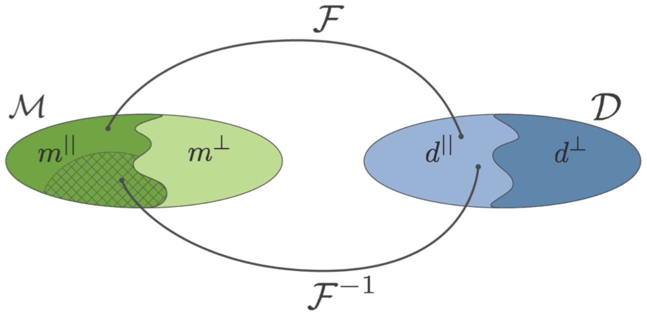
Field Scale

What have we done so far?

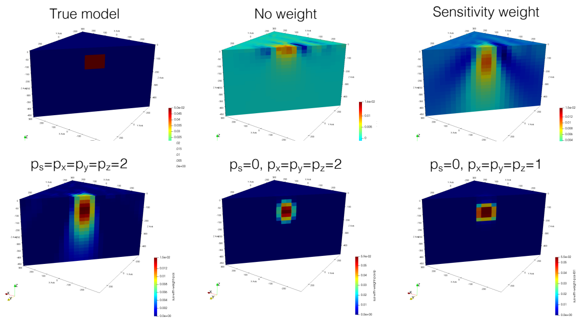
- Methodology and understanding for solving the inverse problem



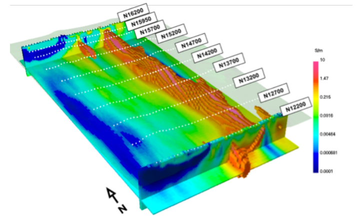
- Solve a non-linear inverse problem
  - Generally voxel-based
  - PDE
  - Integral equation

$$\nabla \cdot \sigma \nabla V = I\delta(r) = q$$

- Incorporate different types of information
  - Regularization function and bounds



- Case histories that show the utility

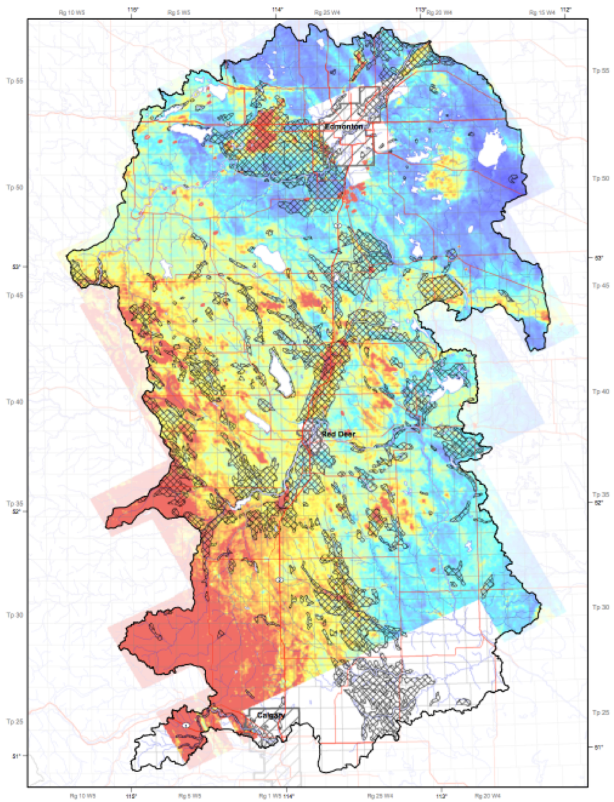


Issues for practical applications

- Inversion in large scale problems
  - Airborne EM data
  - Forward modelling
  - Inversion
  - Field application (e.g. diamond exploration)

- Addressing Uncertainty
  - Exploring model model space
  - Model parameterization
  - Joint inversion
  - Post-inversion classification (multiple physical properties)
  - PGI (petrophysically guided inversion)

AEM resistivity Alberta Corridor



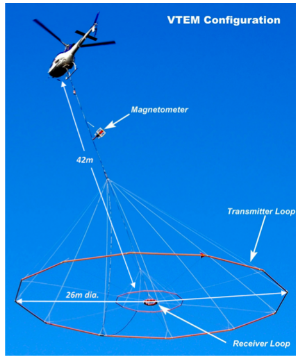
Baker (2011)

Large scale problems: Airborne EM

- Typical airborne EM systems
  - Frequency
  - Time
- Large amount of data
  - ~100,000 soundings
  - multiple frequency or times

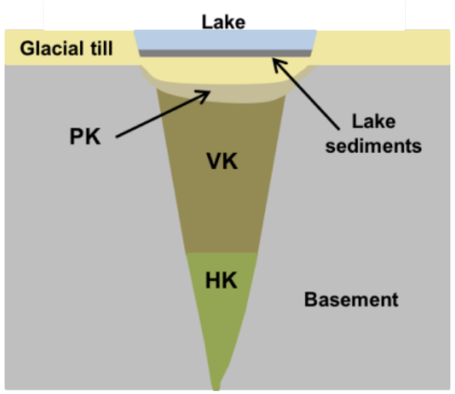
Dighem

VTEM



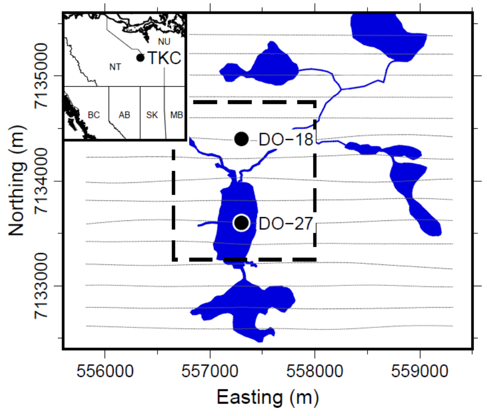
AeroTEM

Apply this to diamond exploration

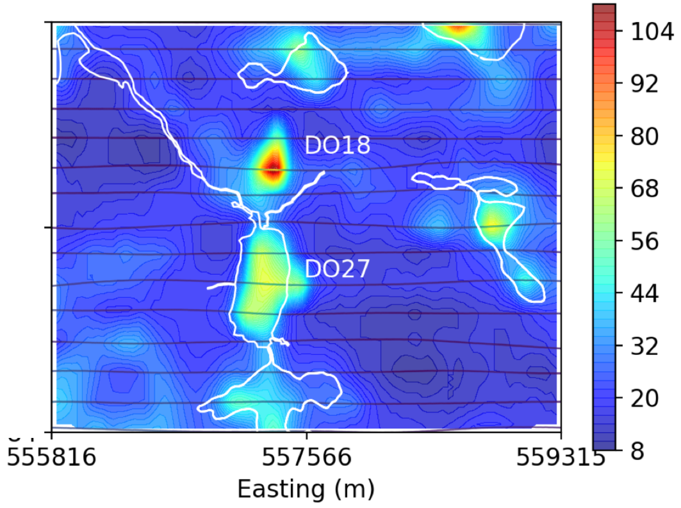


Airborne EM: Tli Kwi Cho (TKC) kimerlites

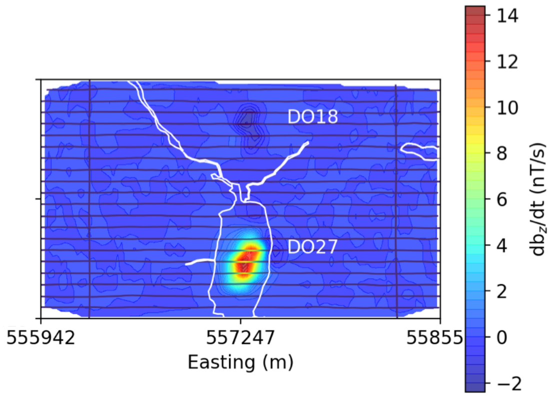
Location map at TKC



DIGHEM (Quadrature 56kHz)



VTEM (90 μs)



DIGHEM (1992)

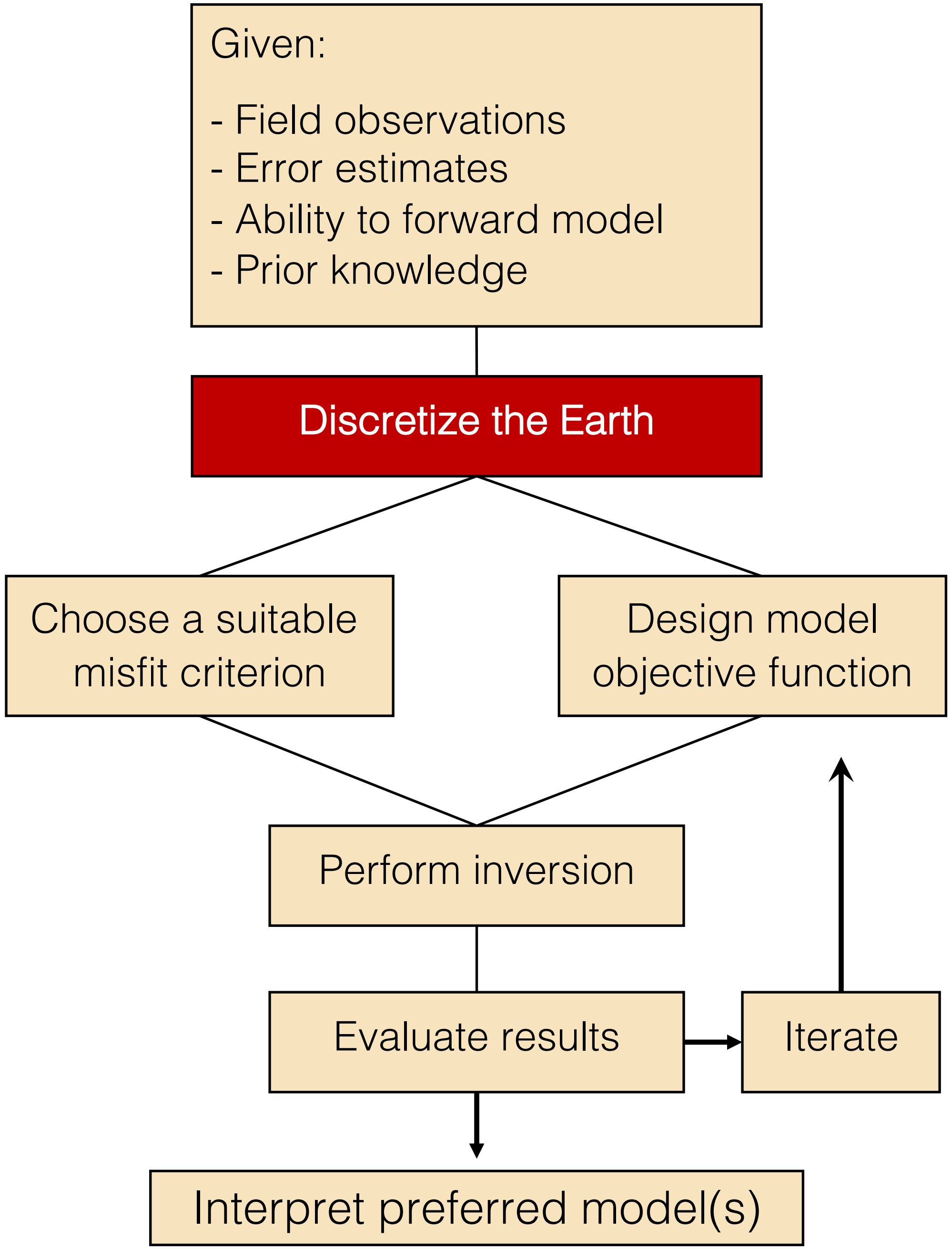
Configuration	HCP
Frequency	900Hz-56kHz
Data unit	ppm
Line spacing	200 m
Line km	52 km
# of sounding	6274

VTEM (2003)

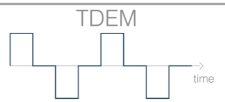
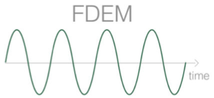
Configuration	Colocated-loop
Off time channel	90-6340 (μs)
Data unit	pV/A-m^4
Line spacing	75 m
Line km	39 km
# of sounding	26342

Flow chart for inverse problem

Need to solve forward problem



Basic Equations

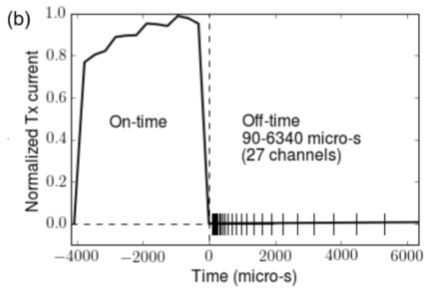
	Time 	Frequency 
Faraday's Law	$\nabla \times \mathbf{e} = -\frac{\partial \mathbf{b}}{\partial t}$	$\nabla \times \mathbf{E} = -i\omega \mathbf{B}$
Ampere's Law	$\nabla \times \mathbf{h} = \mathbf{j} + \frac{\partial \mathbf{d}}{\partial t}$	$\nabla \times \mathbf{H} = \mathbf{J} + i\omega \mathbf{D}$
No Magnetic Monopoles	$\nabla \cdot \mathbf{b} = 0$	$\nabla \cdot \mathbf{B} = 0$
Constitutive Relationships (non-dispersive)	$\mathbf{j} = \sigma \mathbf{e}$ $\mathbf{b} = \mu \mathbf{h}$ $\mathbf{d} = \varepsilon \mathbf{e}$	$\mathbf{J} = \sigma \mathbf{E}$ $\mathbf{B} = \mu \mathbf{H}$ $\mathbf{D} = \varepsilon \mathbf{E}$

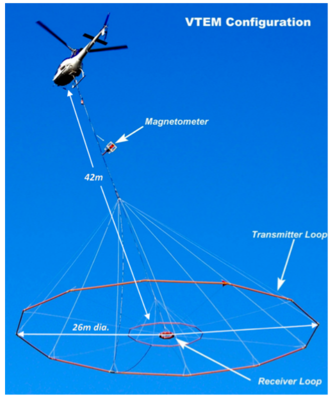
* Solve with sources and boundary conditions

Forward Problem

- Discretize in frequency or time
- Discretize in space:
- Solve system of equations
- Many transmitters

Time (TDEM)	Frequency (FDEM)
$\nabla \times \mathbf{e} = -\dfrac{\partial \mathbf{b}}{\partial t}$	$\nabla \times \mathbf{E} = -i\omega \mathbf{B}$
$\nabla \times \mathbf{h} = \mathbf{j} + \dfrac{\partial \mathbf{d}}{\partial t}$	$\nabla \times \mathbf{H} = \mathbf{J} + i\omega \mathbf{D}$
$\nabla \cdot \mathbf{b} = 0$	$\nabla \cdot \mathbf{B} = 0$
$\mathbf{j} = \sigma \mathbf{e}$	$\mathbf{J} = \sigma \mathbf{E}$
$\mathbf{b} = \mu \mathbf{h}$	$\mathbf{B} = \mu \mathbf{H}$
$\mathbf{d} = \varepsilon \mathbf{e}$	$\mathbf{D} = \varepsilon \mathbf{E}$





Time Domain: Mathematical Setup

Maxwell's equations

$$\nabla \times \mathbf{e} + \frac{\partial \mathbf{b}}{\partial t} = 0$$

$$\nabla \times \mu^{-1}\mathbf{b} - \sigma\mathbf{e} = \mathbf{s}(t)$$

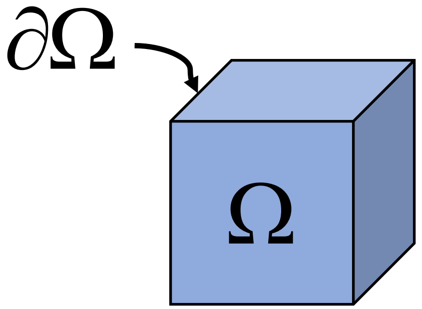
Boundary conditions

$$\mathbf{n} \times \mathbf{b} = 0$$

Initial conditions

$$\mathbf{e}(x, y, z, t = 0) = \mathbf{e}_0$$

$$\mathbf{b}(x, y, z, t = 0) = \mathbf{b}_0$$



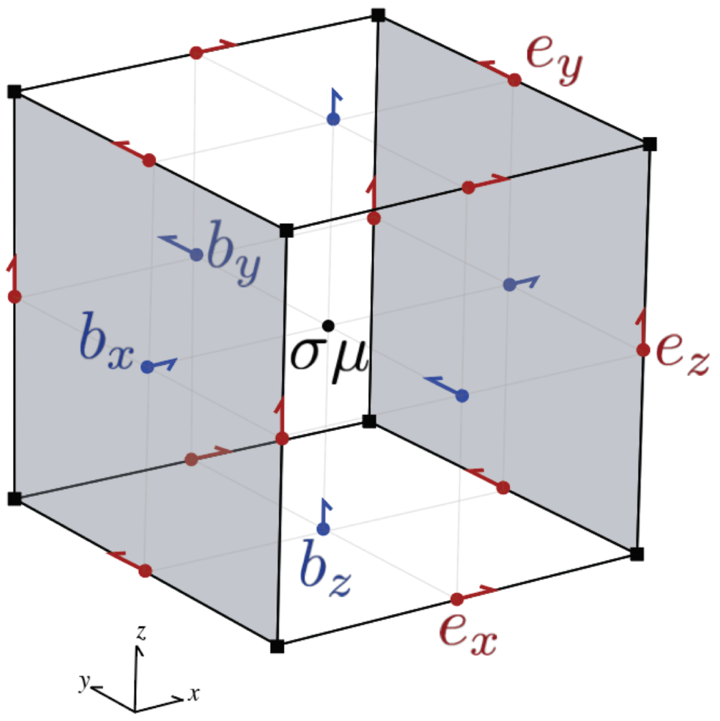
time: $[0, t_f]$

Need to solve in space and time

Semi-discretization in space

Staggered Grid

- Physical properties: cell centers
- Fields: edges
- Fluxes: faces



Continuous second-order equations

$$\nabla \times \mu^{-1} \nabla \times \mathbf{e} + \sigma \frac{\partial \mathbf{e}}{\partial t} = -\frac{\partial \mathbf{s}}{\partial t}$$

Semi-discretized second order equations

$$\mathbf{C}^\top \mathbf{M}^f_{\mu^{-1}} \mathbf{C} \mathbf{e} + \mathbf{M}^e_\sigma \frac{\partial \mathbf{e}}{\partial t} = -\frac{\partial \mathbf{s}}{\partial t}$$

Discretizing in time

First order backwards difference (implicit)

- $\mathbf{e}^{n+1}$ depends upon $\mathbf{e}^{n}$

$$\mathbf{C}^\top \mathbf{M}^f_{\mu^{-1}} \mathbf{C}\mathbf{e} + \mathbf{M}^e_\sigma \frac{\partial \mathbf{e}}{\partial t} = -\frac{\partial \mathbf{s}}{\partial t} \qquad \Delta t = t_{n+1} - t_n$$

- Time-step:

$$\left(\mathbf{C}^\top \mathbf{M}^f_{\mu^{-1}} \mathbf{C} + \frac{1}{\Delta t}\mathbf{M}^e_\sigma\right)\mathbf{e}^{n+1} = -\frac{\mathbf{s}^{n+1} - \mathbf{s}^n}{\Delta t} + \frac{1}{\Delta t}\mathbf{M}^e_\sigma \mathbf{e}^n$$

Solve system at each time step

$$\mathbf{A}_{n+1}\mathbf{u}_{n+1} = -\mathbf{B}_n\mathbf{u}_n + \mathbf{q}_{n+1}$$

Factor $\mathbf{A}_{n+1} = \mathbf{L}\mathbf{L}^\top$

Solving a TDEM Problem

Solve with forward elimination $\mathbf{u}_0$

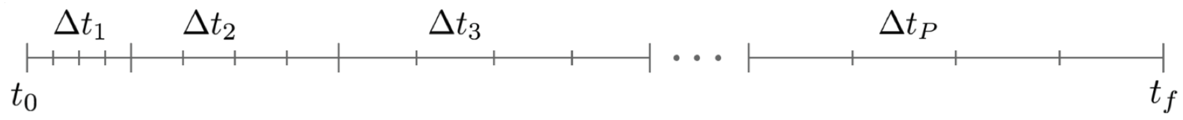
- Initial conditions provide
- To propagate forward, solve

$$\mathbf{A}_{n+1}\mathbf{u}_{n+1} = -\mathbf{B}_n\mathbf{u}_n + \mathbf{q}_{n+1}$$

$$\begin{pmatrix} \mathbf{A}_0 & & & & & \\ \mathbf{B}_1 & \mathbf{A}_1 & & & & \\ & \mathbf{B}_2 & \mathbf{A}_2 & & & \\ & & \ddots & \ddots & & \\ & & & \mathbf{B}_{n-1} & \mathbf{A}_{n-1} & \\ & & & & \mathbf{B}_n & \mathbf{A}_n \end{pmatrix} \begin{pmatrix} \mathbf{u}_0 \\ \mathbf{u}_1 \\ \mathbf{u}_2 \\ \vdots \\ \mathbf{u}_{n-1} \\ \mathbf{u}_n \end{pmatrix} = \begin{pmatrix} \mathbf{q}_0 \\ \mathbf{q}_1 \\ \mathbf{q}_2 \\ \vdots \\ \mathbf{q}_{n-1} \\ \mathbf{q}_n \end{pmatrix}$$

Some details of solving system

- Refactor only if $\mathbf{A}_{n+1}(\sigma, \triangle t)$ changes
- Divide modelling time into P partitions



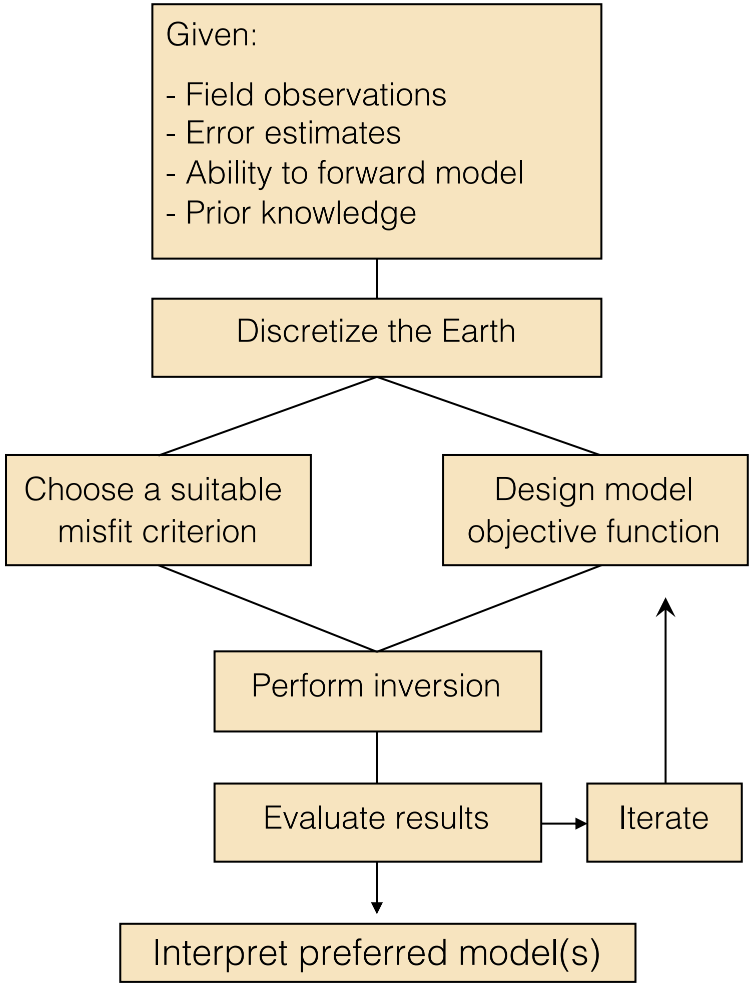
- Total computation time:

$$T = P(N_{\Delta t}N_{TX}t_{\text{solve}} + t_{\text{factor}})$$

Time to solve factored system

Time to factor system

Inverse problem



$$\text{minimize} \quad \phi(\mathbf{m}) = \phi_d(\mathbf{m}) + \beta\phi_m(\mathbf{m})$$

$$\text{subject to} \quad \mathbf{m}_{lower} < \mathbf{m} < \mathbf{m}_{upper}$$

Data misfit $$\phi_d(\mathbf{m}) = \frac{1}{2}||\mathbf{W}_{\mathrm{d}}(F[\mathbf{m}] - \mathbf{d}_{\mathrm{obs}})||_2^2.$$

