

# Petrophysically and Geologically Guided Geophysical Inversion

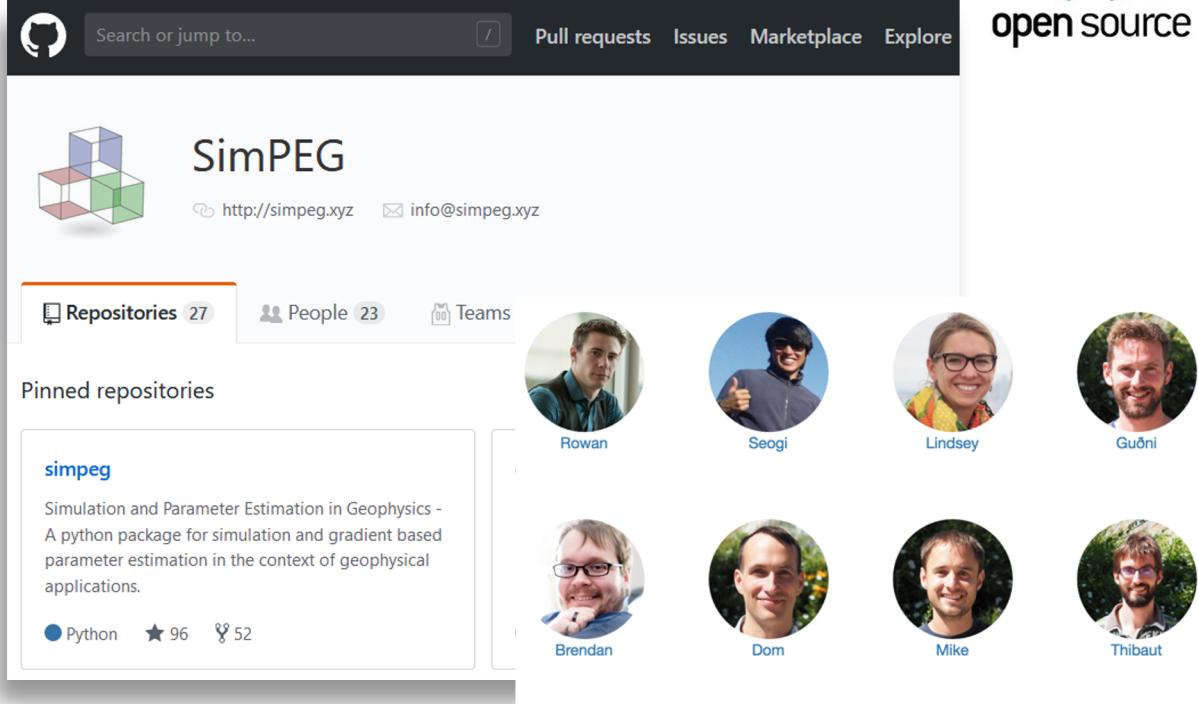
Making inversion more geologic

Thibaut Astic, Douglas Oldenburg  
University of British Columbia,  
Geophysical Inversion Facility (UBC-GIF)



# SimPEG is: open source software simulation inversion in Python

The GitHub interface for the SimPEG repository. At the top, there's a search bar with placeholder text "Search or jump to...", and navigation links for "Pull requests", "Issues", "Marketplace", and "Explore". Below the header, the repository name "SimPEG" is displayed next to a 3D cube icon. It includes a link to the website "http://simpeg.xyz" and an email address "info@simpeg.xyz". A navigation bar below shows "Repositories 27", "People 23", and "Teams". The "Pinned repositories" section features a card for "simpeg", which describes it as a "Simulation and Parameter Estimation in Geophysics - A python package for simulation and gradient based parameter estimation in the context of geophysical applications." It indicates the language is "Python", has 96 stars, and 52 forks. To the right of the repository details, a grid of circular profile pictures and names for the team members: Rowan, Seogi, Lindsey, Guðni, Brendan, Dom, Mike, Thibaut, Adam, Doug, and a "Contributors" icon.



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SimPEG

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Repositories 27 People 23 Teams

Pinned repositories

simpeg

Simulation and Parameter Estimation in Geophysics - A python package for simulation and gradient based parameter estimation in the context of geophysical applications.

