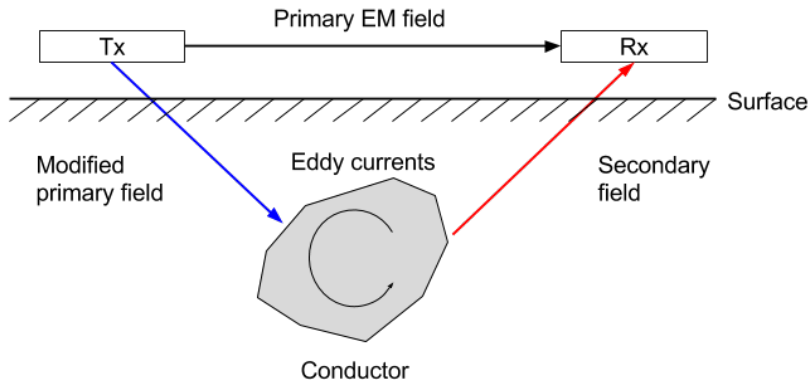
$$\mathbf{B}_p(t) \sim I_p \cos(\omega t)$$

- Target: Secondary

$$\begin{aligned} EMF &= -\frac{\partial \phi_{\mathbf{B}}}{\partial t} \\ &= -\frac{\partial}{\partial t} (\mathbf{B}_p \cdot \hat{\mathbf{n}}) A \end{aligned}$$



Circuit model of EM induction

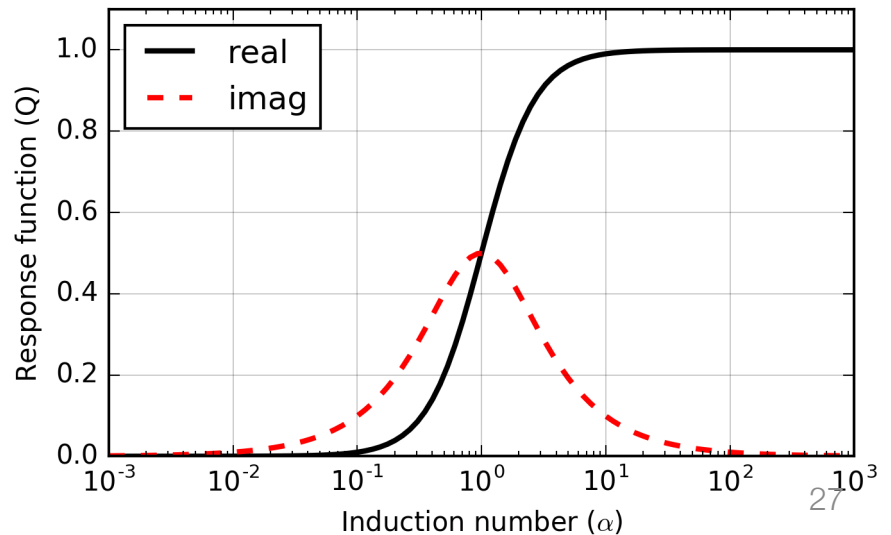


Magnetic field at the receiver

$$\frac{H^s}{H^p} = -\frac{M_{12}M_{23}}{M_{13}L} \underbrace{\left[\frac{\alpha^2 + i\alpha}{1 + \alpha^2} \right]}_Q$$

Induction Number

- Depends on properties of target $\alpha = \frac{\omega L}{R}$



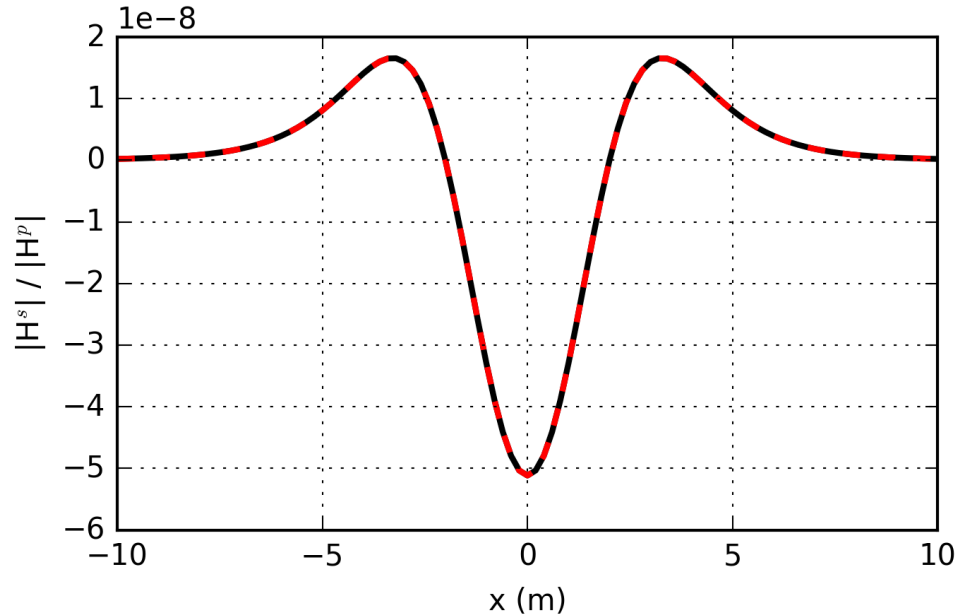
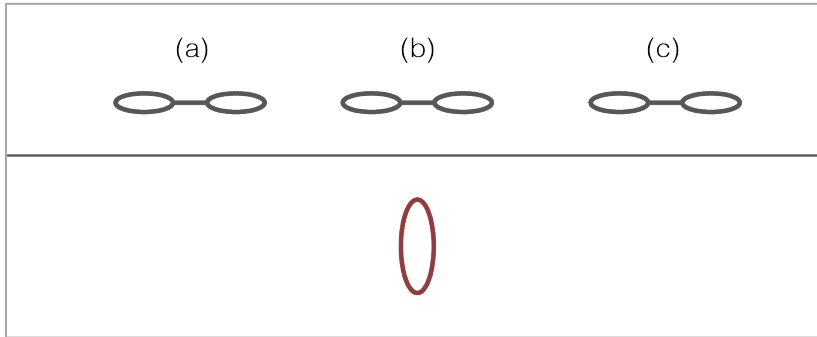
Coupling coefficient

- Depends on geometry

$$M_{12} = \frac{\mu_0}{4\pi} \oint \oint \frac{dl_1 \cdot dl_2}{|\mathbf{r} - \mathbf{r}'|^2}$$

Conductor in a resistive earth: Frequency

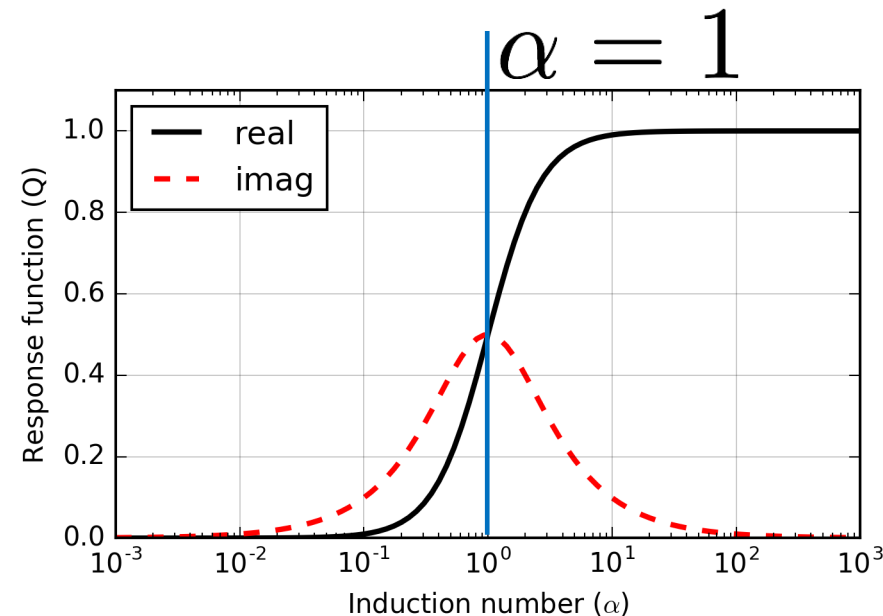
Profile over the loop



- Induction number

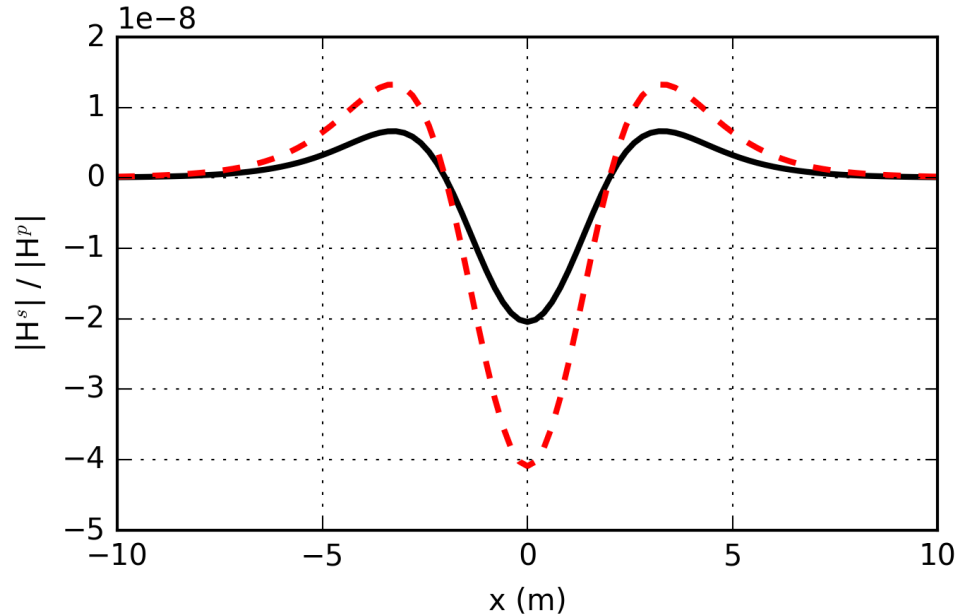
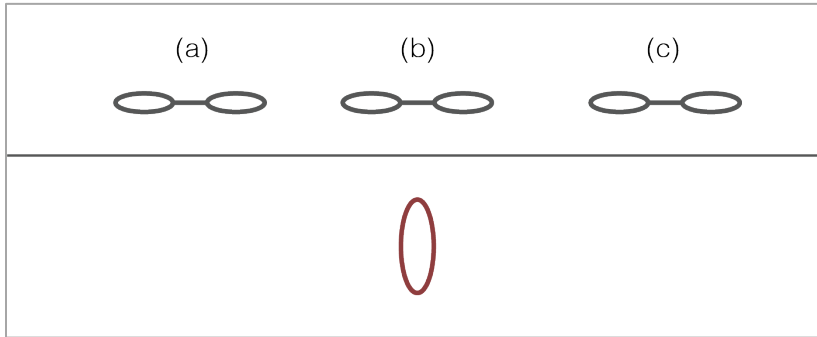
$$\alpha = \frac{\omega L}{R}$$

- When $\alpha = 1$
 - Real = Imag



Conductor in a resistive earth: Frequency

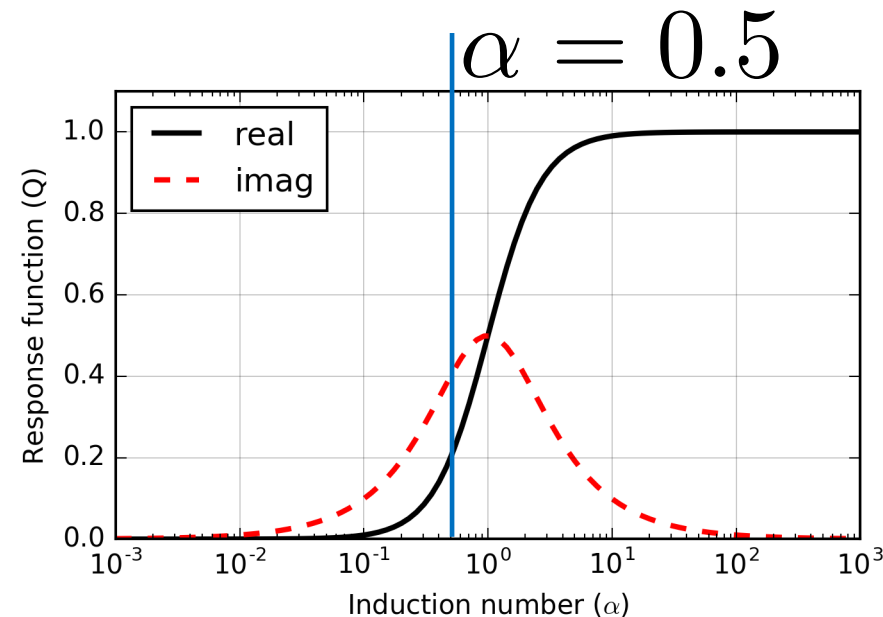
Profile over the loop



- Induction number

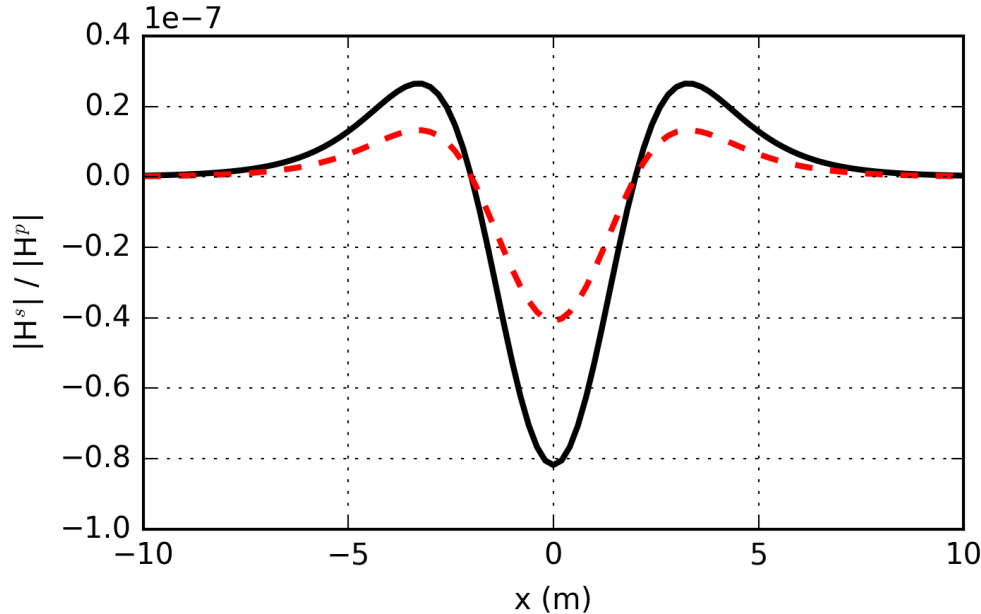
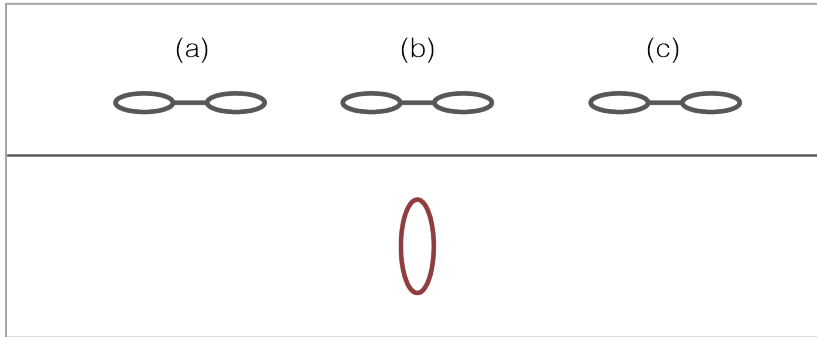
$$\alpha = \frac{\omega L}{R}$$