Regularization $$\phi_m(\mathbf{m}) = \frac{1}{2}||\mathbf{W}_{\mathrm{m}}(\mathbf{m} - \mathbf{m}_{\mathrm{ref}})||_2^2.$$

$$\begin{cases} d\text{: data for all transmitters} \\ \mathbf{m} = log(\boldsymbol{\sigma}) \\ M\text{: number of cells} \end{cases}$$

Gauss-Newton approach

- Inverse problem
$$\min_{\mathbf{m}} \phi(\mathbf{m}) = \phi_d(\mathbf{m}) + \beta\phi_m(\mathbf{m})$$
$$= \frac{1}{2}\|\mathbf{W}_d(F[\mathbf{m}]) - \mathbf{d}^{obs}\|^2 + \frac{\beta}{2}\|\mathbf{W}_m(\mathbf{m} - \mathbf{m}_{ref})\|^2$$
- Gradient
$$\mathbf{g}(\mathbf{m}) = \mathbf{J}^\top \mathbf{W}_d^\top \mathbf{W}_d(F[\mathbf{m}] - \mathbf{d}^{obs}) + \beta \mathbf{W}_m^\top \mathbf{W}_m(\mathbf{m} - \mathbf{m}_{ref})$$
- Taylor expand: Gauss Newton equation
$$(\mathbf{J}^\top \mathbf{W}_d^\top \mathbf{W}_d \mathbf{J} + \beta \mathbf{W}_m^\top \mathbf{W}_m)\delta\mathbf{m} = -\mathbf{g}(\mathbf{m})$$
- Use inexact PCG to solve for model update (N_{CG} iterations)
$$\mathbf{m}_{k+1} = \mathbf{m}_k + \delta\mathbf{m}$$

Number of forward modellings: $2(N_{CG} + 1) \sim 20$

Gauss-Newton approach

$$\min_{\mathbf{m}} \phi(\mathbf{m}) = \phi_d(\mathbf{m}) + \beta\phi_m(\mathbf{m})$$

$$= \frac{1}{2}\|\mathbf{W}_d(F[\mathbf{m}]) - \mathbf{d}^{obs}\|^2 + \frac{\beta}{2}\|\mathbf{W}_m(\mathbf{m} - \mathbf{m}_{ref})\|^2$$

Choose β_0, $\mathbf{m}_{ref}$

Evaluate $\boldsymbol{\phi}(\mathbf{m}_{ref})$, $\mathbf{g}(\mathbf{m}_{ref})$, matrices $\mathbf{W_d}$, **W...**

for i in range([0, max_beta_iter]):

for k in range([0, max_inner_iterations]):

- IPCG to solve $(\mathbf{J}^\top \mathbf{W}_d^\top \mathbf{W}_d \mathbf{J} + \beta \mathbf{W}_m^\top \mathbf{W}_m)\delta\mathbf{m} = -\mathbf{g}(\mathbf{m})$
- line search for step length α
- Update model $\mathbf{m}_{k+1} = \mathbf{m}_k + \alpha\delta\mathbf{m}$
- Exit if $\phi < \phi_d^*$ or $\frac{\|\mathbf{g}(\mathbf{m}_{k+1})\|}{\|\mathbf{g}(\mathbf{m}_k)\|} < \text{tol}$

Reduce β

Tally up the computations

Number of transmitters	1000
Number of time steps	50
Solving a GN step	20
Number of GN iterations	20

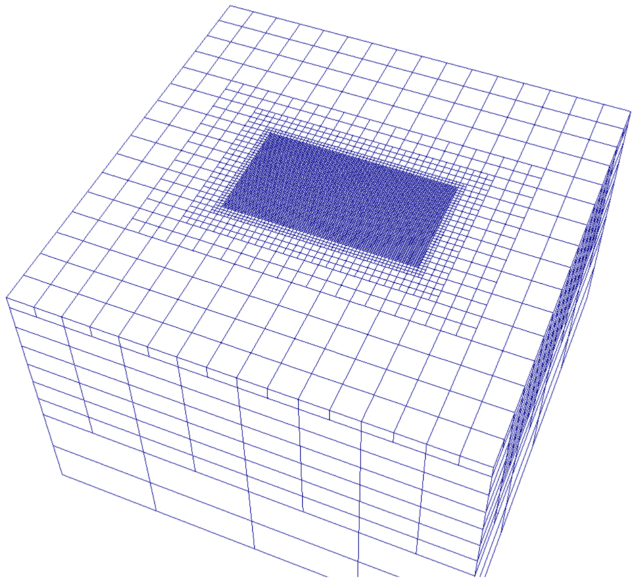
- Total number of Maxwell solutions is 20,000,000
- Suppose: t_{factor}=1 sec
  - 100 processors: 55 hours
  - 1000 processors 5.5 hours

Need:
- Fast forward modelling
- Multiple cpu

Mesh

- Trade off (accuracy vs. computation)
- Consider a 3D airborne EM simulation (1000 sources)

Octree mesh



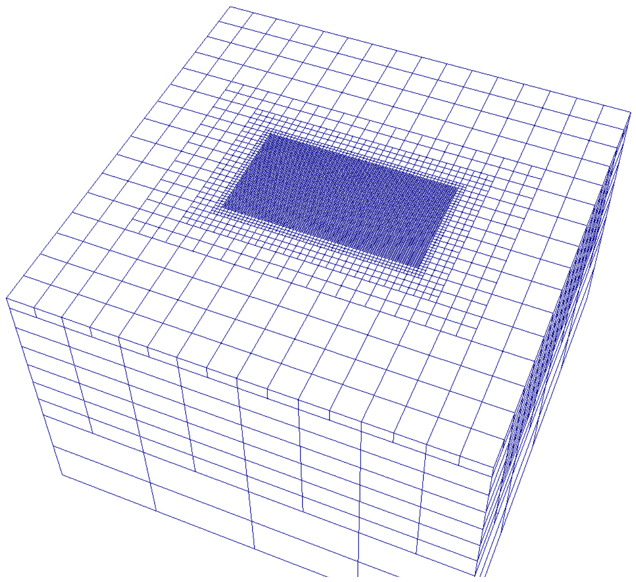
> 1,000,000 cells (this is big!)

How do we tackle this?

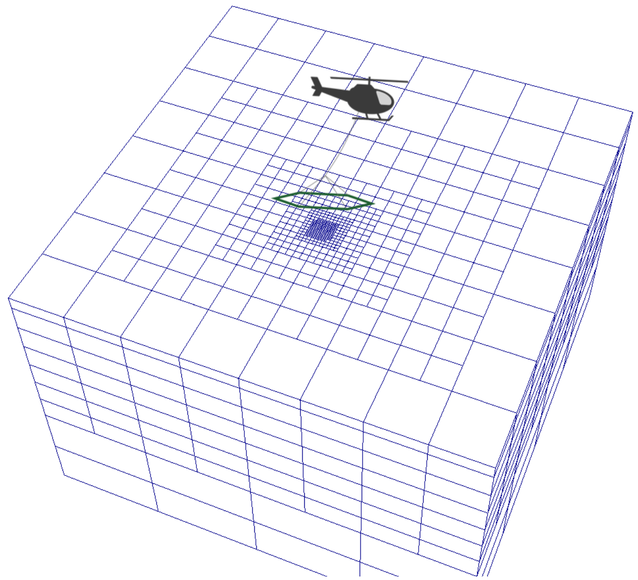
Mesh decomposition

- Separate forward modelling mesh for each transmitter

Global mesh



Local mesh



Mesh decomposition

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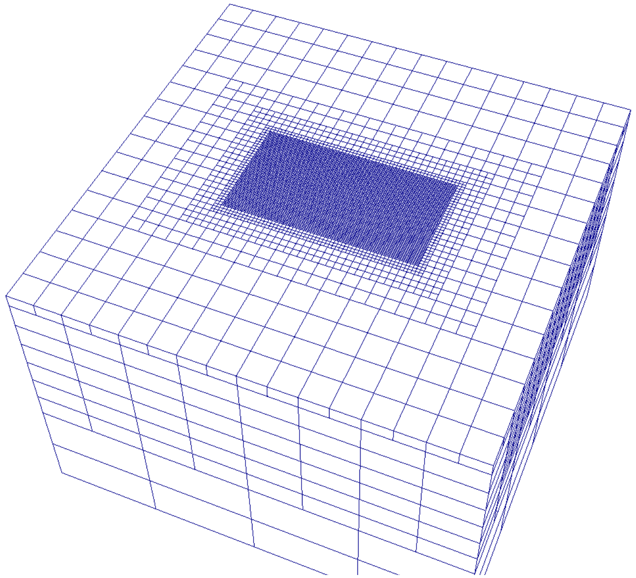
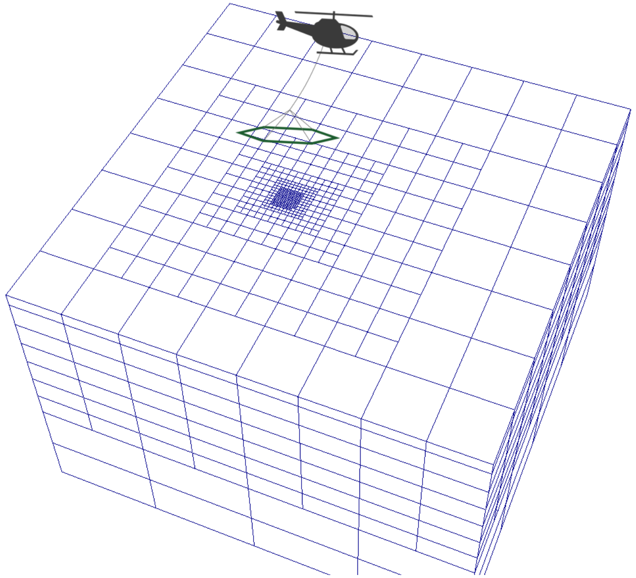
Global mesh



Local mesh



Mesh decomposition

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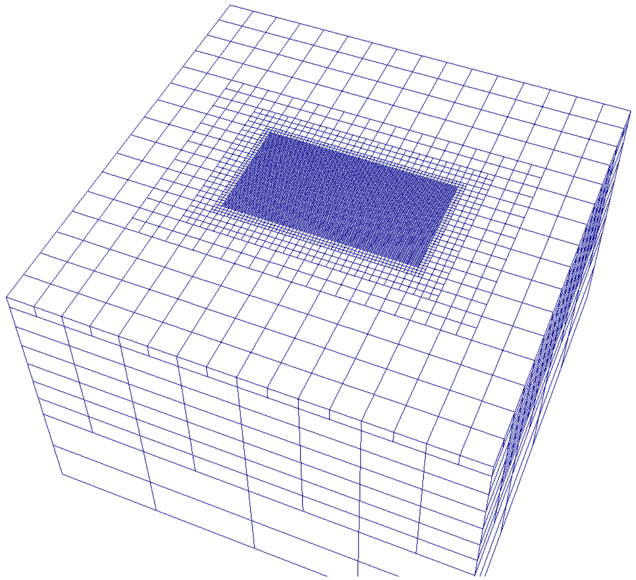
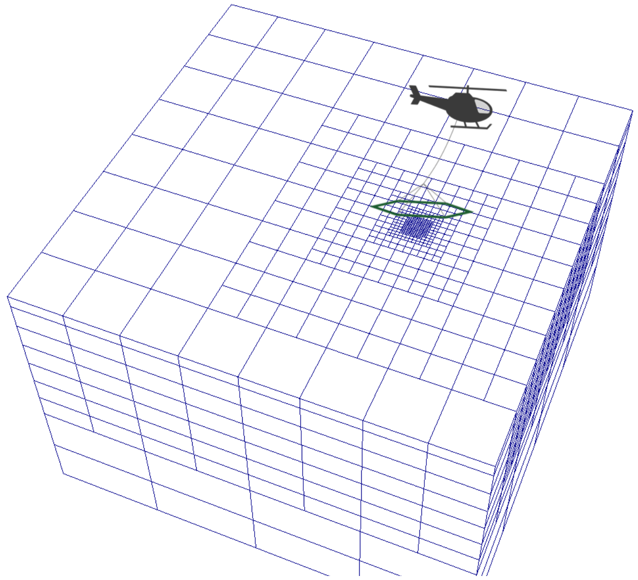
Global mesh

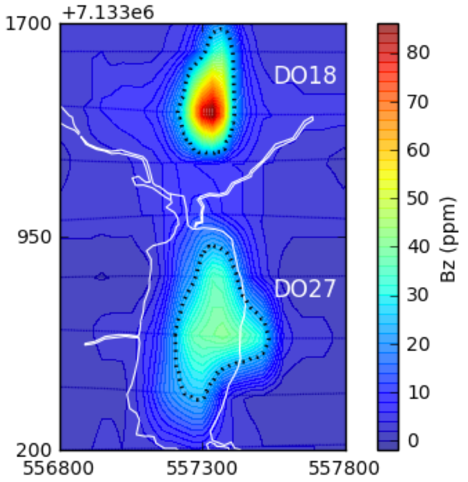


Local mesh

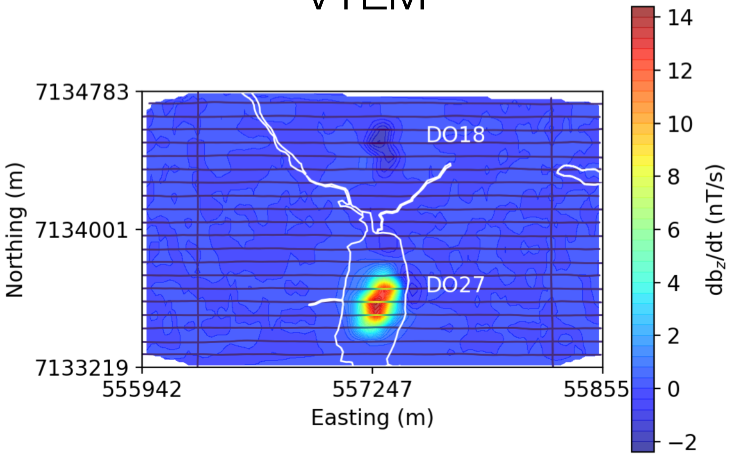


3D inversion (TKC)

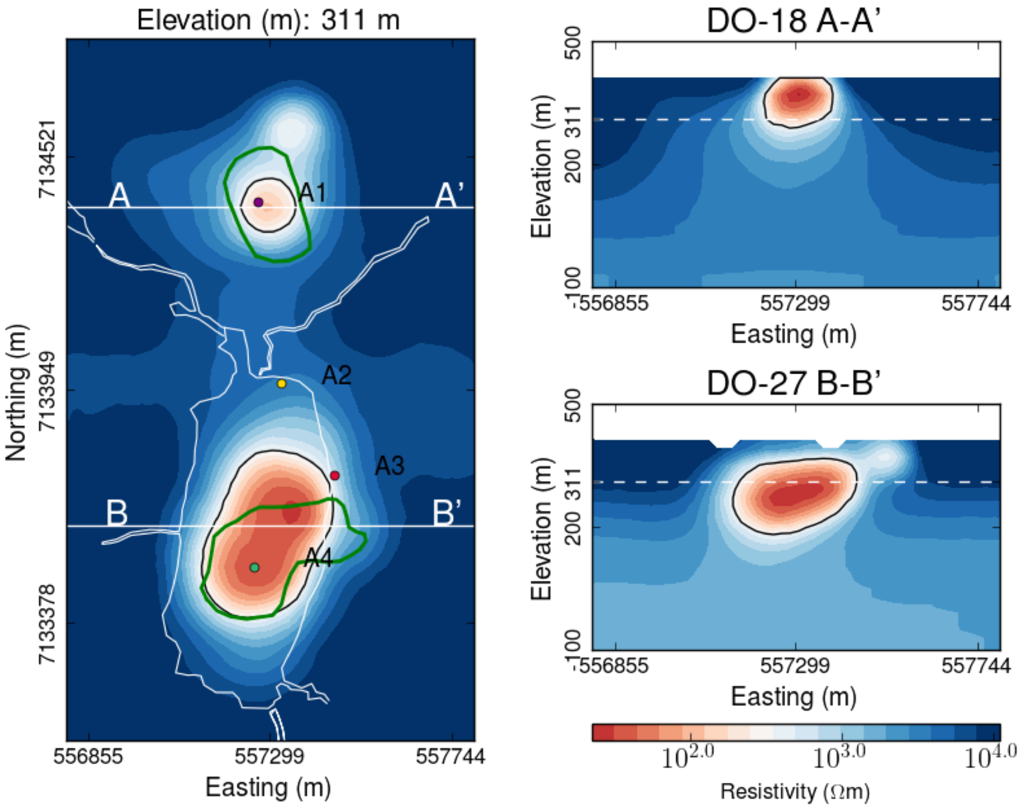
DIGHEM



VTEM



Recovered 3D conductivity



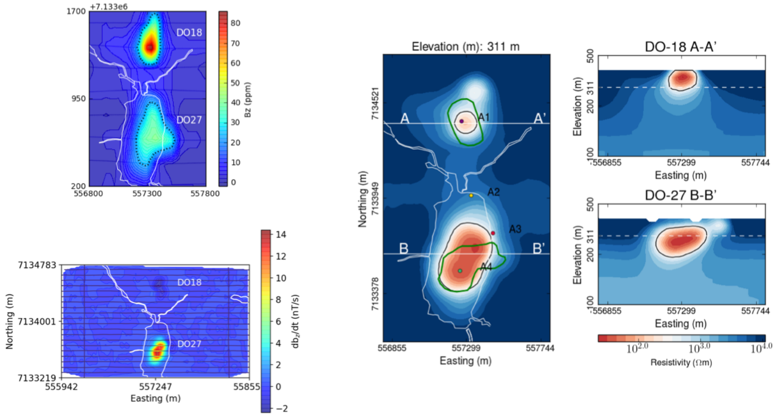
Outline of two pipes

Summary: Large-scale Inversions

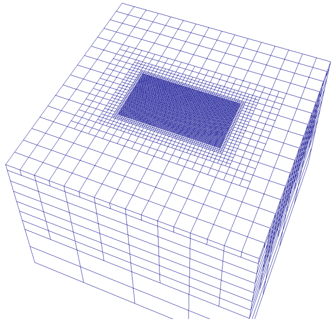
Same methodology as small-scale

Advances in scientific computing

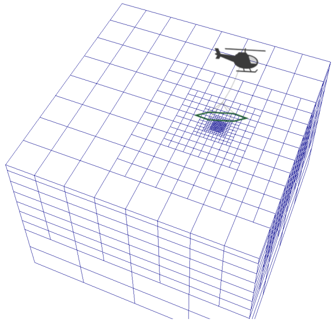
- Direct solvers (factor Maxwell operator)
- Semi-structured meshes (OcTree)
- Separate forward and inverse meshes
- Handling the sensitivity matrix
- Access to multi-cores



Global mesh

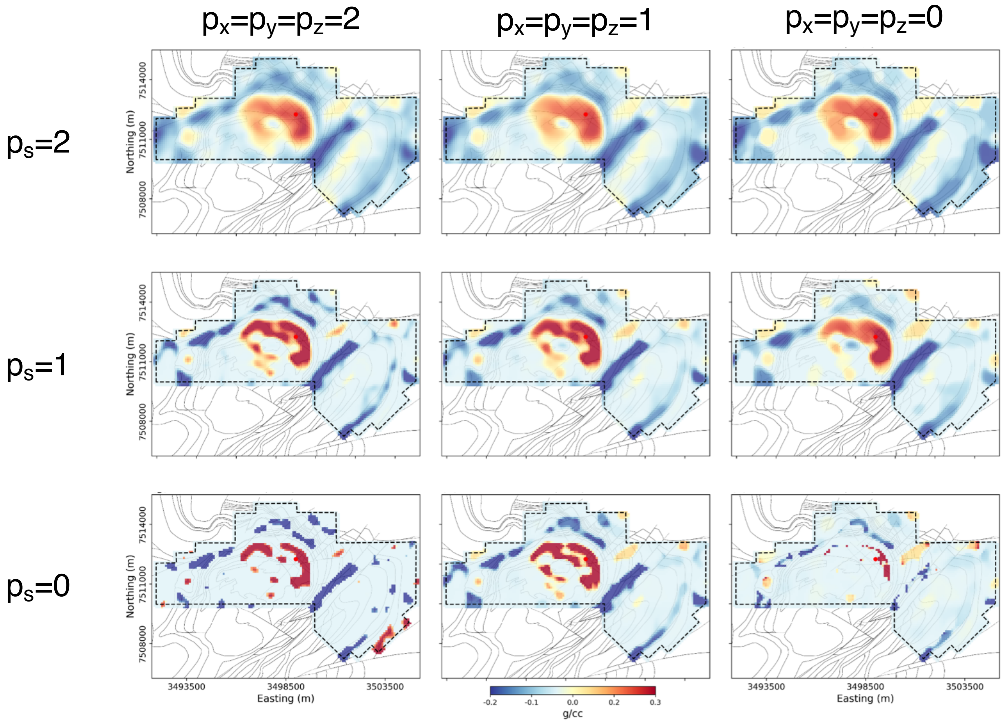


Local mesh



Addressing Uncertainty

- Exploring model space
  - Solutions with different character
  - Hypothesis testing
  - DOI

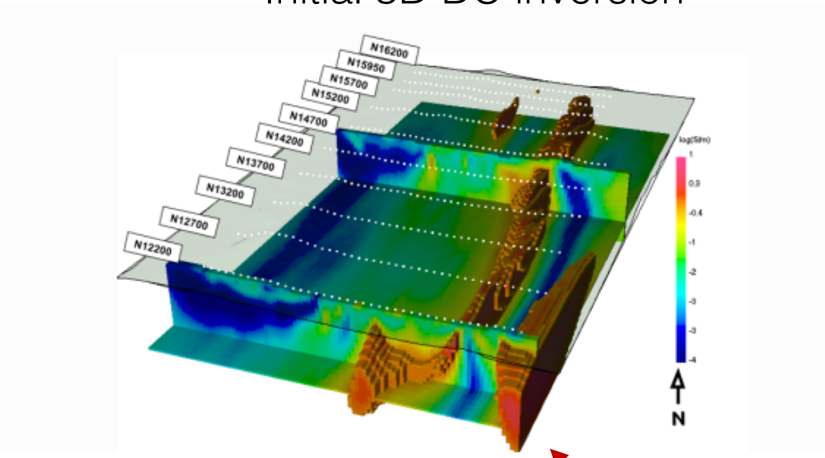


- Model parameterization
- Joint inversion
- Post-inversion classification (multiple physical properties)
- PGI (petrophysically guided inversion)

Hypothesis testing

- Generate “best” model
- Analyze for features of interest

Initial 3D DC inversion



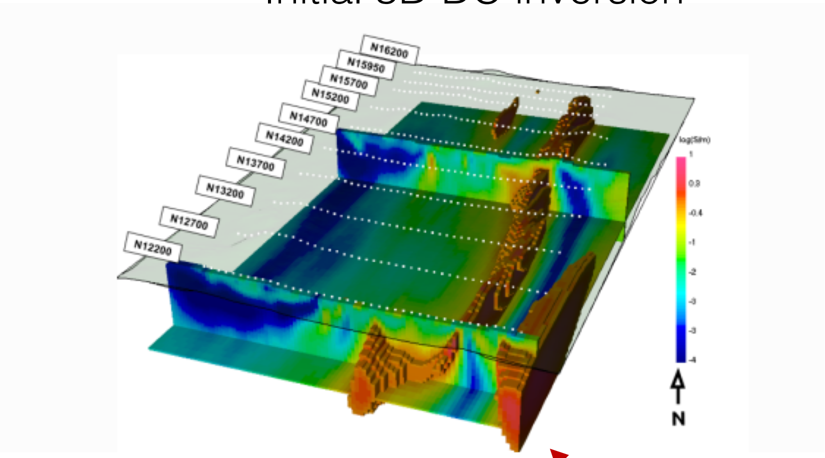
Suspicious feature

Hypothesis testing

- Generate “best” model
- Analyze for features of interest
- Test existence: Generate a counter example.
- Use a weighted reference model that doesn’t have feature.

