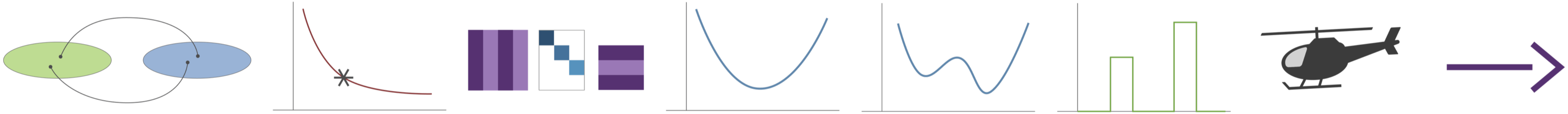
Wrap-up and Future

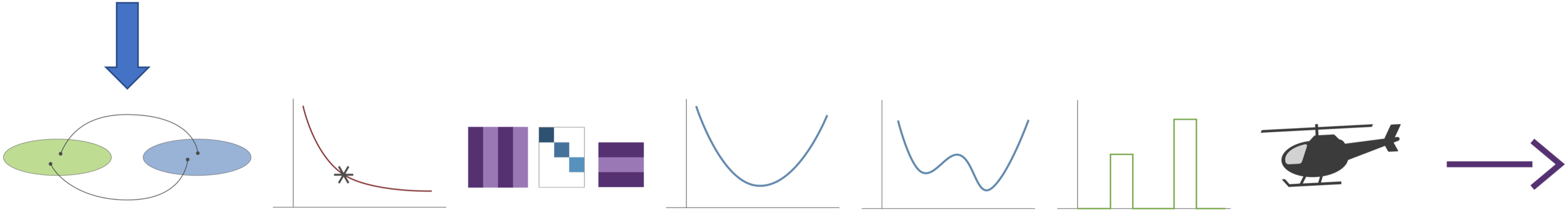
Major components for the week



Overview | Tikhonov | SVD | Linear | Nonlinear | Lp-norms | Field scale | The future

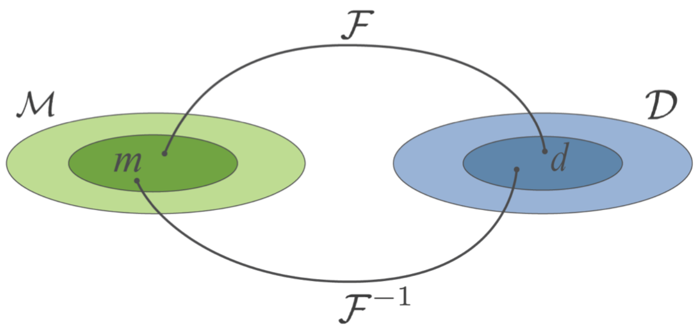
https://courses.geosci.xyz/lapis2019

Major components for the week



Overview Tikhonov SVD Linear Nonlinear Lp-norms Field scale The future

Summary for inverse problem



Non-unique
Ill-conditioned

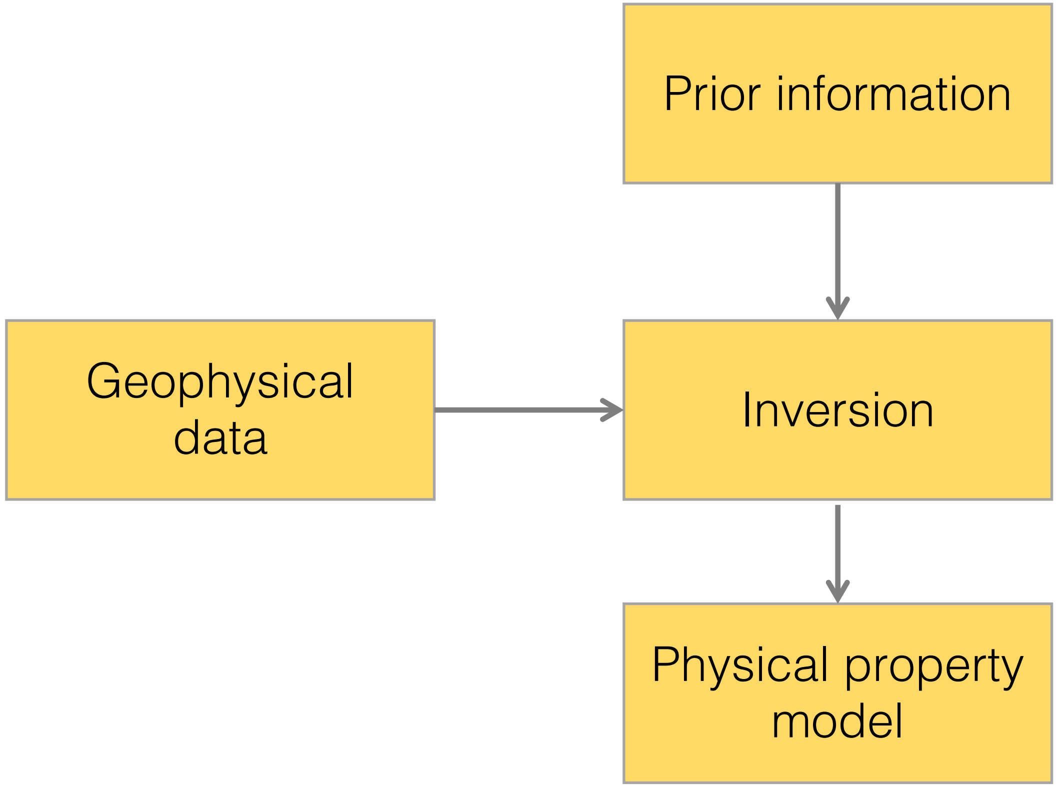
The Inverse Problem is ill-posed

Approach to solving the inverse problem address these issues

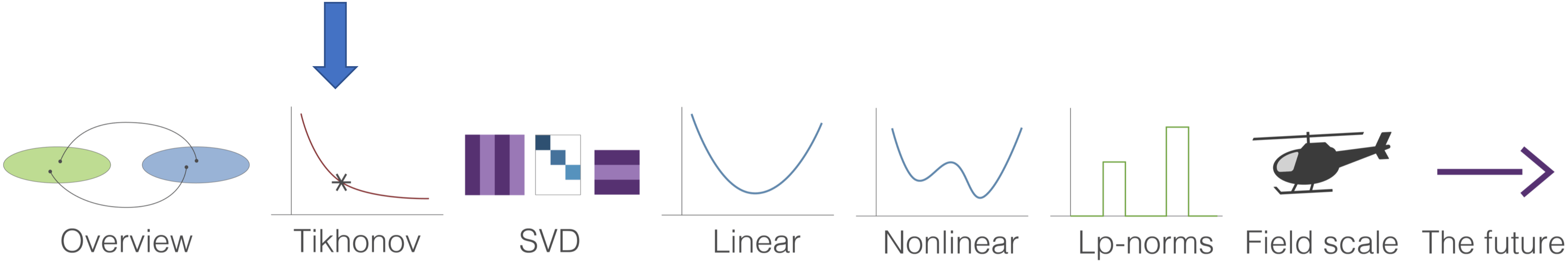
Constraining the inversion

What information is available?

- Geologic structure
- Geologic constraints
- Reference model
- Bounds
- Multiple data sets
- Physical property measurements



Major components for the week



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Tikhonov approach

- Define misfit: $\phi_d = \sum_{j=1}^{N} \Big(\frac{d_j - d_j^{obs}}{\epsilon_j} \Big)^2$