- When $\alpha < 1$
 - Real < Imag



Conductor in a resistive earth: Frequency

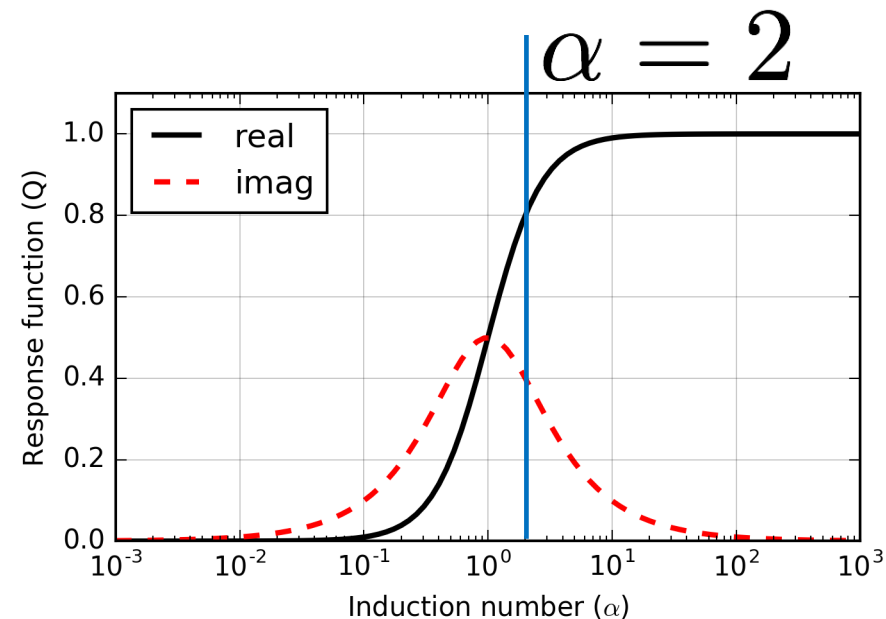
Profile over the loop



- Induction number

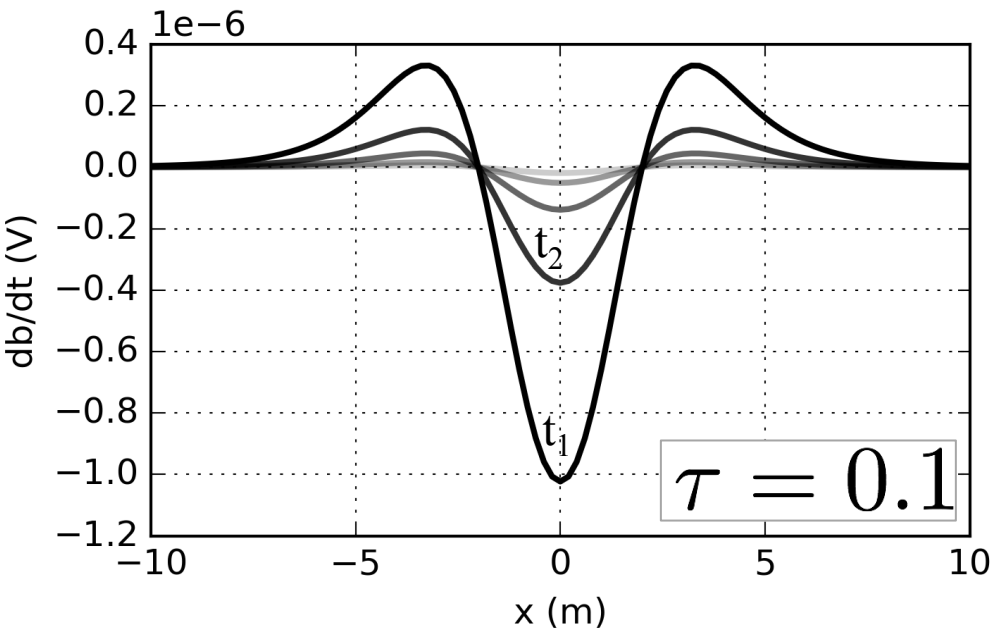
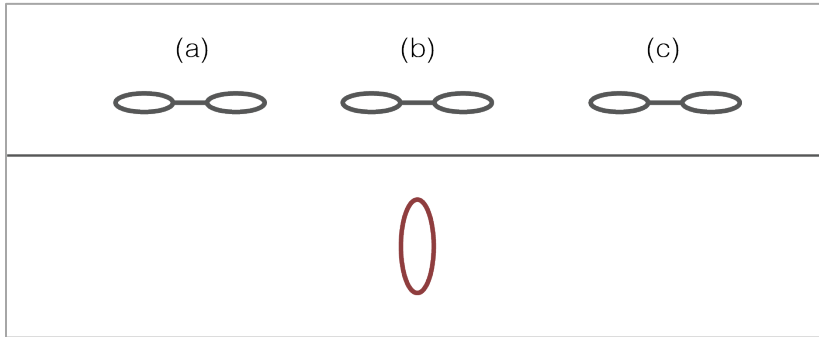
$$\alpha = \frac{\omega L}{R}$$

- When $\alpha > 1$
 - Real > Imag



Conductor in a resistive earth: Transient

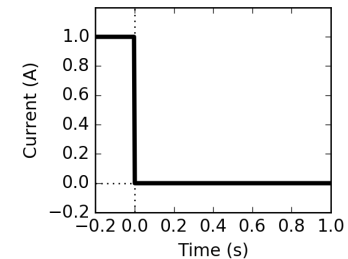
Profile over the loop



- Time constant

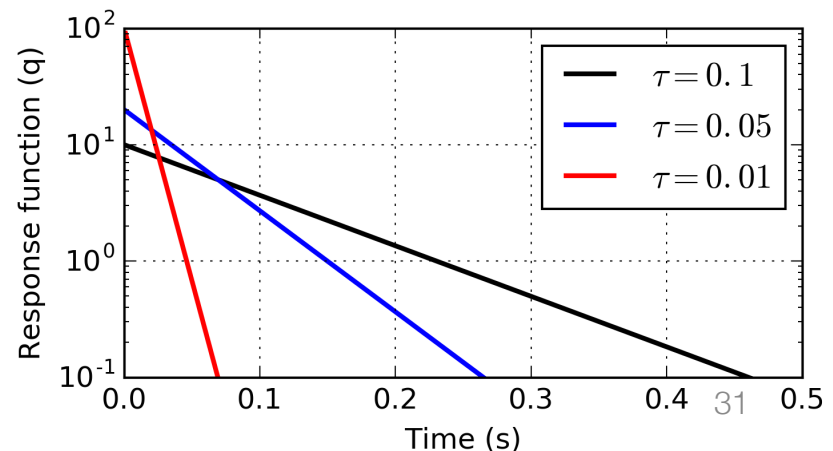
$$\tau = L/R$$

- Step-off current in Tx



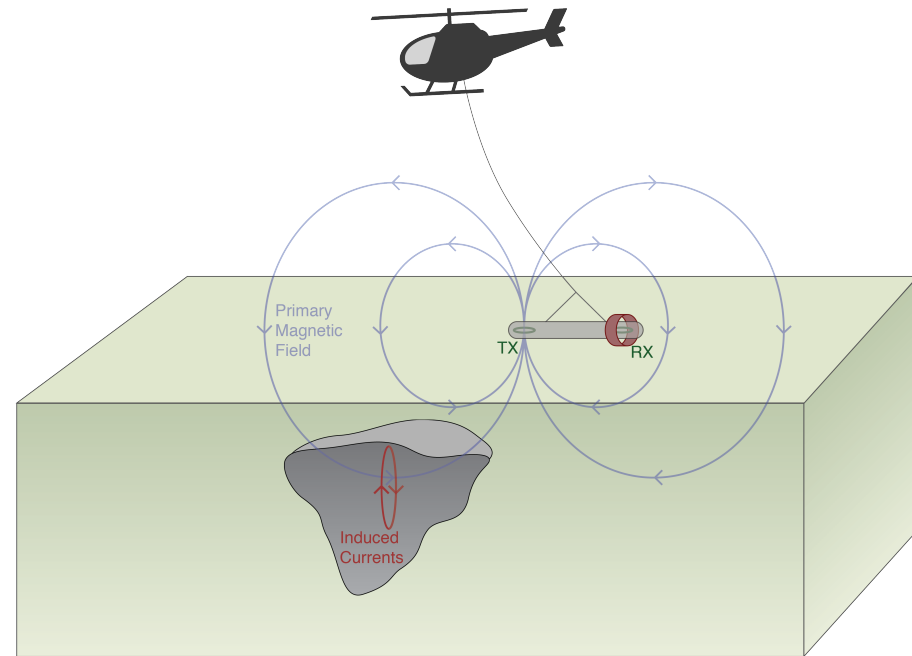
- Response function depends on time, τ

$$q(t) = e^{-t/\tau}$$



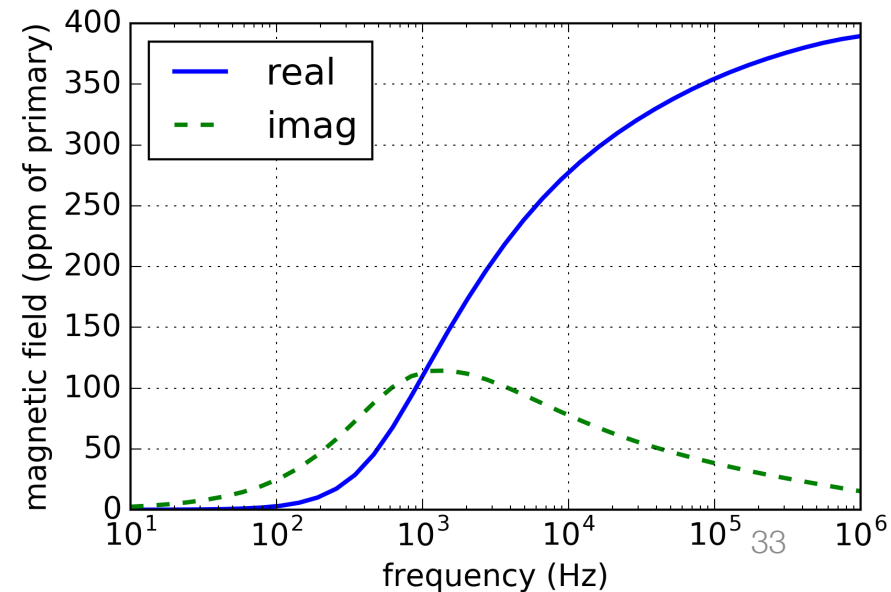
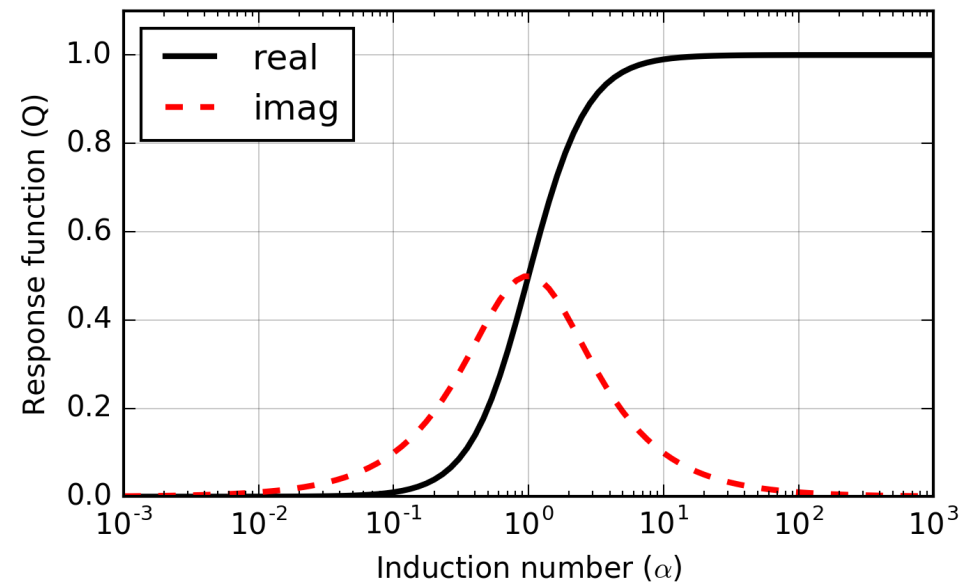
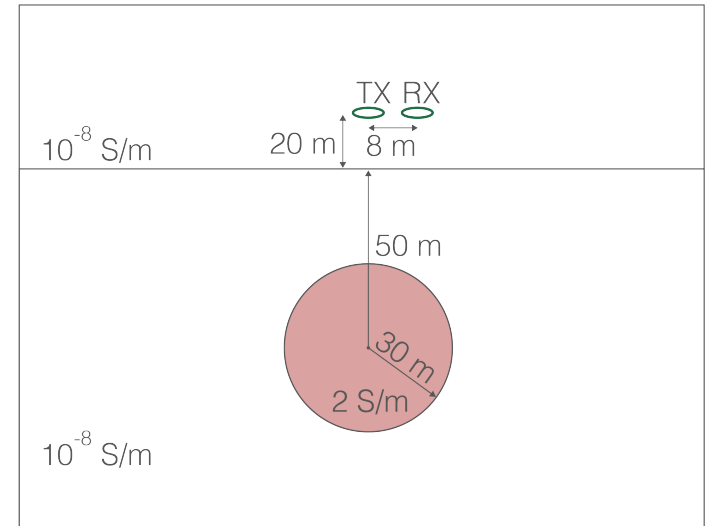
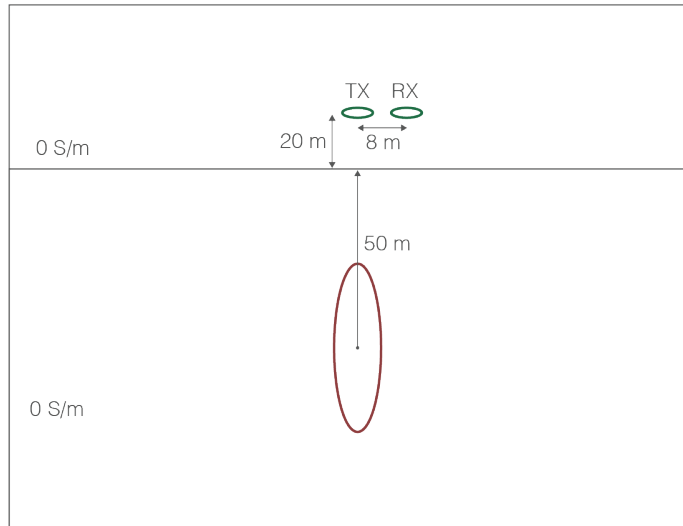
Recap: what have we learned?