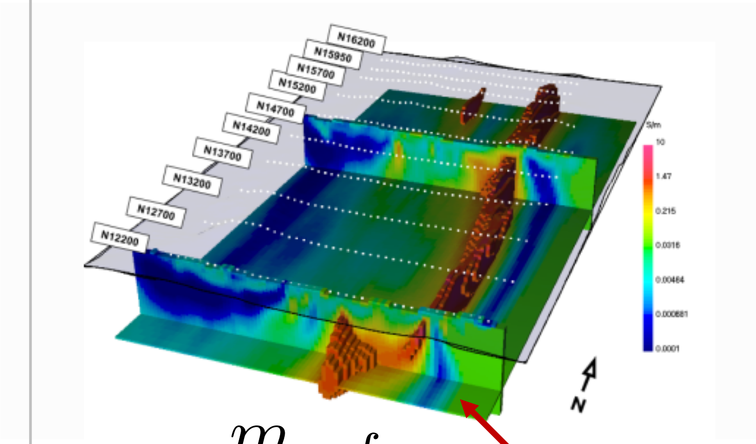
$$\phi_s = \int_V w_s (m - m_{ref})^2 dv$$

Initial 3D DC inversion



Suspicious feature

Reference model
Without south-east conductor



m_{ref}

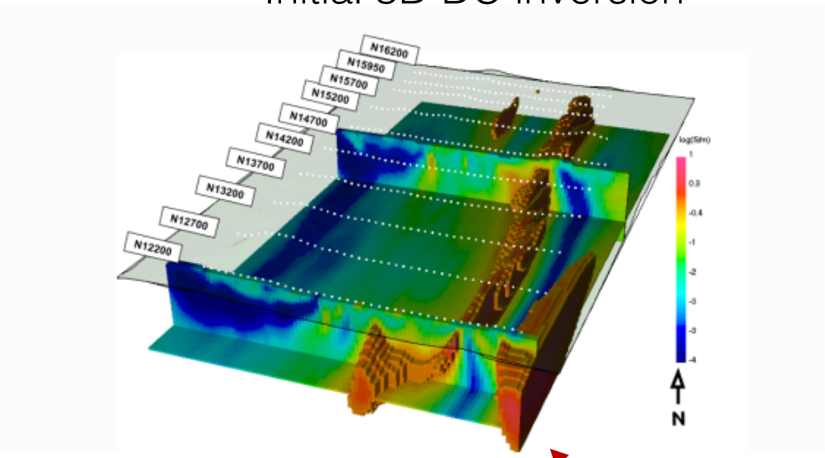
Assign high $w_s(x, y, z)$

Hypothesis testing

- Generate “best” model
- Analyze for features of interest
- Test existence: Generate a counter example.
- Use a weighted reference model that doesn’t have feature.

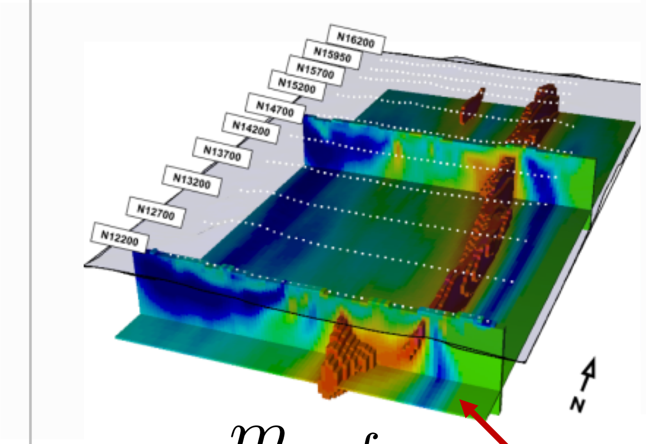
$$\phi_s = \int_V w_s(m - m_{ref})^2 dv$$

Initial 3D DC inversion



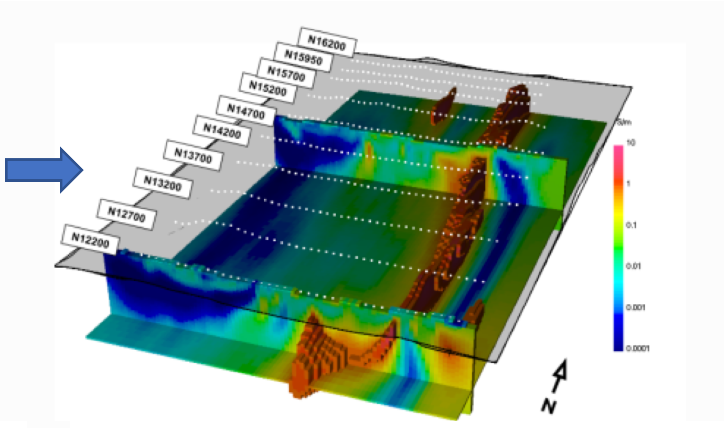
Suspicious feature

Reference model
Without south-east conductor



m_{ref}

Final resistivity model



Assign high $w_s(x, y, z)$

DOI (Depth of Investigation)

Model objective function for 1D AEM inversion:

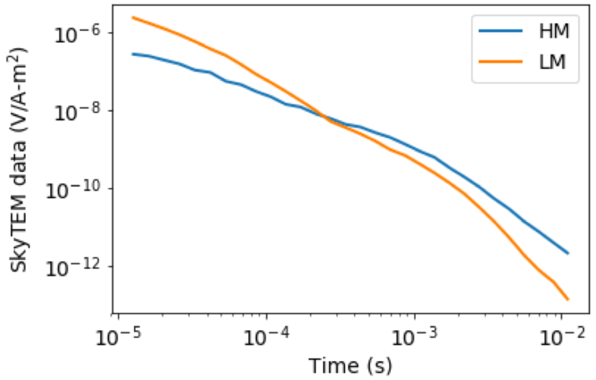
$$\phi_m = \phi_s + \phi_z$$

Reference model in smallness term:

$$\phi_s = \int_V (m - m_{ref})^2 dv$$

- Carry out inversion with two different reference models

AEM data



DOI (Depth of Investigation)

Model objective function for 1D AEM inversion:

$$\phi_m = \phi_s + \phi_z$$

Reference model in smallness term:

$$\phi_s = \int_V (m - m_{ref})^2 dv$$

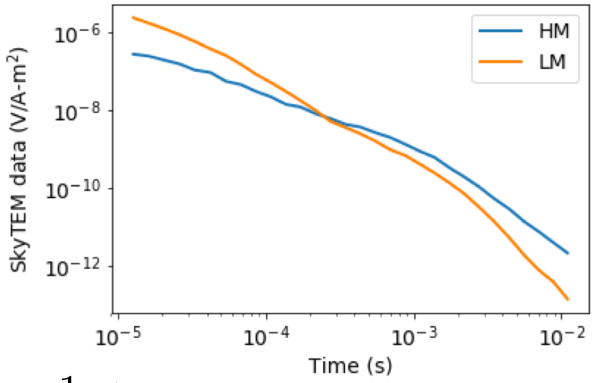
- Carry out inversion with two different reference models

- Compute DOI:

$$\text{DOI index} = \left| \frac{m^1 - m^2}{m^1_{ref} - m^2_{ref}} \right|$$

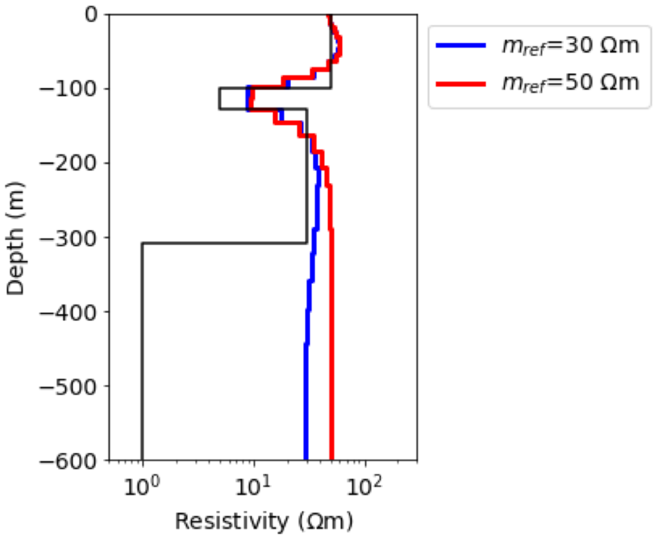
$$\begin{cases} m^1\text{: inversion model with } m^1_{ref} \\ m^2\text{: inversion model with } m^2_{ref} \end{cases}$$

AEM data



$\mathcal{F}^{-1}$

Recovered models



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Model objective function for 1D AEM inversion:

$$\phi_m = \phi_s + \phi_z$$

Reference model in smallness term:

$$\phi_s = \int_V (m - m_{ref})^2 dv$$

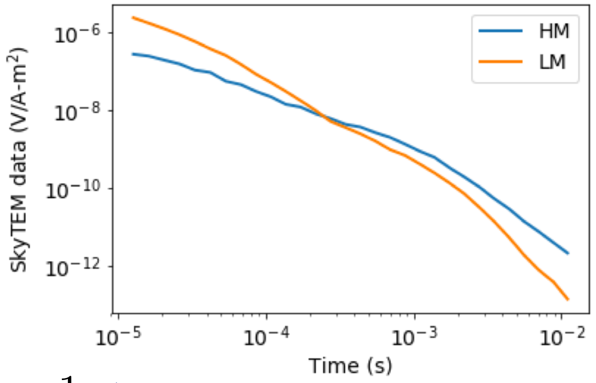
- Carry out inversion with two different reference models

- Compute DOI:

$$\text{DOI index} = \left| \frac{m^1 - m^2}{m^1_{ref} - m^2_{ref}} \right|$$

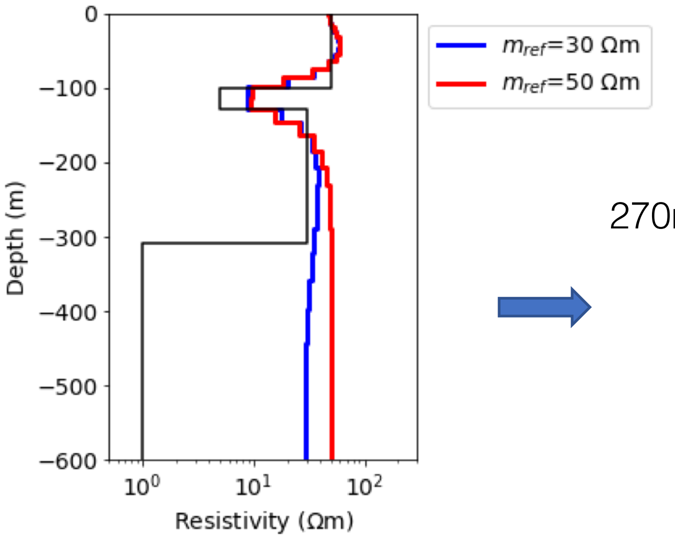
$$\begin{cases} m^1\text{: inversion model with } m^1_{ref} \\ m^2\text{: inversion model with } m^2_{ref} \end{cases}$$

AEM data

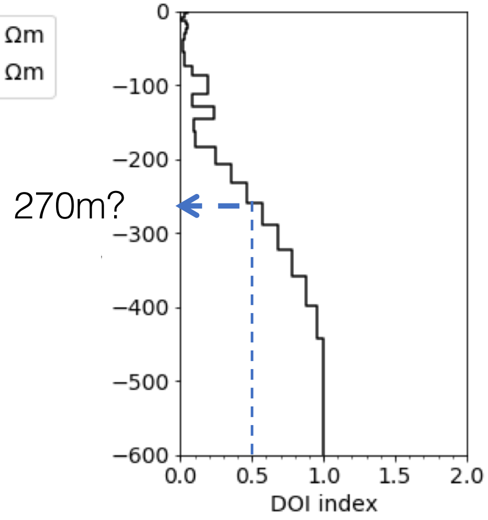


$\mathcal{F}^{-1}$

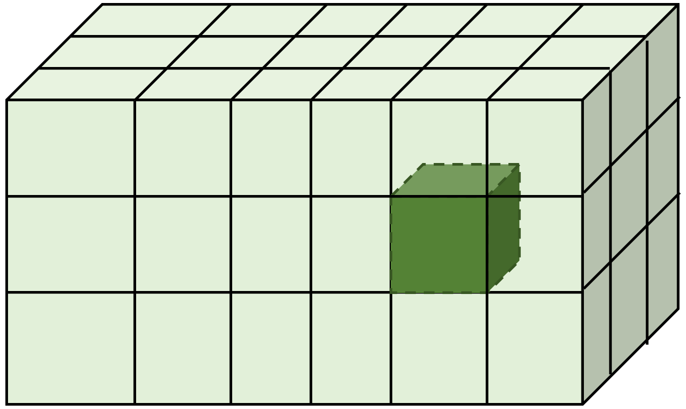
Recovered models



DOI index



Parameterizations



Use voxel model: M ~10^5-10^7 cells

May always need this for solving forward problem

Inverse Problem: Options to parameterize

(1) Geologic Knowledge: Isolated body

- $\vec{m}$ for inversion
  - (x, y, z) location of body
  - parameters of body
  - orientation
  - Physical property

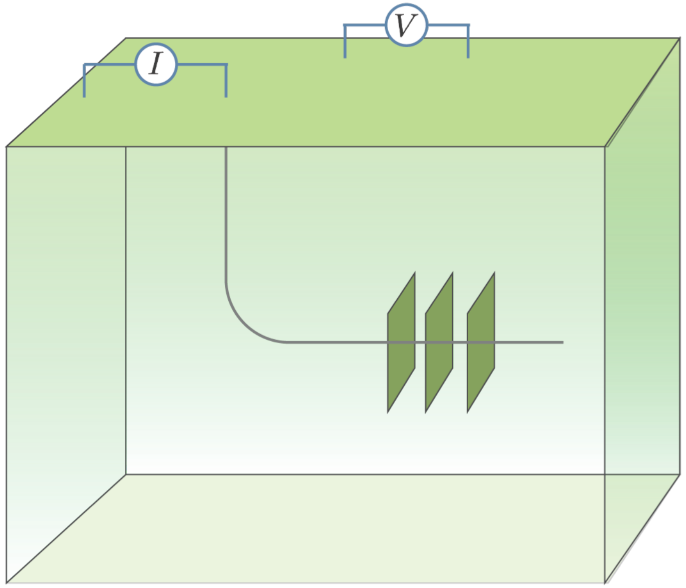
  e.g. sphere

$$\vec{m} = (x, y, z, r, \sigma_{sphere}, \sigma_b)$$

Parameterizations

(2) Resolution in certain locations doesn't justify fine scale inversion

Fracturing problem



Field objective: where are the fractures?

- Model the fracture plane as a plate

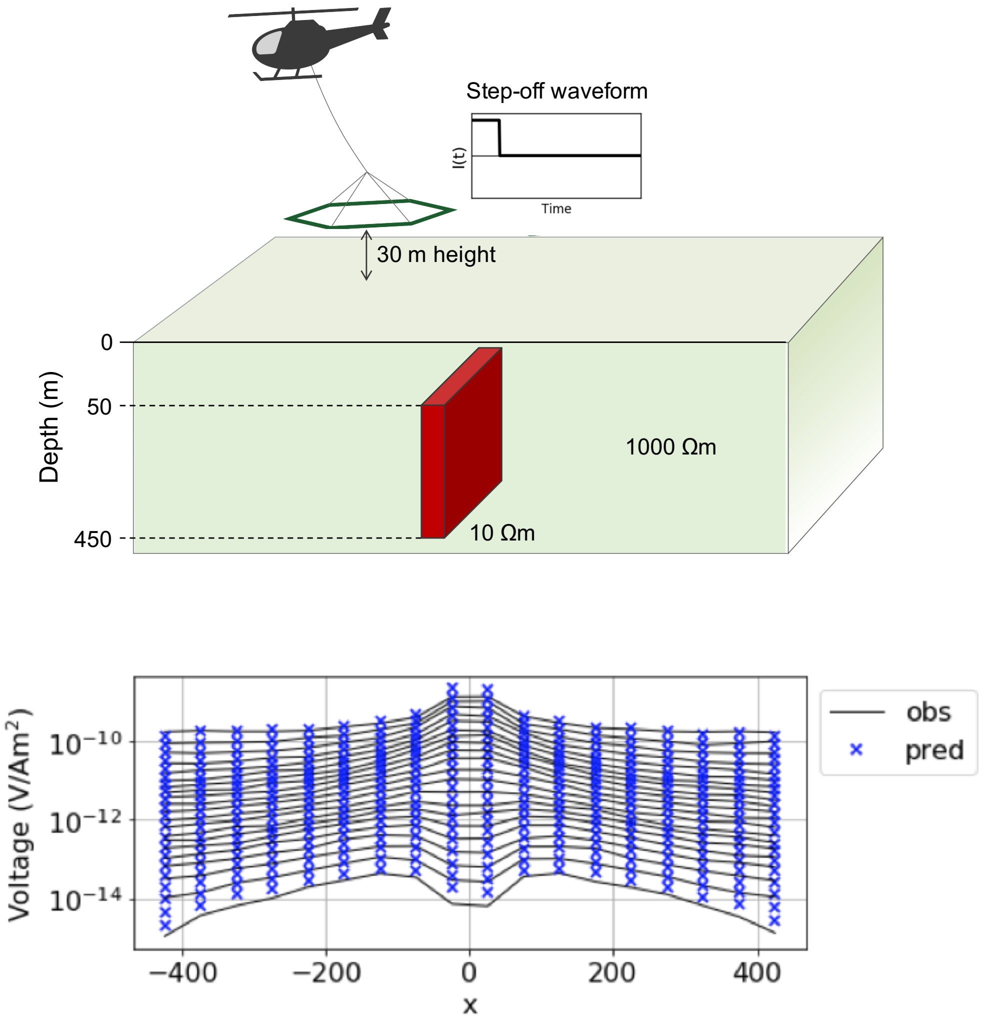
Note: inverse problem has completely changed

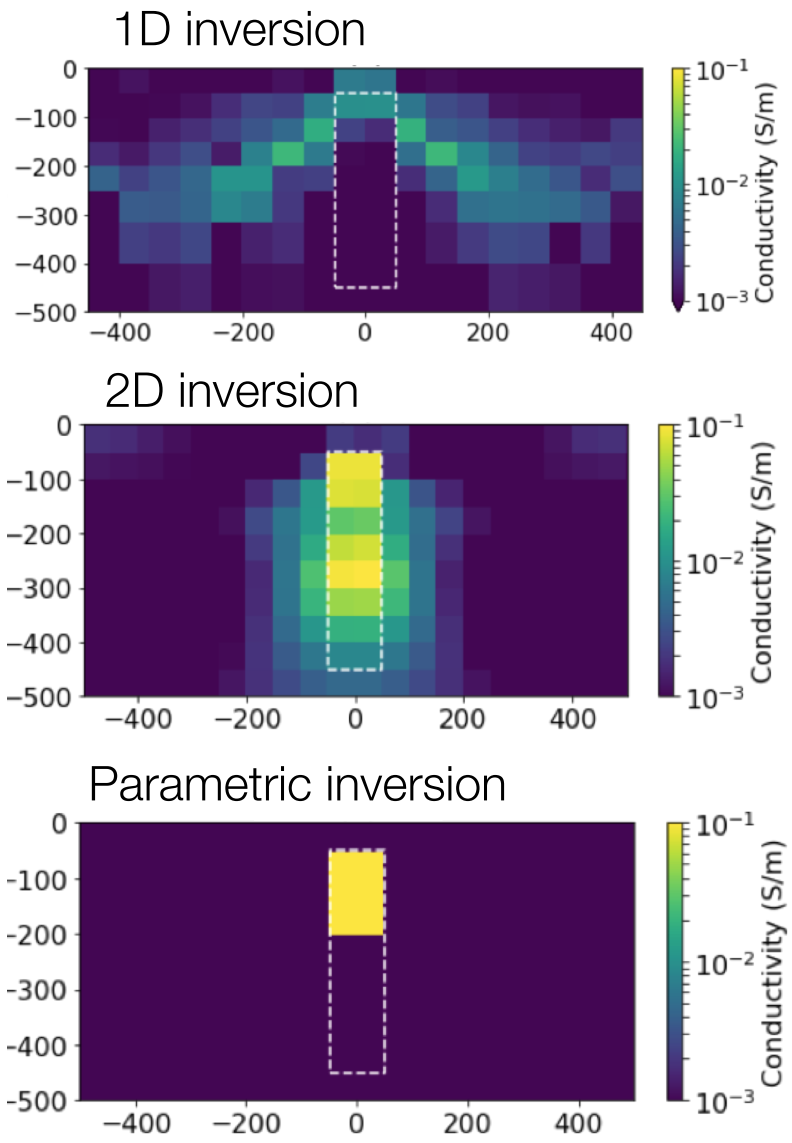
- Only search for a few parameters
- Objective function

$$\phi(\mathbf{m}) = \phi_d(\mathbf{m}) + \beta\phi_m(\mathbf{m})$$
$$= \|\mathbf{W}_d(\mathcal{F}[\mathbf{m}] - \mathbf{d})\|^2$$

Same gradient methods can still be used, but final solution can be more sensitive to starting values