- Define a ruler form measuring size

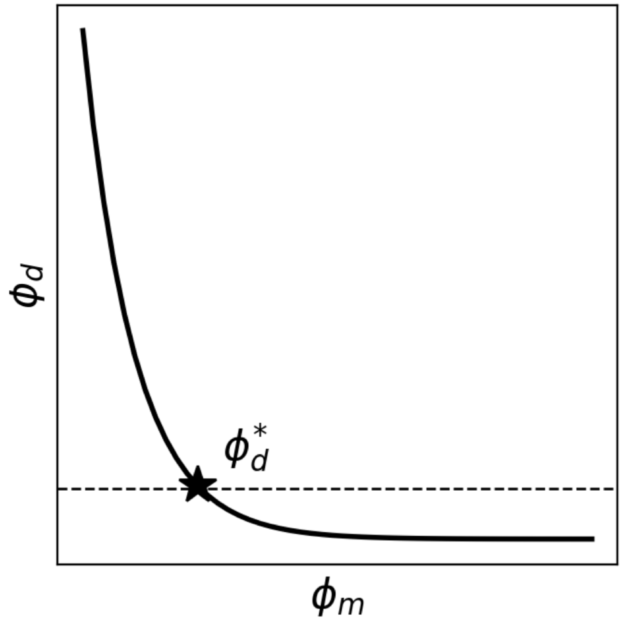
$$\phi_m = \int (m - m_{ref})^2 dx$$

- Minimize

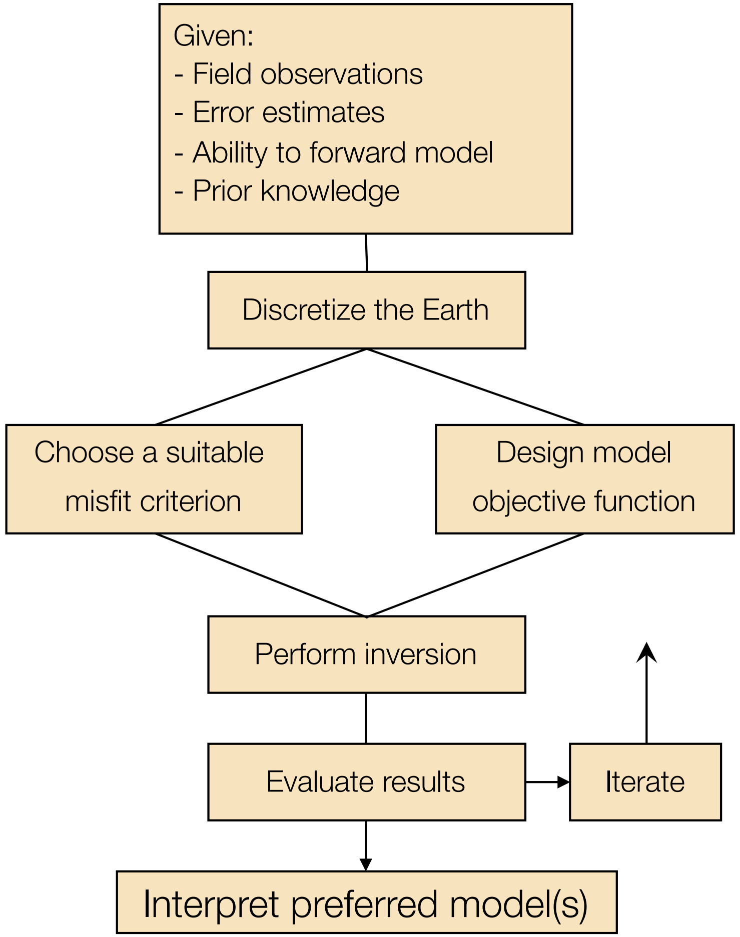
$$\phi = \phi_d + \beta \phi_m$$

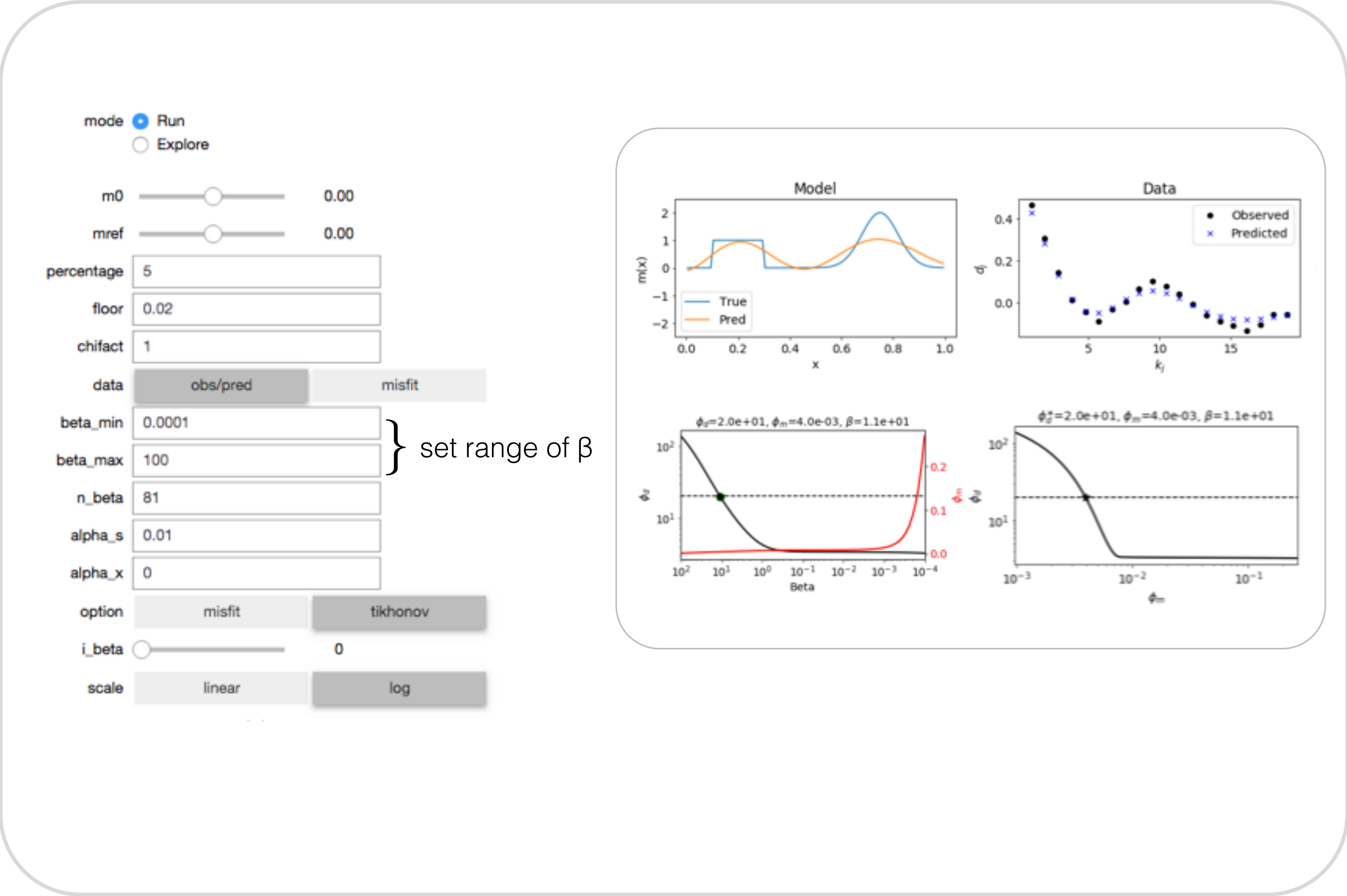
- β is regularization parameter

Tikhonov curve

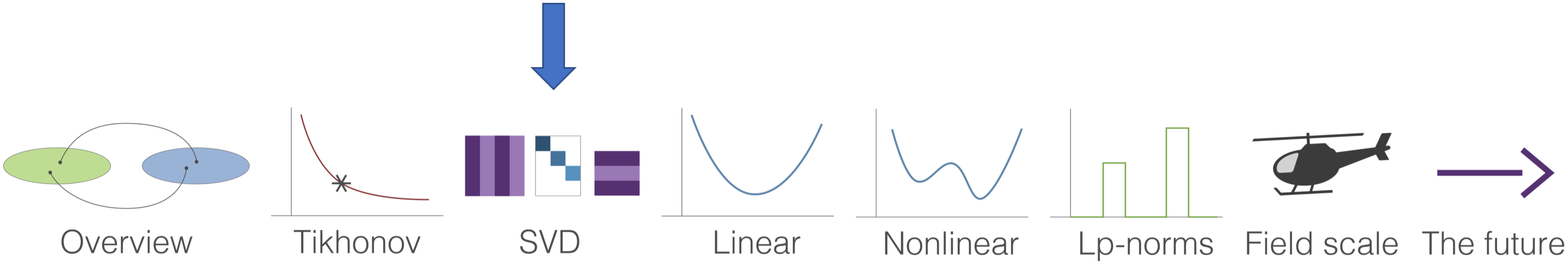


Inversion Workflow with app





Major components for the week



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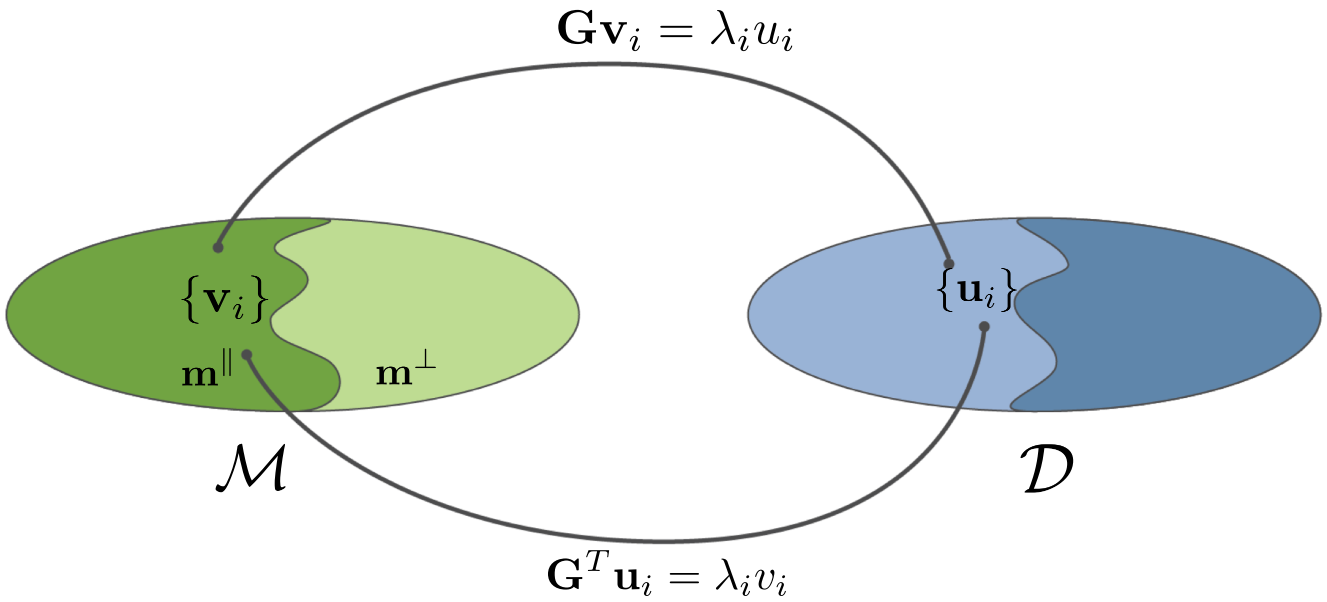
Singular Value Decomposition (SVD)

$$\mathbf{Gm} = \mathbf{d}$$

$$\mathbf{G} = \mathbf{U}_p \boldsymbol{\Lambda} \mathbf{V}_p^T$$



$\mathbf{m}^{\parallel}$: activated portion of model space
$\mathbf{m}^{\perp}$:annihilator space

Truncated SVD

- If data are inaccurate, noise is also amplified by $1/\lambda_i$

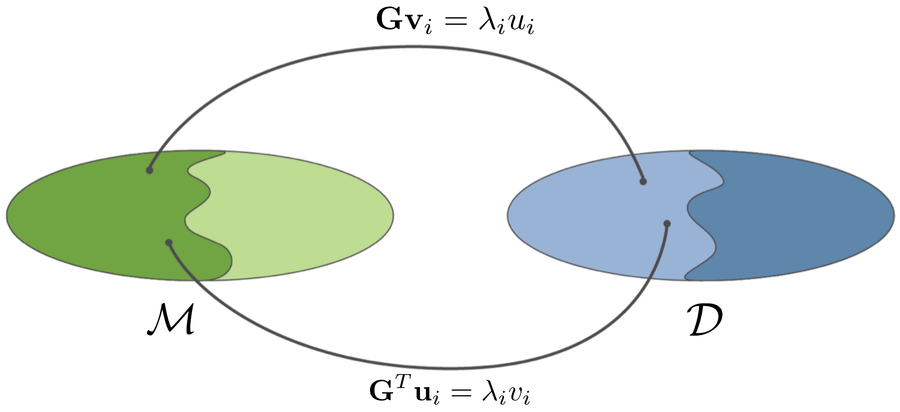
$$\mathbf{m}_c = \sum_{i=1}^{q} \left(\frac{\mathbf{u}_i^T \mathbf{d}}{\lambda_i} \right) \mathbf{v}_i + \underbrace{\sum_{i=q+1}^{N} \left(\frac{\mathbf{u}_i^T \mathbf{d}}{\lambda_i} \right) \mathbf{v}_i}_{\text{Cause more harm than good}}$$

- So

$$\mathbf{m}_c = \sum_{i=1}^{q} \left(\frac{\mathbf{u}_i^T \mathbf{d}}{\lambda_i} \right) \mathbf{v}_i$$

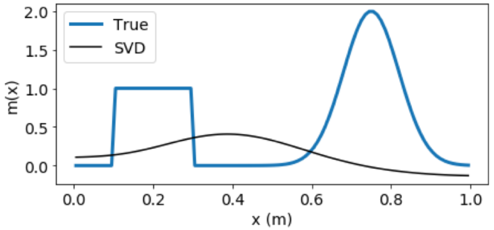
- Solution lies in a small sub-space

- Treats non-uniqueness and ill-conditioning

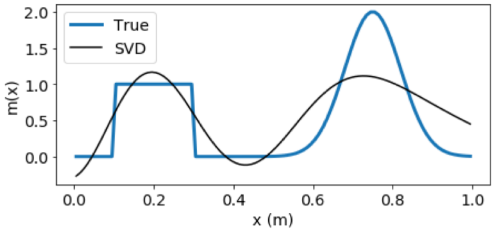


Plot Tikhonov curve

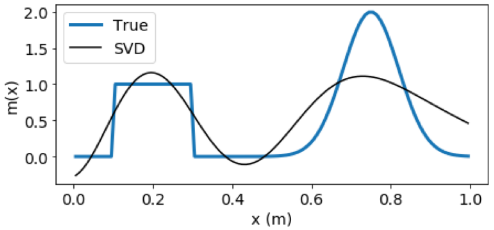
$q = 3$



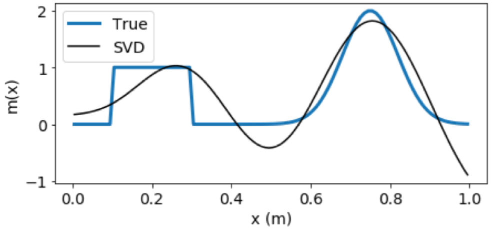
$q = 4$

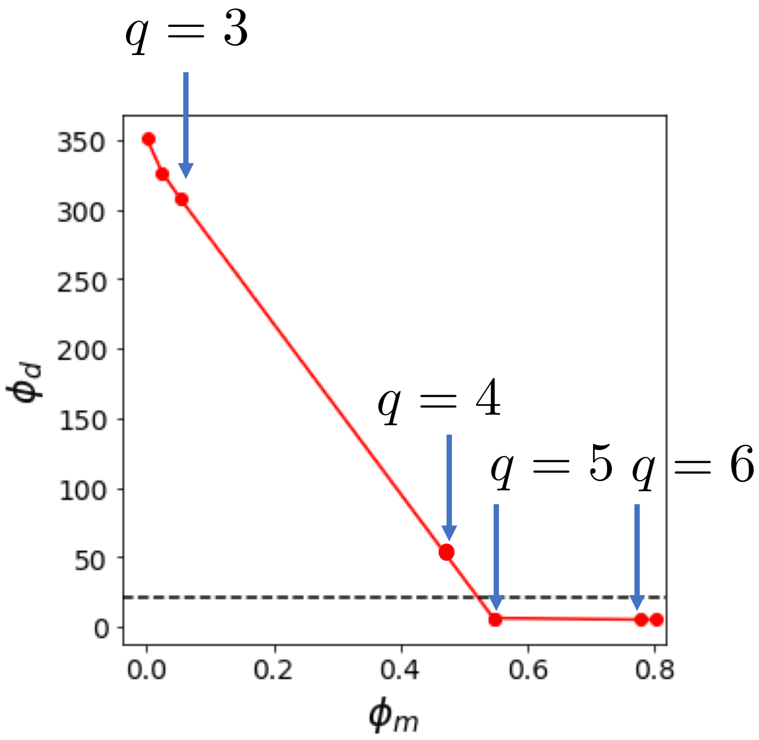


$q = 5$

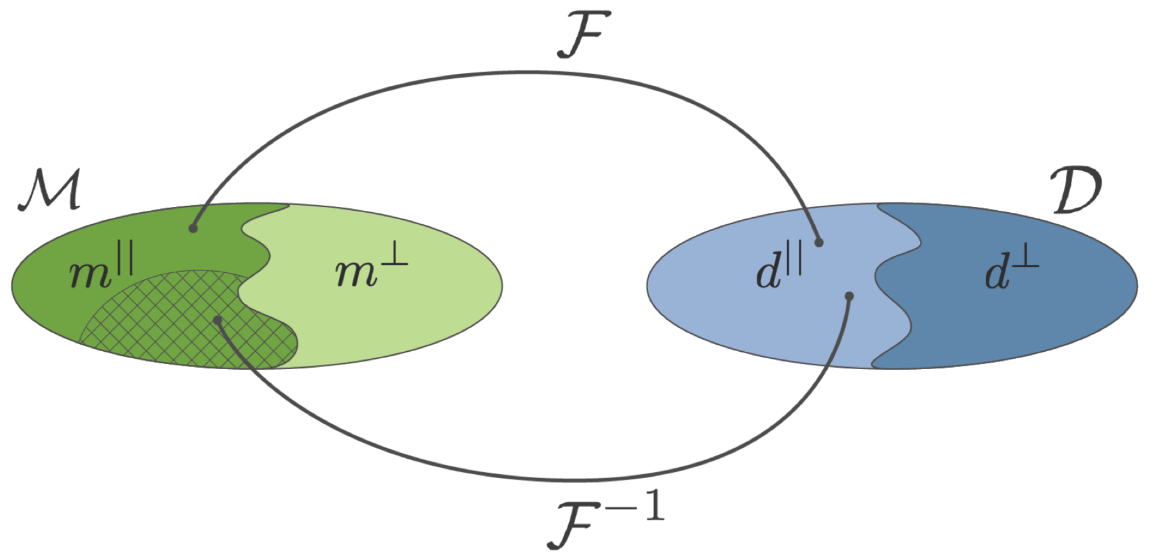


$q = 6$





What does the solution tell us

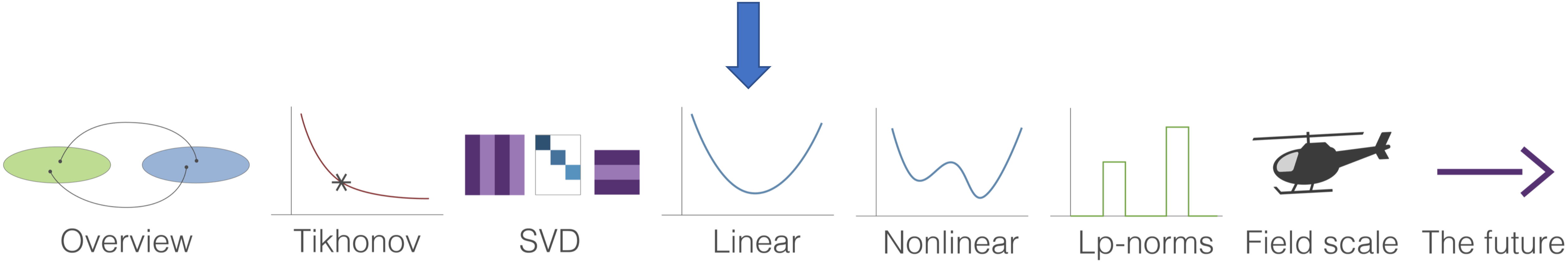


- Full data set can recover information about $\mathbf{m}^{||}$
- Regularized solution is in a reduced region of model space