- Basics of EM induction
- Response functions
- Mutual coupling
- Data for frequency or time domain systems
- Circuit model provides representative results
 - Applicable to geologic targets?



Sphere in a resistive background

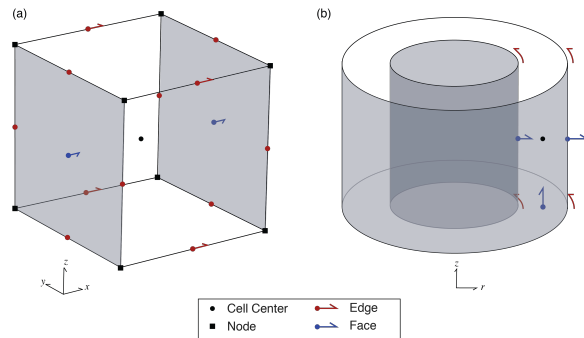
How representative is a circuit model?



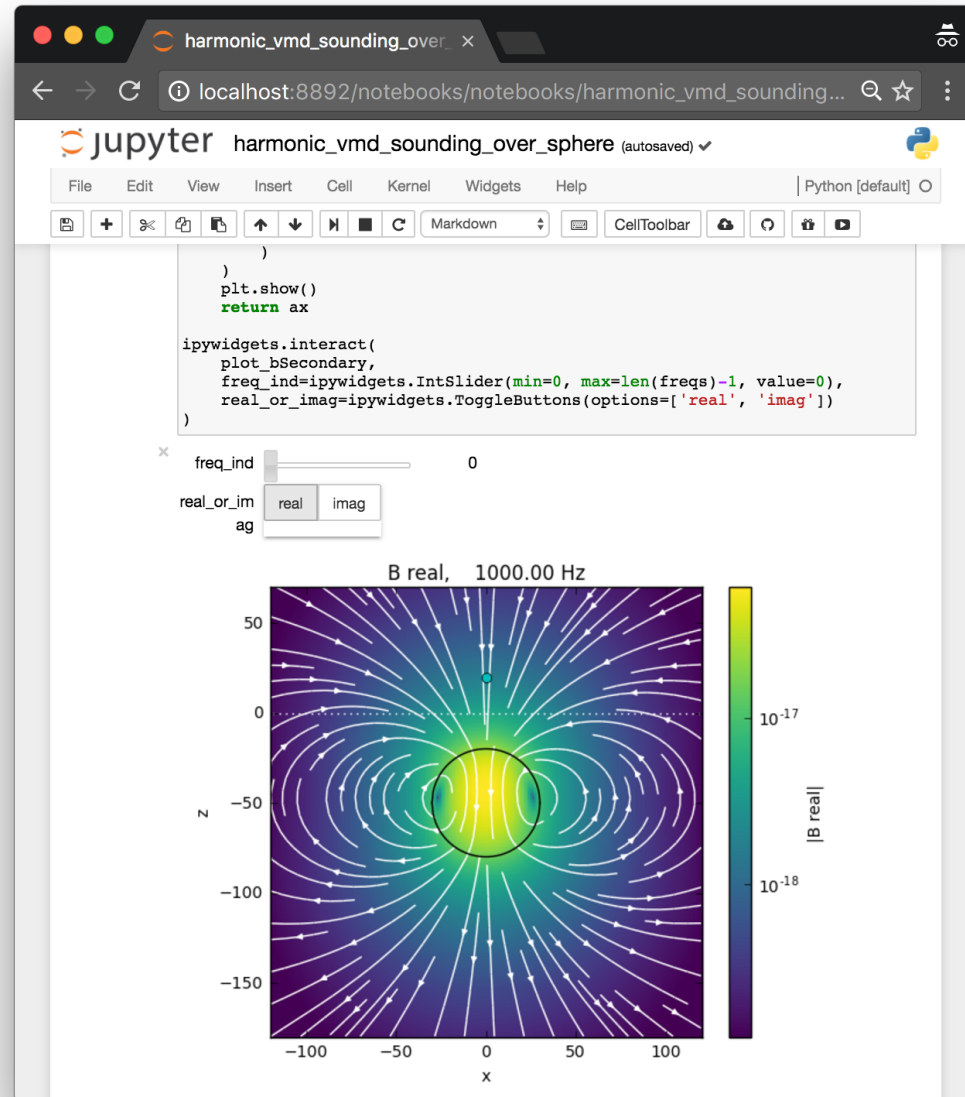
Cyl Code



- Finite Volume EM
 - Frequency and Time



- Built on SimPEG
- Open source, available at:
<http://em.geosci.xyz/apps.html>
- Papers
 - [Cockett et al, 2015](#)
 - [Heagy et al, 2017](#)



Recap: what have we learned?

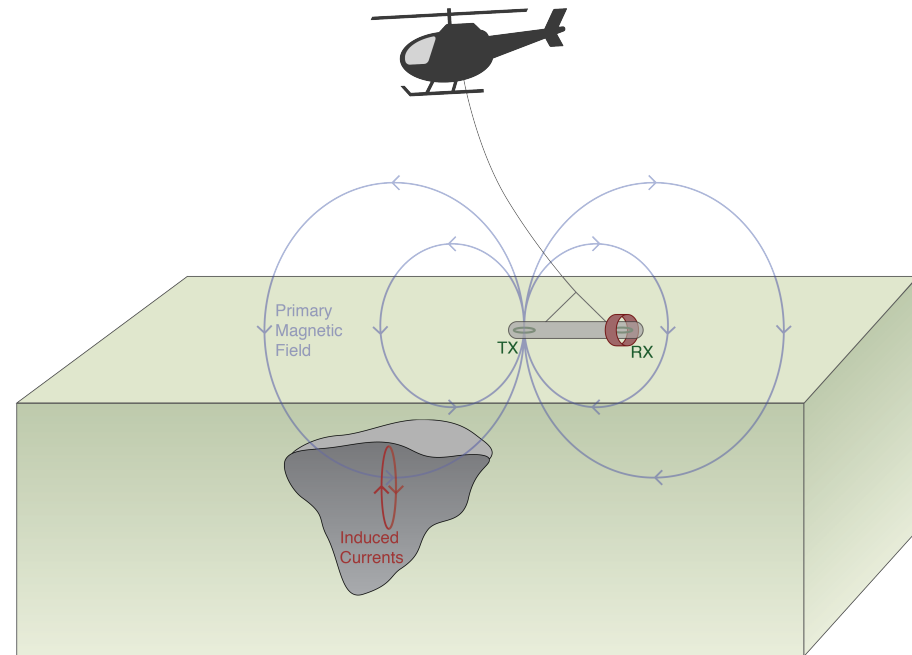
- Basics of EM induction
- Response functions
- Mutual coupling
- Data for frequency or time domain systems
- Circuit model is a good proxy

2-Coil Apps

- Frequency domain
- Time domain

Major item not yet accounted for...

- Propagation of energy from
 - Transmitter to target
 - Target to receiver



How do EM fields and fluxes behave in a
conductive background?

Revisit Maxwell's equations

First order equations

$$\begin{aligned}\nabla \times \mathbf{e} &= -\frac{\partial \mathbf{b}}{\partial t} & \mathbf{j} &= \sigma \mathbf{e} \\ \nabla \times \mathbf{h} &= \mathbf{j} + \frac{\partial \mathbf{d}}{\partial t} & \mathbf{b} &= \mu \mathbf{h} \\ & & \mathbf{d} &= \epsilon \mathbf{e}\end{aligned}$$

Second order equations

$$\nabla^2 \mathbf{h} - \underbrace{\mu\sigma \frac{\partial \mathbf{h}}{\partial t}}_{\text{diffusion}} - \underbrace{\mu\epsilon \frac{\partial^2 \mathbf{h}}{\partial t^2}}_{\text{wave propagation}} = 0$$

In frequency

$$\begin{aligned}\nabla^2 \mathbf{H} + k^2 \mathbf{H} &= 0 \\ k^2 &= \omega^2 \mu \epsilon - i\omega \mu \sigma\end{aligned}$$

Plane waves in a homogeneous media

In frequency

$$\nabla^2 \mathbf{H} + k^2 \mathbf{H} = 0$$

$$k^2 = \omega^2 \mu \epsilon - i \omega \mu \sigma$$

Quasi-static

$$\frac{\omega \epsilon}{\sigma} \ll 1$$

even if...

$$\sigma = 10^{-4} \text{ S/m}$$

$$f = 10^4 \text{ Hz}$$

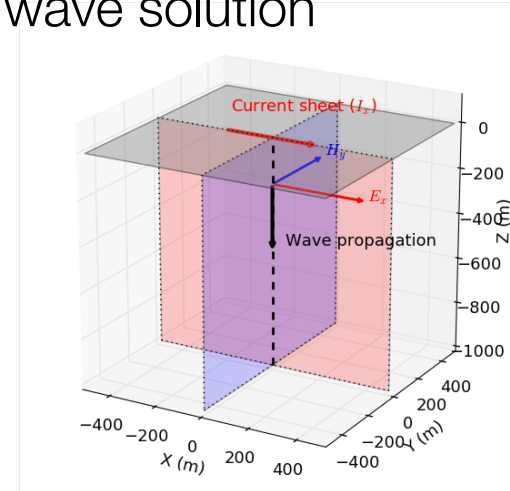
then

$$\frac{\omega \epsilon}{\sigma} \sim 0.005$$

$$k = \sqrt{-i \omega \mu \sigma} = (1 - i) \sqrt{\frac{\omega \mu \sigma}{2}}$$

$$\equiv \alpha - i \beta$$

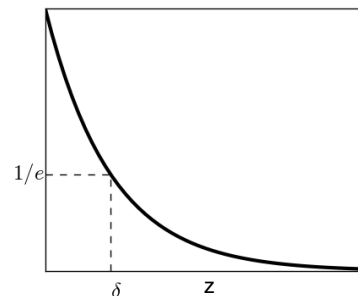
Plane wave solution



$$\mathbf{H} = \mathbf{H}_0 \underbrace{e^{-\alpha z}}_{\text{attenuation}} \underbrace{e^{-i(\beta z - \omega t)}}_{\text{phase}}$$

Skin depth

δ : skin depth



$$\delta = \sqrt{\frac{2}{\omega \mu \sigma}} = 503 \sqrt{\frac{1}{\sigma f}}$$

Plane waves in a homogeneous media

In time

$$\nabla^2 \mathbf{h} - \mu\epsilon \frac{\partial^2 \mathbf{h}}{\partial t^2} - \mu\sigma \frac{\partial \mathbf{h}}{\partial t} = 0$$

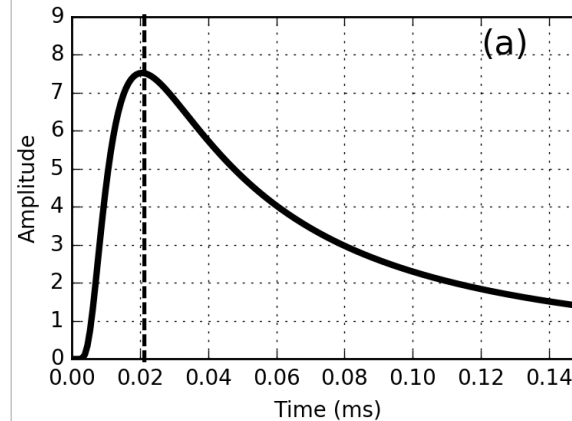
$$\mathbf{h}(t = 0) = \mathbf{h}_0 \delta(t)$$