EM inversions

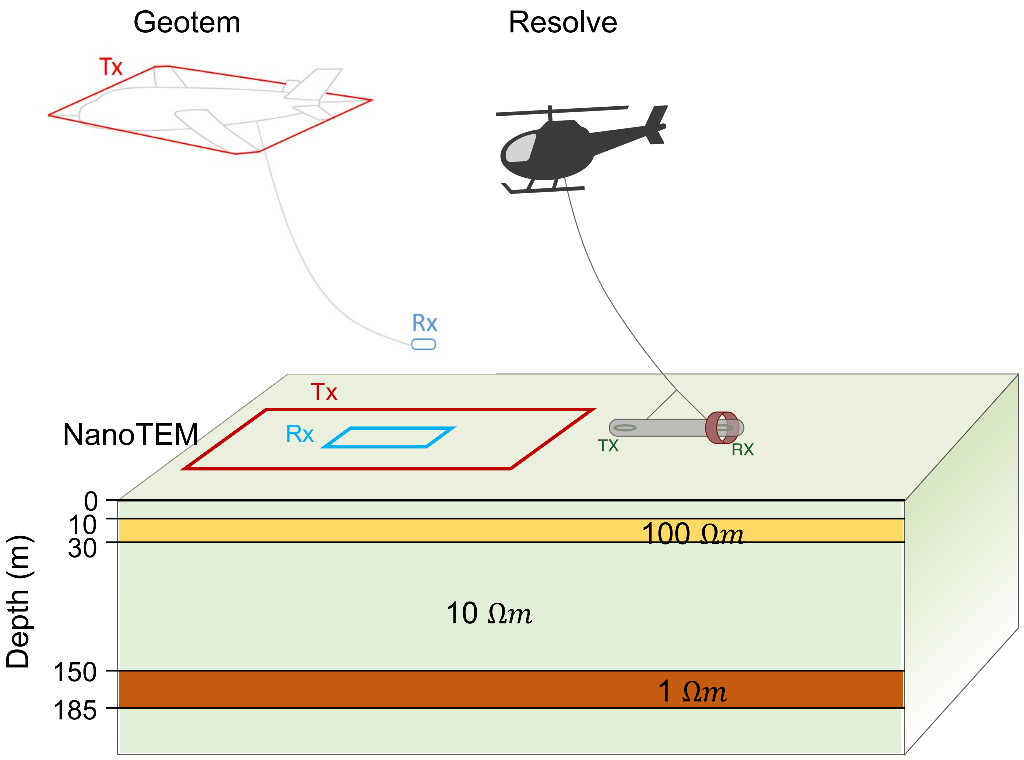




Multiple EM surveys

- Resolve (frequency domain)
- Geotem (airborne TDEM)
- NanoTEM (ground TDEM)

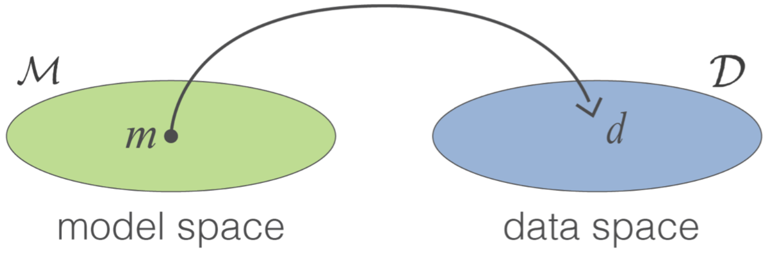
We can invert each survey in 3D
Joint inversion is a challenge



Setting up the inversion

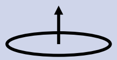
Modular: Each datum has own forward mapping

$$d_i = F_i[\mathbf{m}]$$



- For jointly inverting multiple EM data sets:

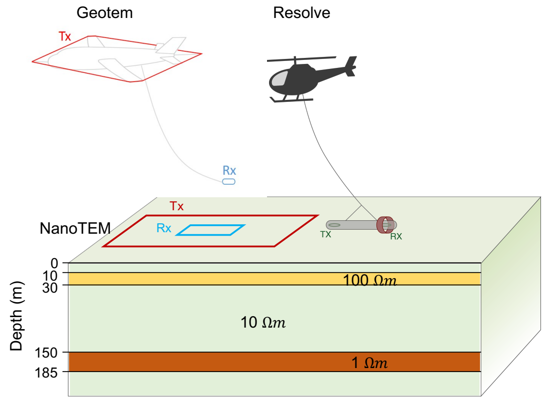
Data misfit: $\phi_d = \Sigma\left(d_i^{obs} - F_i[\mathbf{m}]\right)^2$

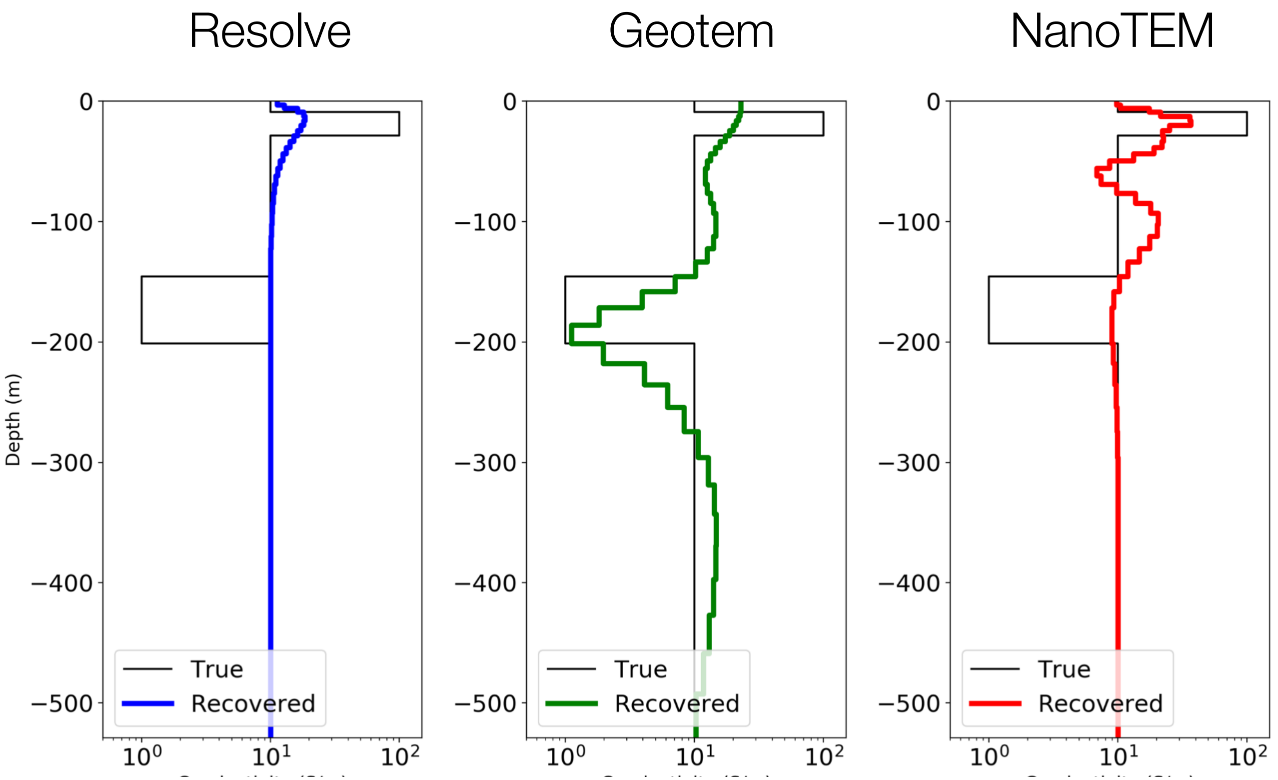
System	Maxwell's Eqs.	Source	Time or frequency range	Depth of investigation
Resolve	Frequency		400Hz-130kHz	~70m
Geotem	Time		100μs-10ms	~300m
NanoTEM	Time		1μs-1ms	~100m

+ Mesh options: 1D, 2D, 3D, parametric

Separate inversions

$$\phi_{\rm m}(\mathbf{m}) = \frac{1}{2}\|\mathbf{W}_m(\mathbf{m}-\mathbf{m}_{\rm ref})\|_2$$

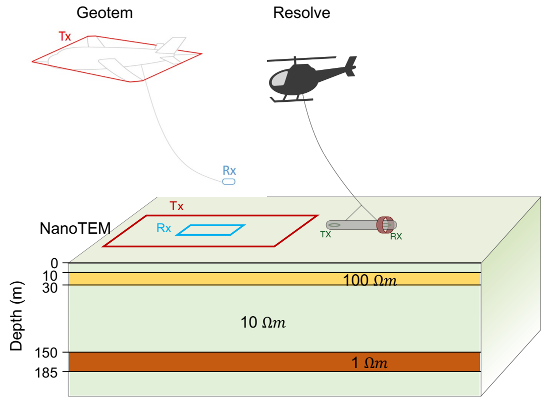


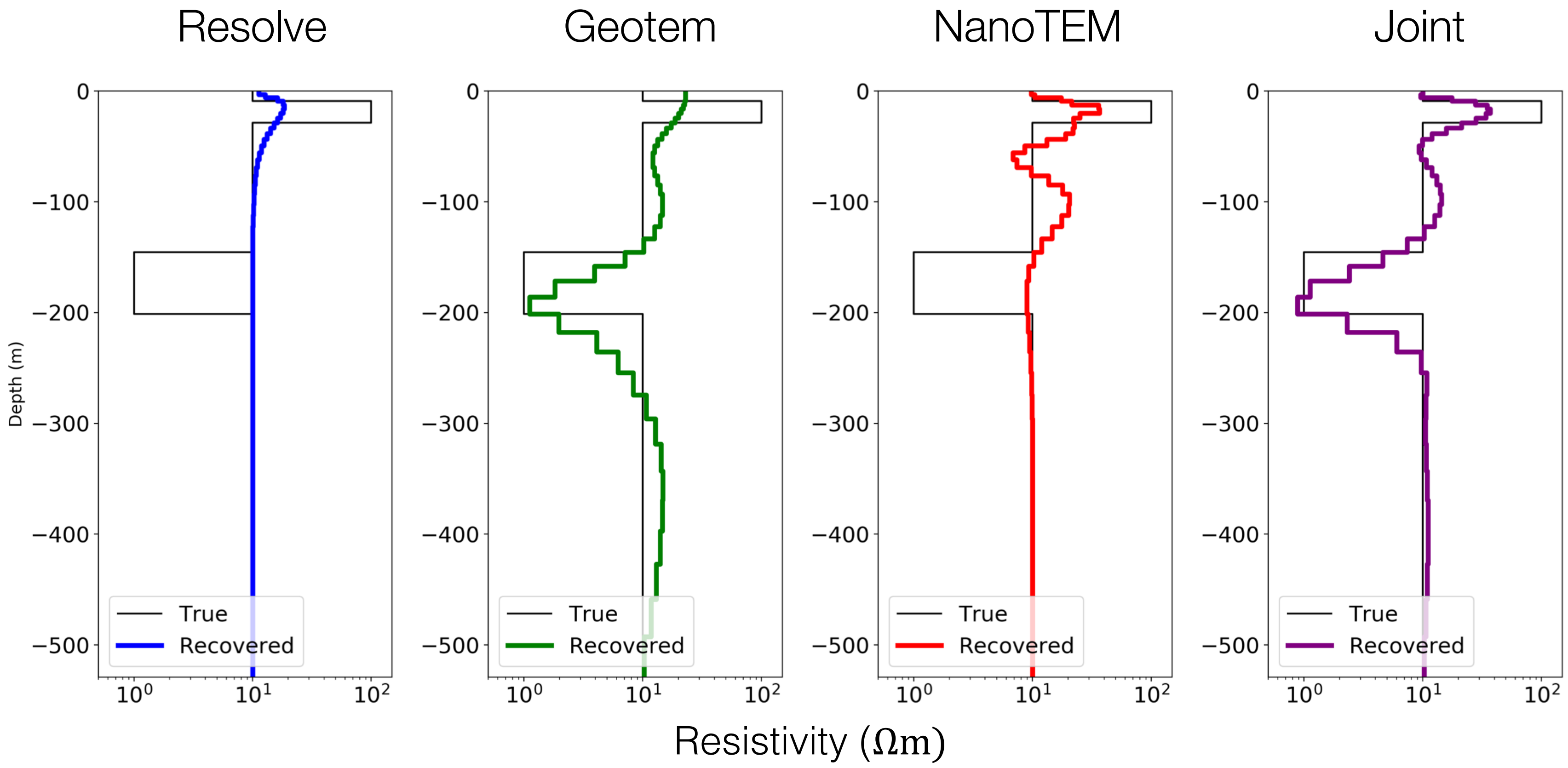


Resistivity (Ωm)

Joint inversion

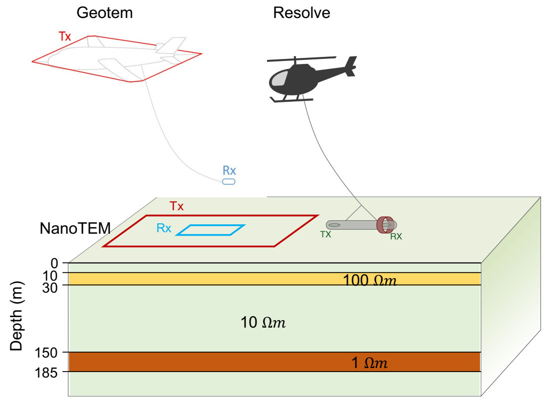
$$\phi_{\mathrm{m}}(\mathbf{m}) = \frac{1}{2}\|\mathbf{W}_m(\mathbf{m} - \mathbf{m}_{\mathrm{ref}})\|_2$$

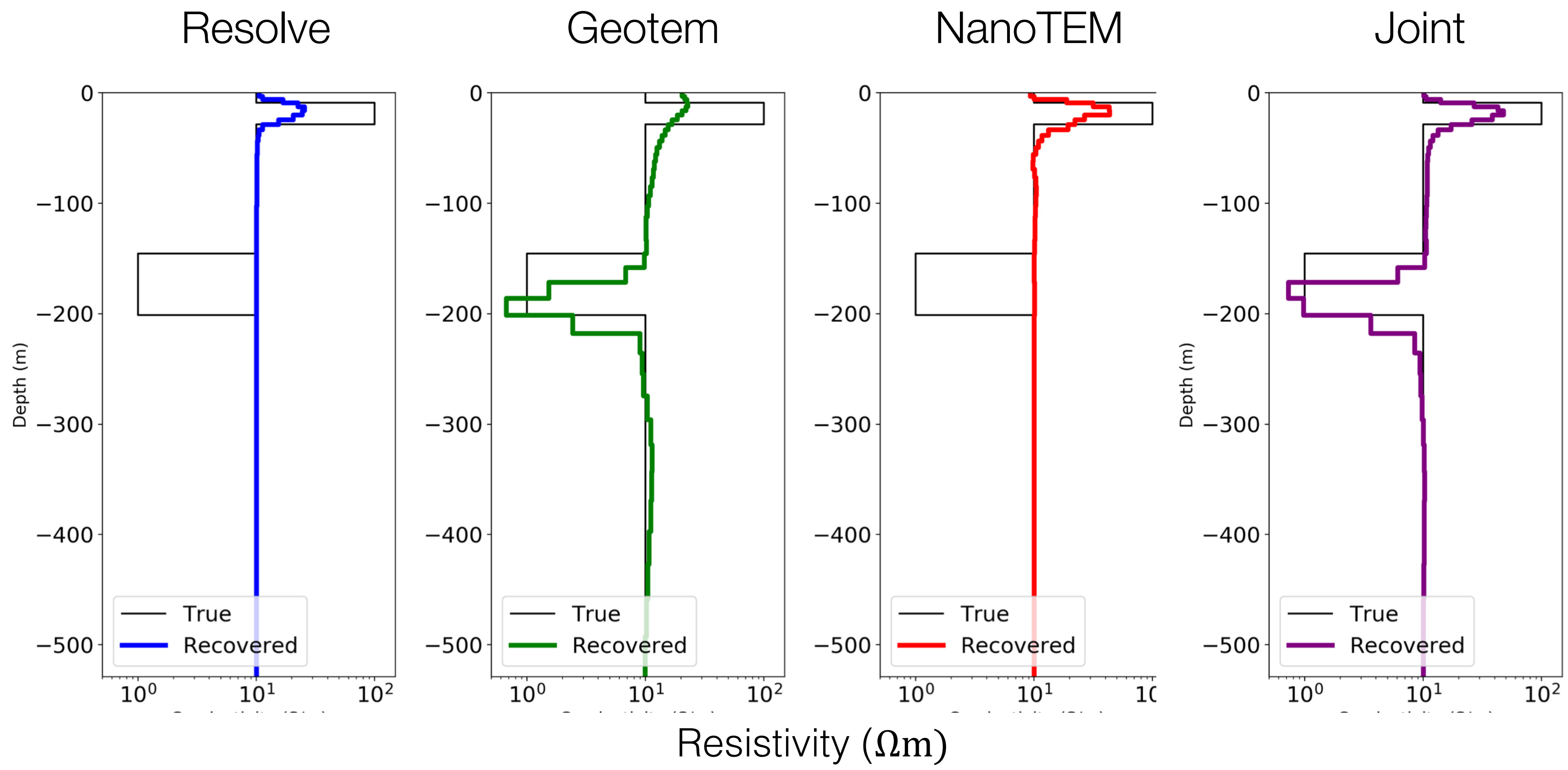




L_2 vs L_0 norms

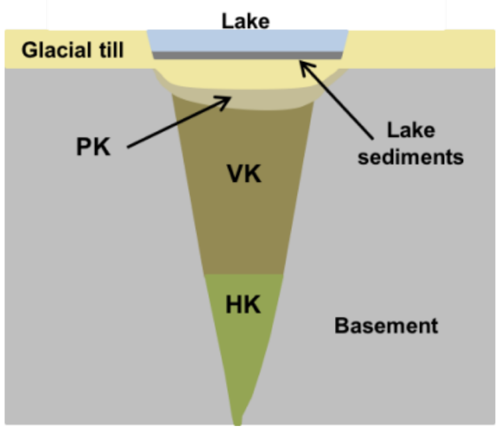
$$\phi_{\mathrm{m}}(\mathbf{m}) = \frac{1}{2}\|\mathbf{W}_m(\mathbf{m}-\mathbf{m}_{\mathrm{ref}})\|_0$$





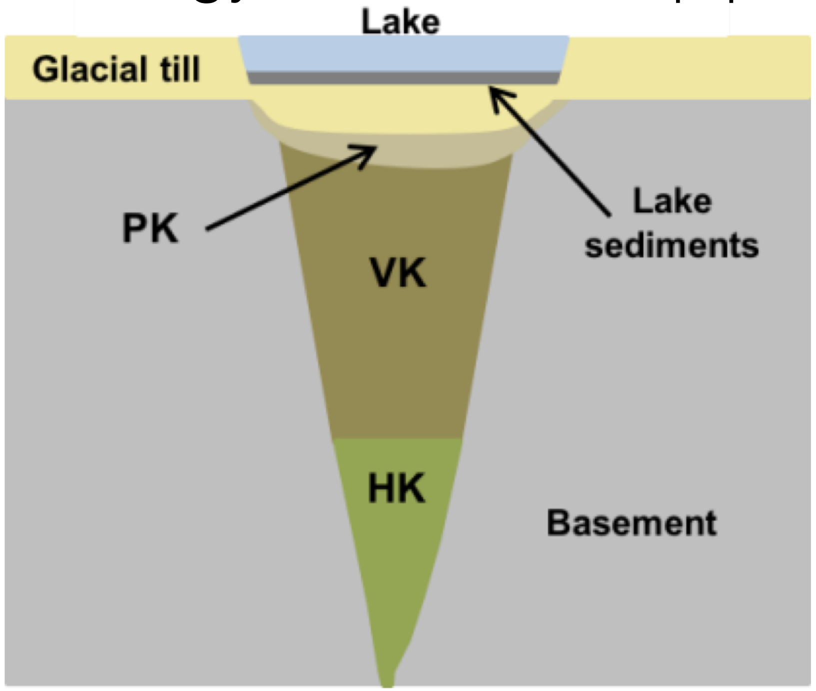
Joint Inversion: Multiple physical properties

- What do we want to extract from the data?
- Physical property values are often not of intrinsic interest.
- Multiple Properties: How are they connected?
  - Analytic relationship (e.g. seismic velocity and density)
  - Structurally: Properties change at the same location
  - Connection through physical property values
    - Geologic insight from posterior inference: TKC kimberlite



Physical Properties

Geology of a diamond pipe



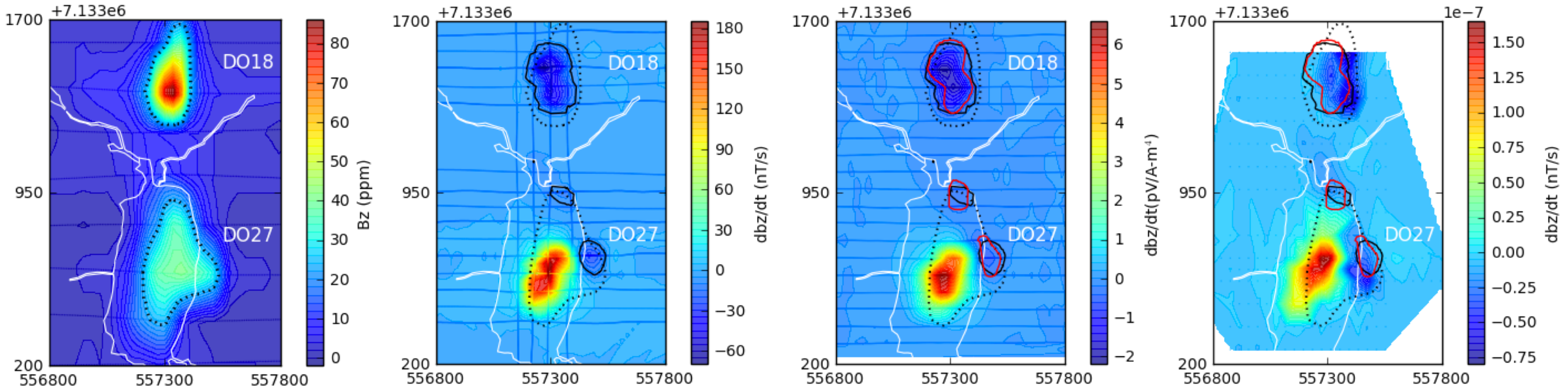
Physical property table

Rock type	Glacial till	Host rock	HK	VK	PK
Density	Moderate	Moderate	Low	Low	Low
Susceptibility	None	None	High	Low-moderate	Low-moderate
Conductivity	Moderate-high	Low	Low-moderate	Moderate-high	Moderate-high
Chargeability	Low	Low	?	?	?