Geophysical model lies outside of this region

To explore that we need to incorporate more information

Major components for the week



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L_2 inversion

- Misfit $\phi_d = \sum_{j=1}^{N} \left(\frac{F_j(m) - d_j^{obs}}{\epsilon_j} \right)$

- Model norms

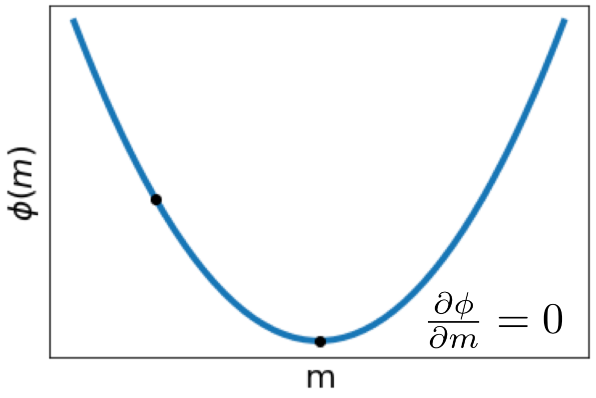
$$\phi_m = \alpha_s \|\mathbf{W}_s(\mathbf{m} - \mathbf{m}_{ref})\|_2^2 + \alpha_x \|\mathbf{W}_x(\mathbf{m})\|_2^2$$

- Minimize

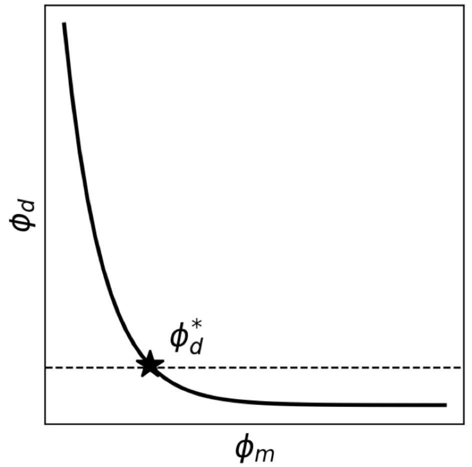
$$\phi = \phi_d + \beta \phi_m$$

- Quadratic function (Solution obtained in one step)

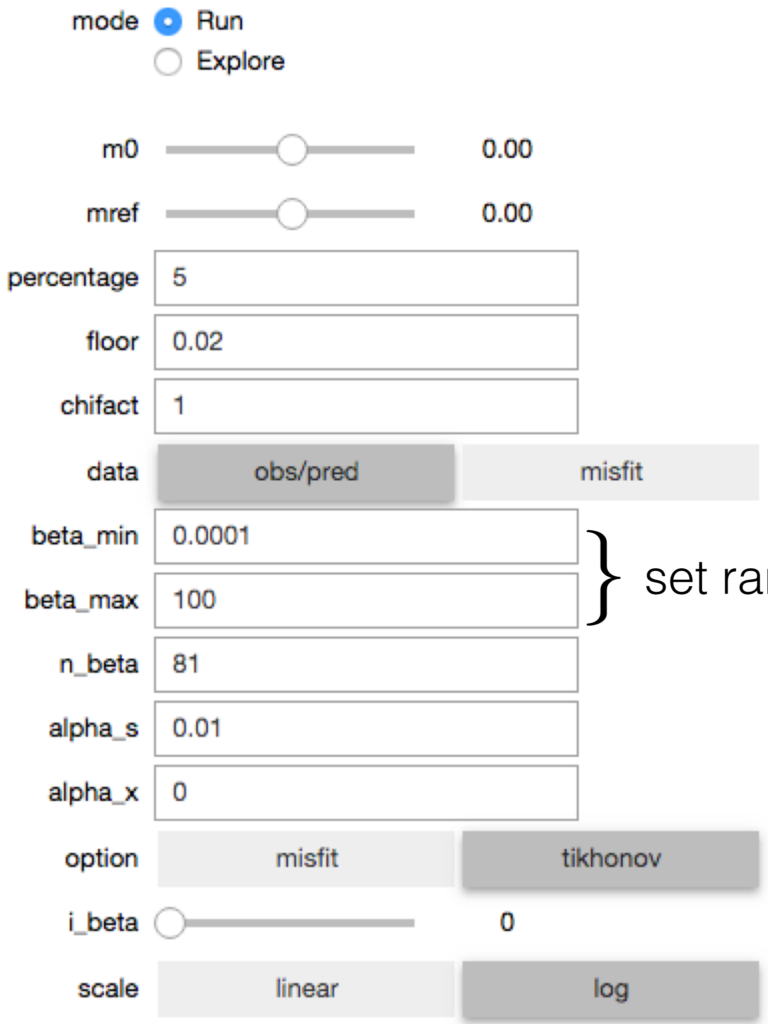
- Need to choose β



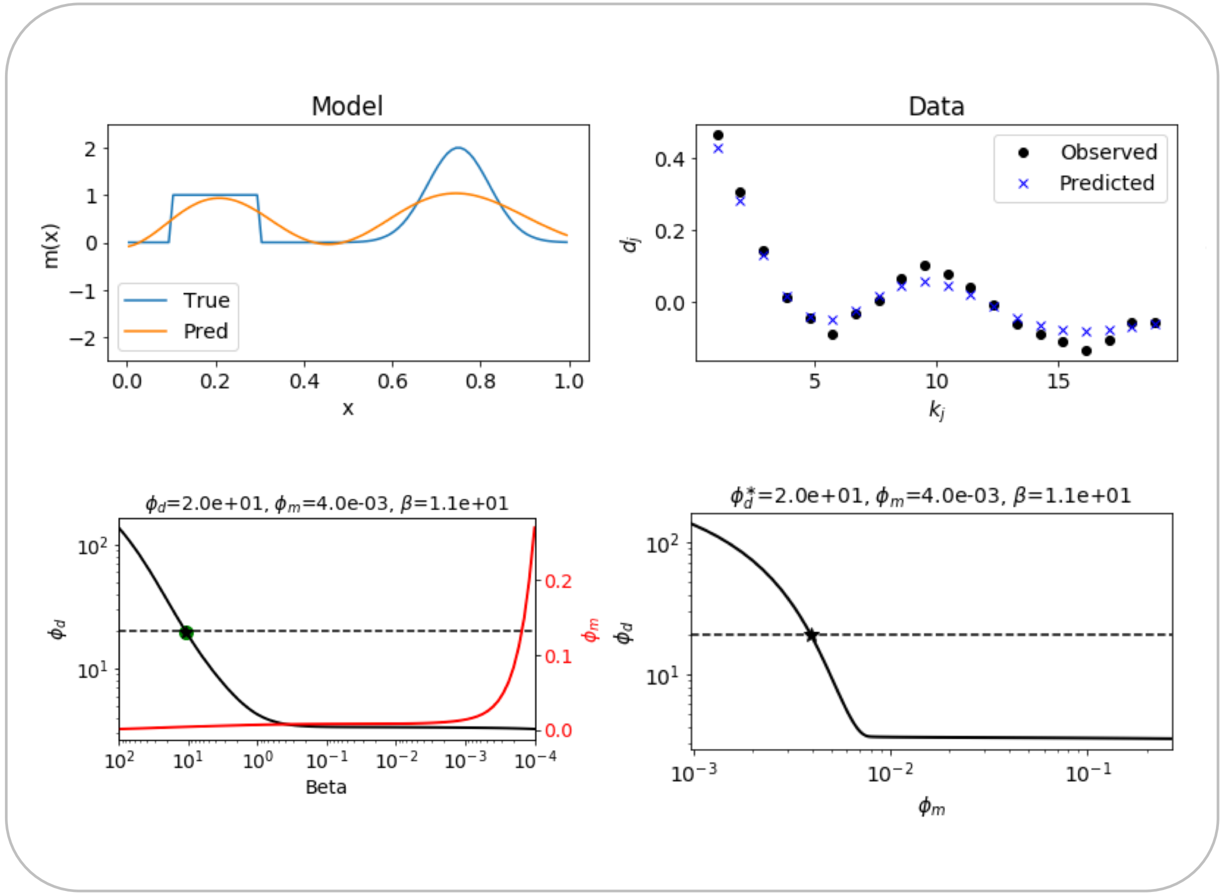
Tikhonov curve



Inversion app



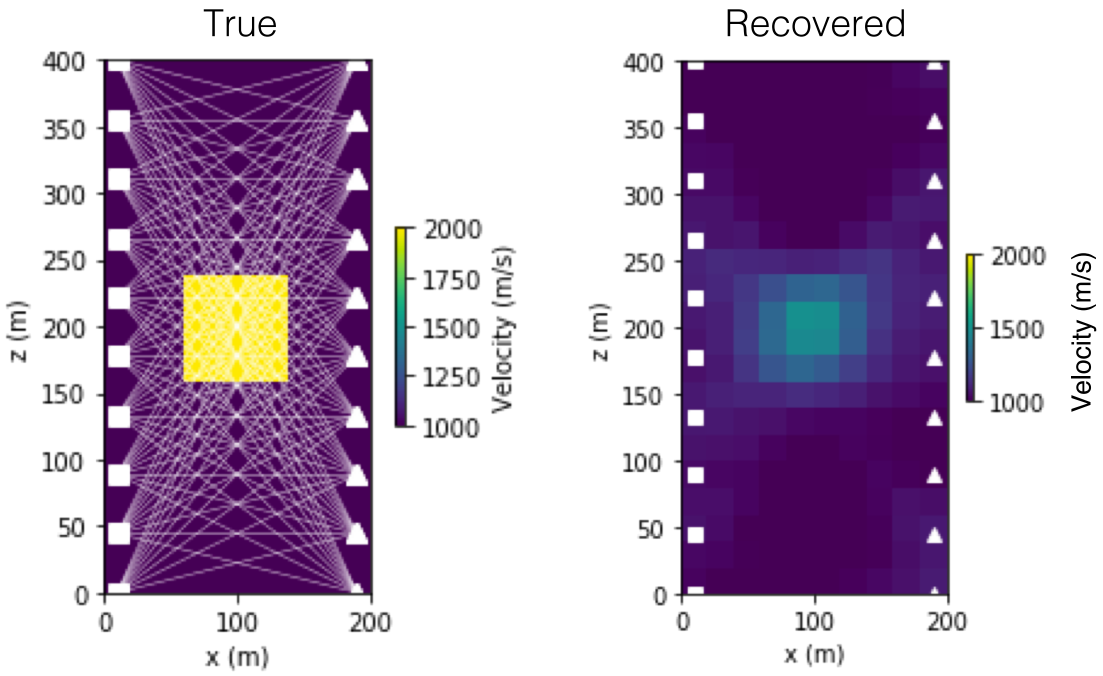
} set range of β

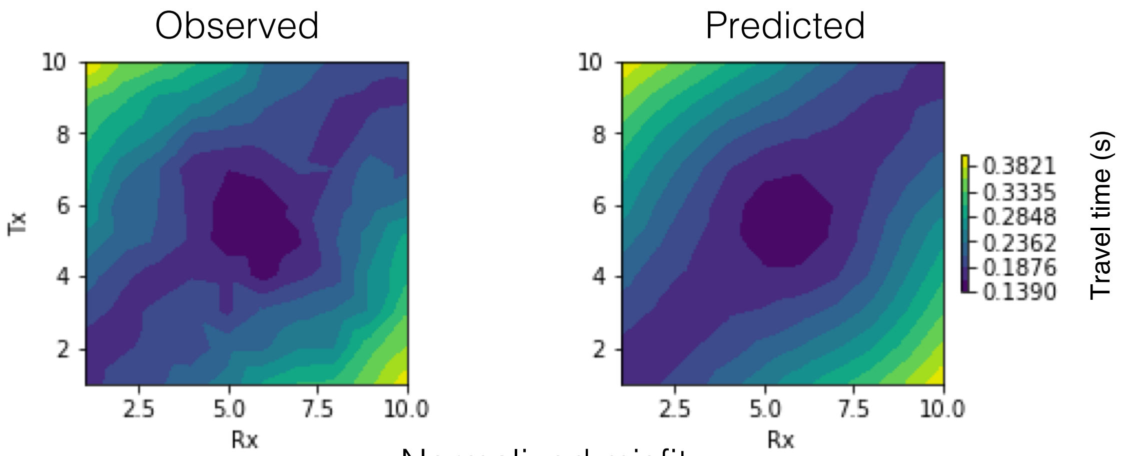


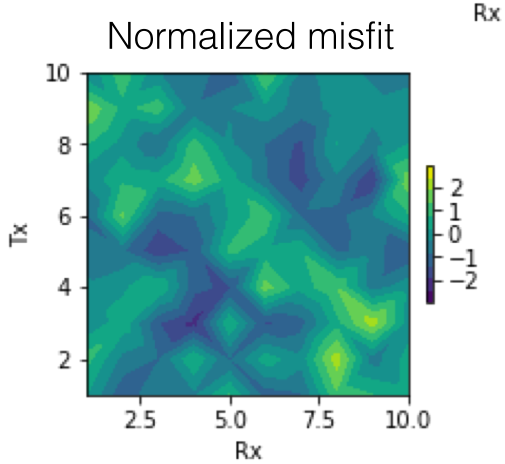
L_2 inversion: 2D tomography

$$\phi_m = \alpha_s \|\mathbf{W}_s(\mathbf{m} - \mathbf{m}_{\text{ref}})\|^2 + \alpha_x \|\mathbf{W}_x \mathbf{m}\|^2 + \alpha_z \|\mathbf{W}_z \mathbf{m}\|^2$$