Solution for quasi-static

$$\mathbf{h}(t) = -\frac{(\mu\sigma)^{1/2} z}{2\pi^{1/2} t^{3/2}} e^{-\mu\sigma z^2 / (4t)}$$

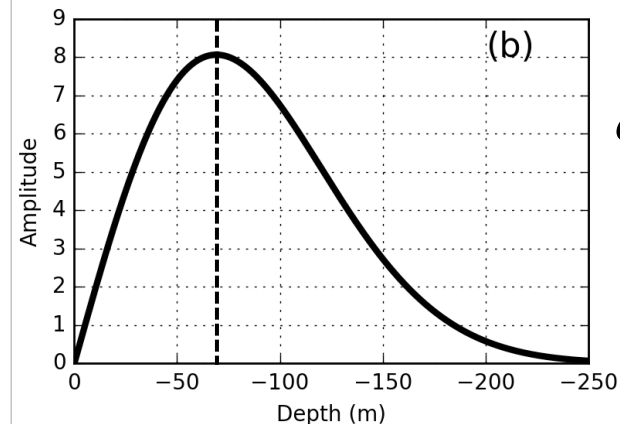
z : depth (m)

Peak time:



$$t_{max} = \frac{\mu\sigma z^2}{6}$$

Diffusion distance



$$d = \sqrt{\frac{2t}{\mu\sigma}}$$

$$\approx 1260 \sqrt{\frac{t}{\sigma}}$$

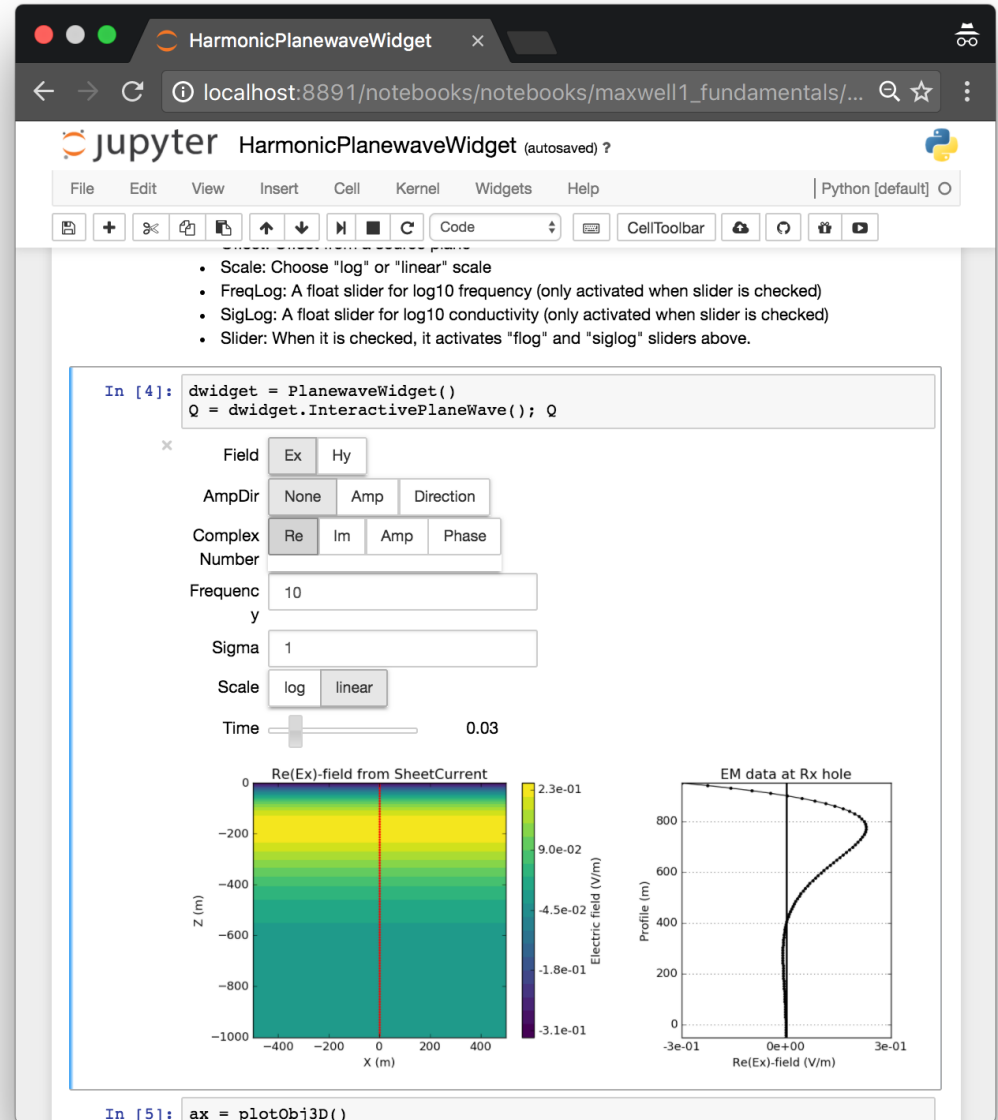
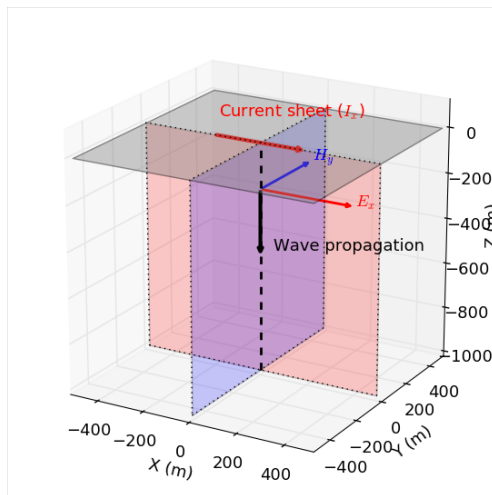
Plane Wave apps

- 2 apps:
 - Transient

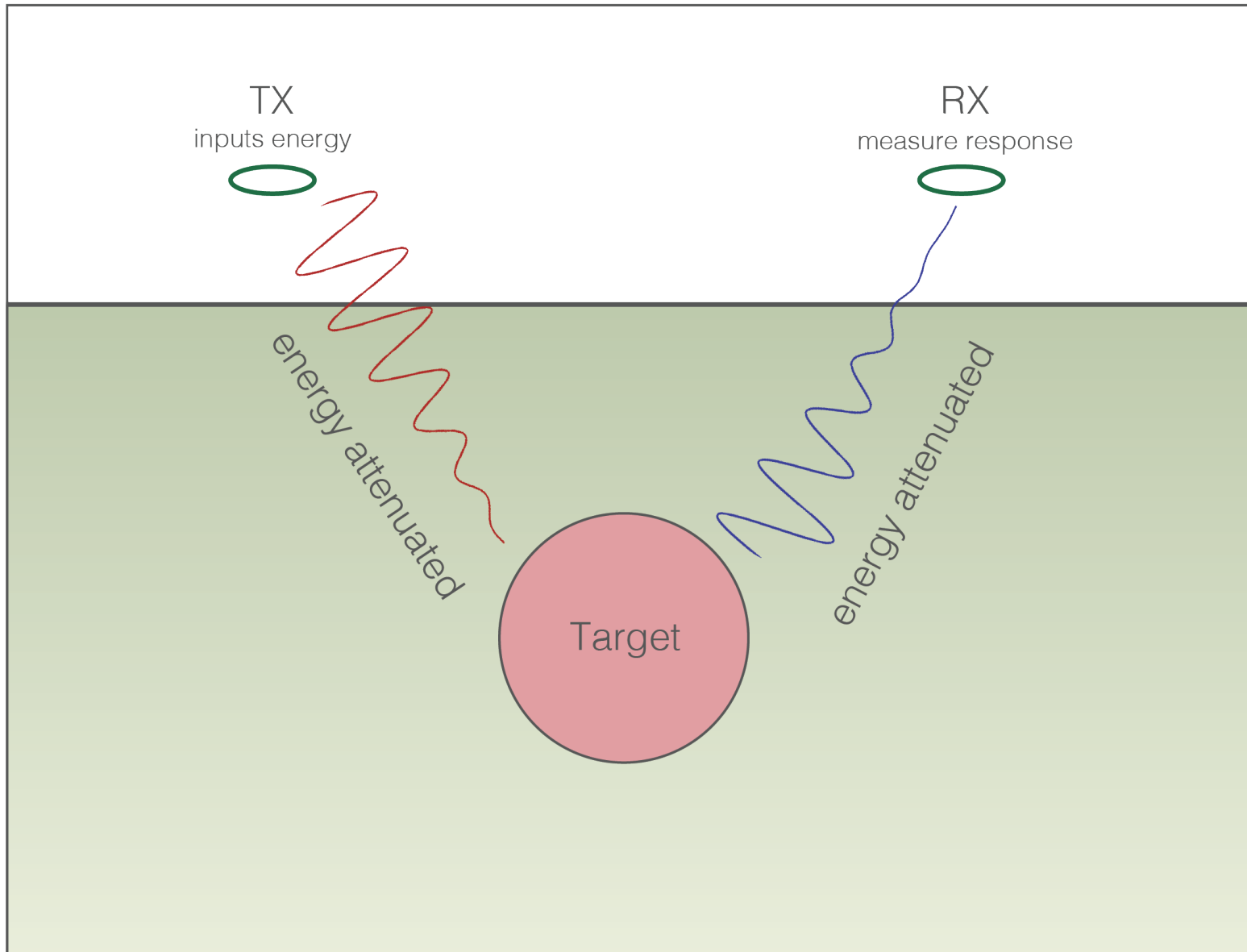
$$\mathbf{h}(t) = -\frac{(\mu\sigma)^{1/2}z}{2\pi^{1/2}t^{3/2}}e^{-\mu\sigma z^2/(4t)}$$

- Harmonic

$$\mathbf{H} = \mathbf{H}_0 \underbrace{e^{-\alpha z}}_{\text{attenuation}} \underbrace{e^{-i(\beta z - \omega t)}}_{\text{phase}}$$

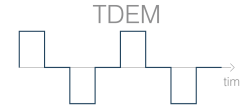


Effects of background resistivity

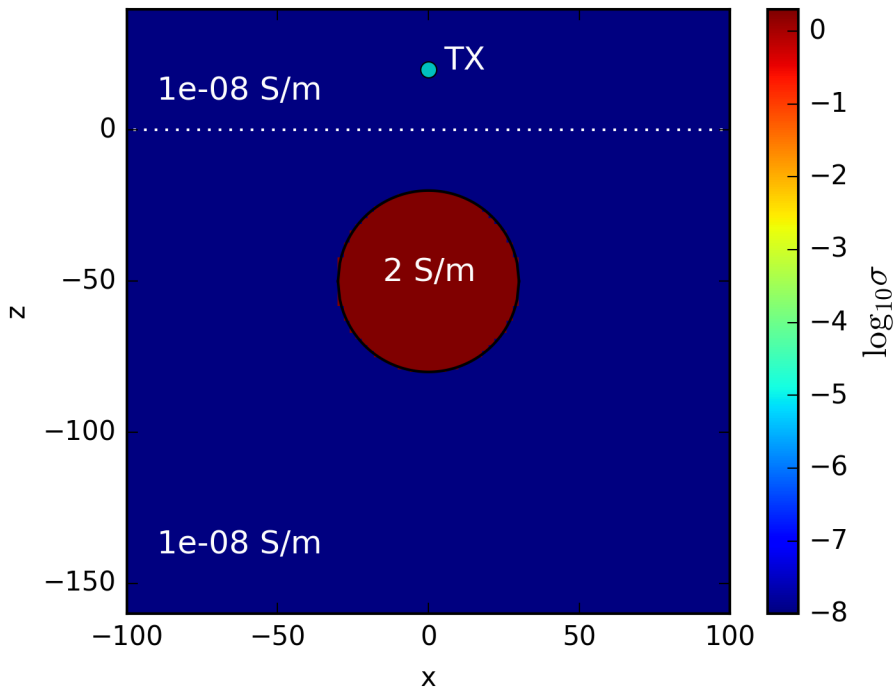


Effects of background resistivity: Time

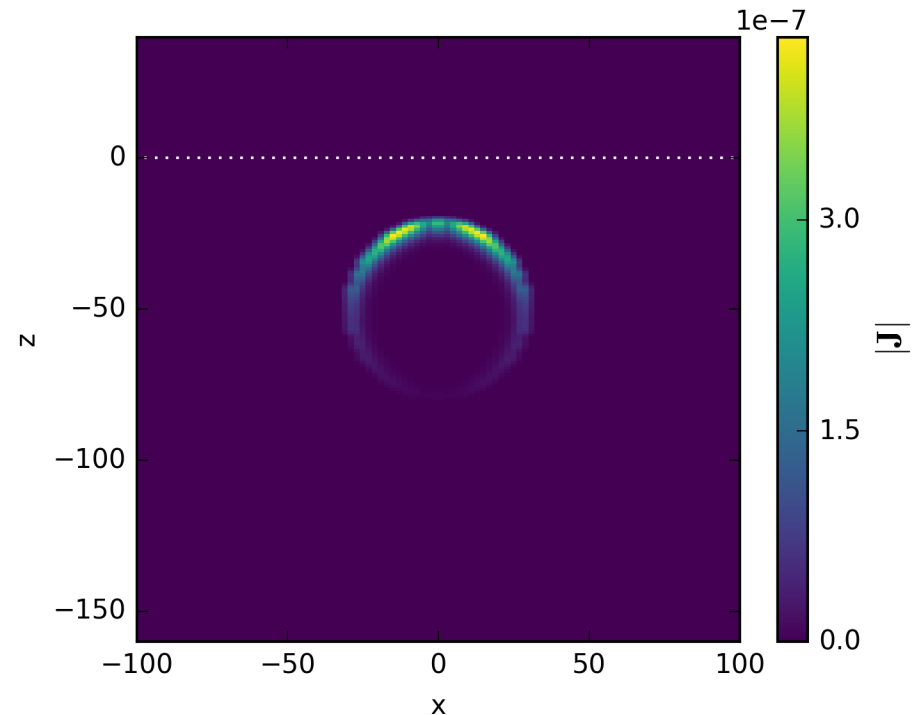
- Buried, conductive sphere
- Vary background conductivity
- Time: 10^{-5} s