- Kimberlite rocks: low density
- HK: high susceptibility
- VK and PK:
    - Low-moderate susceptibility
    - Moderate-high conductivity

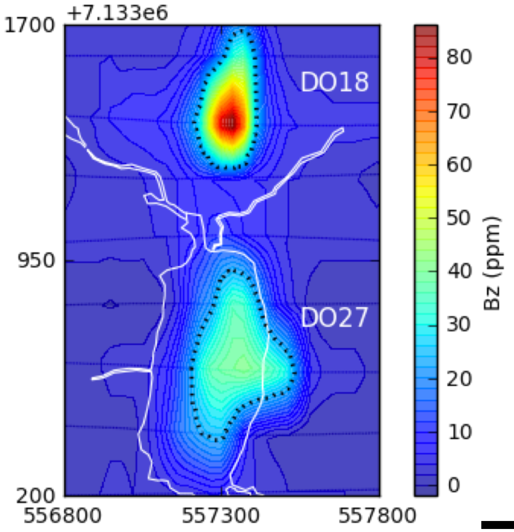
EM data for TKC

Dighem (1992) | AeroTEMII (2003) | VTEM (2004) | NanoTEM (1993)

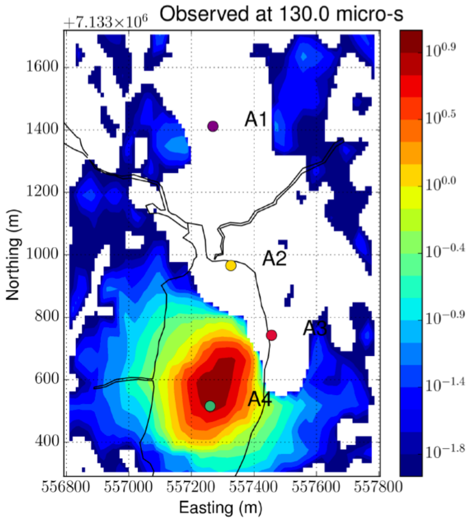


Joint conductivity inversion

DIGHEM

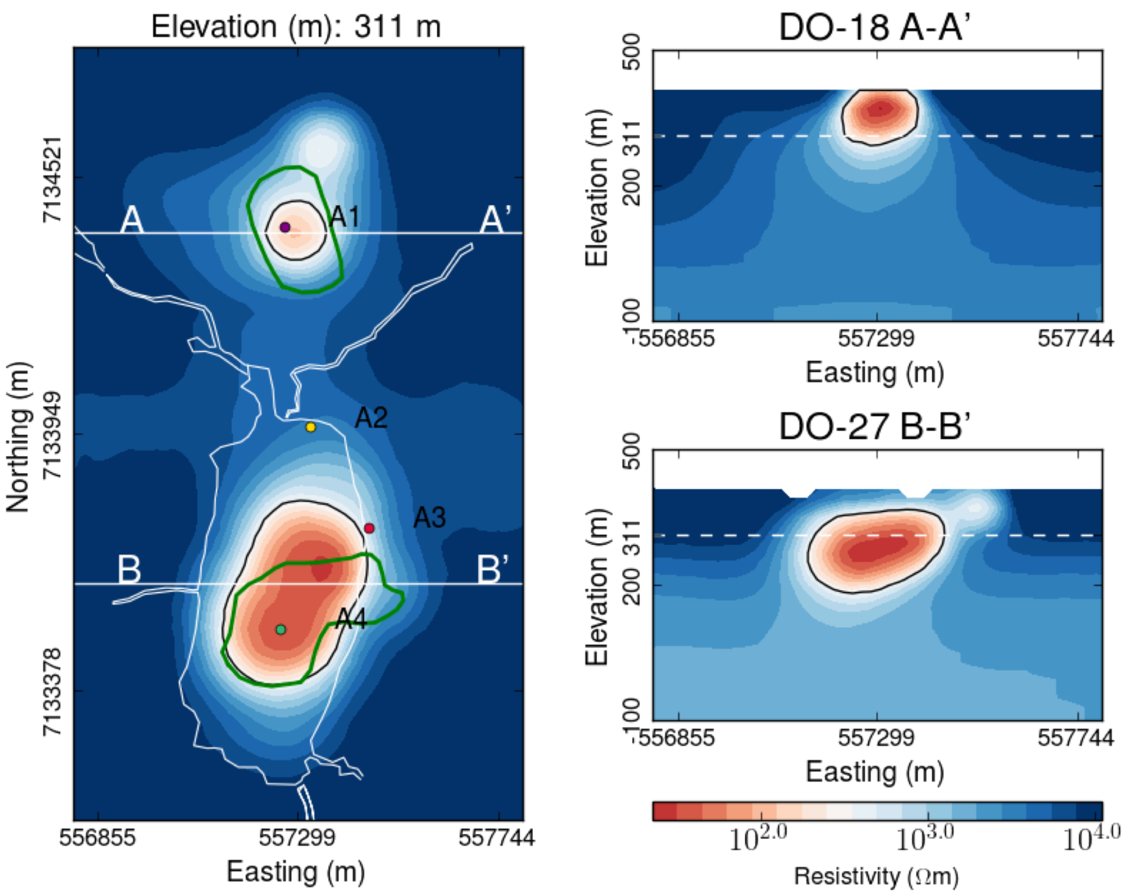


Positive VTEM (EM-dominant)



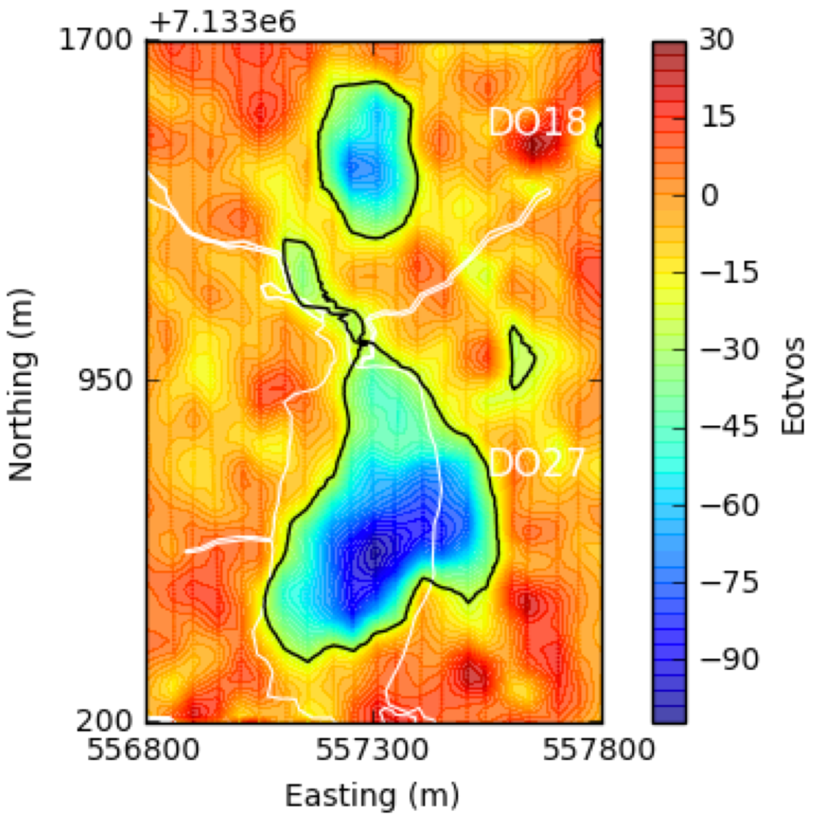
$\mathcal{F}^{-1}$

Recovered 3D model

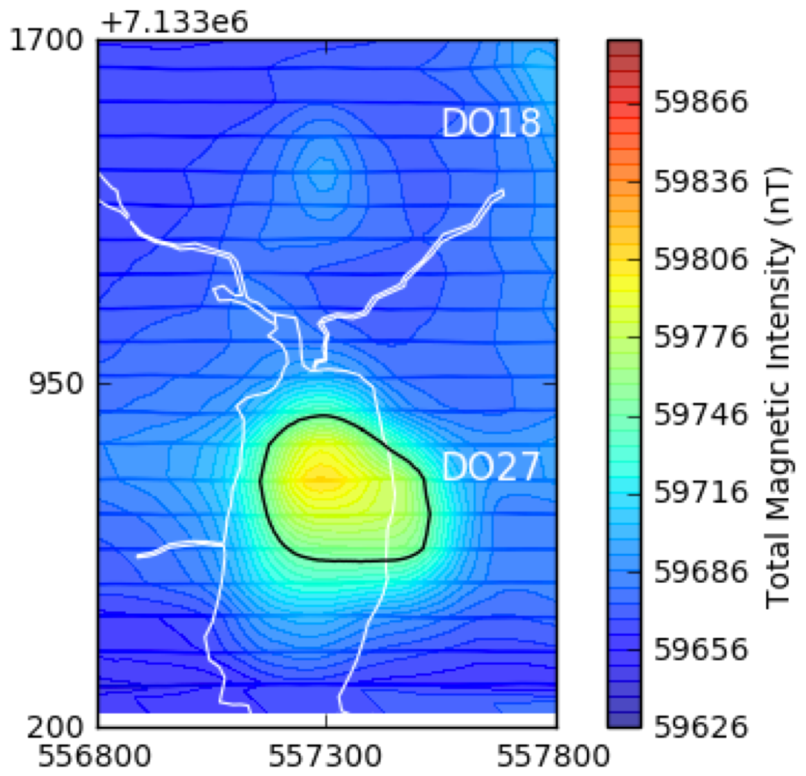


Potential field data at TKC

Gravity gradiometry data

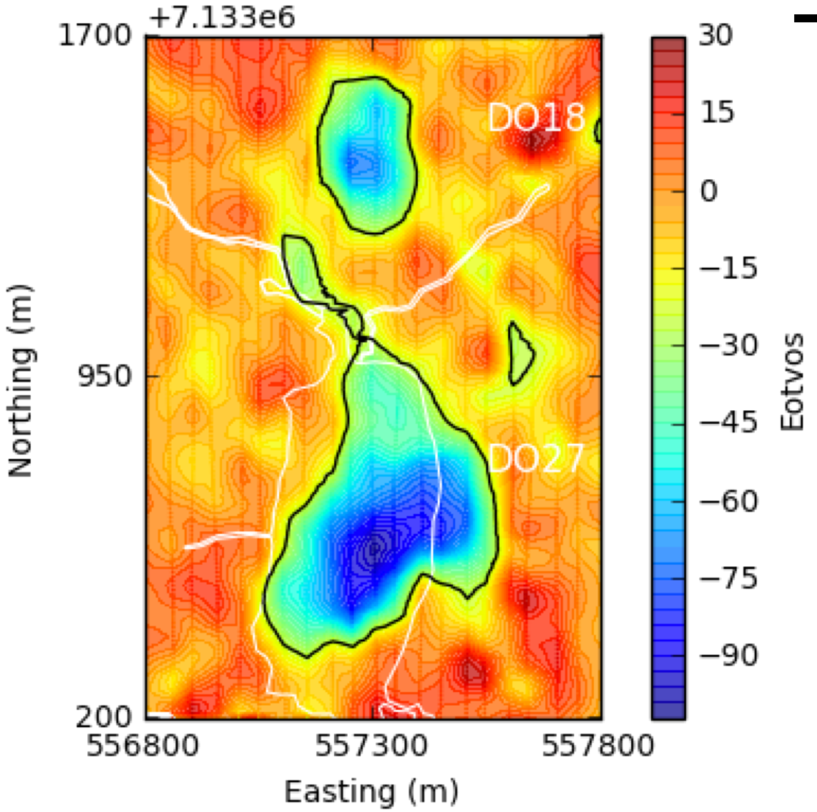


VTEM mag data



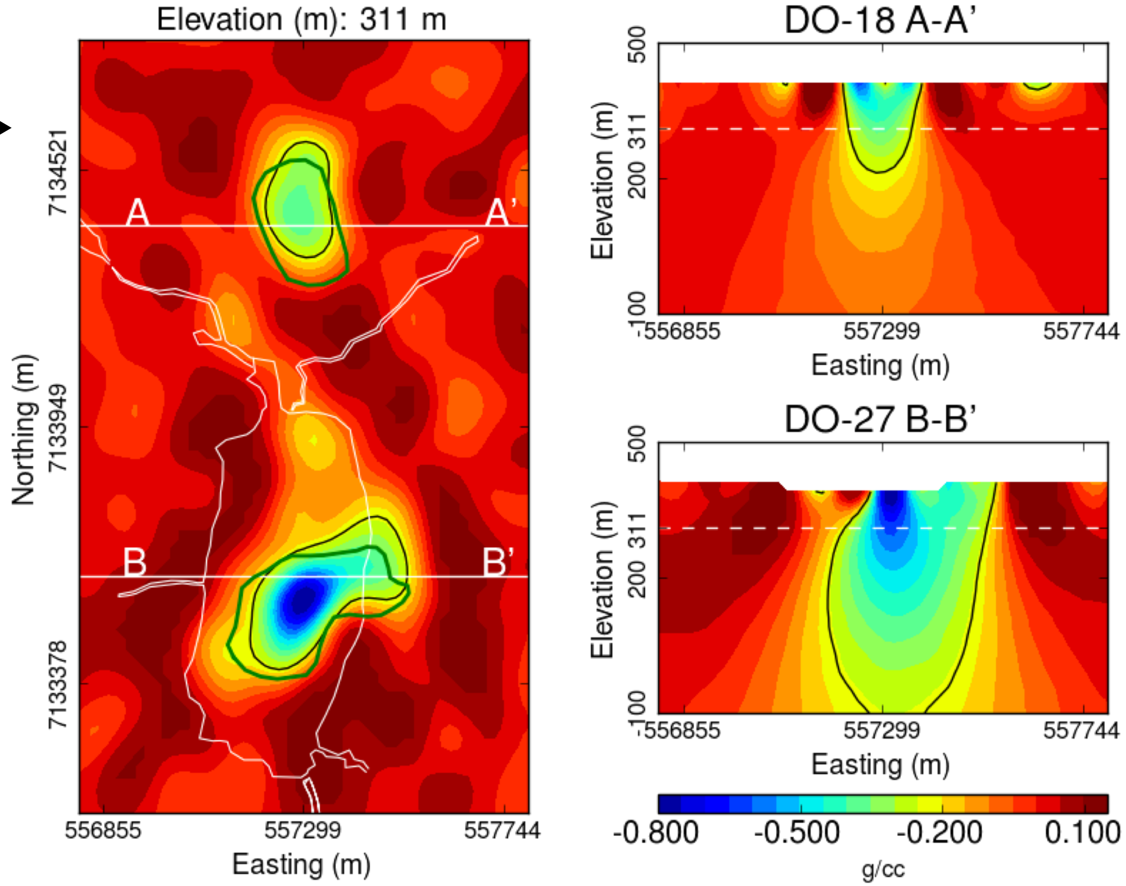
Density: ρ

Gravity gradiometry data



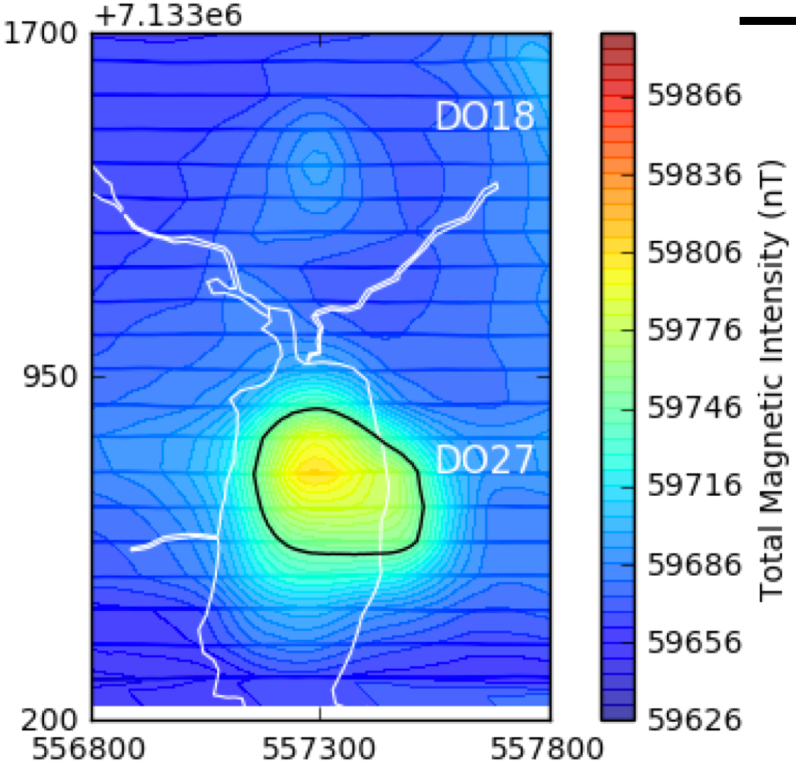
$\mathcal{F}^{-1}$

Recovered 3D model



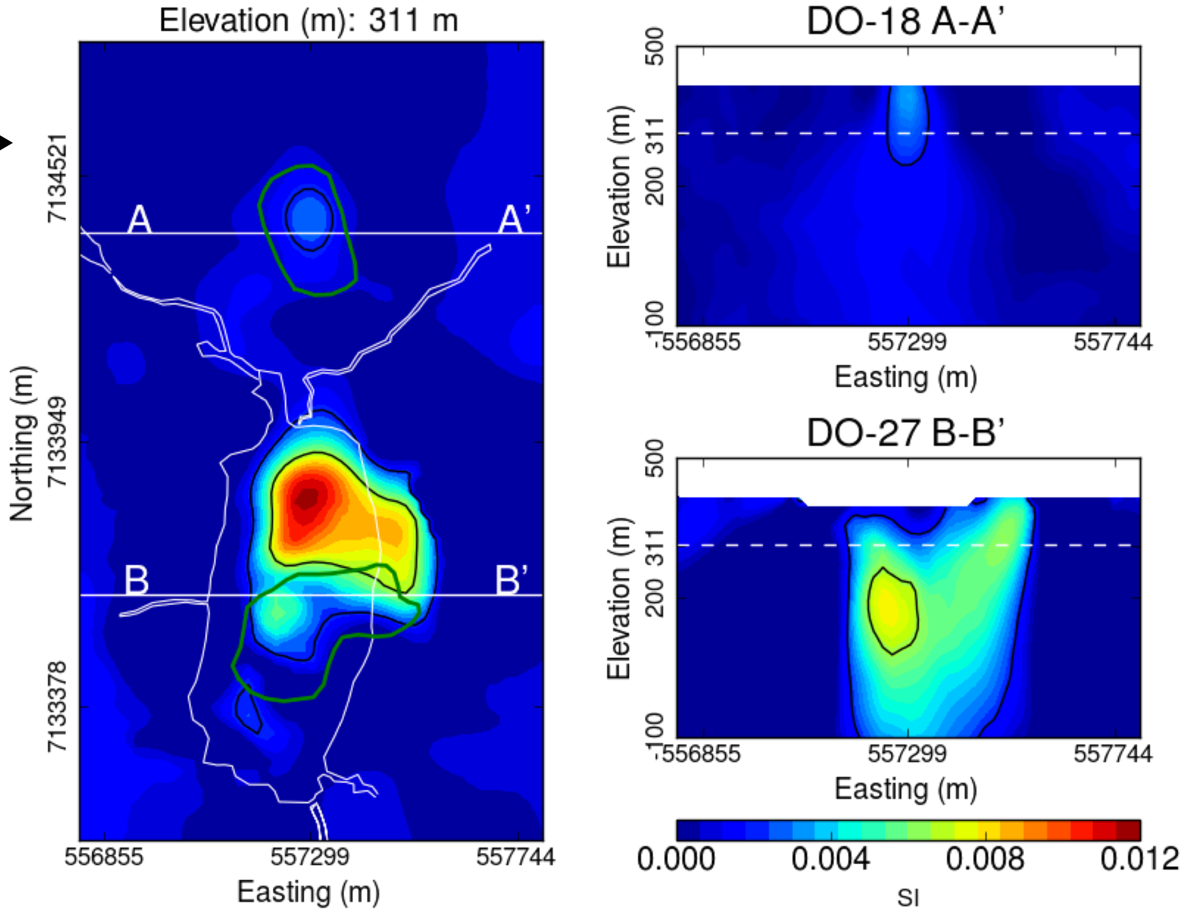
Susceptibility: κ

VTEM Mag data



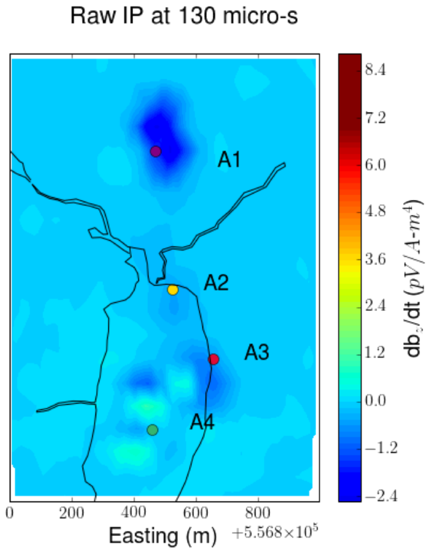
$\mathcal{F}^{-1}$

Recovered 3D model



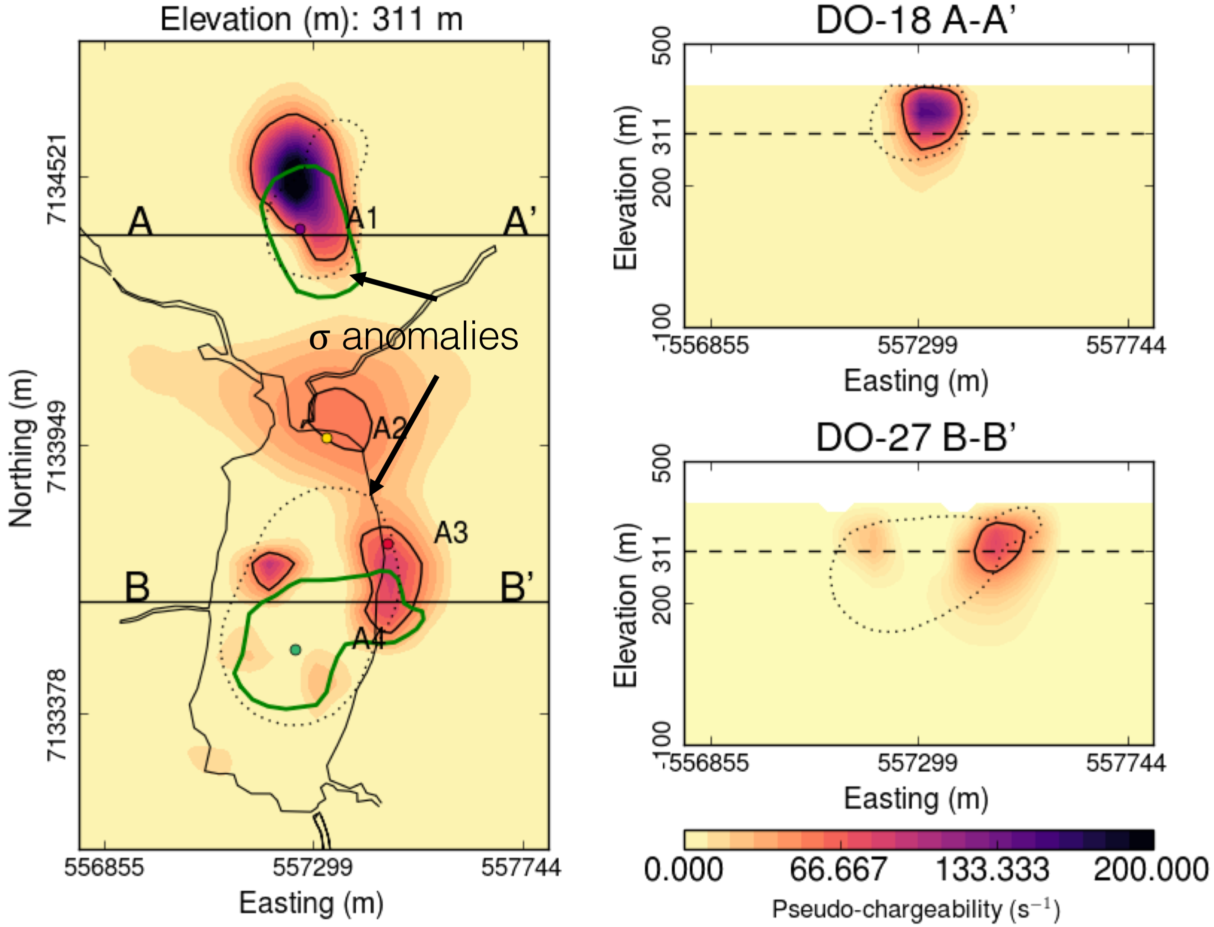
Pseudo-chargeability (Early): $\tilde{\eta}_E$