Choose
$$\mathbf{m}_{\text{ref}} = 1/1000$$
$$\alpha_s = 1/1000$$
$$\alpha_x = \alpha_z = 1$$

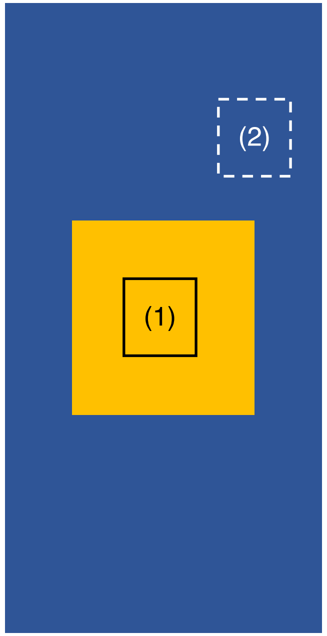






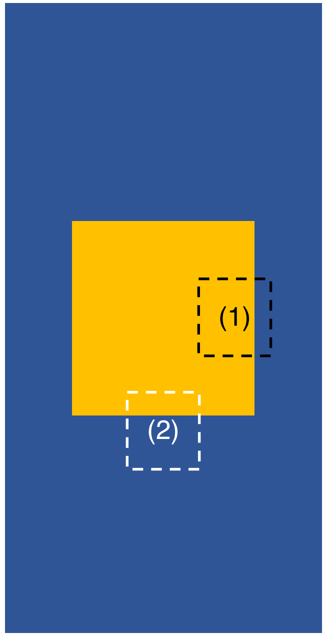
Enhanced functionality using ϕ_m

$$\phi_m = \alpha_s \int_v w_s(\mathbf{m} - \mathbf{m}_{\mathrm{ref}})^2 dv + \alpha_x \int_v w_x \left(\frac{dm}{dx}\right)^2 dx + \alpha_z \int_v w_z \left(\frac{dm}{dz}\right)^2 dz$$



$w_s(x,z)$: local confidence in m_{ref}

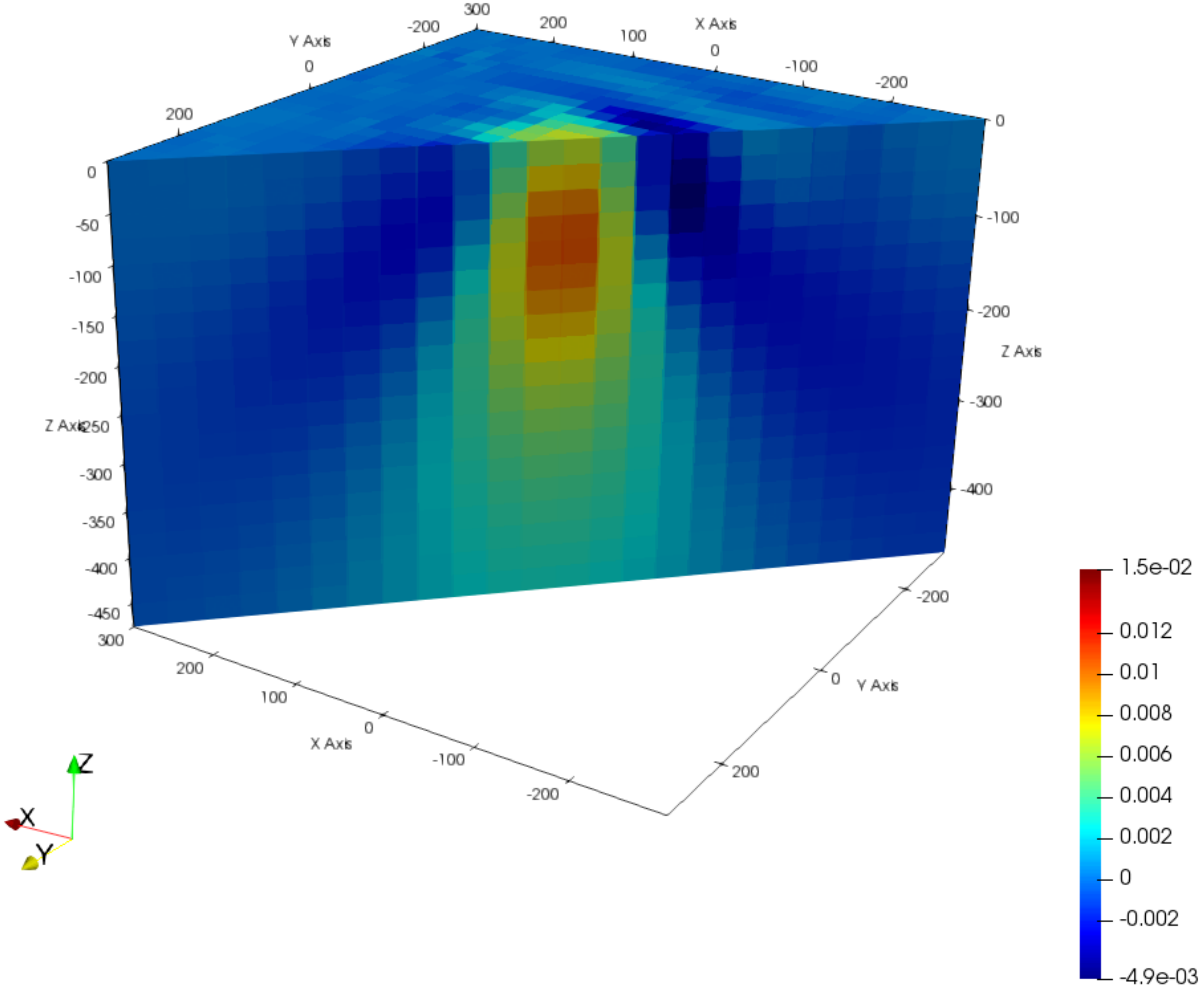
(1) w_s: small
(2) w_s: large



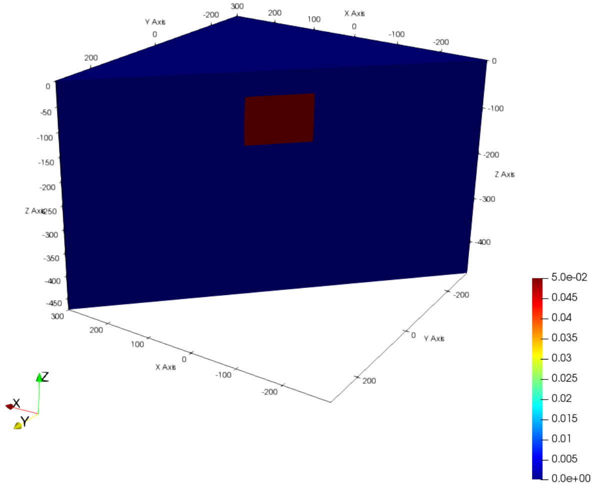
(1) w_x: small, w_z: large → allows jump in x-direction

(2) w_x: large, w_z: small → allows jump in z-direction

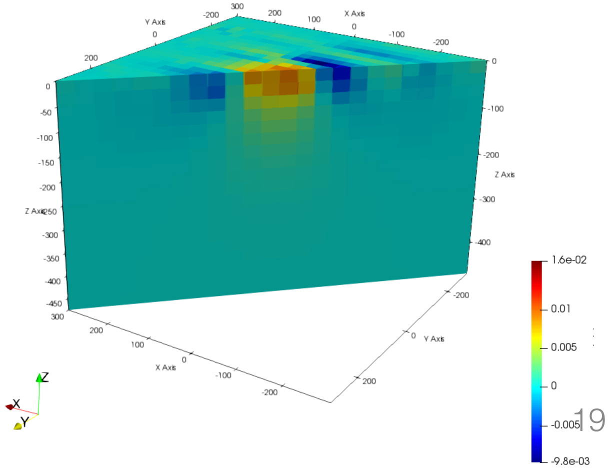
Inversion with sensitivity weighting



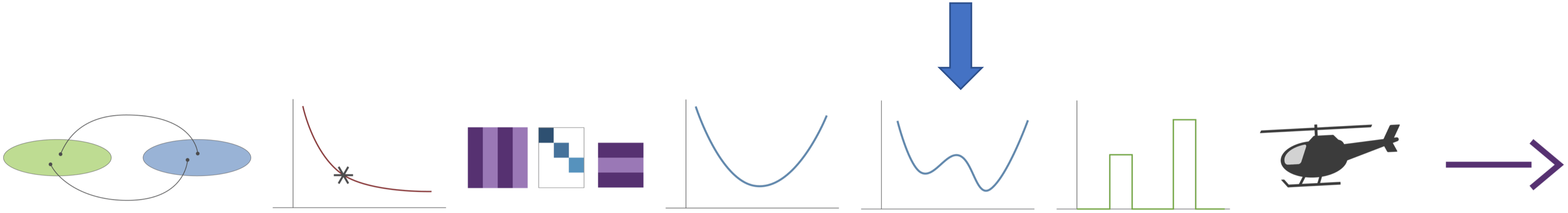
True model



Without sensitivity weight



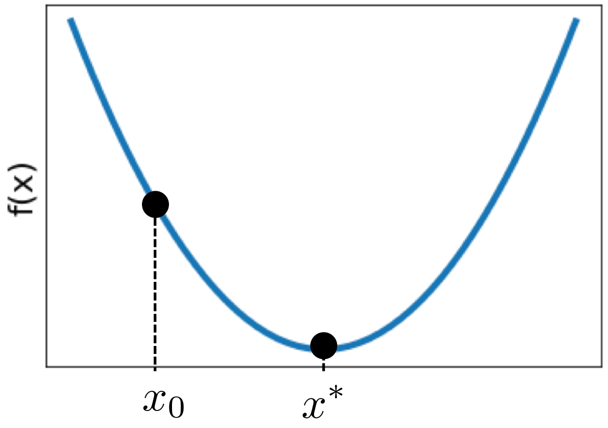
Next up …



Overview Tikhonov SVD Linear Nonlinear Lp-norms Field scale The future

Summary: Newton's method

Linear

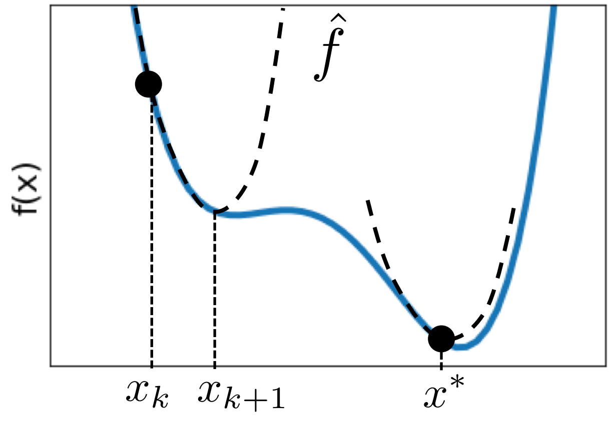


$$f''(x)\delta x = -f'(x)$$

$$x^* = -\frac{f'(x)}{f''(x)}$$

Solution in one step

Non-linear



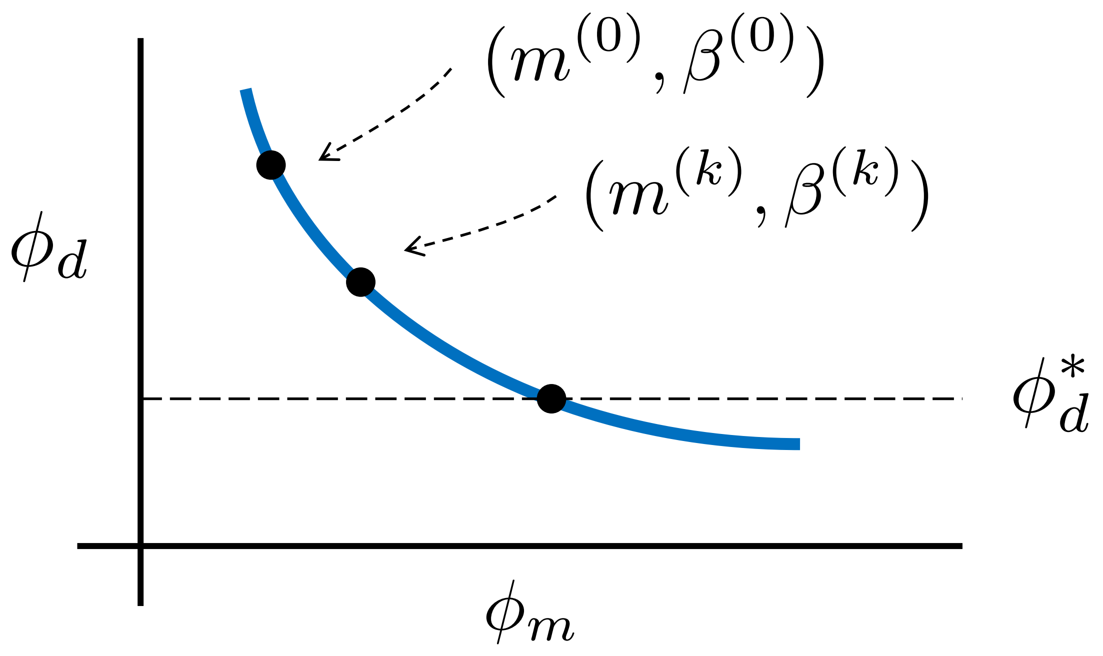
$$f''(x_k)\delta x = -f'(x_k)$$

$$\delta x = -\frac{f'(x_k)}{f''(x_k)}$$

$$x_{k+1} = x_k + \alpha \delta x \qquad \alpha < 1$$

Iterate to convergence

General algorithm:



$$\text{minimize } \phi = \phi_d + \beta\phi_m$$

Initialize $m^{(0)}, \beta^{(0)}$

until convergence

$$H\delta m = -g$$

$$m^{(k+1)} = m^{(k)} + \alpha\delta m \;\; \text{(line search)}$$

$$\beta^{(k+1)} = \frac{\beta^{(k)}}{\gamma} \quad \text{(cooling)}$$

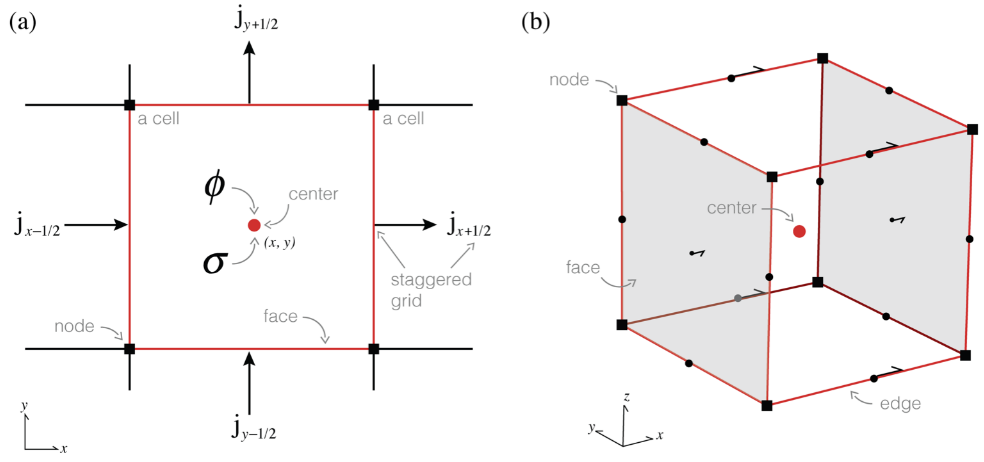
Many variants:

- Solving system
- Cooling rate
- ....

$$(J^T J + \beta)\delta m = -(J^T \delta d + \beta m)$$

$$\delta d = \mathcal{F}[m] - d^{obs}$$

Forward problem



Continuous

$$\nabla \cdot \sigma \nabla V = I\delta(r) = q$$

$$\vec{j} = \sigma \vec{e}$$

$$\vec{e} = -\nabla V$$

$$\vec{j} \cdot \hat{n} = 0 \text{ at boundary}$$

Discrete (FV)

$$\mathbf{G}^T \mathbf{M}_\sigma \mathbf{G} \mathbf{u} = \mathbf{q}$$

generically,

$$\mathbf{A}(\mathbf{m})\mathbf{u} = \mathbf{q}$$

$\mathbf{G}$: gradient matrix
$\mathbf{M}_\sigma$: conductivity inner product matrix
$\mathbf{q}$: source term

Examine sensitivity

$$J = -PA(m)^{-1}\mathcal{G}(m,u)$$

P: projection matrix
$A(m)^{-1}$: forward modelling

- For DC problem

$$\mathbf{A} = \mathbf{G}^T\mathbf{M}_\sigma\mathbf{G}$$

$$\mathbf{M}_\sigma = \mathbf{diag}\big(\mathbf{A}_v^T\mathbf{diag}(\boldsymbol{\sigma}\odot\mathbf{vol})\big)$$

$$\mathcal{G}(m,u) = \nabla_m\big[A(m)u(m)_{\text{fixed}}\big] = \mathbf{G}^T\mathbf{diag}\big(\mathbf{Gu}\big)\mathbf{Av}^T\mathbf{diag}(\mathbf{vol})$$

$\mathbf{Av}$: averaging matrix
$\mathbf{vol}$: volume of cells

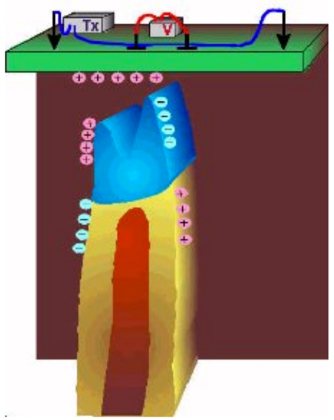
Putting everything together

- Solve $(J^T J + \beta)\delta m = -(J^T \delta d + \beta m)$

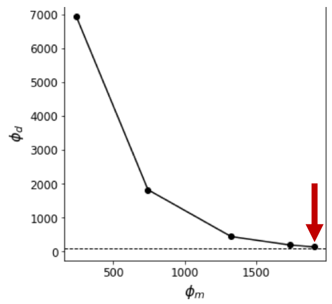
- Using CG we need: $J^T y \qquad y \in \mathbb{R}^N$
  $$Jv \qquad v \in \mathbb{R}^M$$

- but, $J = -PA(m)^{-1}\mathcal{G}(m,u)$ all sparse matrices

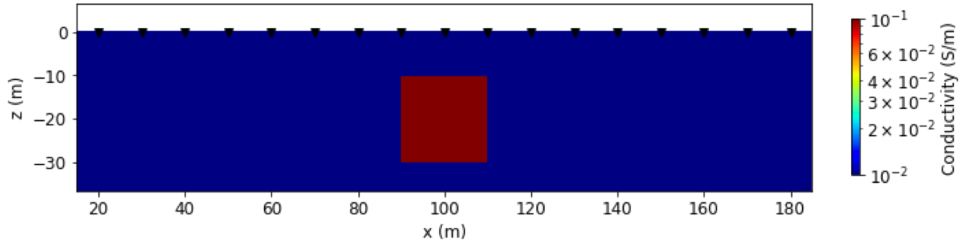
2D DC inversion



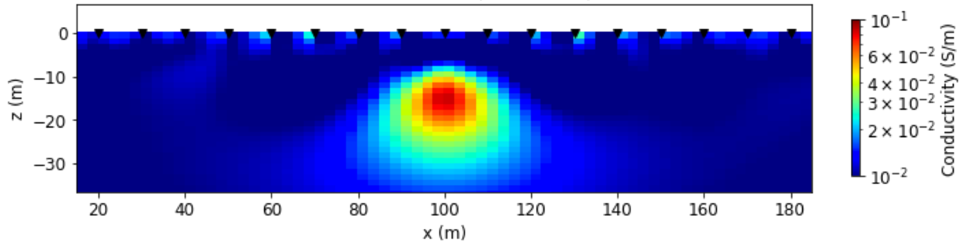
Tikhonov curve



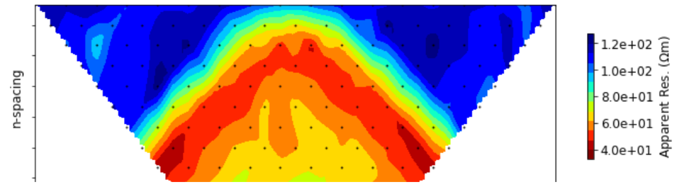
True model



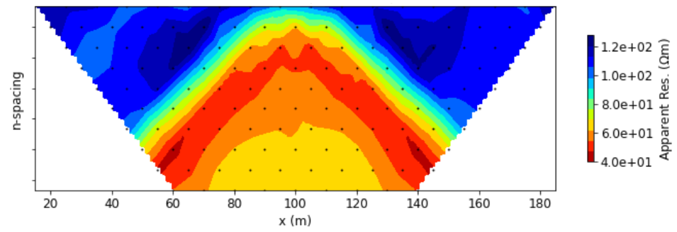
Predicted model



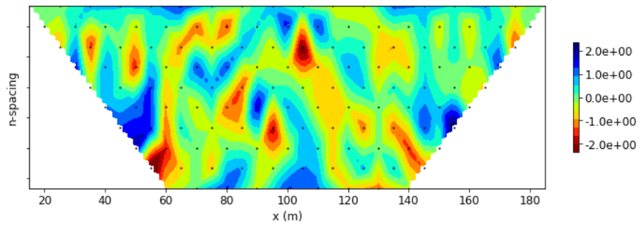
Observed data



Predicted data



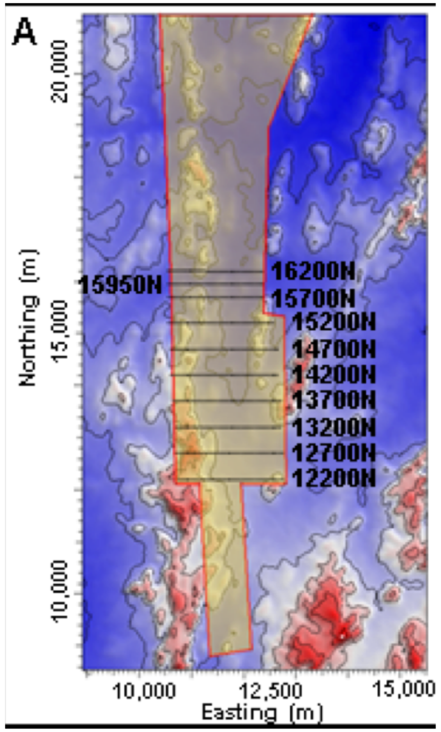
Normalized misfit



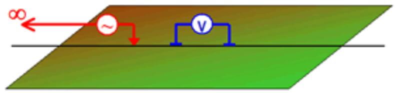
3D DC survey and data

- Eight survey lines
- Two survey configurations.

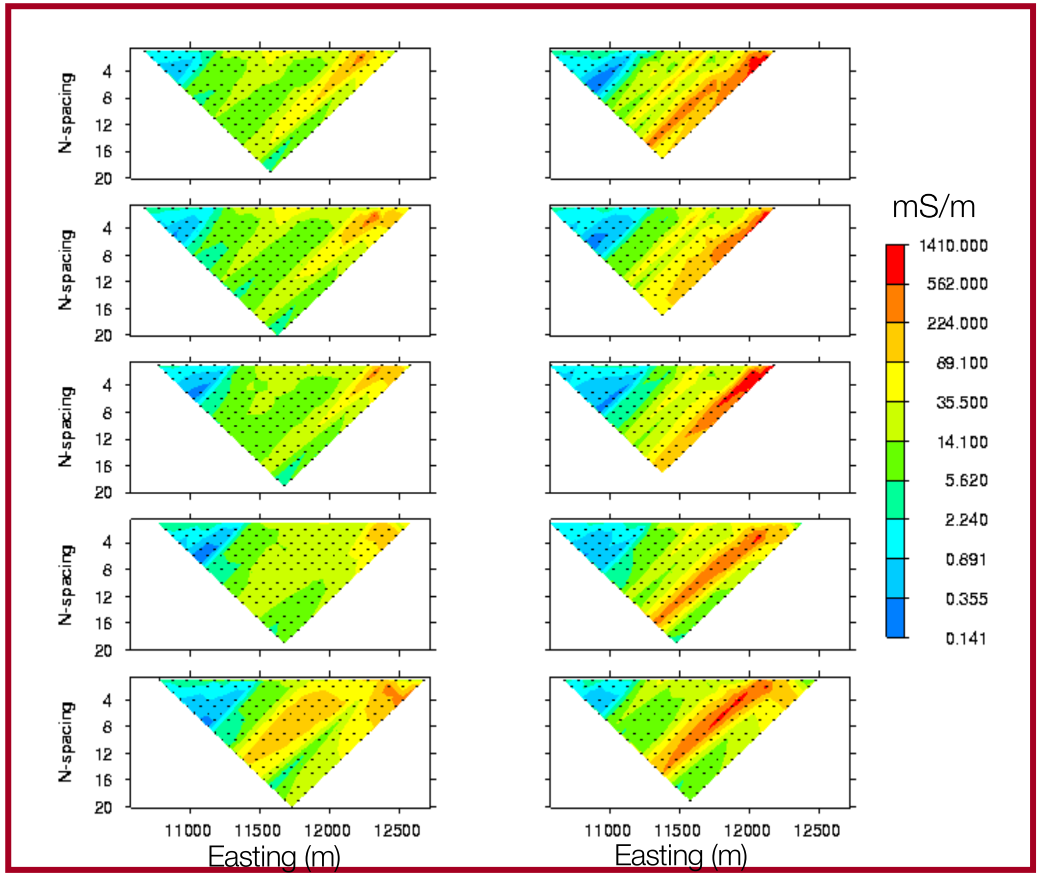
Surface topography



Data set #1:

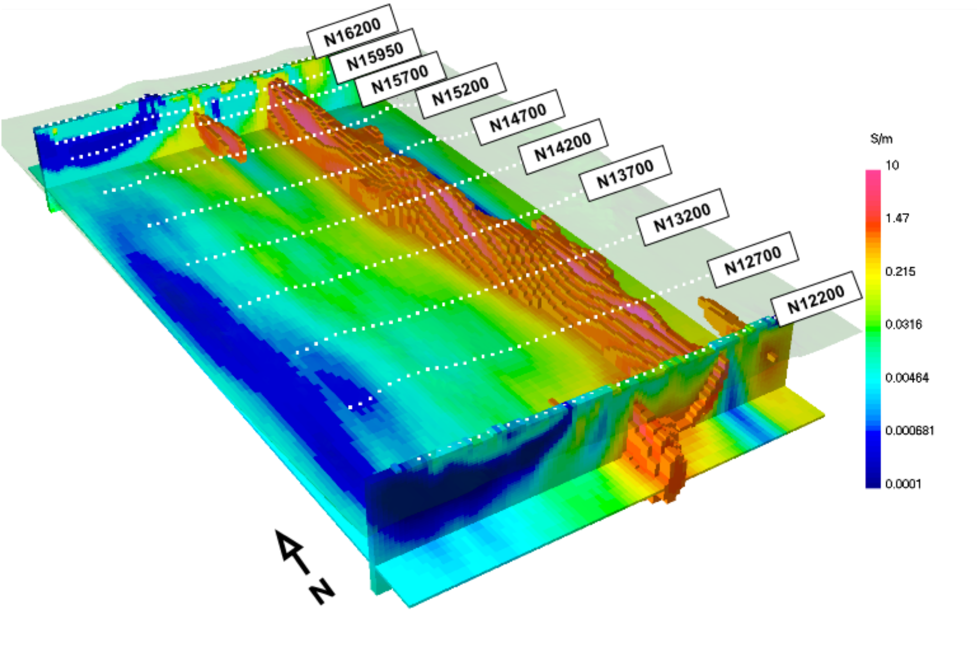


Apparent resistivity, pole - dipole.