10^{-8} S/m background

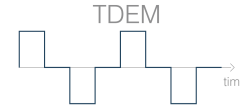


Current Density

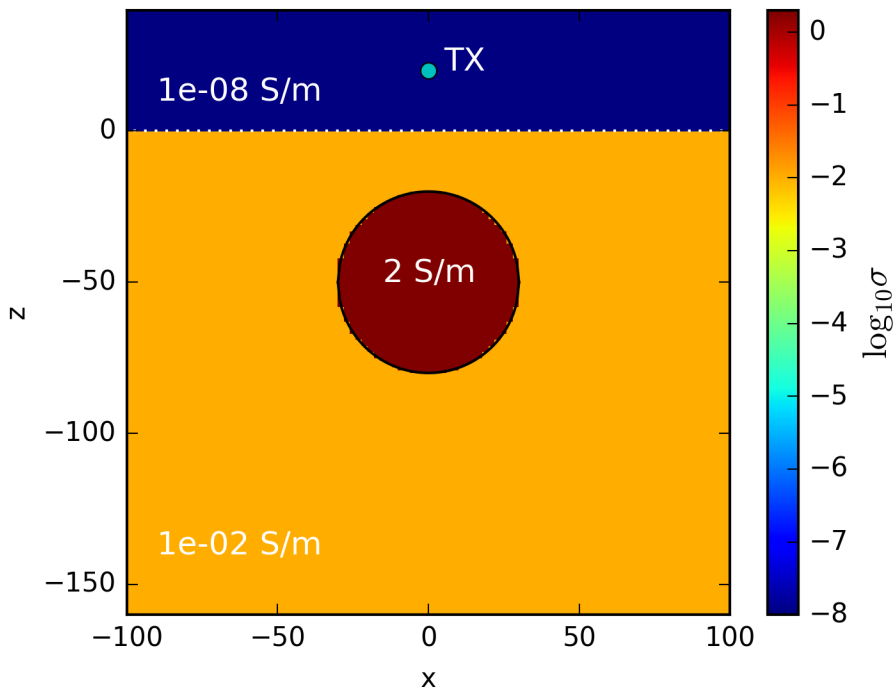


Effects of background resistivity: Time

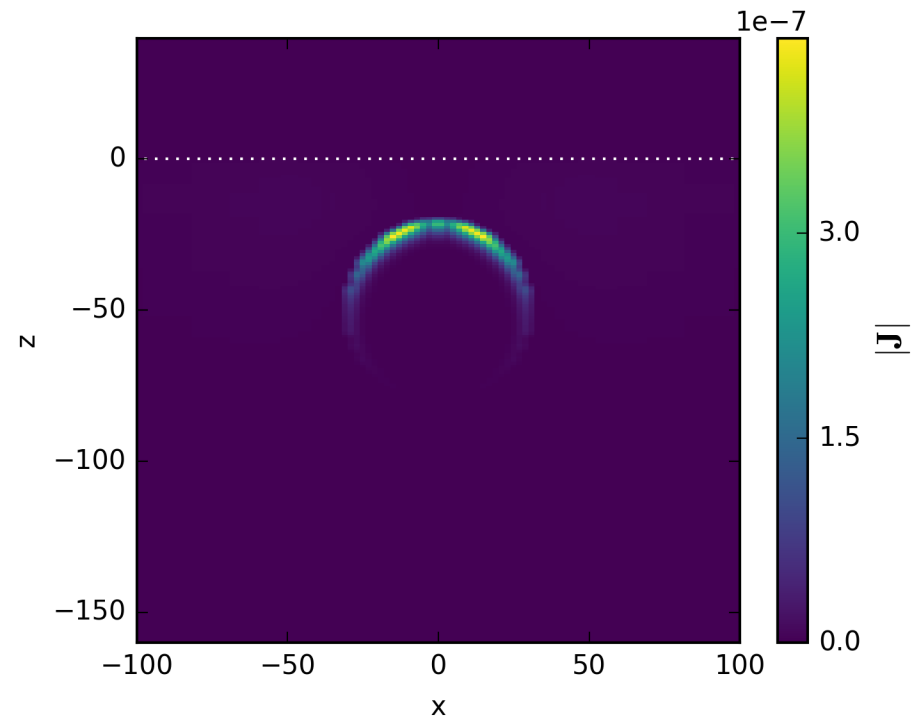
- Buried, conductive sphere
- Vary background conductivity
- Time: 10^{-5} s



10^{-2} S/m background

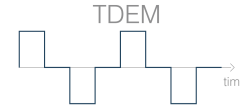


Current Density

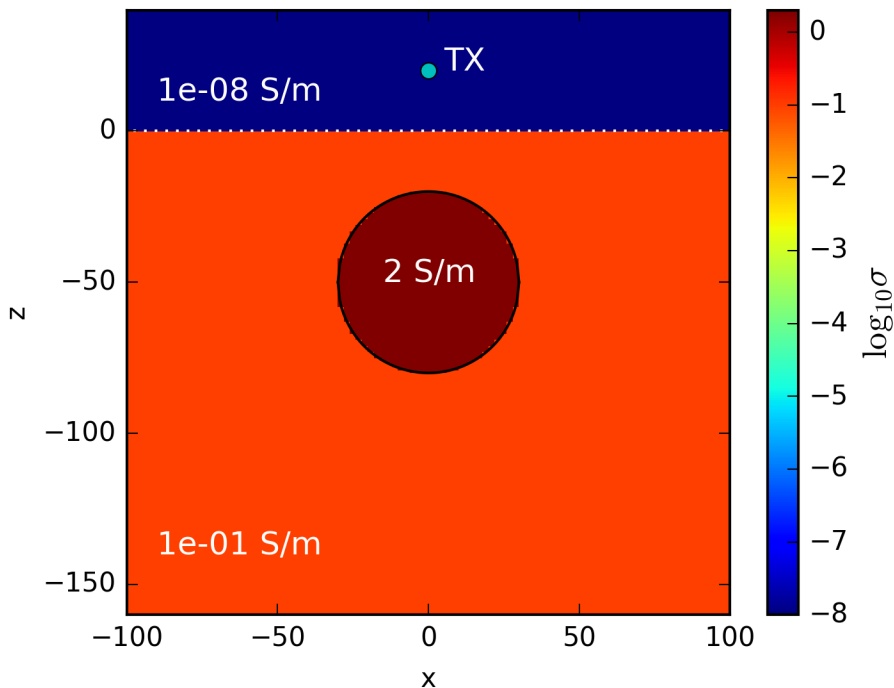


Effects of background resistivity: Time

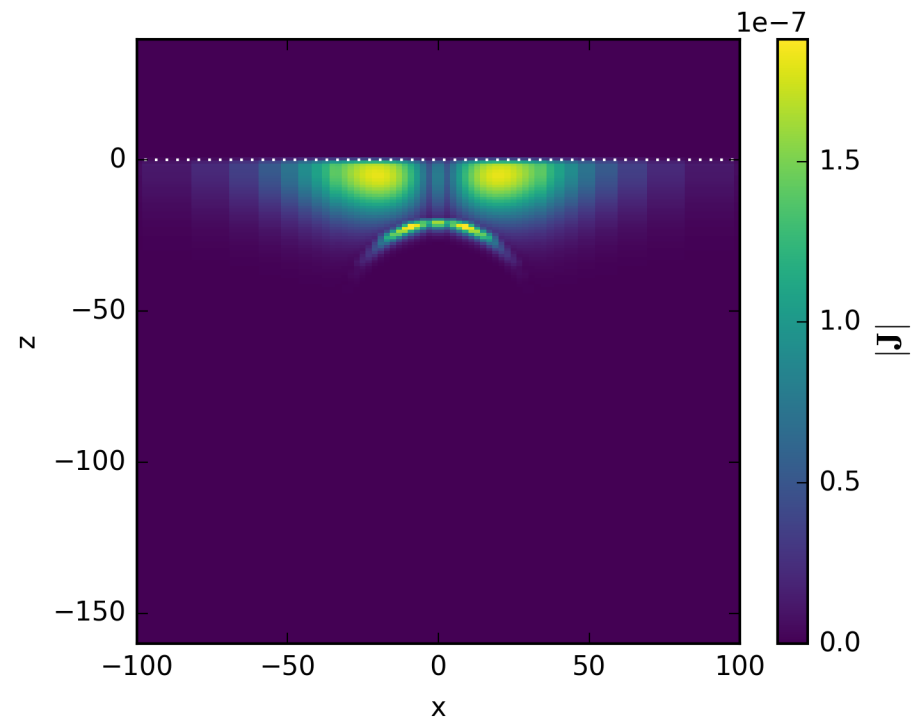
- Buried, conductive sphere
- Vary background conductivity
- Time: 10^{-5} s



10^{-1} S/m background

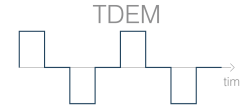


Current Density

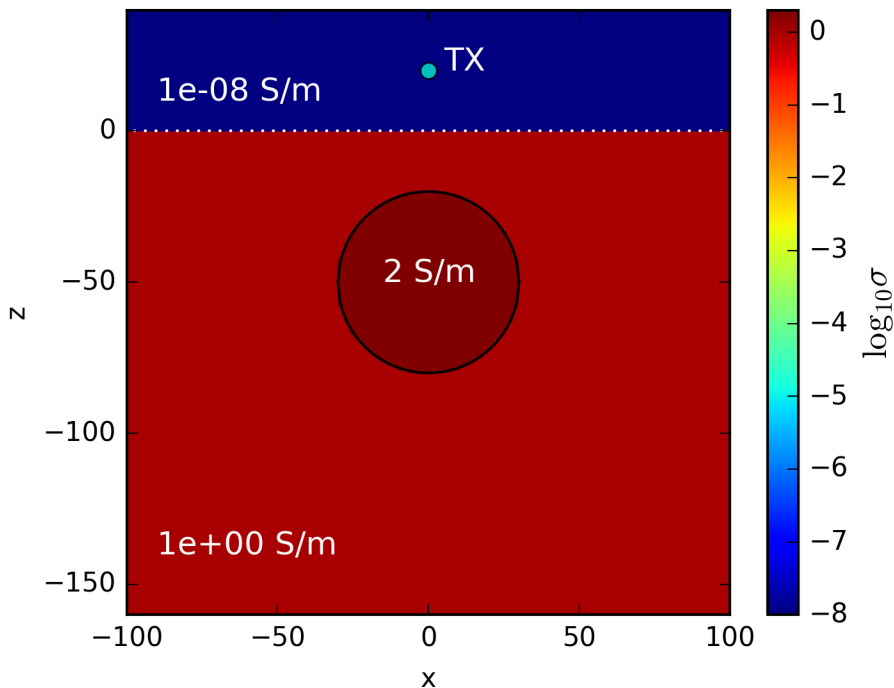


Effects of background resistivity: Time

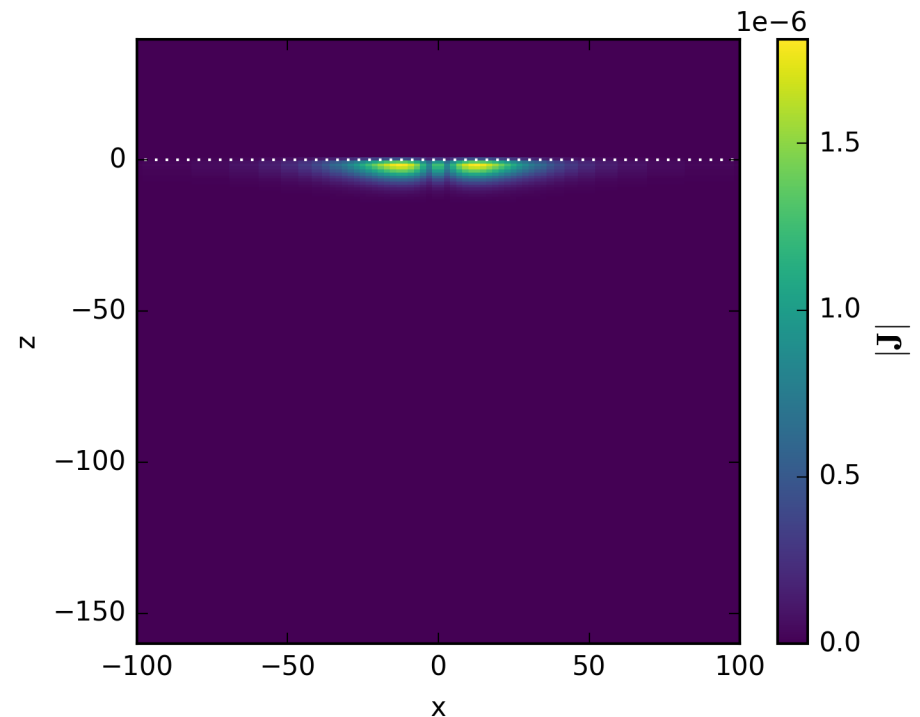
- Buried, conductive sphere
- Vary background conductivity
- Time: 10^{-5} s