IP data



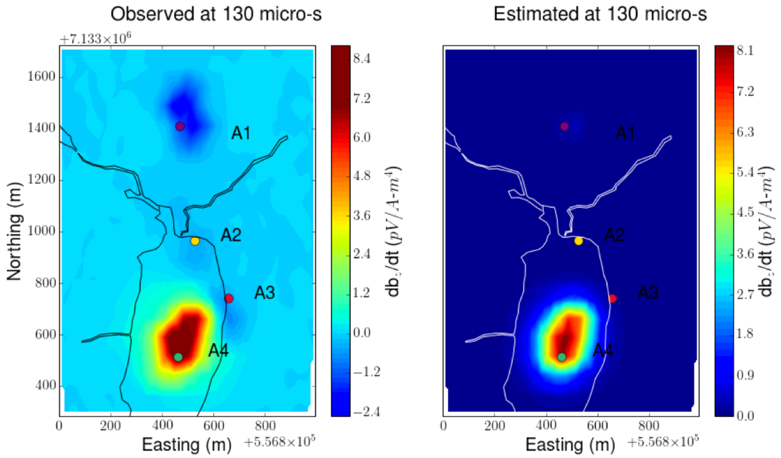
$\mathcal{F}^{-1}$

Recovered 3D model



Observation

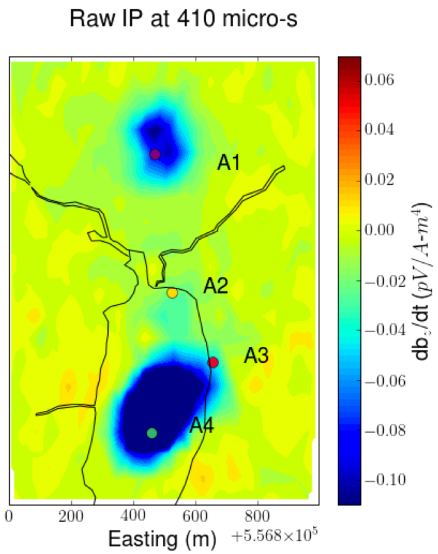
Fundamental



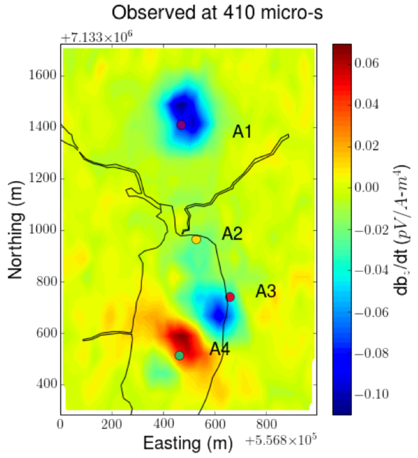
IP = Observation - Fundamental

Pseudo-chargeability (Late): $\tilde{\eta}_L$

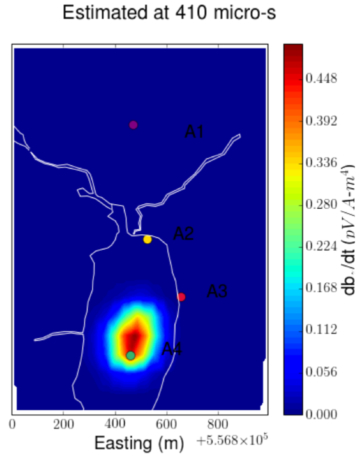
IP data



Observation



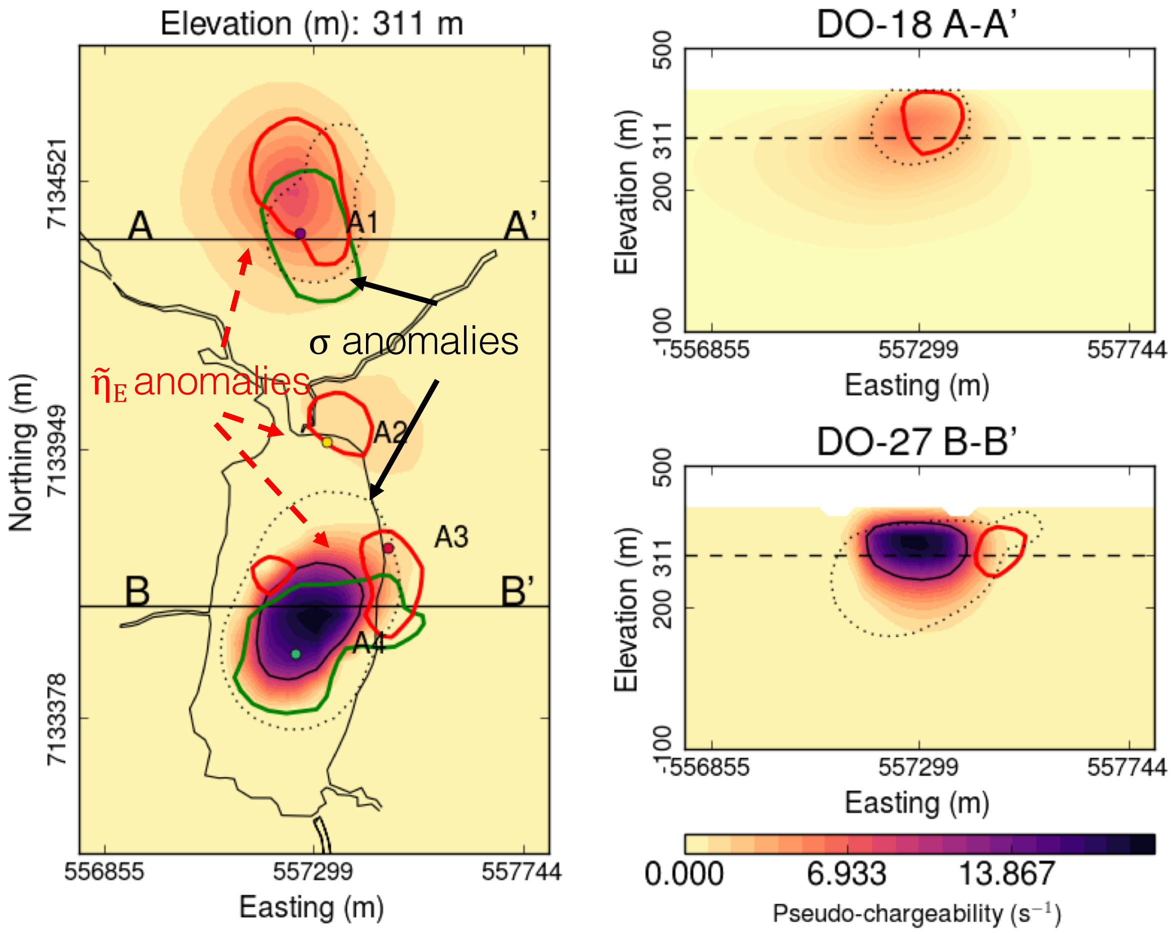
Fundamental



IP = Observation - Fundamental

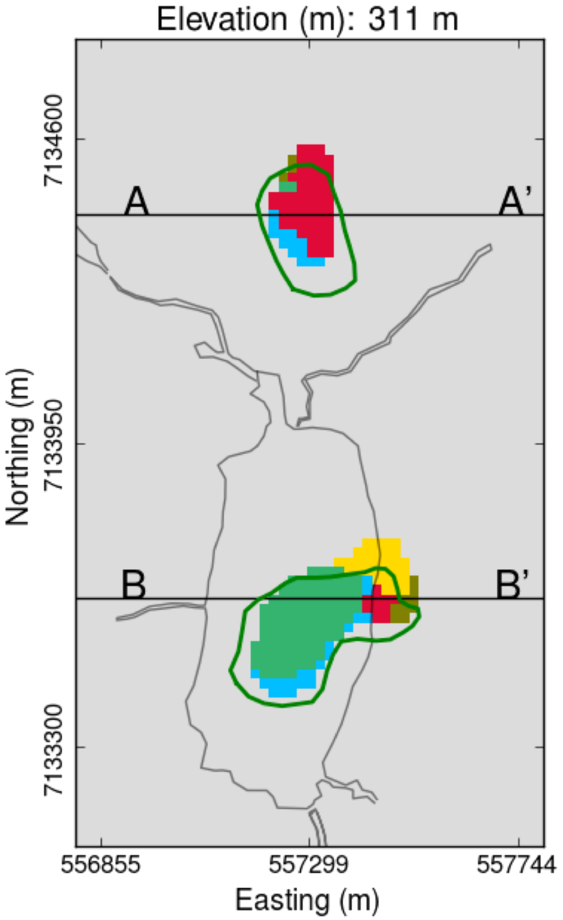
$\mathcal{F}^{-1}$

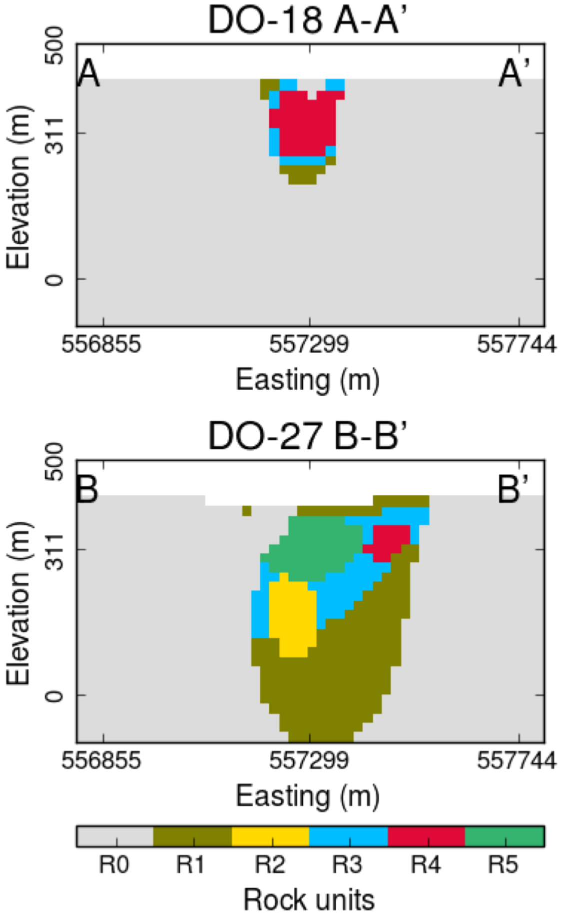
Recovered 3D model



Interpretation

Petrophysical model





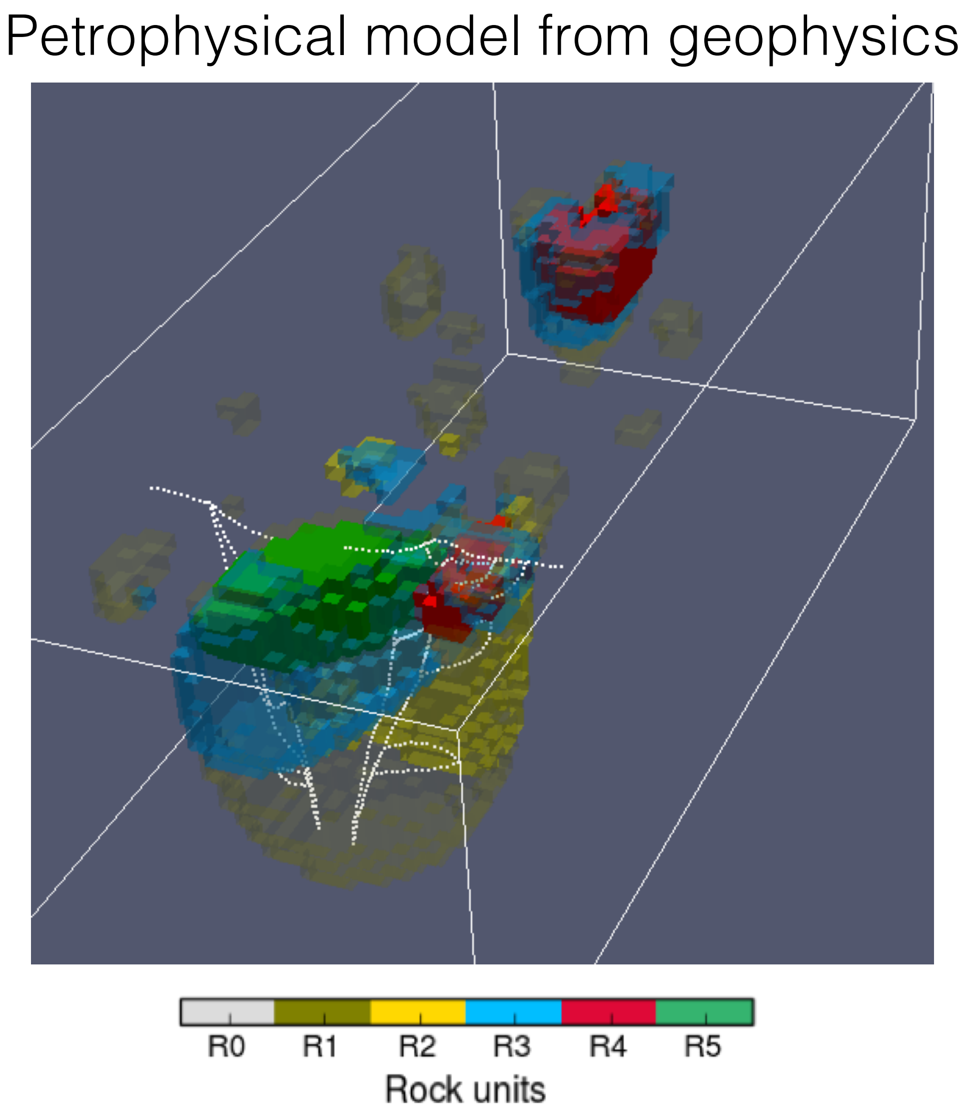
Distinction of PK and VK

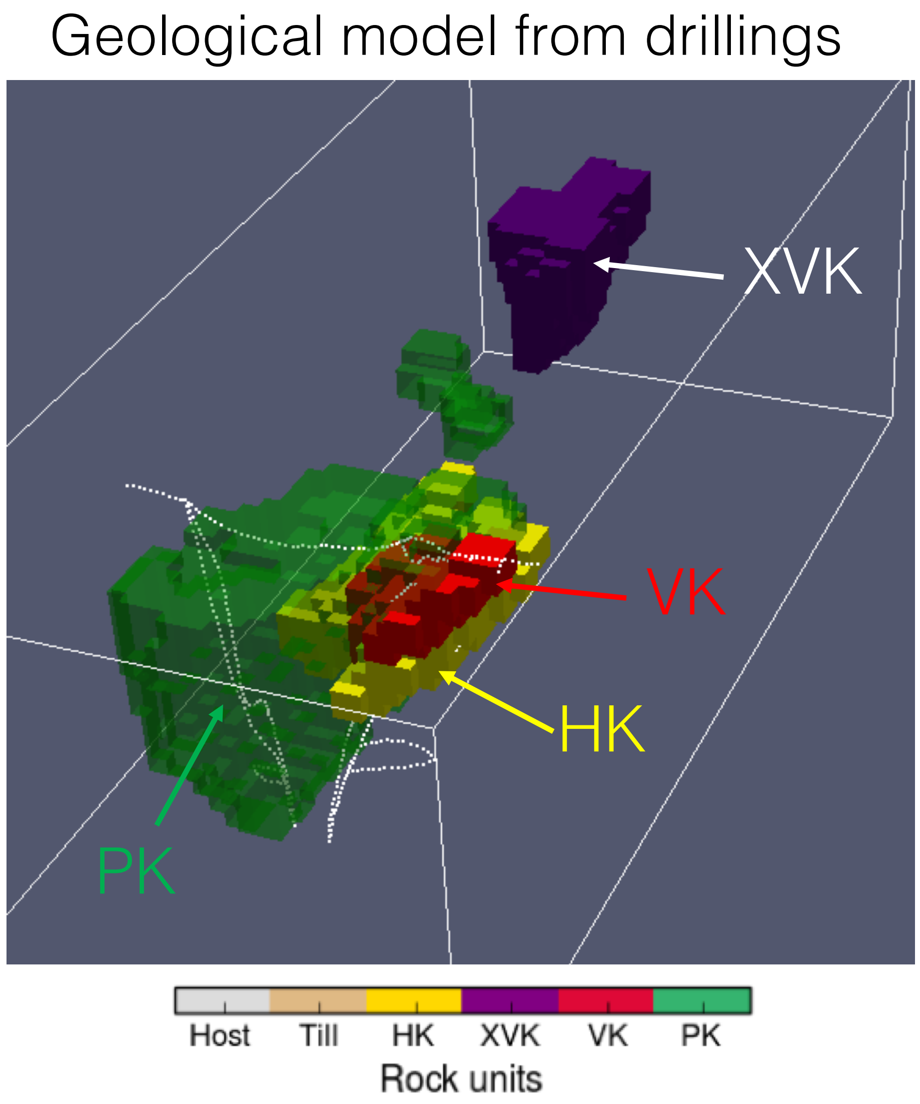
- PK is deposited after an explosive event
- PK has greater pore size than VK
- Result in greater time constant: τ
- R4 (small τ) – “VK”, R5 (greater τ) – “PK”

Interpreted rock table (R0-R5)

Rock Unit	ρ	κ	σ	$\tilde{\eta}_E$	$\tilde{\eta}_L$	τ	Interpretation
R0	Mod.	Low	Low	Low	Low	N/A	Host Rock
R1	Low	Low	Low	Low	Low	N/A	Kimberlite
R2	Low	High	Low	Low	Low	N/A	HK
R3	Low	Mod.	Mod.	Low	Low	N/A	PK or VK
R4	Low	Mod.	Mod.	High	Low	Small	VK
R5	Low	Mod.	Mod.	Low	High	Large	PK

3D cut-off volume





(Towards) Geologic inversion

Petrophysics

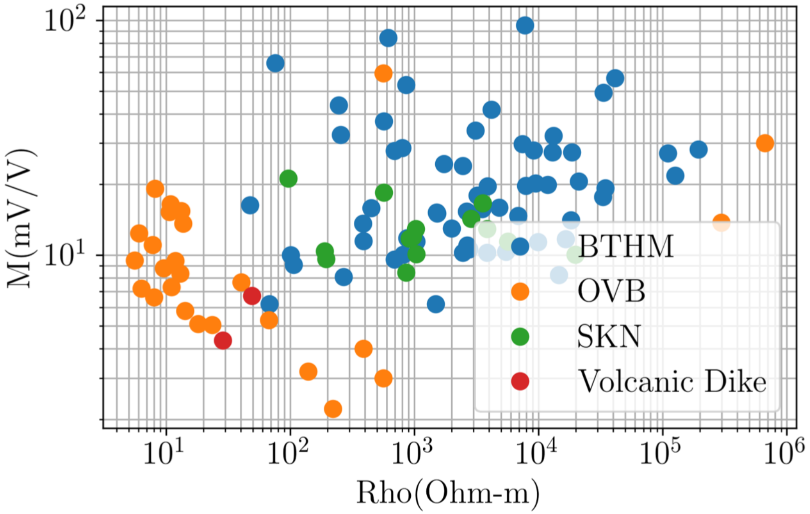
Petrophysics

On-site lab

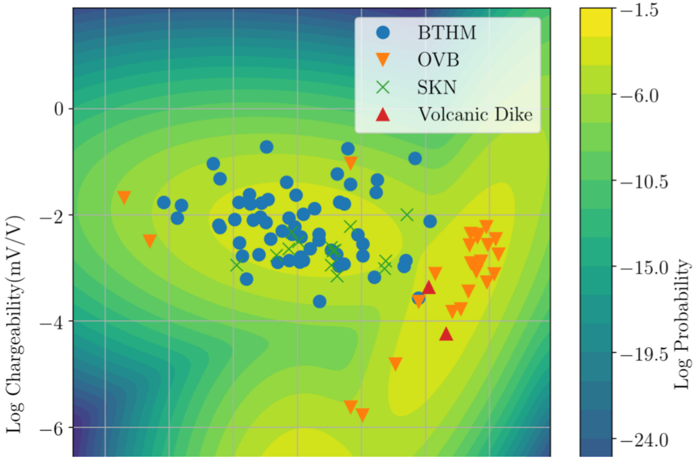
Drill cores

Physical properties measurements

Measured resistivity and chargeability

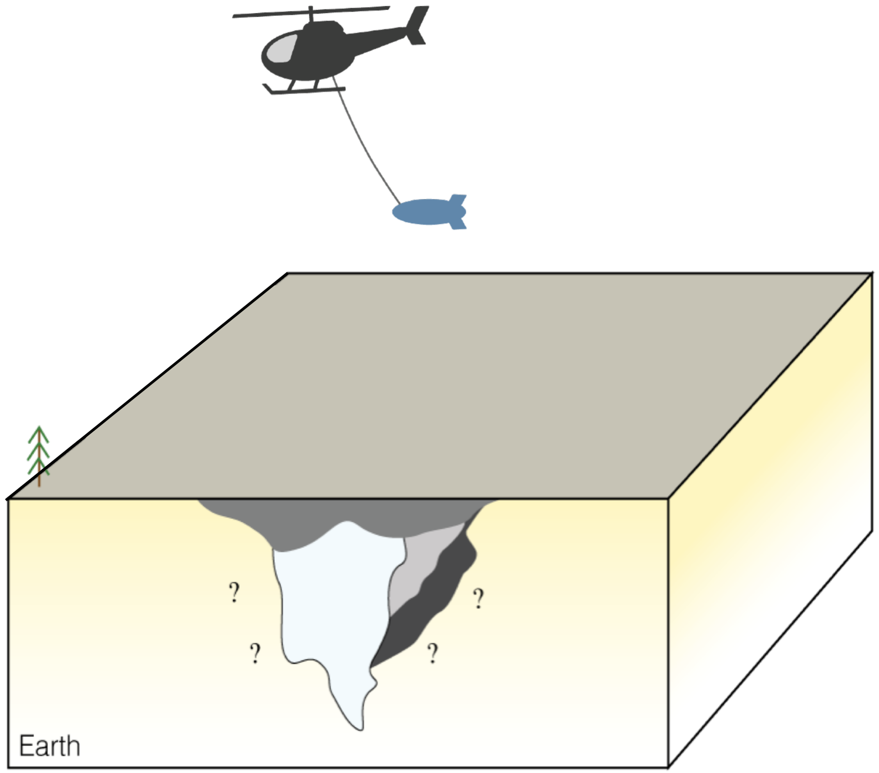


Trends

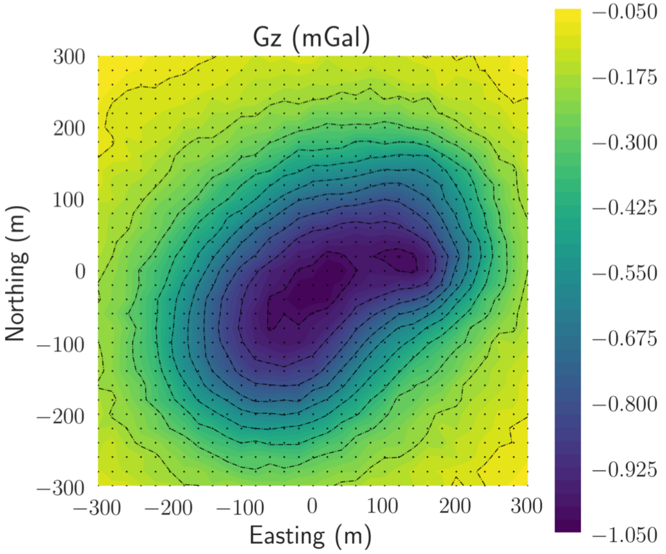


How do we include this in our inversions?