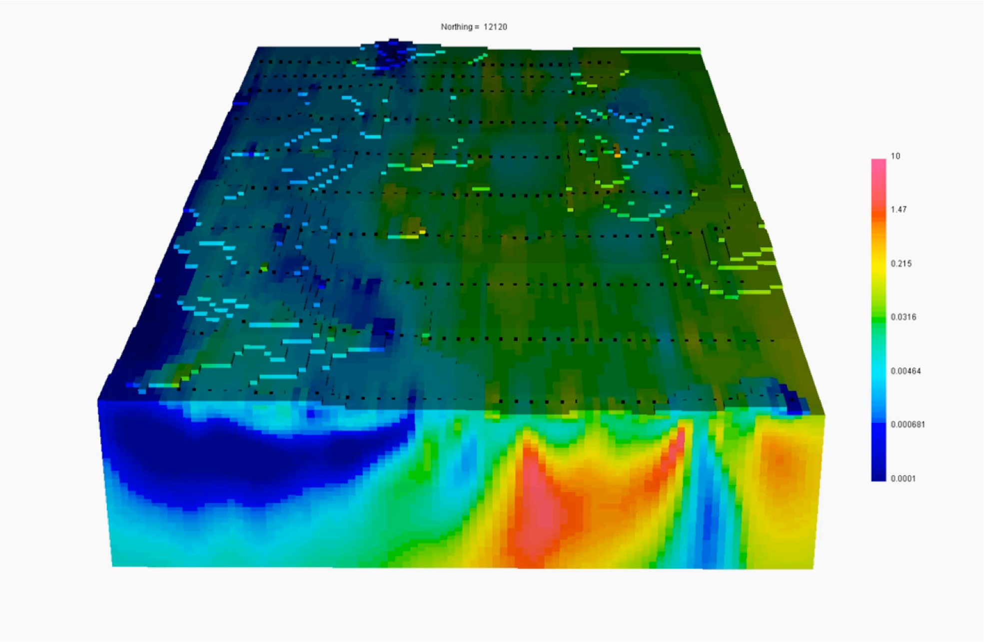


3D DC inversion

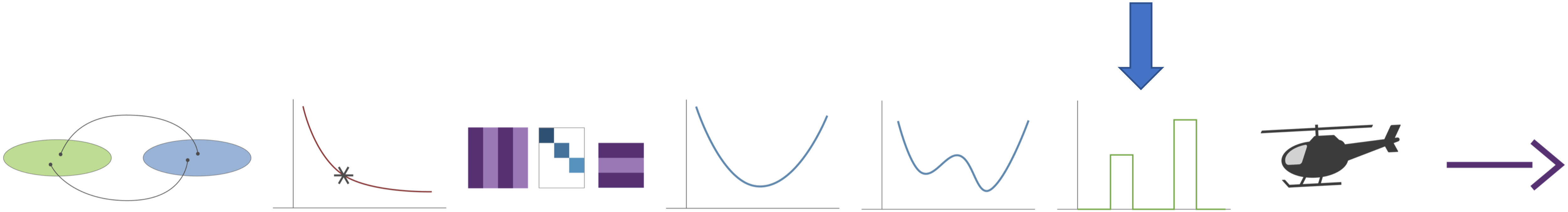
3D resistivity model



Animation



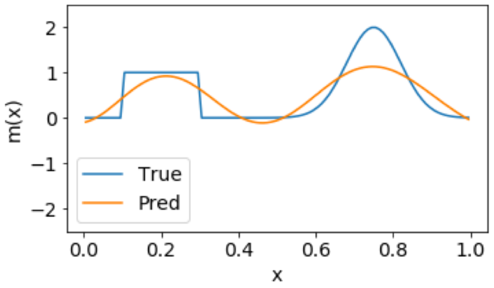
Next up …



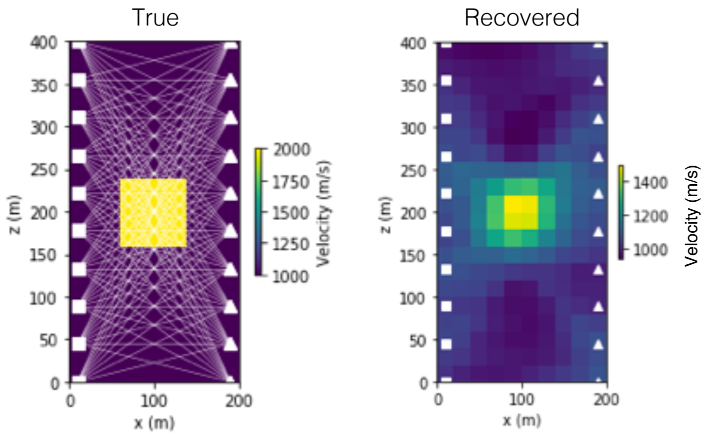
Overview Tikhonov SVD Linear Nonlinear Lp-norms Field scale The future

L_p-norms

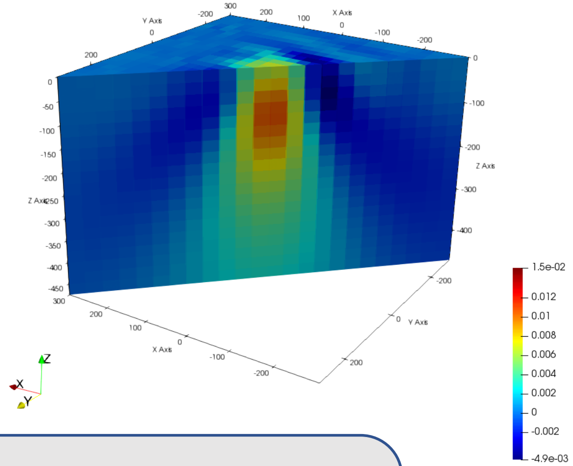
1D



2D



3D



L_2 norms generate smooth boundaries, can have artefacts, can violate physics

General L_p norms

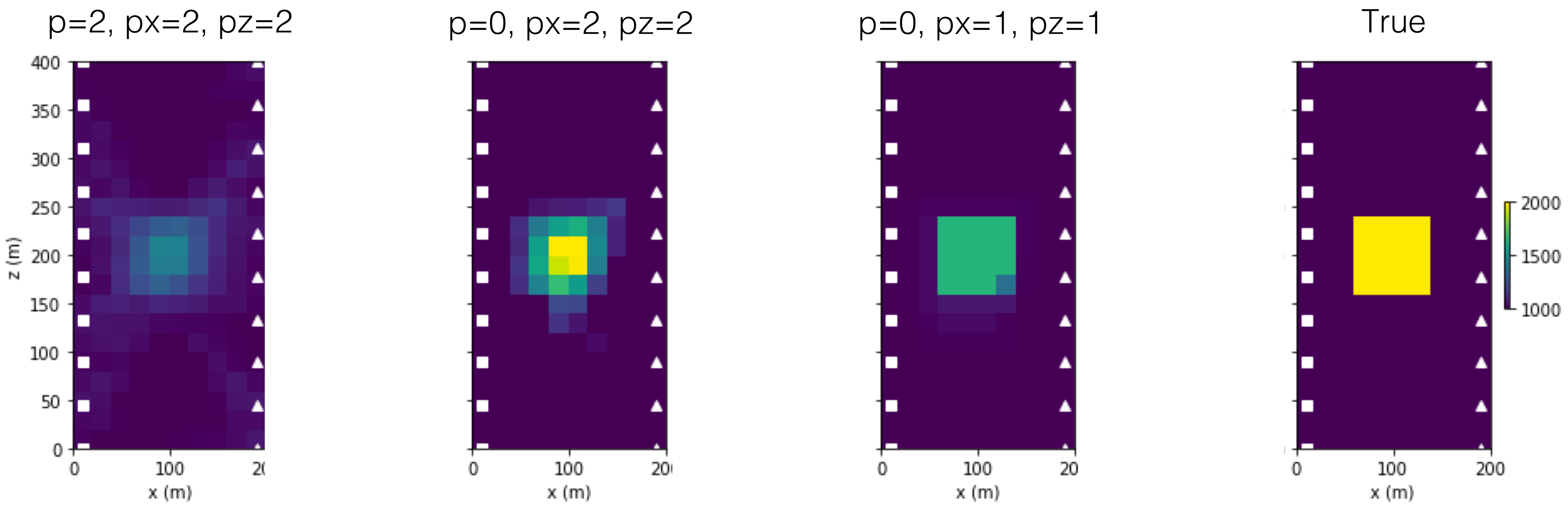
$$\phi_m = \sum_{j=s,x,y,z} \alpha_j \int_v |f_j(m)|^{p_j} dv \qquad f_s(m) = m \quad f_x(m) = \frac{dm}{dx} \quad f_y(m) = \frac{dm}{dy} \quad f_z(m) = \frac{dm}{dz}$$

$$0 \leq p_j \leq 2$$

2D crosswell example

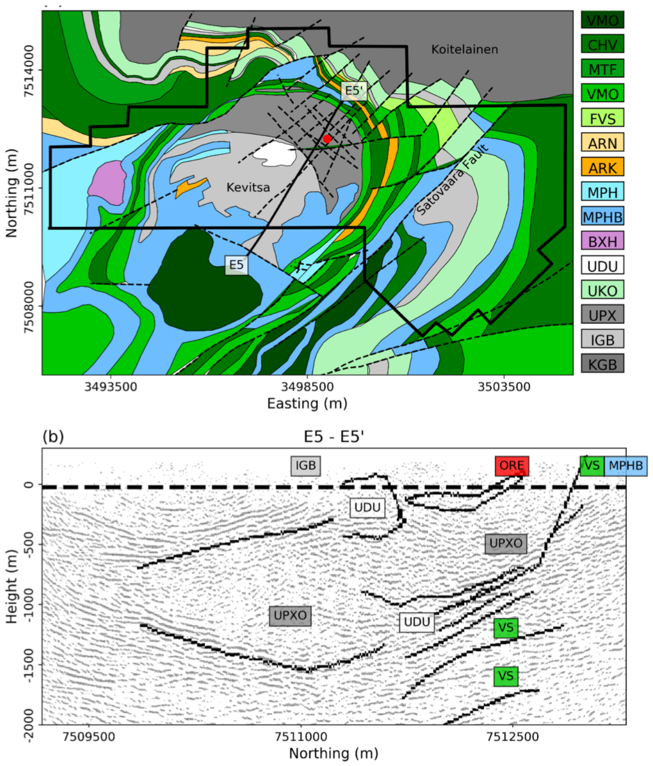
$$\phi_m = \alpha_s \int_v |\mathbf{m} - \mathbf{m}_{\text{ref}}|^{p_s} dv + \alpha_x \int_v \left| \frac{d}{dx}(\mathbf{m} - \mathbf{m}_{\text{ref}}) \right|^{p_x} dx + \alpha_z \int_v \left| \frac{d}{dz}(\mathbf{m} - \mathbf{m}_{\text{ref}}) \right|^{p_z} dz$$

Each term has three adjustable parameters: (α, p, ϵ_m)