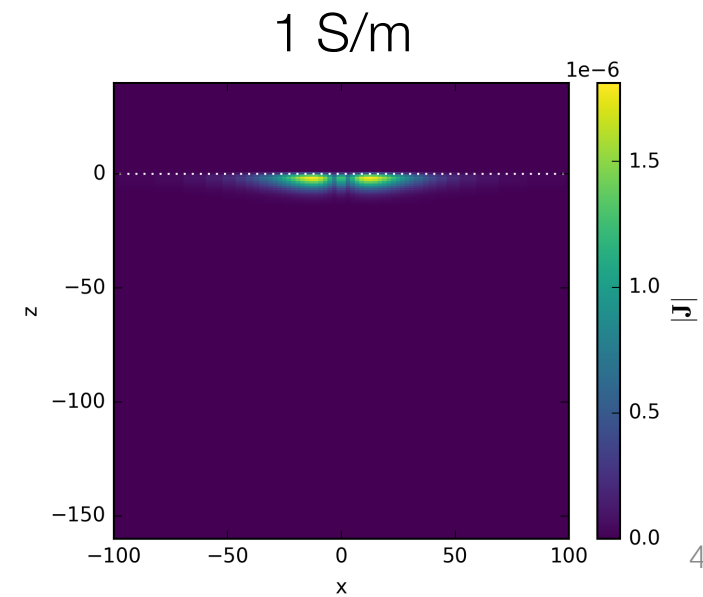
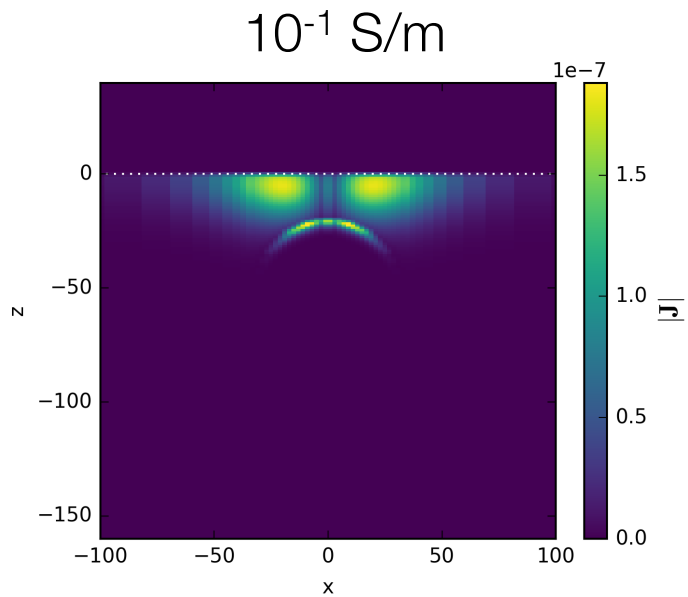
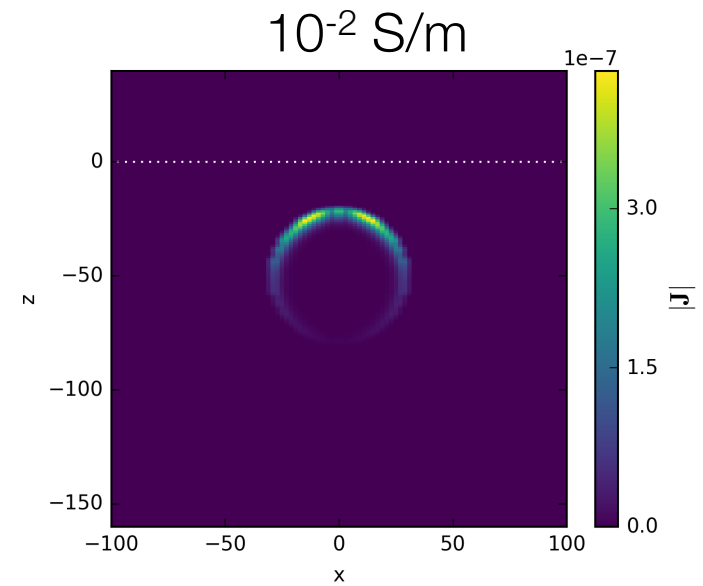
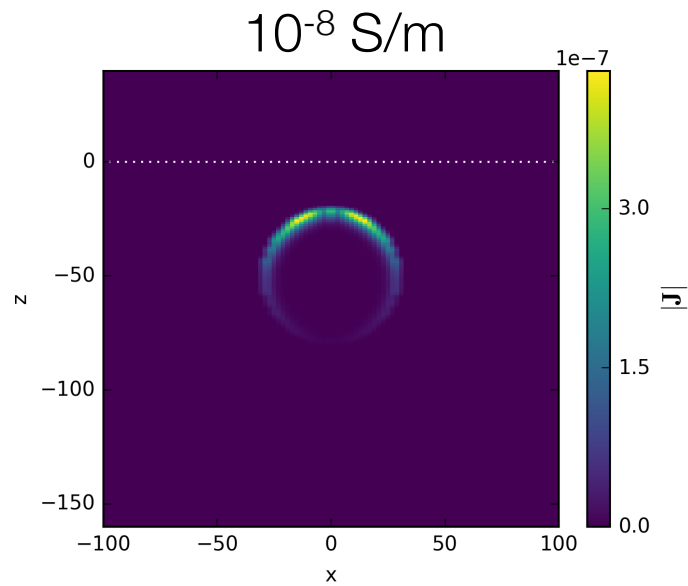
1 S/m background



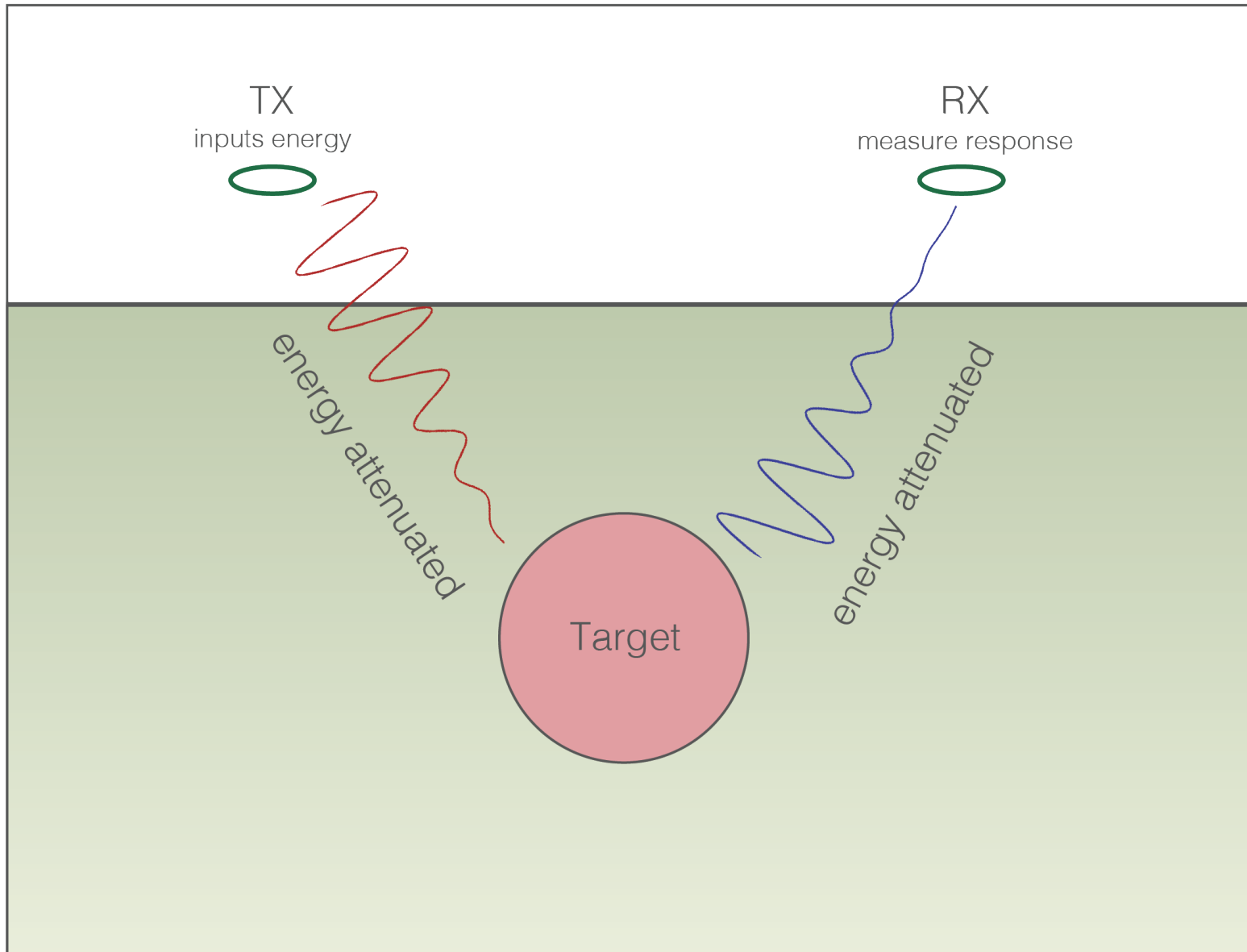
Current Density



Effects of background resistivity: Time

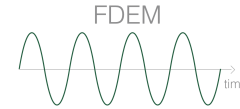


Effects of background resistivity

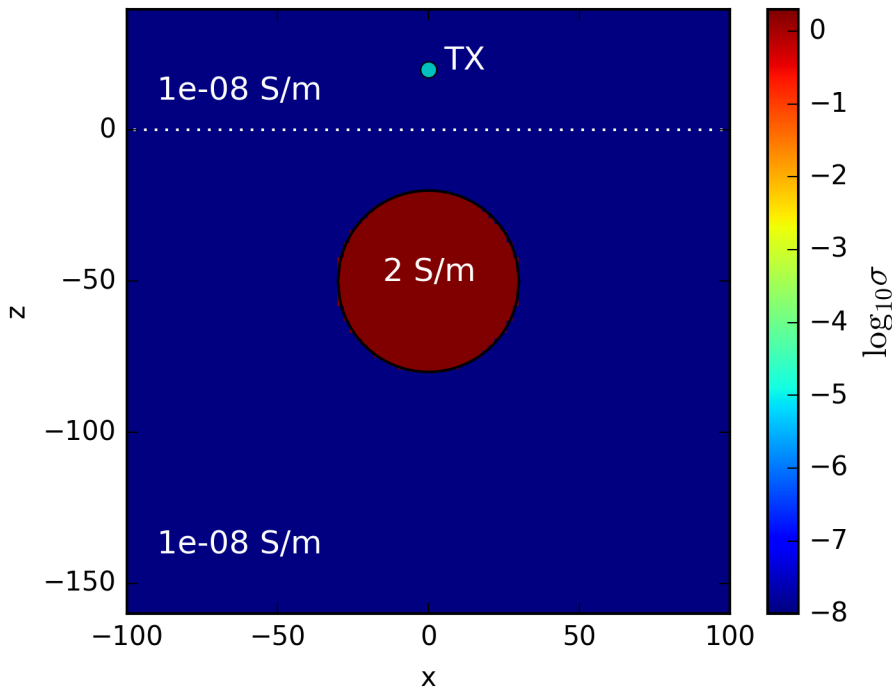


Effects of background resistivity: Frequency

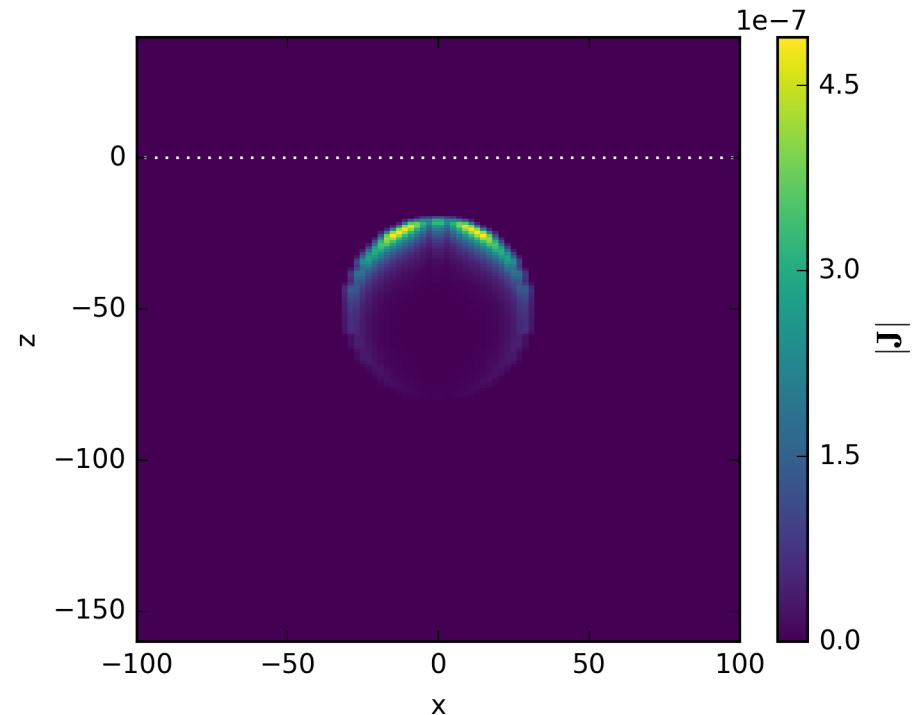
- Buried, conductive sphere
- Vary background conductivity
- Frequency: 10^4 Hz