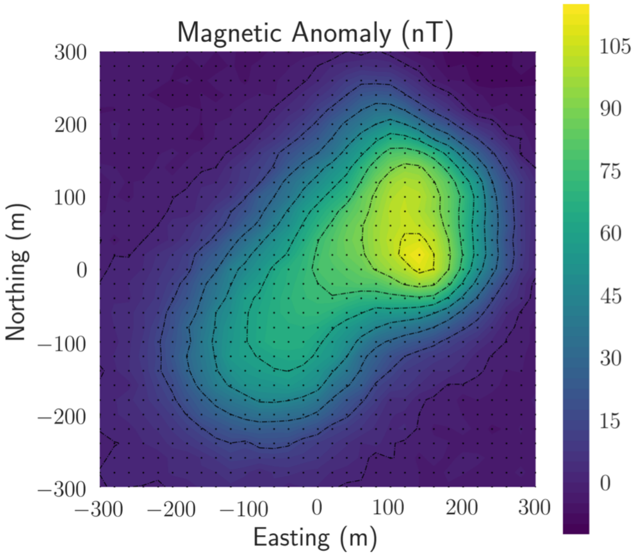
Synthetic diamond deposit



Gravity



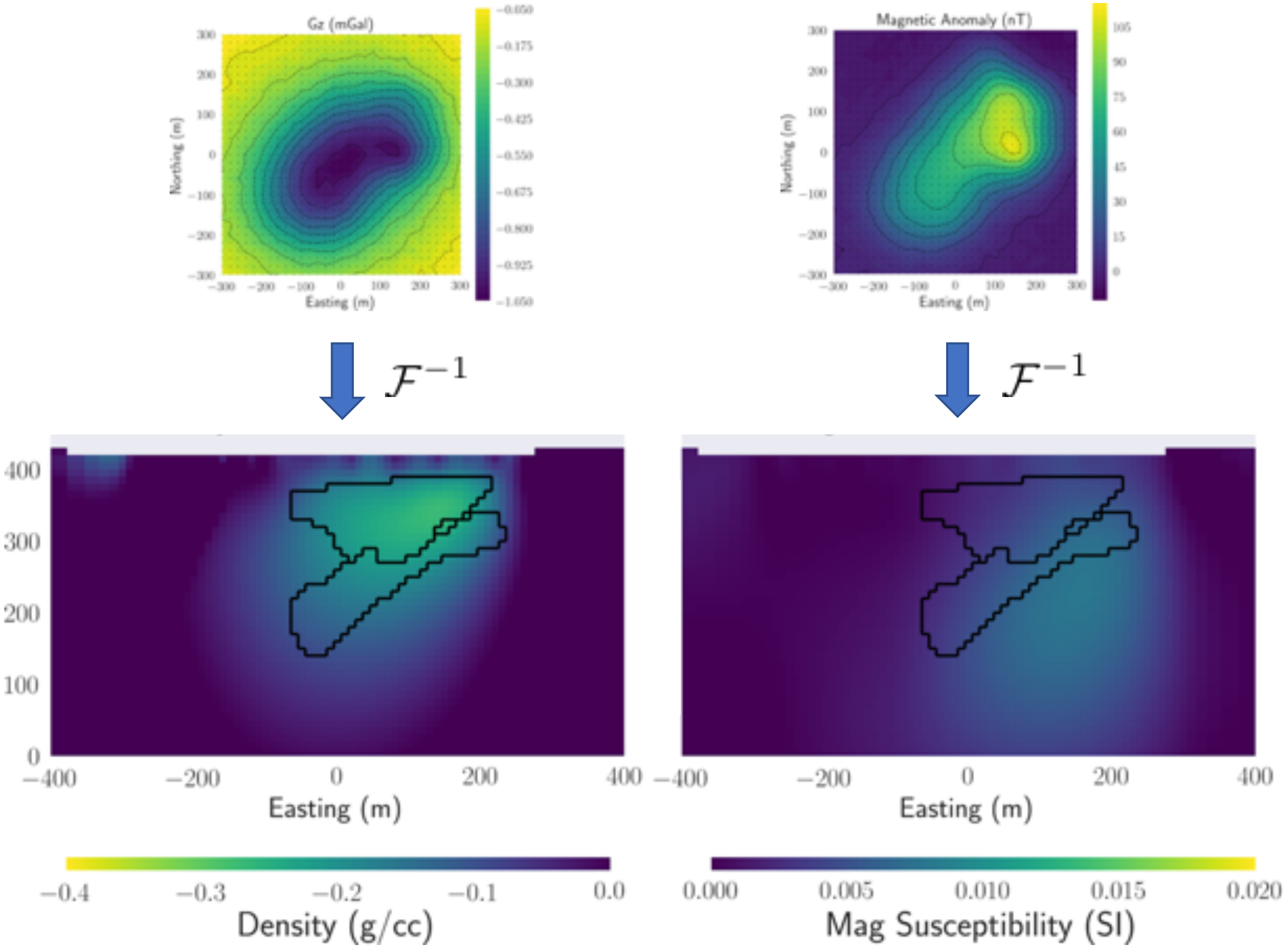
Magnetic



L_2 Inversion

Gravity data

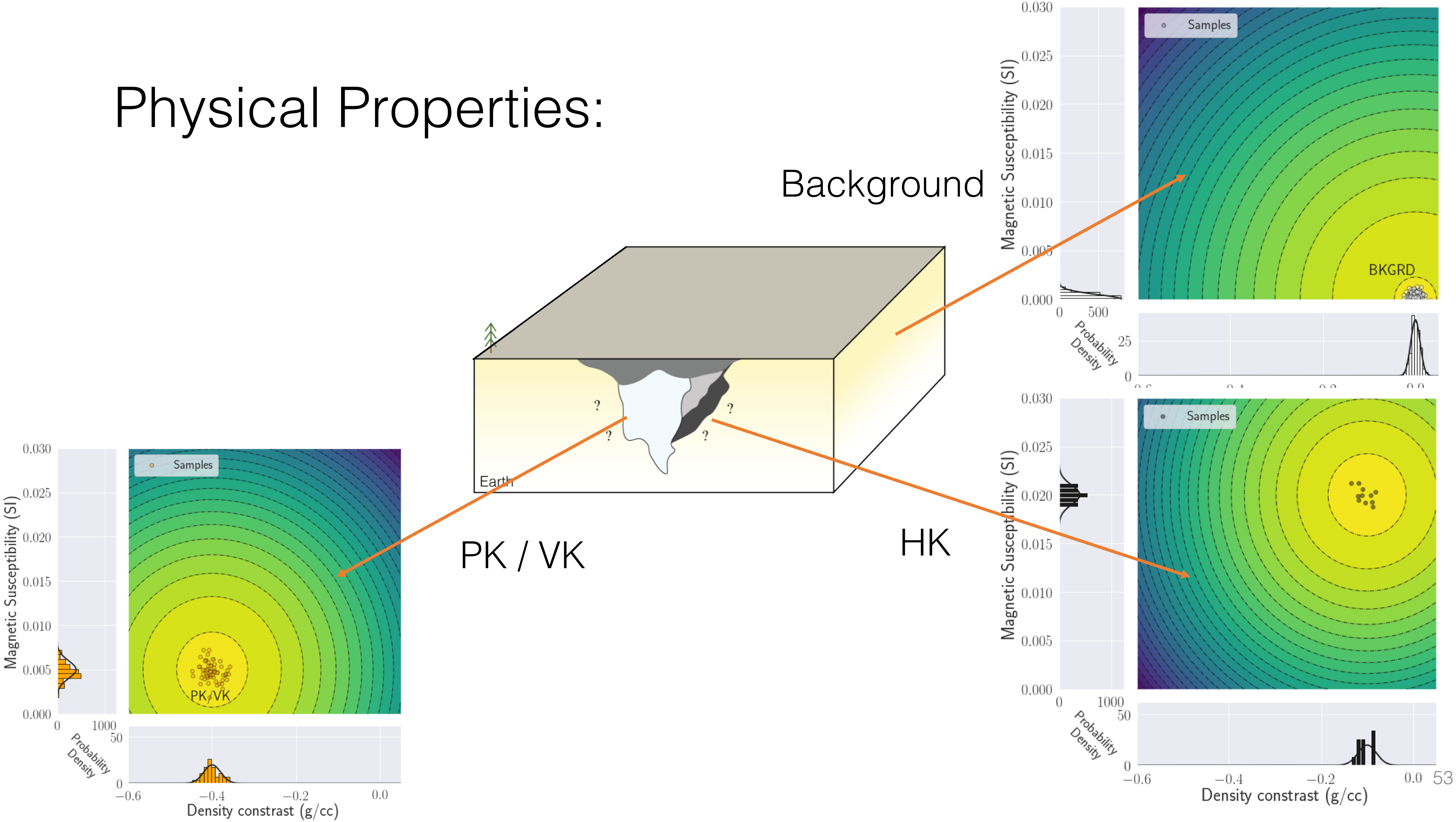
Magnetic data



Density model

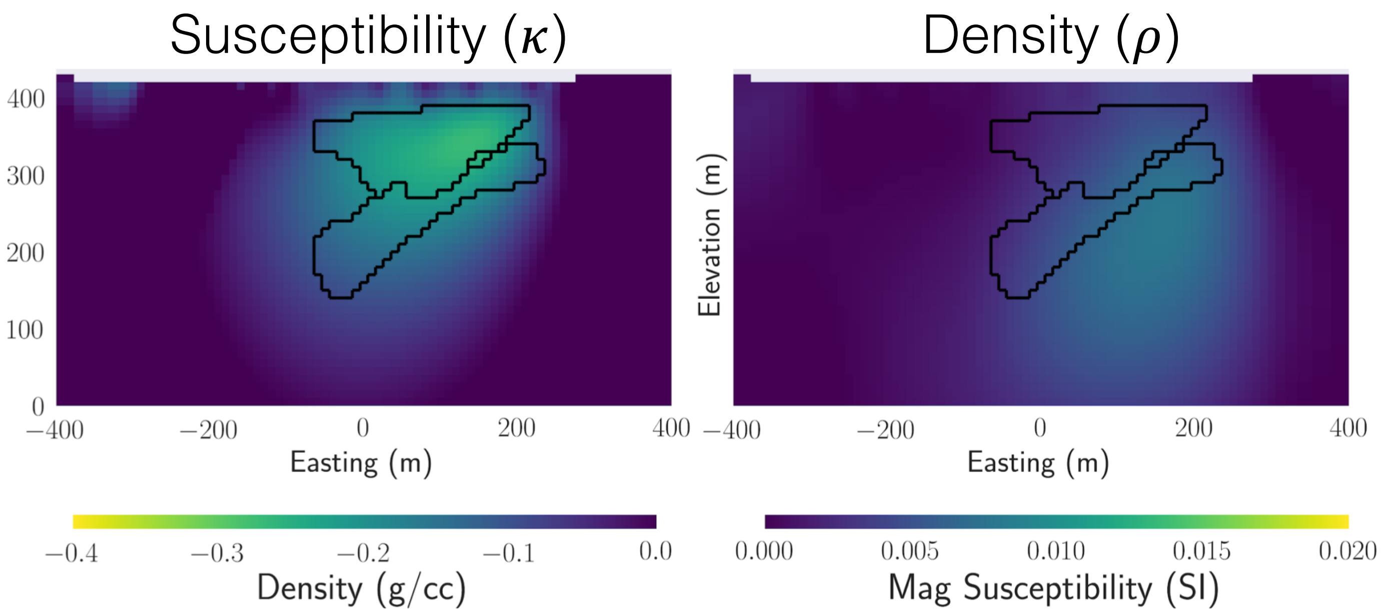
Susceptibility model

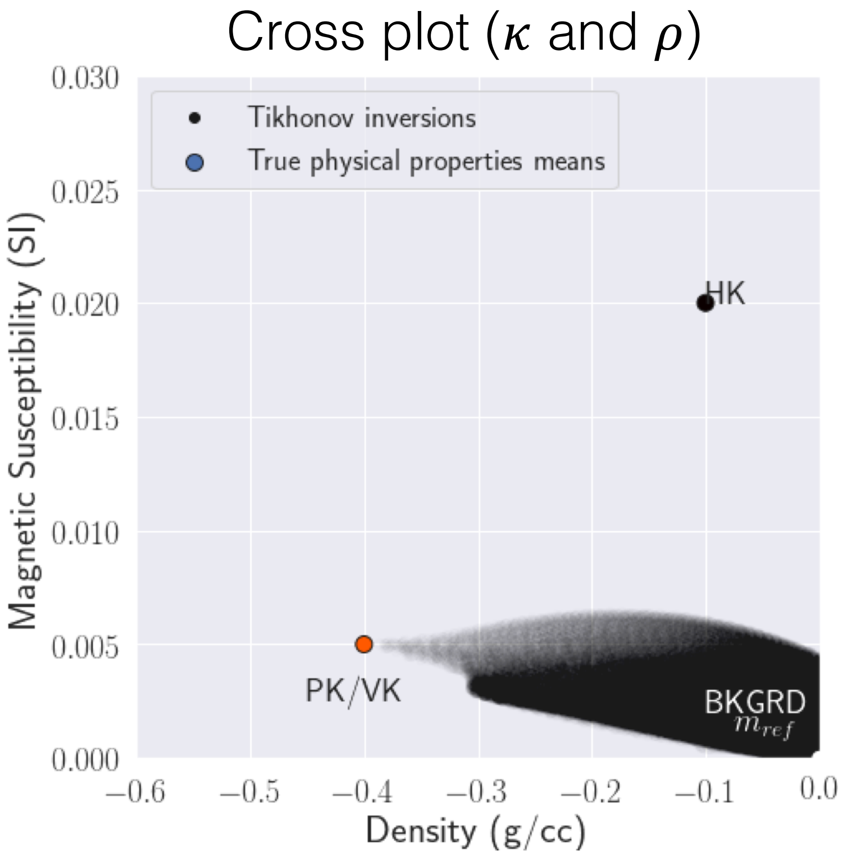
Physical Properties:



Physical Property values from inversion

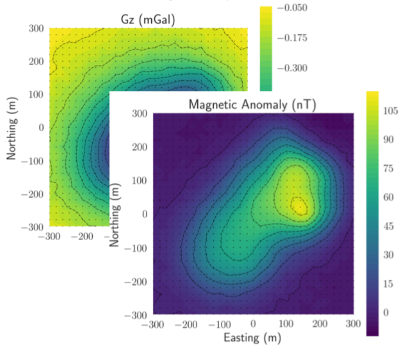
- L_2 inversion (smooth)



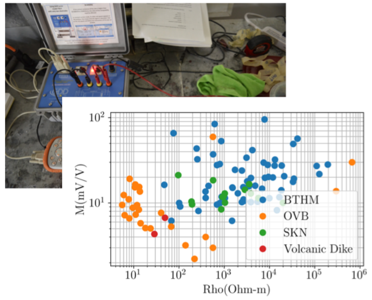


Find a geologic inversion result

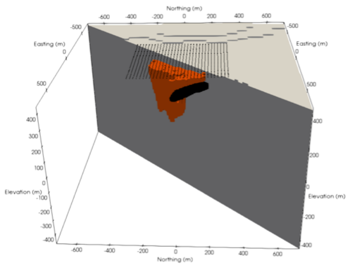
Geophysics



Petrophysics



Geology



Each pixel

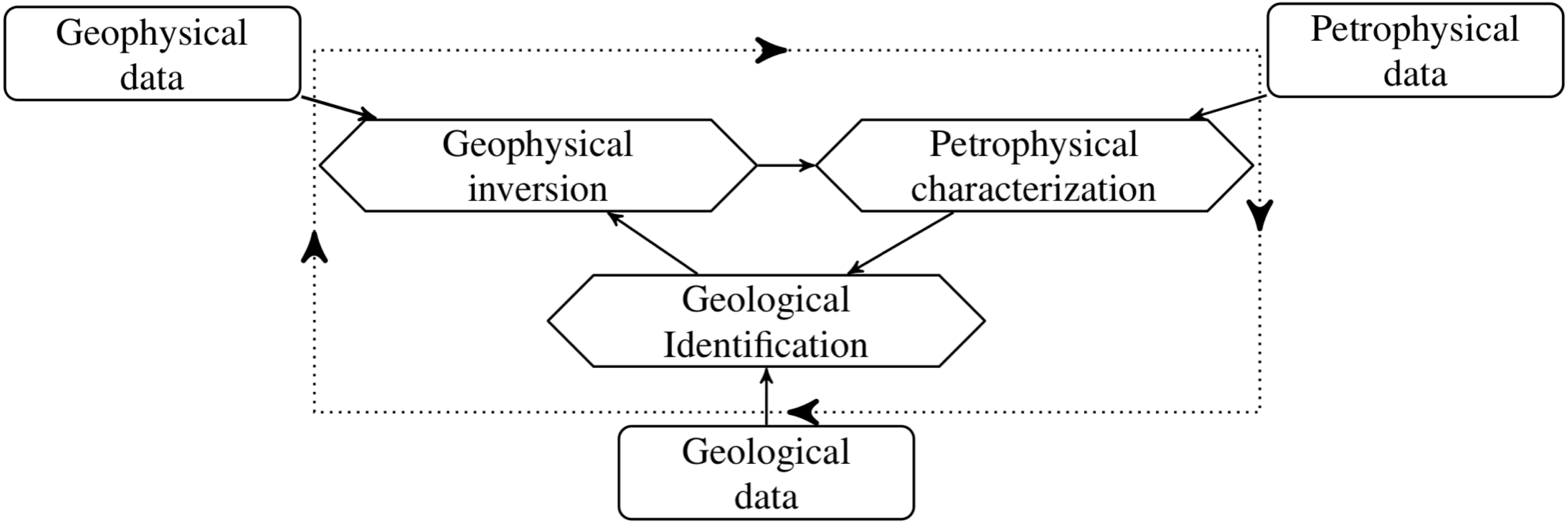
- needs a physical property value that is associated with a viable rock unit
- needs a geologic identifier

Globally

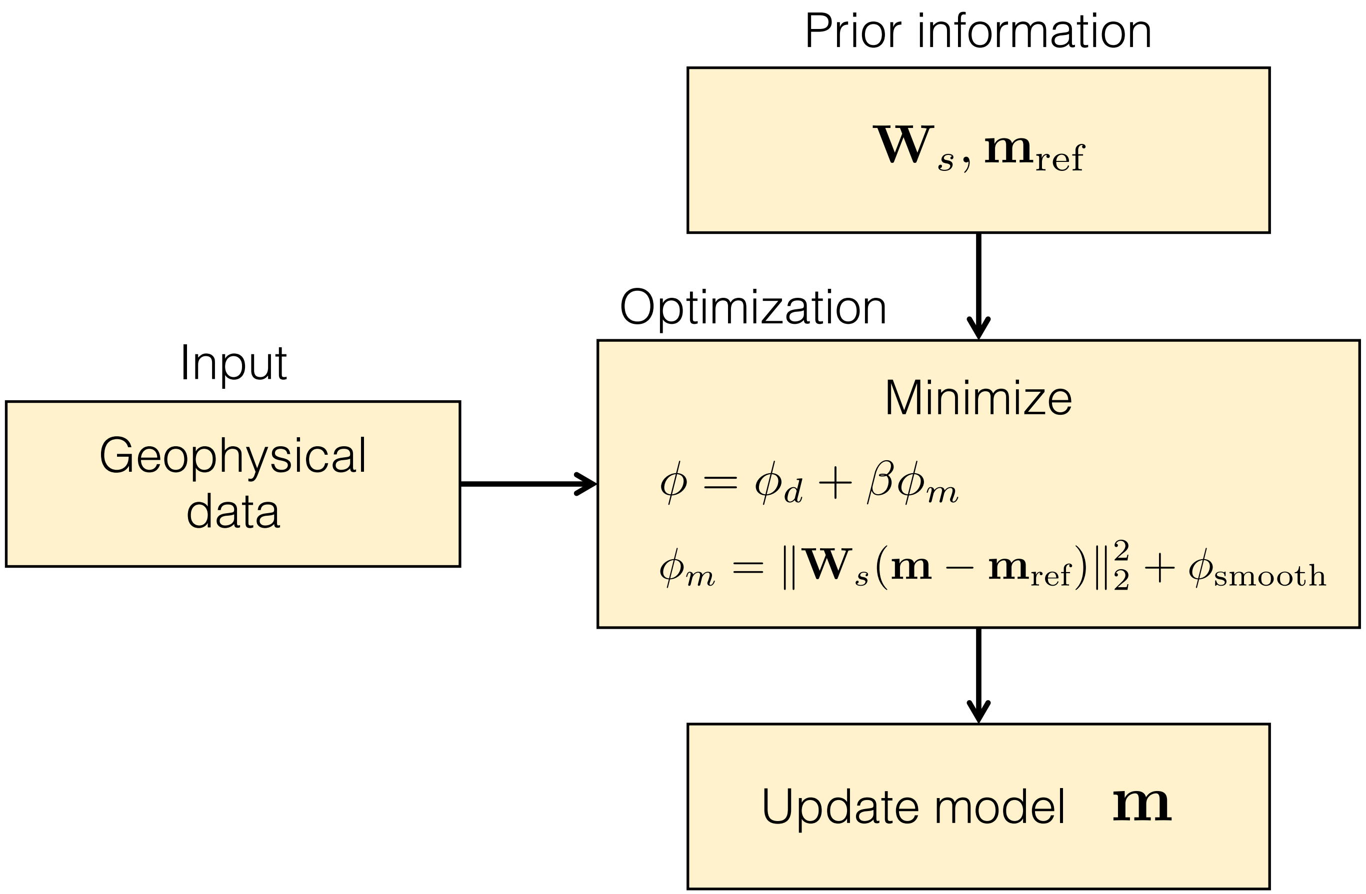
- both geophysical and petrophysical data must be fit

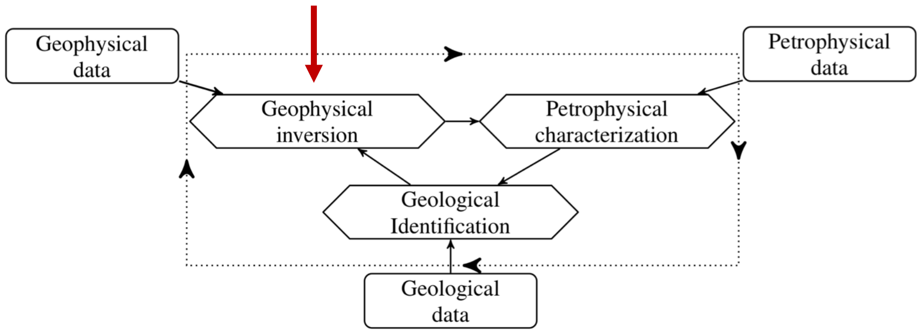
Tie geophysical, petrophysical and geological information together in a single conventional geophysical inversion framework