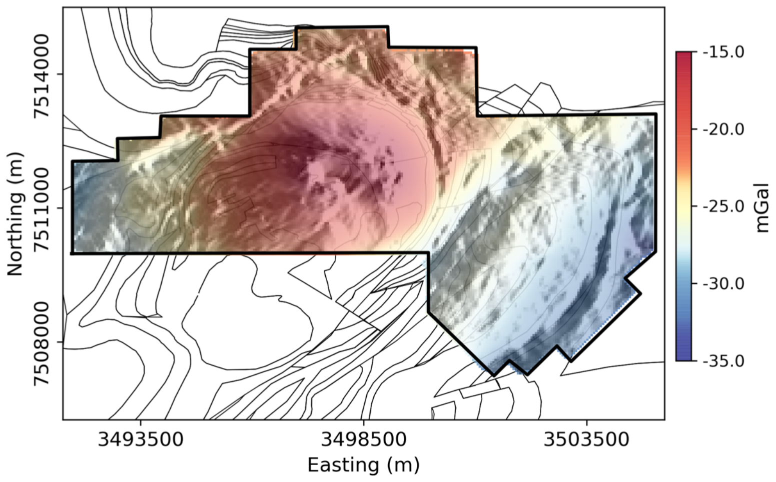


Kevitsa

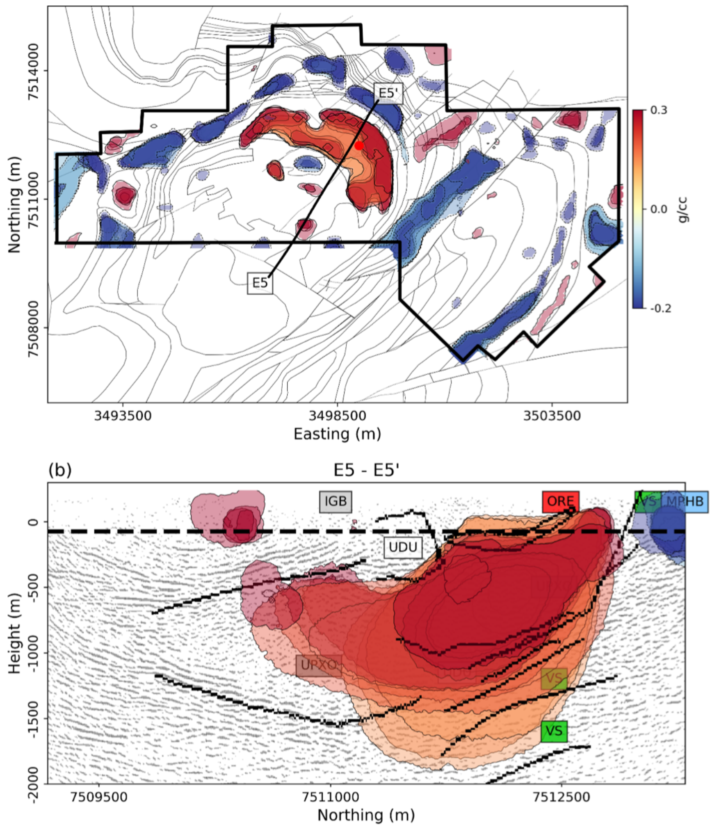
Geology



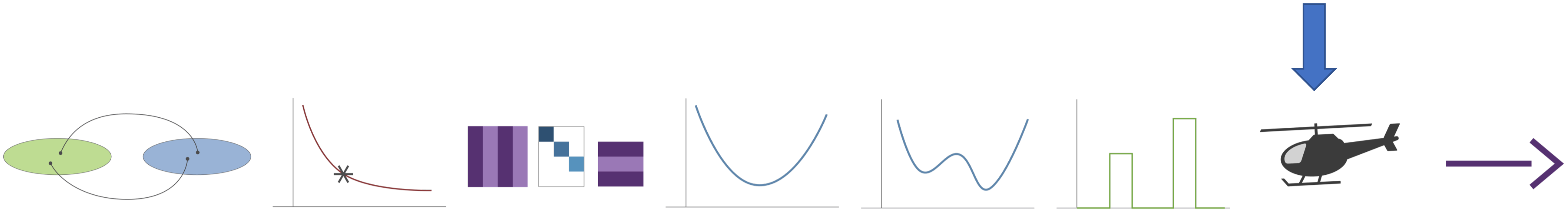
Gravity data



Density model



Next up …



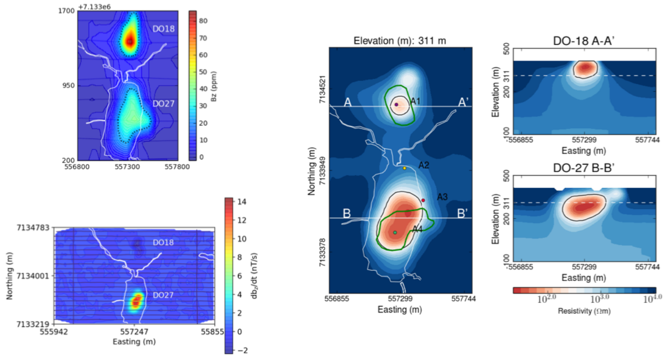
Overview
Tikhonov
SVD
Linear
Nonlinear
Lp-norms
Field scale
The future

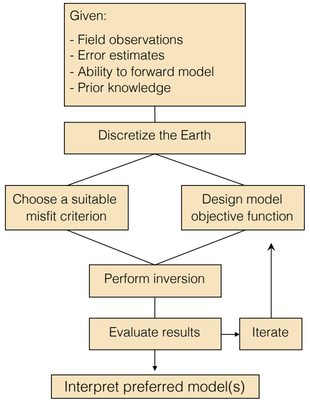
Large-Scale Inversions

Same methodology as small-scale

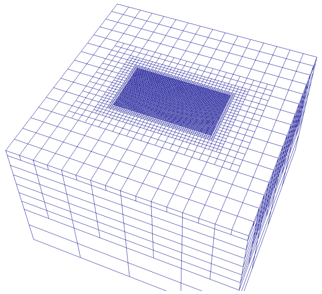
Advances in scientific computing

- Direct solvers (factor Maxwell operator)
- Semi-structured meshes (OcTree)
- Separate forward and inverse meshes
- Handling the sensitivity matrix
- Access to multi-cores

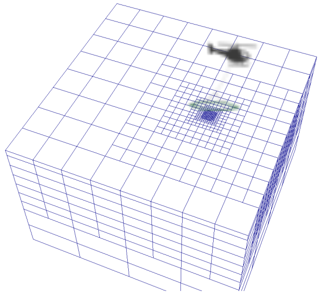




Global mesh

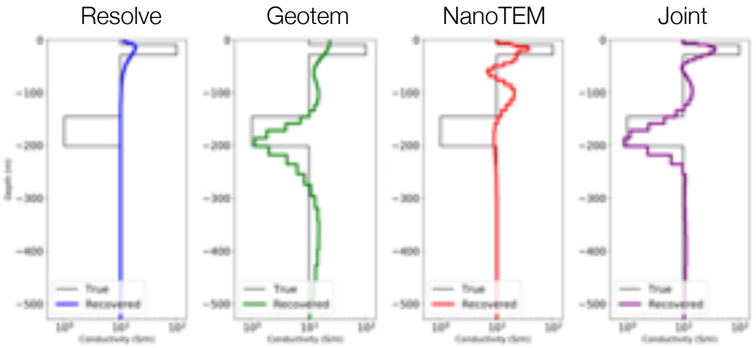


Local mesh

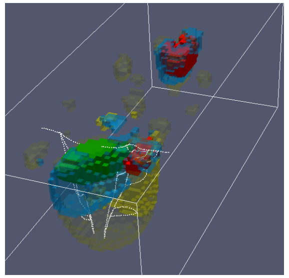


Joint inversion and Petrophysics

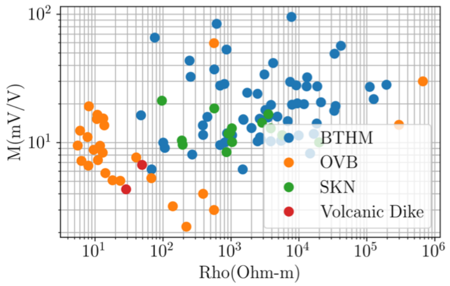
- Joint inversion

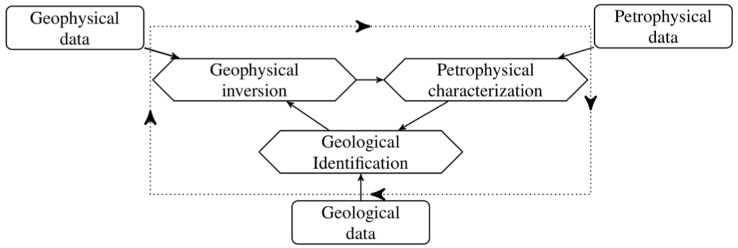


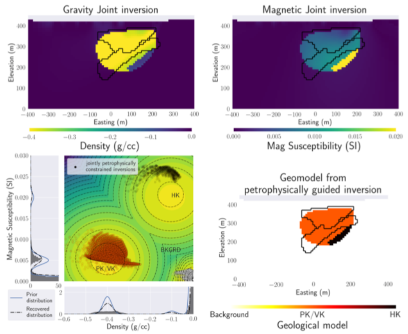
- Multiple surveys and properties



- PGI (petrophysically guided inversion)

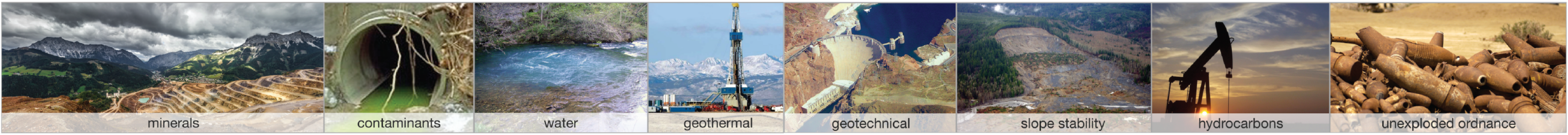






Summary

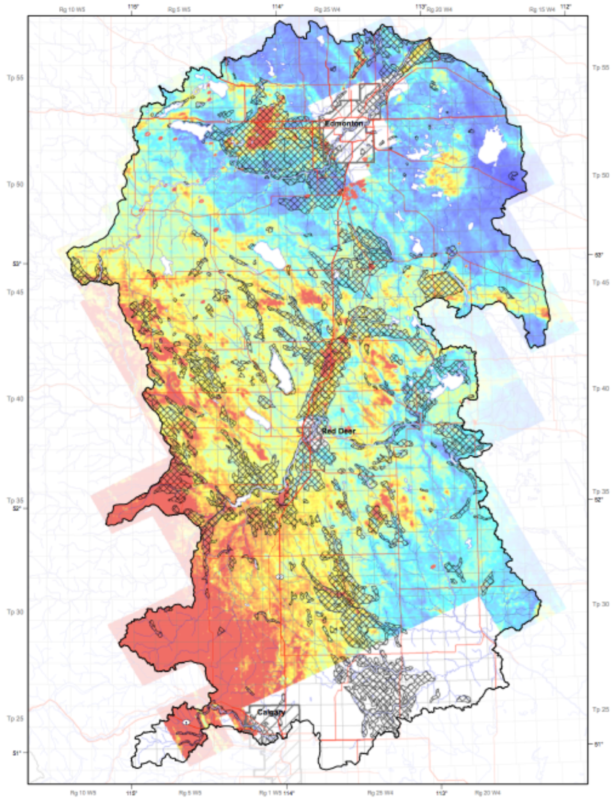
- Basic understanding about inverse problem
- Next is application:
  - Large-scale
  - Multidisciplinary
  - …
- What is involved and where does inversion play a role?



Future problem: groundwater

- Consider Edmonton-Calgary Corridor (ECC)
  - Large scale problem
  - But conductivity itself is not completely informative
- Questions
  - Where are the aquifers and aquitards?
  - What is the water quality (e.g. arsenic, salt water)?
  - What is the storage capacity, flow rate?
  - How are the aquifers recharged (or not)?
  - Are losing water or gaining water? (water balance)
- Stake holder
  - Farmers
  - Government and industry
  - Public
  - Hydrogeologists, engineers, geophysicists