10^{-8} S/m background



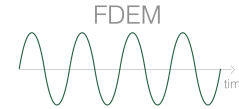
Current Density



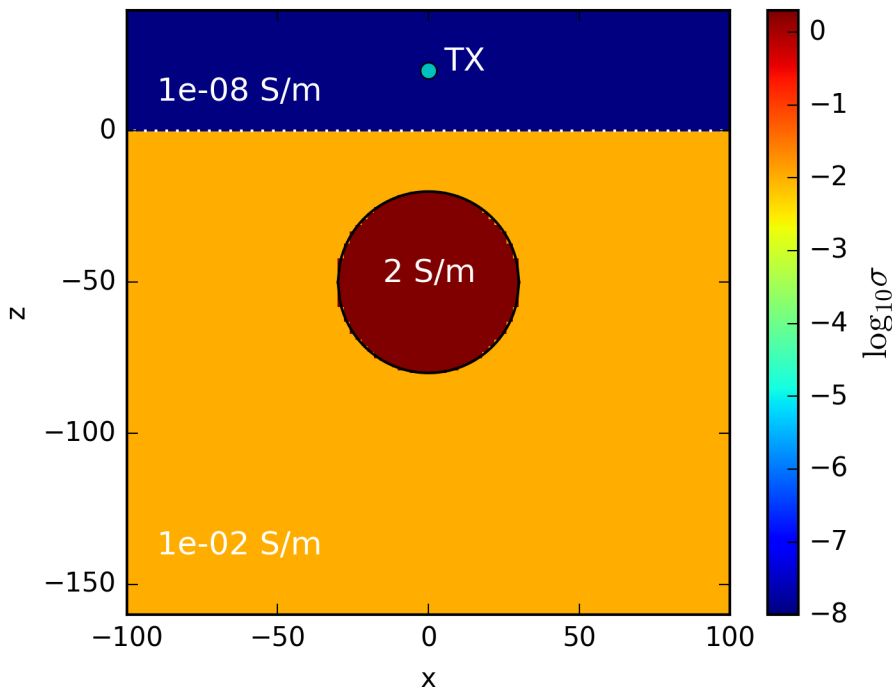
Effects of background resistivity: Frequency

- Buried, conductive sphere
- Vary background conductivity

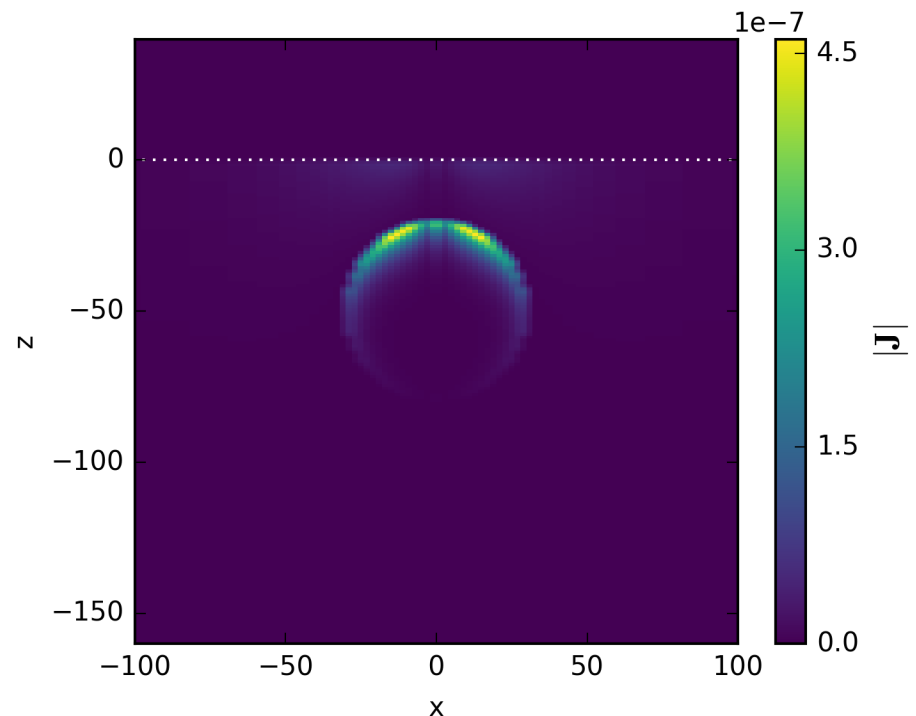
- Frequency: 10^4 Hz



10^{-2} S/m background



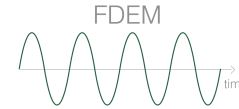
Current Density



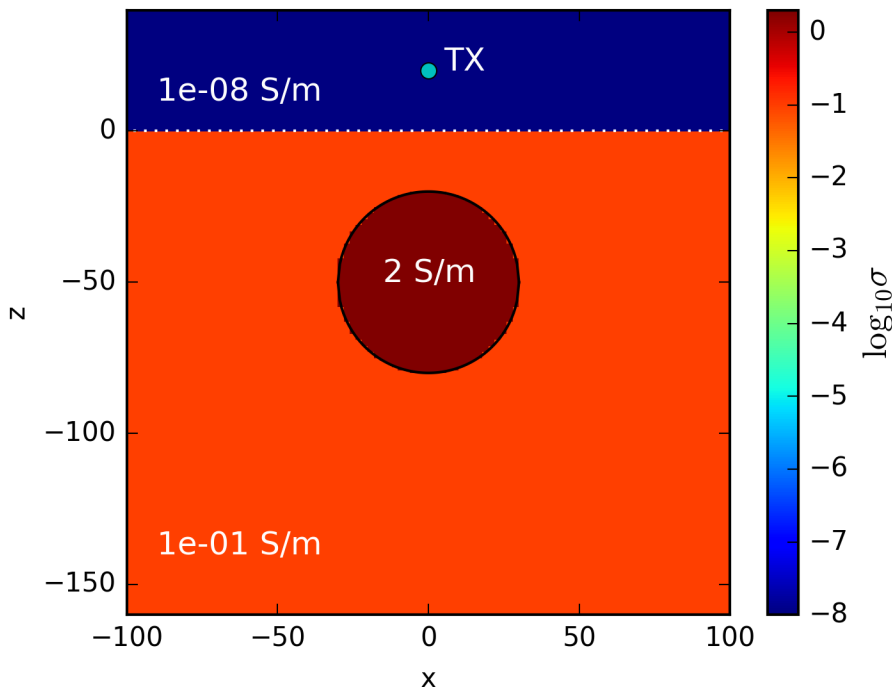
Effects of background resistivity: Frequency

- Buried, conductive sphere
- Vary background conductivity

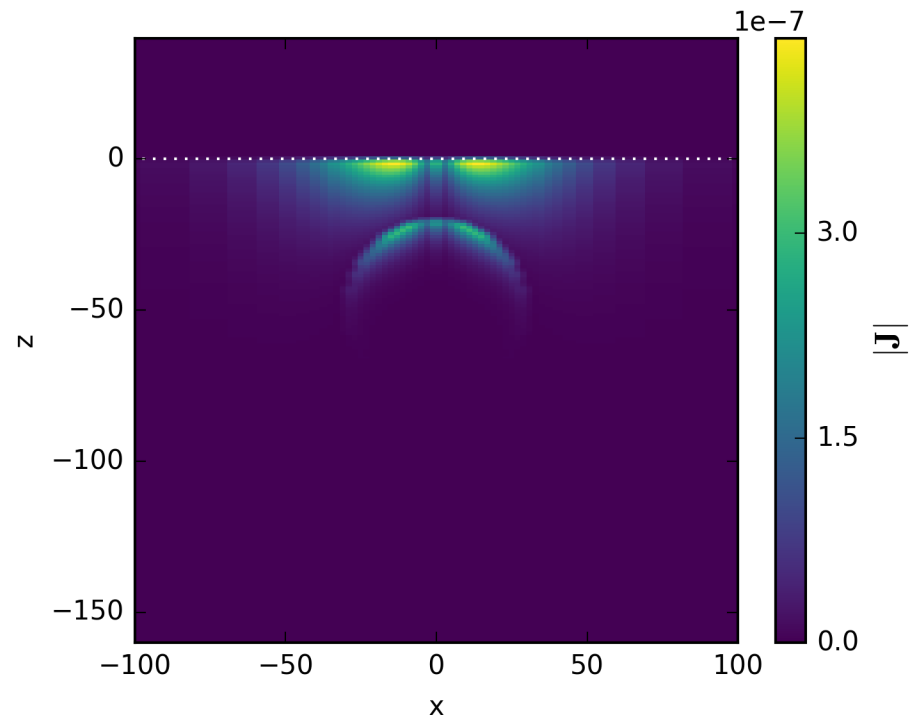
- Frequency: 10^4 Hz



10^{-1} S/m background

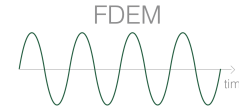


Current Density

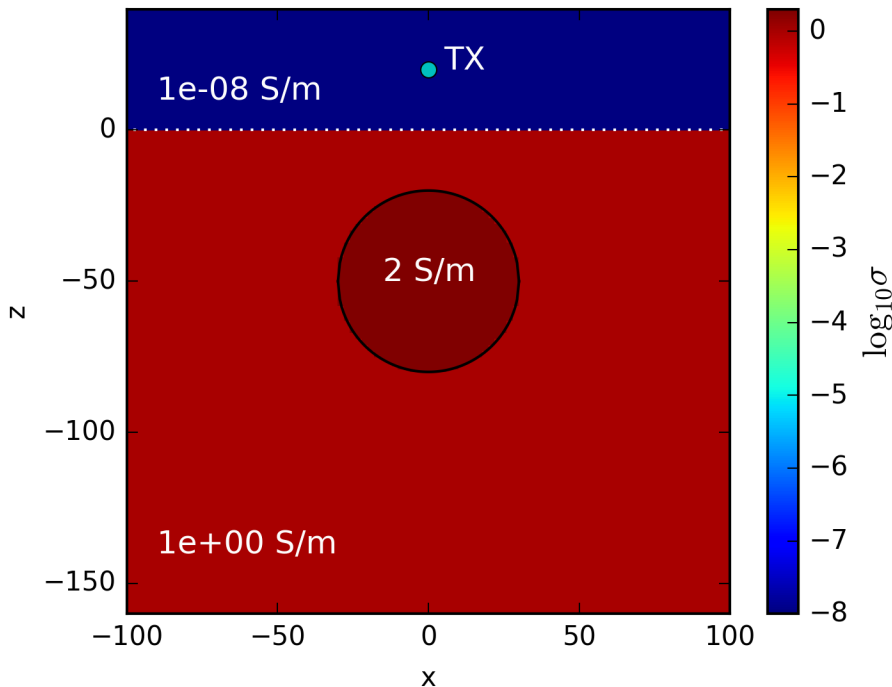


Effects of background resistivity: Frequency

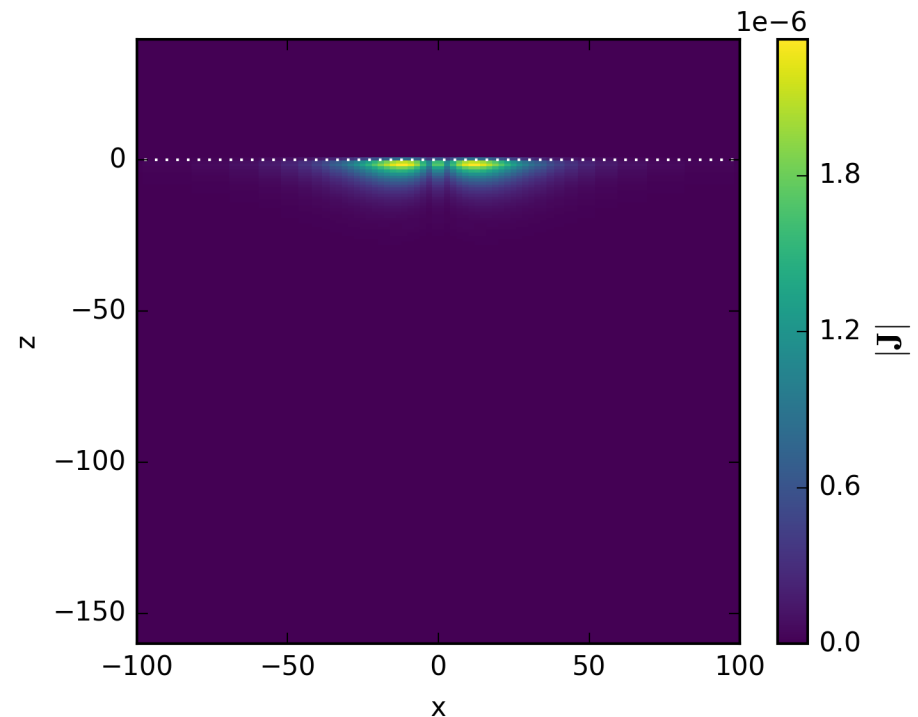
- Buried, conductive sphere
- Vary background conductivity
- Frequency: 10^4 Hz