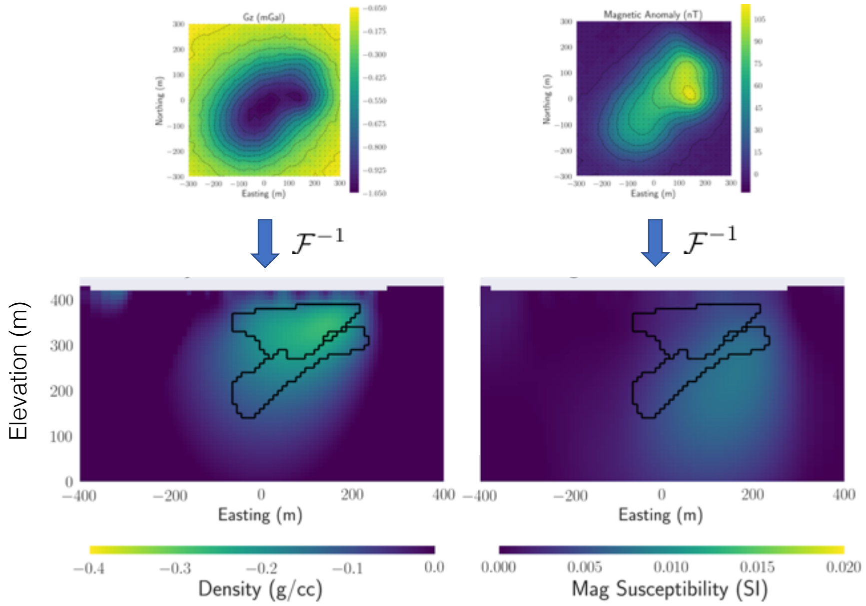
Linking Geophysics, Petrophysics and Geology



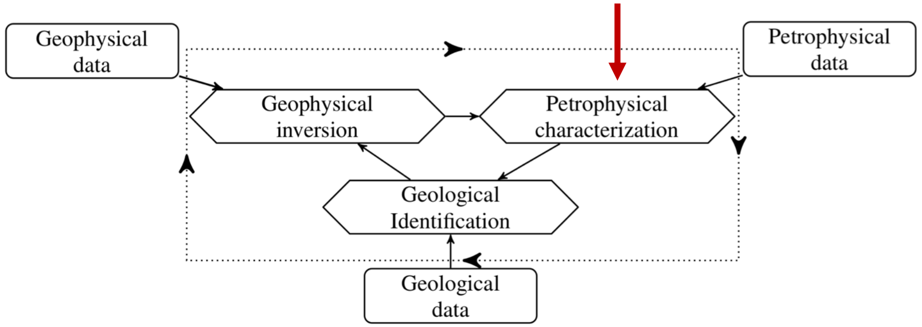
Geophysical Inversion







Petrophysical Inversion



Gaussian Mixture Model (GMM)

$$\mathcal{M}(m|\theta) = \sum_{j=1}^{c} \pi_j \mathcal{N}(m|\mu_j, \Sigma_j)$$

π_j: proportionality
μ_j: mean
Σ_j: convariance

Prior information

GMM
π_j, μ_j, Σ_j

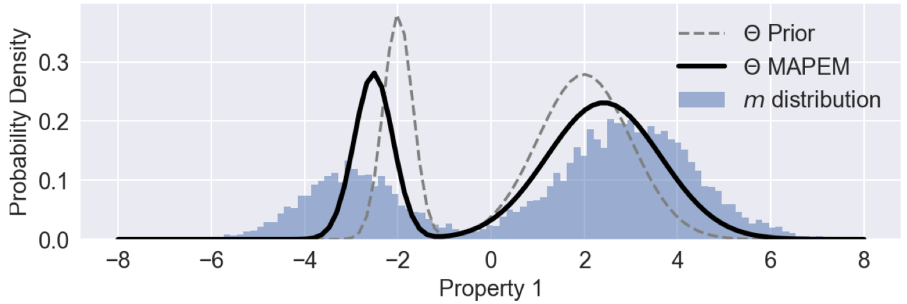
Optimization

Input

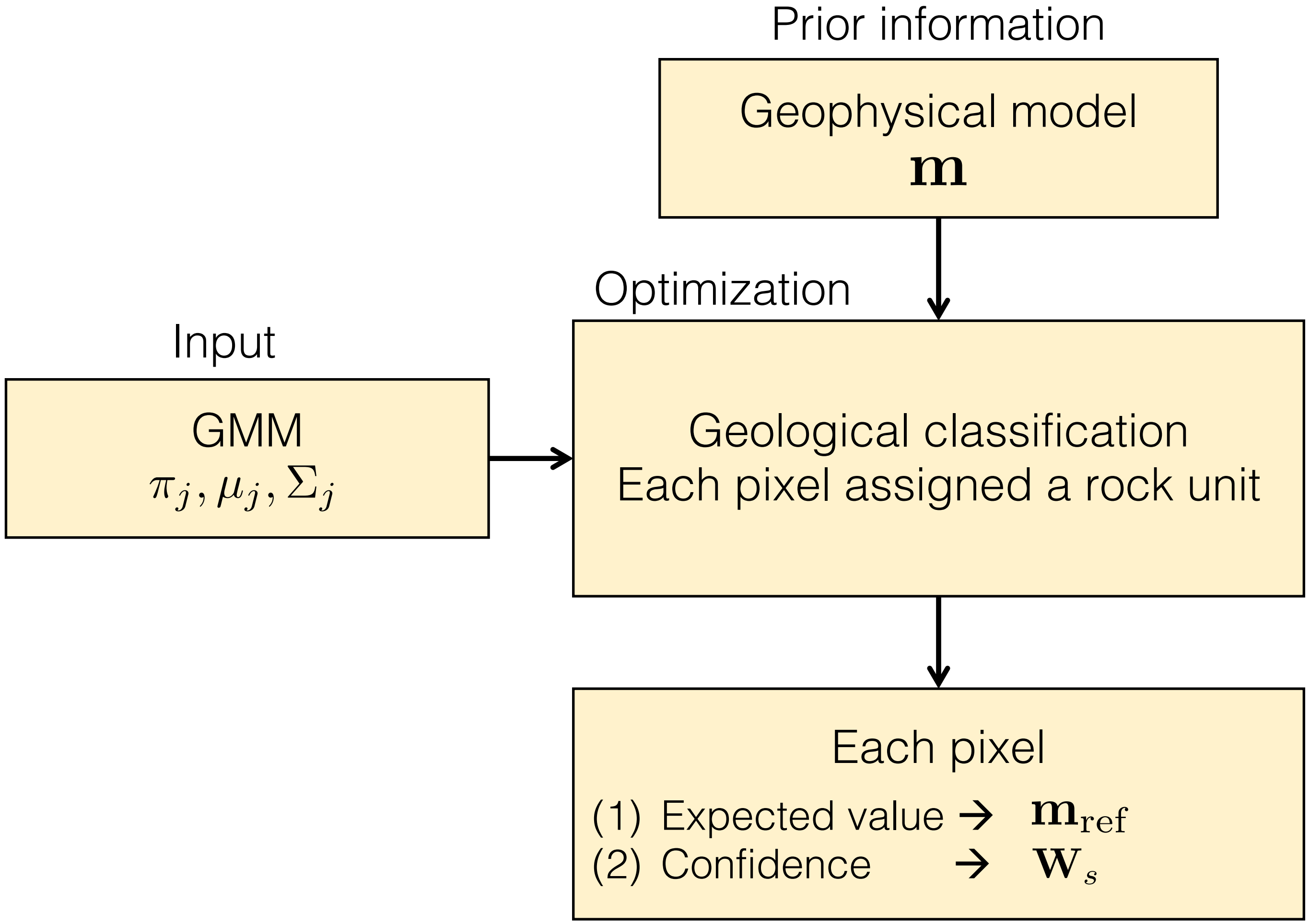
Geophysical model
$\mathbf{m}$

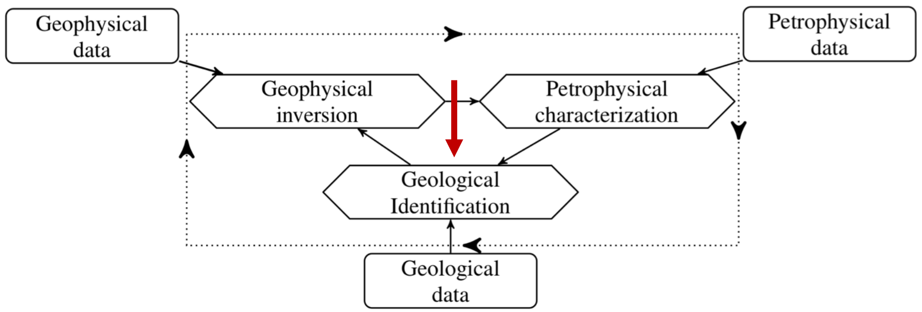
MAP-EM
solution

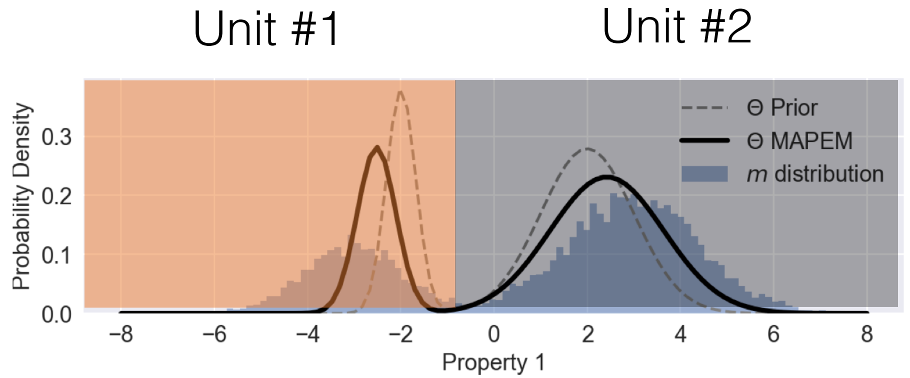
New GMM
update π_j, μ_j, Σ_j



Geologic Characterization



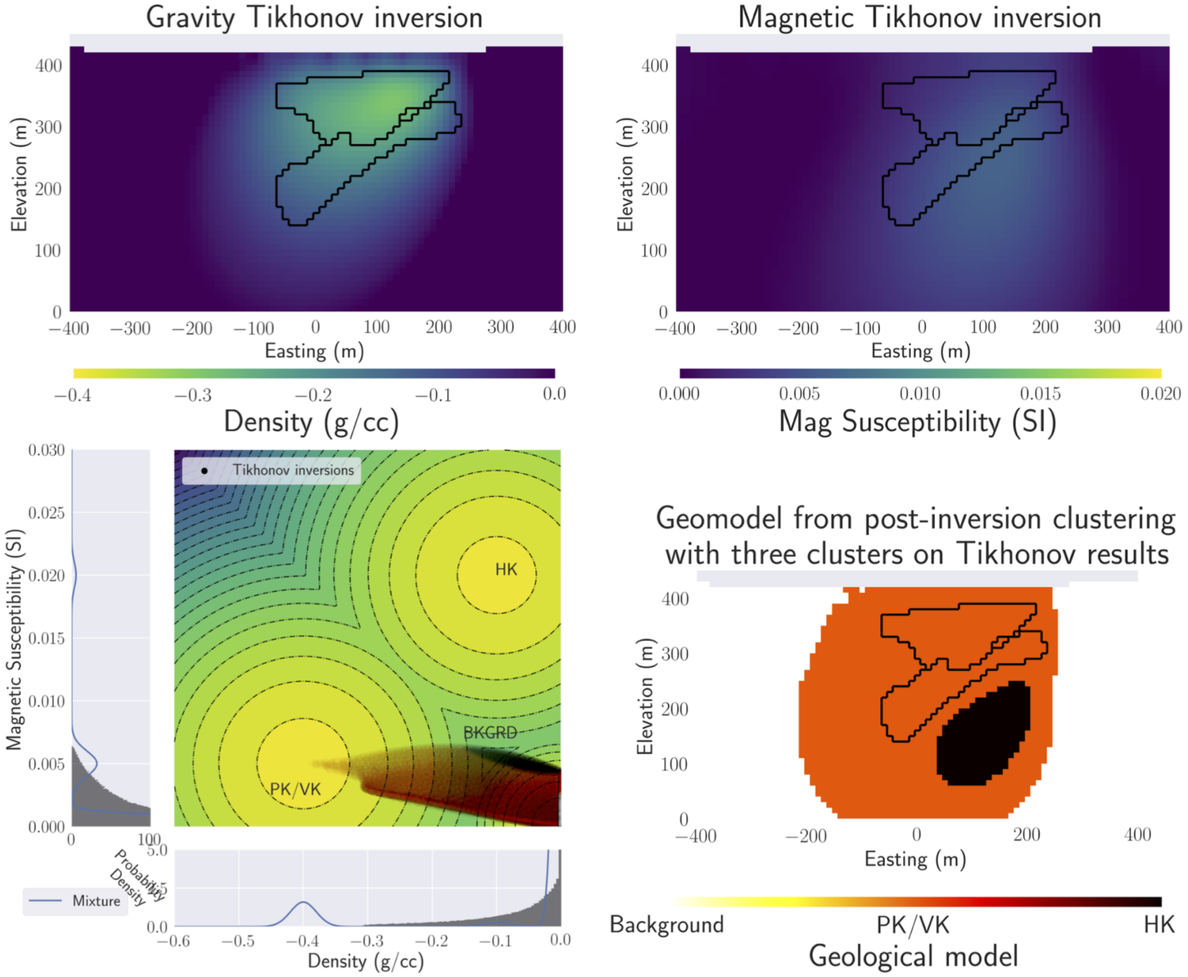




$$\phi_s(\mathbf{m}) = \|\mathbf{W}_s(\mathbf{m} - \mathbf{m}_{\text{ref}})\|_2^2$$

$$\mathcal{P}_s(m) \propto \mathcal{N}\left(\mathbf{m}|\mathbf{m}_{\text{ref}}, (\mathbf{W}_s^T\mathbf{W}_s)^{-1}\right)$$

Separate inversions without petrophysics



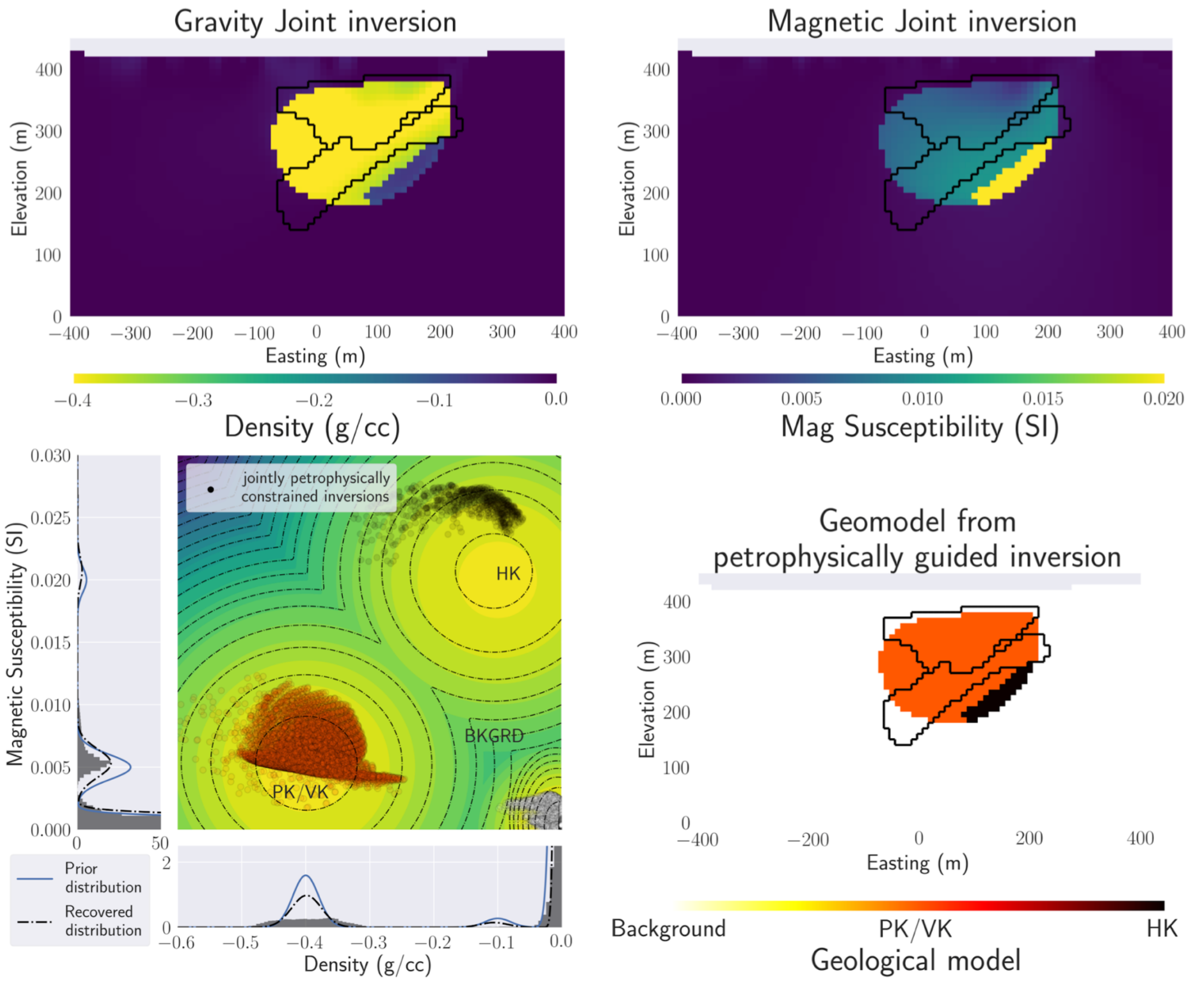
- Invert each data, obtain density and susceptibility model

- Apply rock classification using GMM

$$\mathcal{M}(m|\theta) = \sum_{j=1}^{c} \pi_j \mathcal{N}(m|\mu_j, \Sigma_j)$$

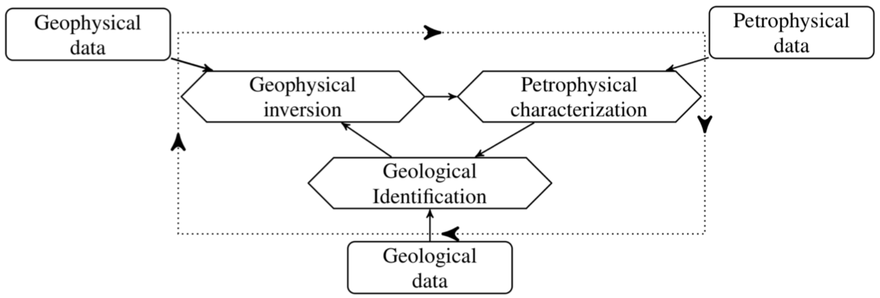
$$\begin{cases} \pi_j: \text{ proportionality} \\ \mu_j: \text{ mean} \\ \Sigma_j: \text{ convariance} \end{cases}$$

Joint inversion with petrophysics



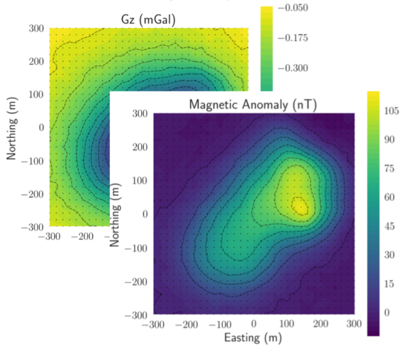
- Use together:
  - Geophysical data
  - Petrophysics
  - Geology

- Petrophysically guided inversion

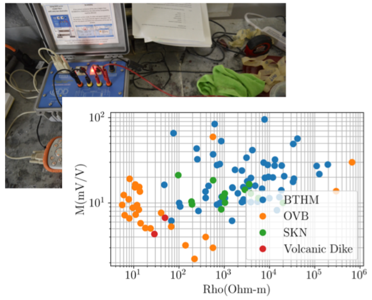


Summary

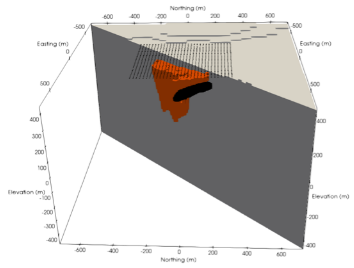
Geophysics



Petrophysics



Geology



Each pixel

- needs a physical property value that is associated with a viable rock unit
- needs a geologic identifier

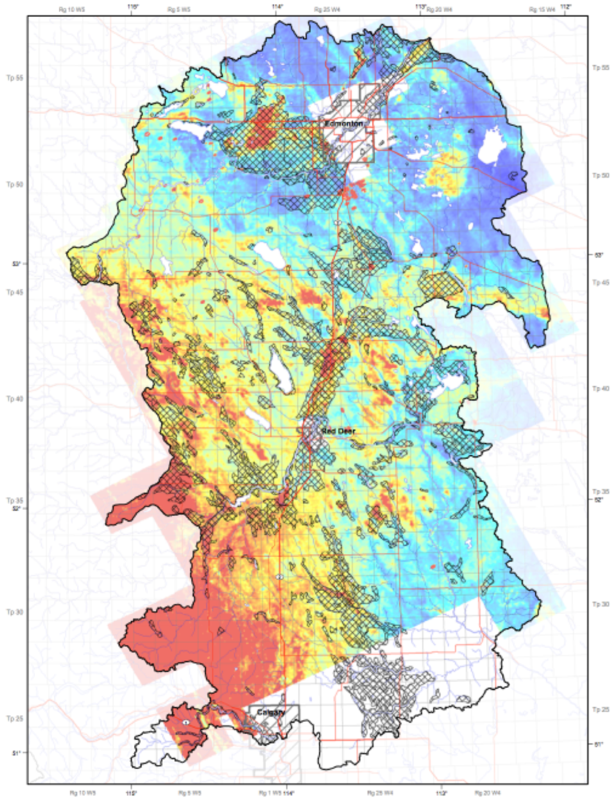
Globally

- both geophysical and petrophysical data must be fit

Tie geophysical, petrophysical and geological information together in a single conventional geophysical inversion framework

Practical needs for field applications

AEM resistivity at Edmonton

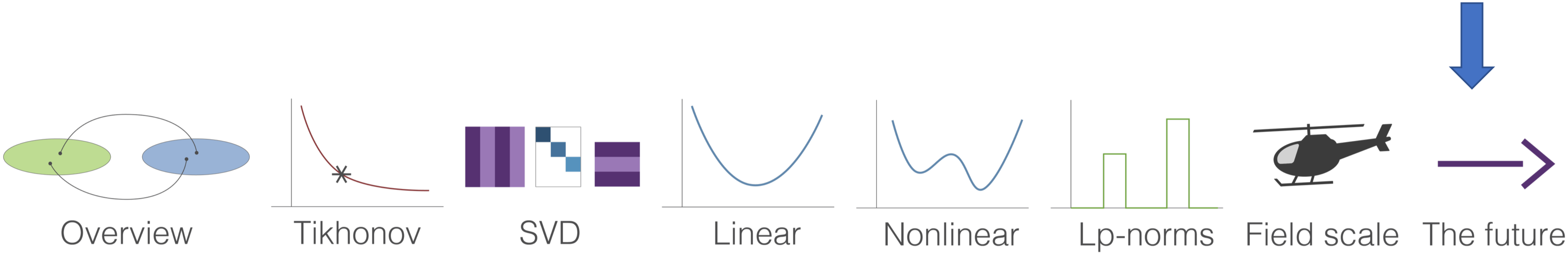


Baker (2011)

- Inversion in large scale problems
  - Airborne EM data
  - Forward modelling
  - Inversion
  - Field application (e.g. diamond exploration)

- Addressing Uncertainty
  - Exploring model model space
  - Model parameterization
  - Joint inversion
  - Post-inversion classification (multiple physical properties)
  - PGI (petrophysically guided inversion)

Next up …


Overview
Tikhonov
SVD
Linear
Nonlinear
Lp-norms
Field scale
The future

The end