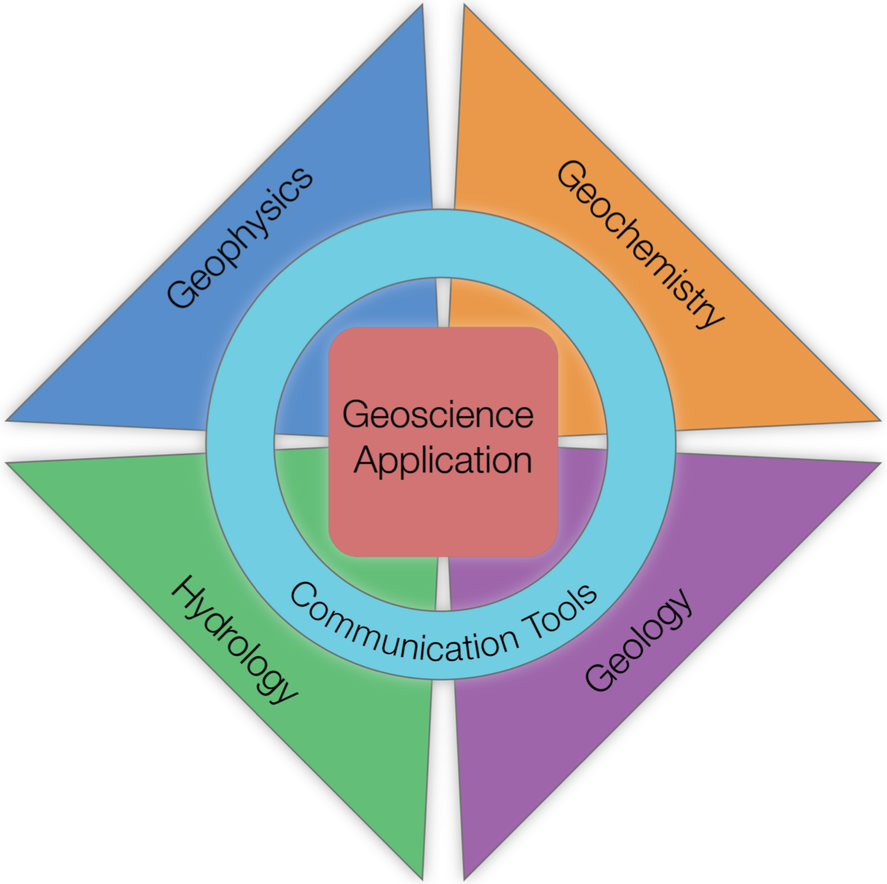
AEM resistivity Alberta Corridor



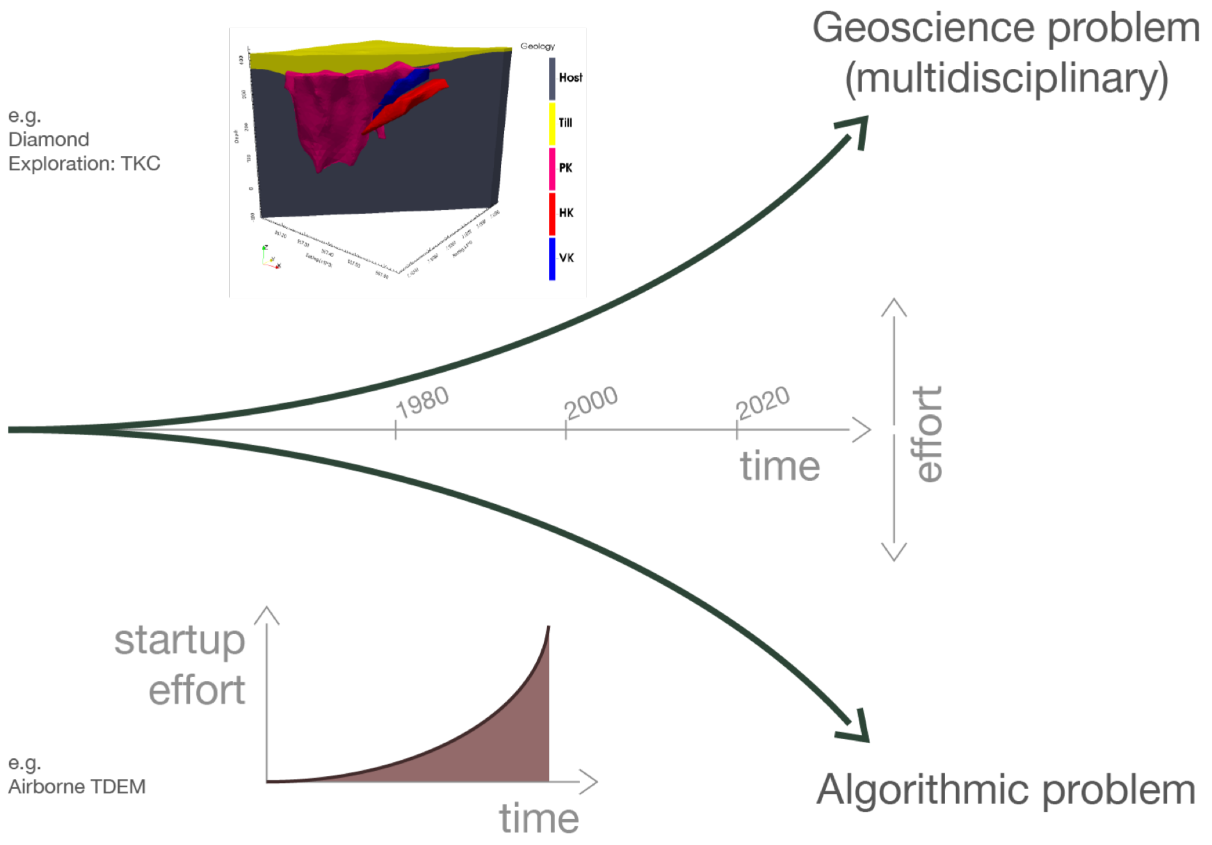
Baker (2011)

Next Generation of Geoscience Problems

- Multi-disciplinary
- Geophysics has a support role
- Inversion needed in multiple places
- Interaction is needed



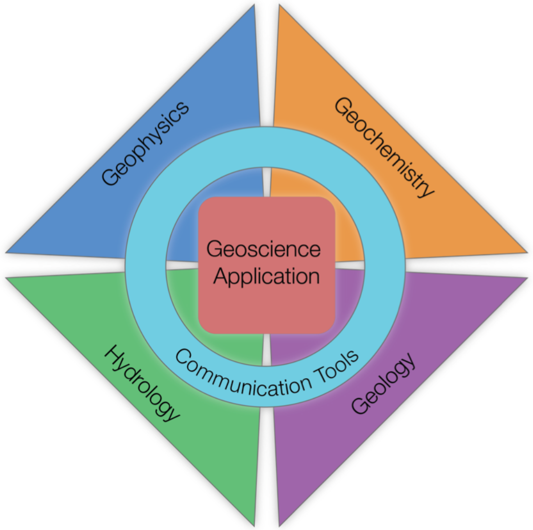
Research challenges keep increasing



Next Generation of Geoscience Problems

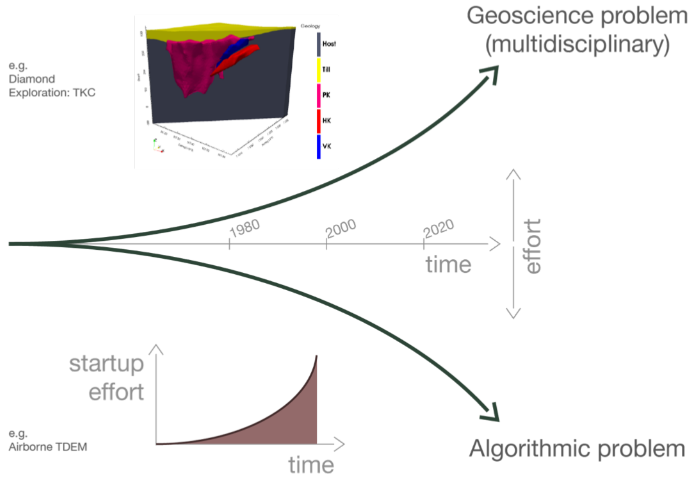
- How to extract information about physical properties from data

- How to integrate that to help solve the geoscience problem?



- Who are the researchers?
  - Industry
  - Academia

- Tools for cooperation: Open Source



Open Source

Open source communities already doing this:

- Collaboration
  - Development of software
  - Implementing and applying
- Development practices
  - Shared repository
  - Version control
  - Automated testing
  - User and developer documentation
  - Peer review of code
  - Issue tracking
  - Attribution for contributors
  - Licensing

astropy
A Community Python Library for Astronomy
253 contributors

scikit learn
machine learning in Python
1,095 contributors

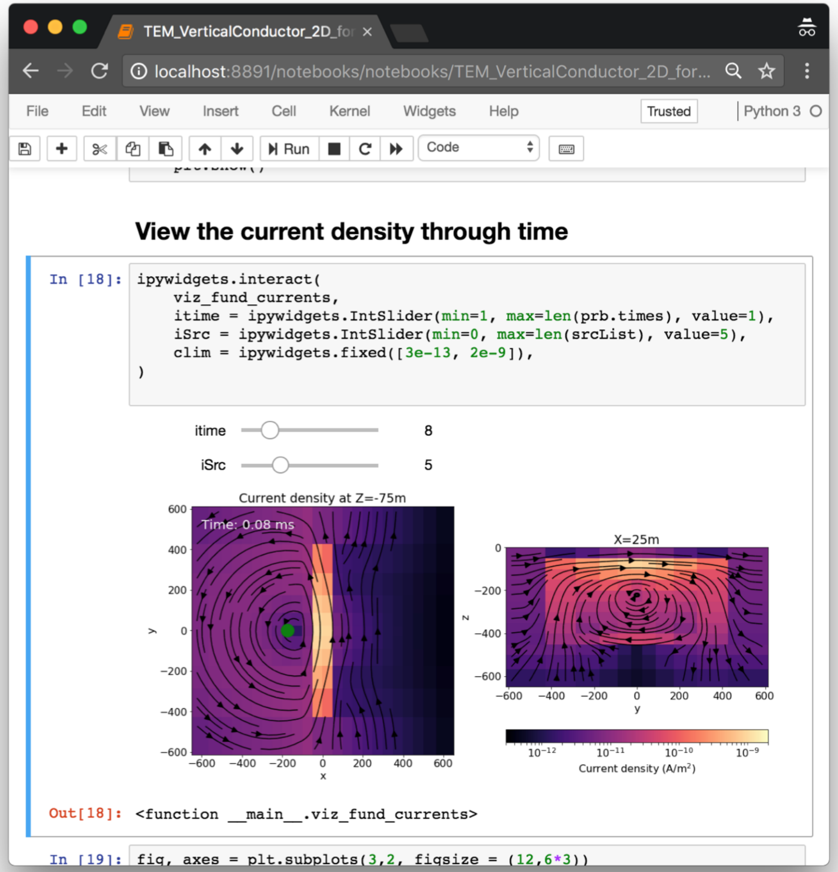
SciPy
Scientific Computing with Python
612 contributors

Sampling of modern open-source projects

- For EM
  - empymod
  - jInv
  - Geoscience Australia
  - pyGIMLi
  - Fatiando
  - SimPEG
  - …
- They differ in objectives, capabilities, structure, interactivity, license, and language

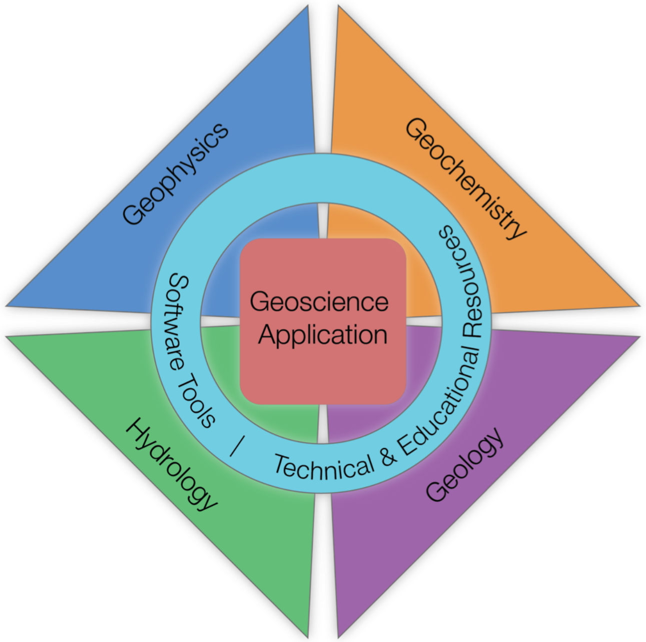
simpeg

- Modular framework for simulation and inversion of geophysical data
  - gravity, magnetics, vadose flow, DC/IP, FDEM, TDEM
- Open source
- Written in Python
- Specific to electromagnetics
  - Quasi-static Maxwell
  - Tensor, OcTree, Curvilinear and Cylindrical meshes
  - Easily visualize fields, fluxes, charges

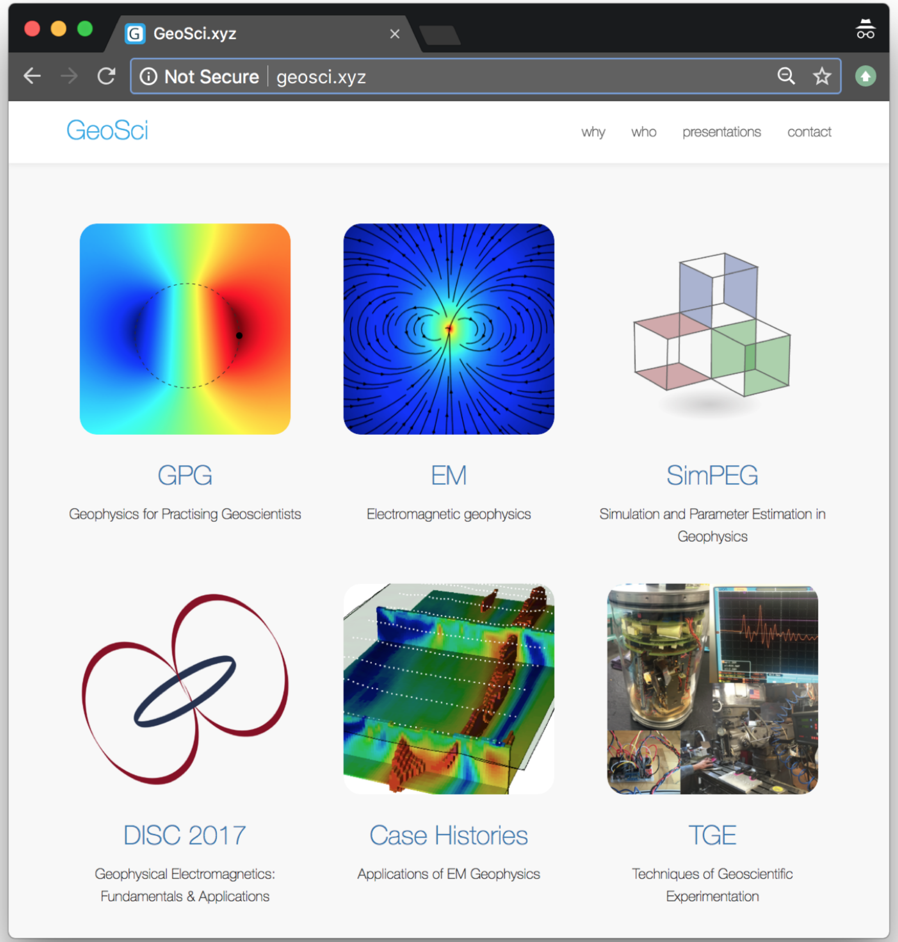


http://simpeg.xyz

GeoSci.xyz



Geophysics
Geochemistry
Hydrology
Geology
Software Tools / Technical & Educational Resources
Geoscience Application



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EM
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DISC 2017
Geophysical Electromagnetics: Fundamentals & Applications
Case Histories
Applications of EM Geophysics
TGE
Techniques of Geoscientific Experimentation

Thank You

Resources

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SimPEG Team