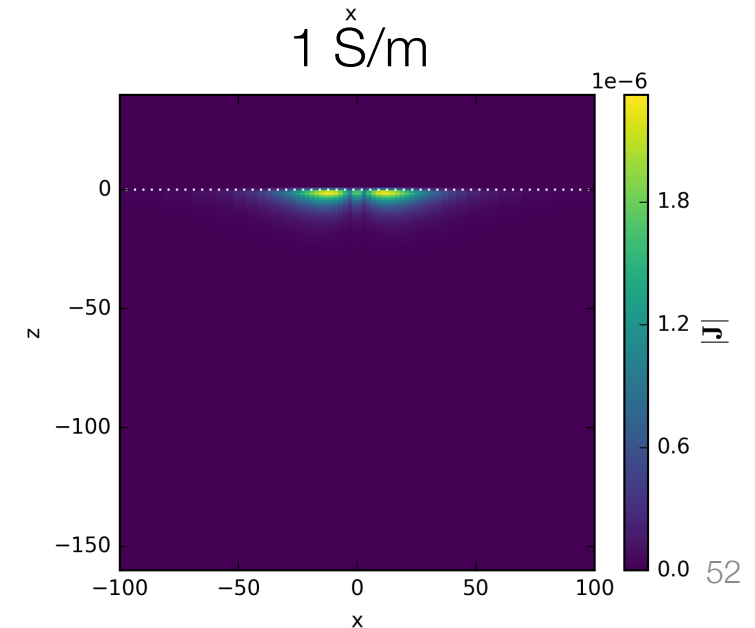
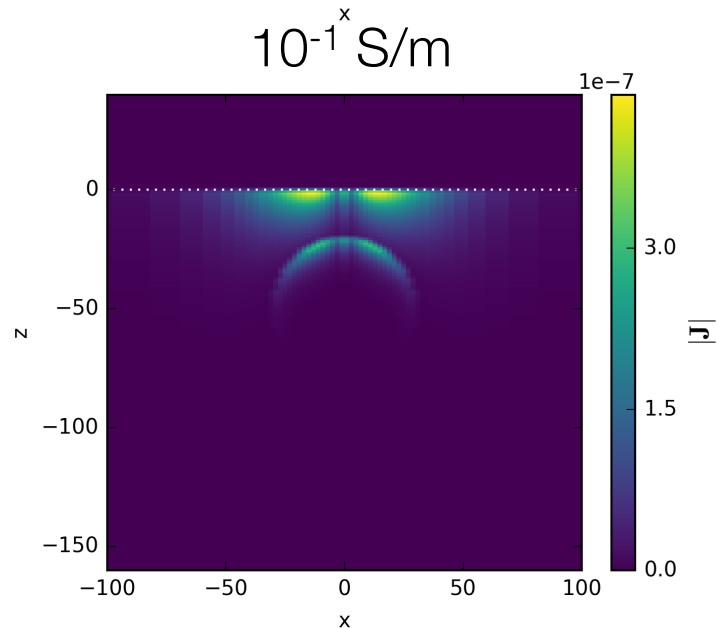
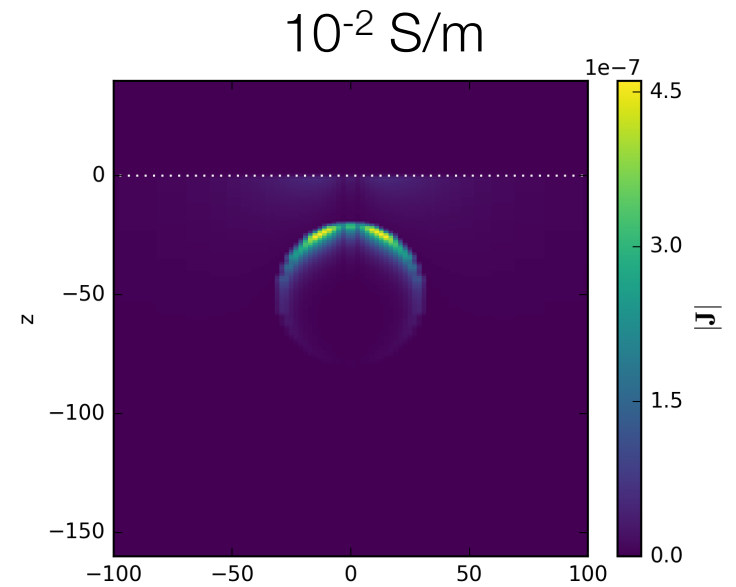
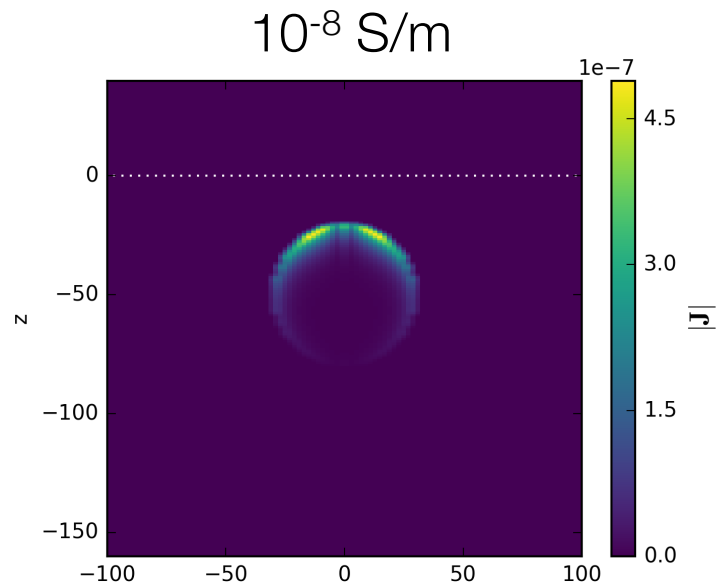
1 S/m background



Current Density

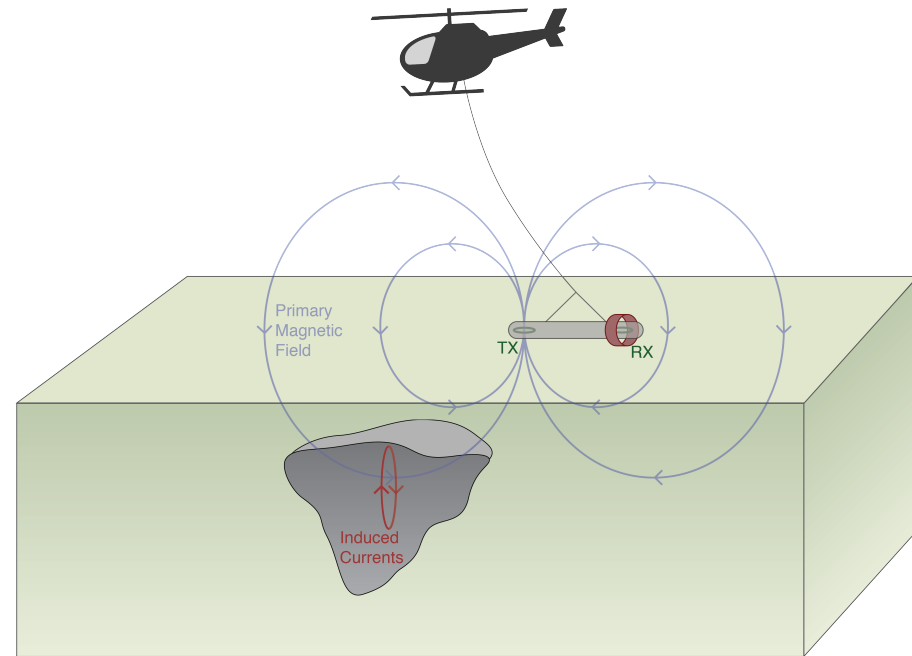


Effects of background resistivity: Frequency

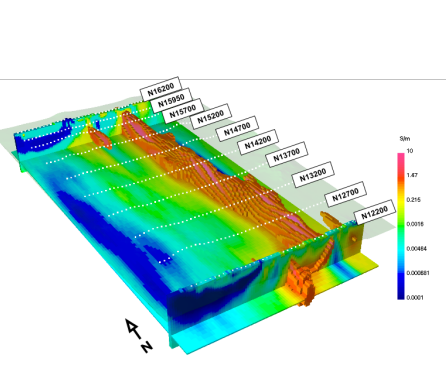


Recap: what have we learned?

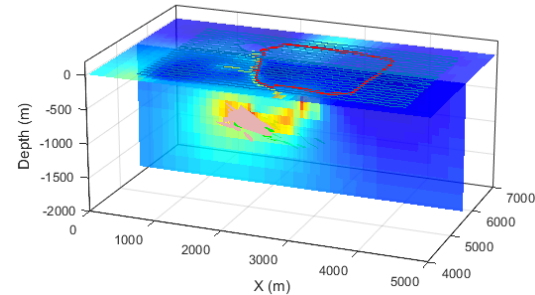
- Basics of EM induction
- Response functions
- Mutual coupling
- Data for frequency or time domain systems
- Circuit model is a good proxy
- Need to account for energy losses
- Ready to look at some field examples



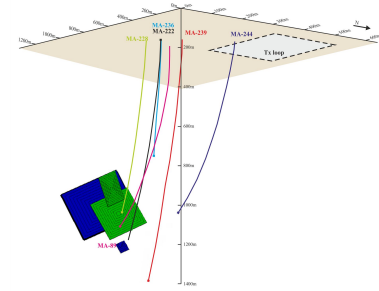
Today's Case Histories



Kasted, Denmark:
mapping paleochannels

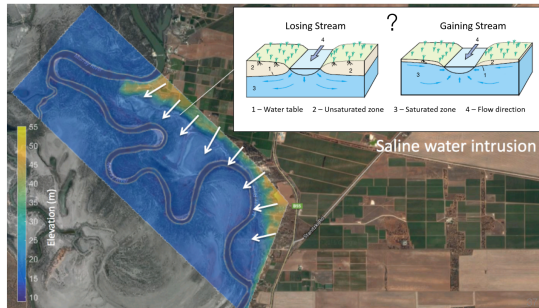


HeliSAM at Lalore:
Minerals

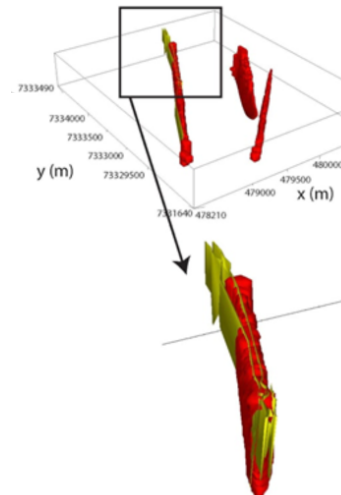


La Magdalena:
Minerals

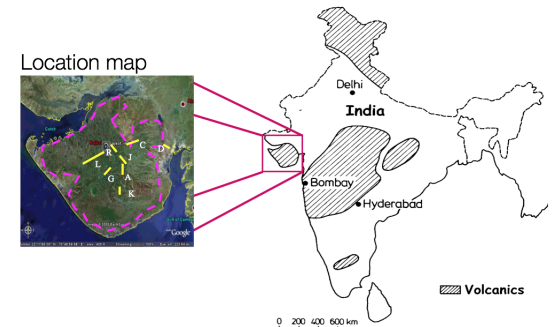
Mt. Isa, Australia:
Mineral Exploration



Bookpurnong, Australia:
diagnosing river
salinization

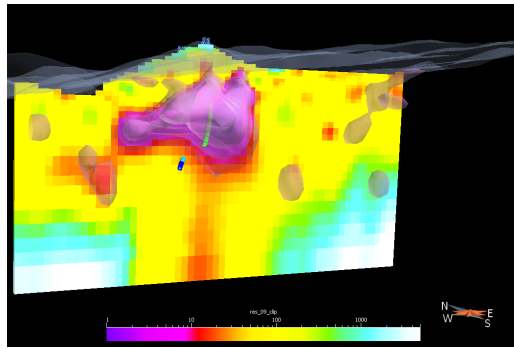


West Plains, Canada:
Mineral exploration

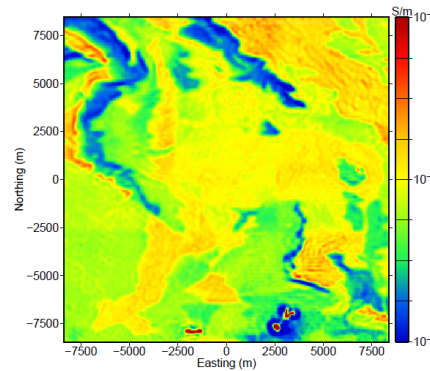


Deccan Traps, India:
mapping sediment beneath
basalt

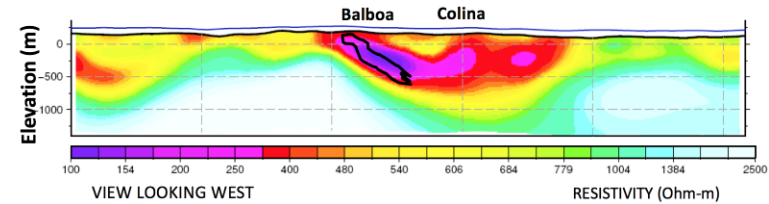
Today's Case Histories



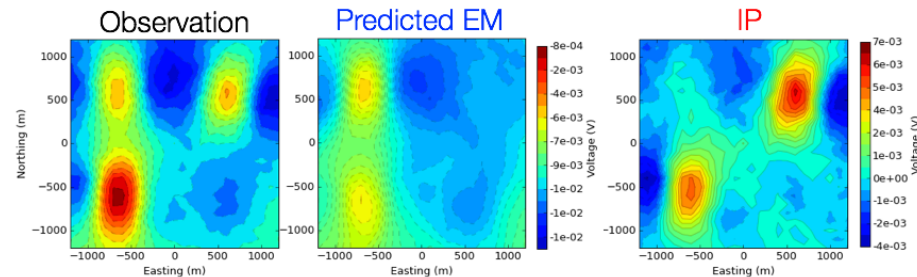
Santa Cecilia, Chile:
Mineral Exploration



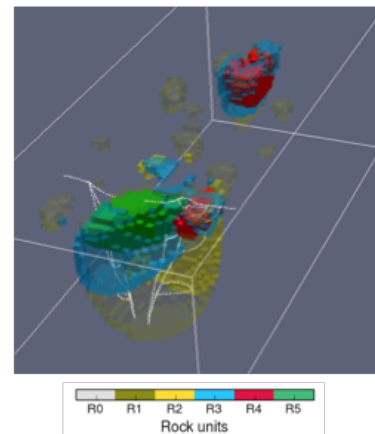
Noranda, Canada:
Geologic Mapping



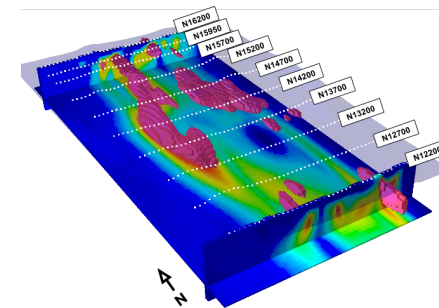
Balboa, Panama:
Mineral Exploration



EM – IP Inversion
(decoupling)



TKC, Canada:
Mineral Exploration



Mt. Isa, Australia:
Mineral Exploration

End of EM Fundamentals