Python ★ 96 ⚡ 52

Rowan Seogi Lindsey Guðni

Brendan Dom Mike Thibaut

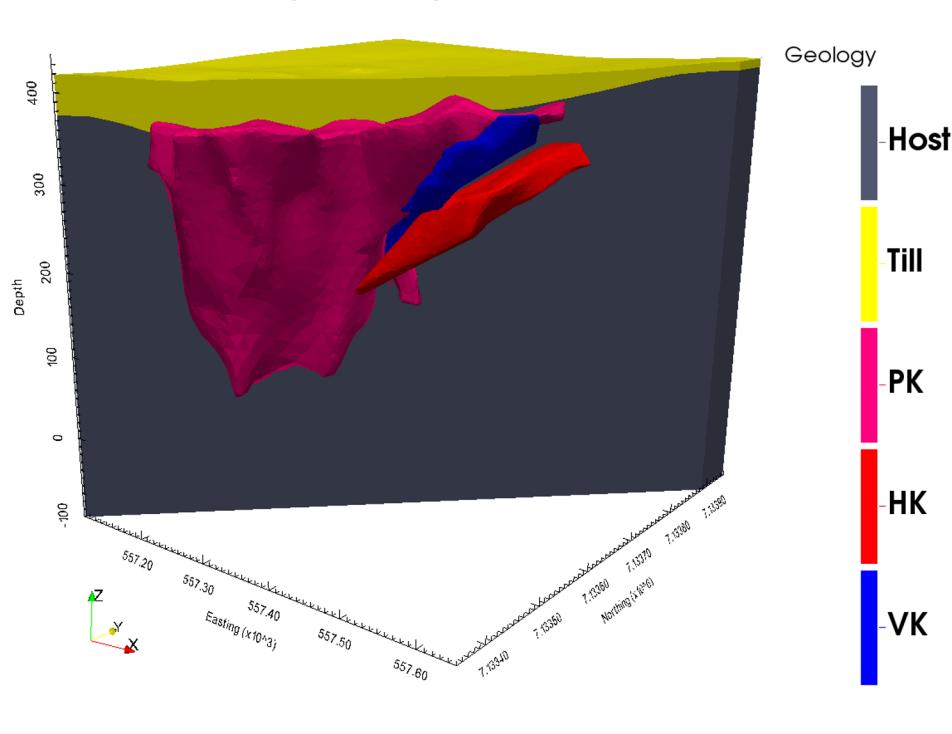
Adam Doug Contributors

# Objective:

Tie together  
geophysical, petrophysical and geological  
information in a single conventional  
geophysical inversion framework

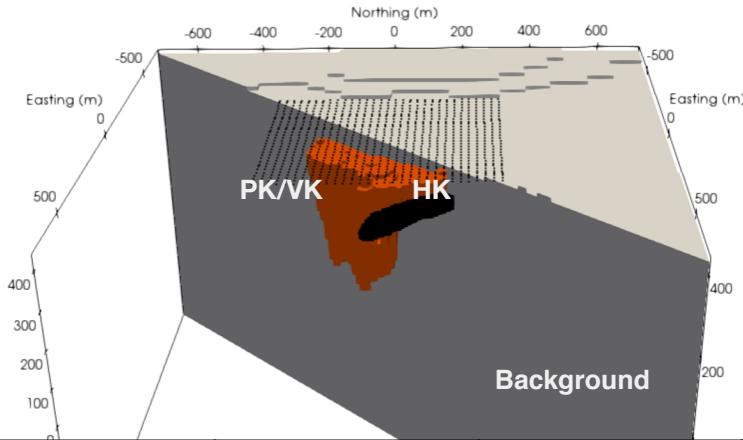


# Motivation Example: Tli Kwi Cho DO-27 kimberlite

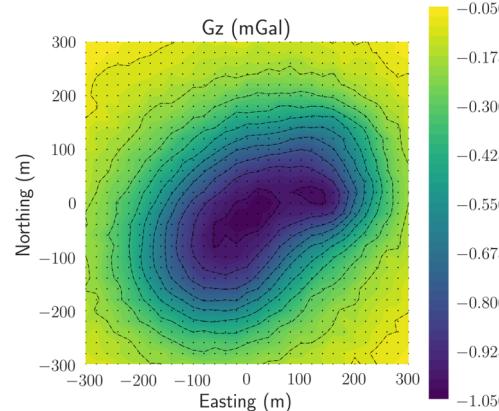


- Diamondiferous kimberlite in the Northwest Territories, Canada
- Well studied:
  - Devriese, S. G. R., K. Davis, and D. W. Oldenburg, 2017, Inversion of airborne geophysics over the do-27/do-18 kimberlites - part 1: Potential fields: Interpretation, 5, T299–T311.

# TKC: Setup

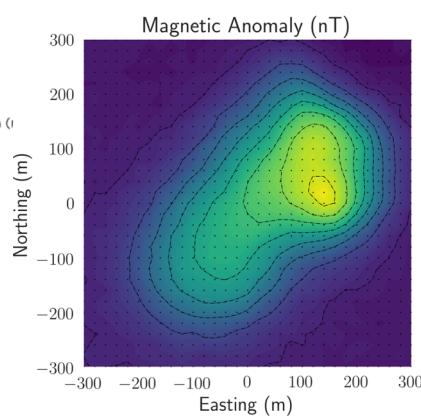


Unit	Density (g/cc)	Mag. Susc. (SI)
BCKGRD	0	0
PK & VK	-0.4	0.005
HK	-0.1	0.02



Gravity inverted model

Inversion code  
GIF



Magnetic inverted model

Inversion code  
GIF  
SEG18  
ANAHEIM, CA



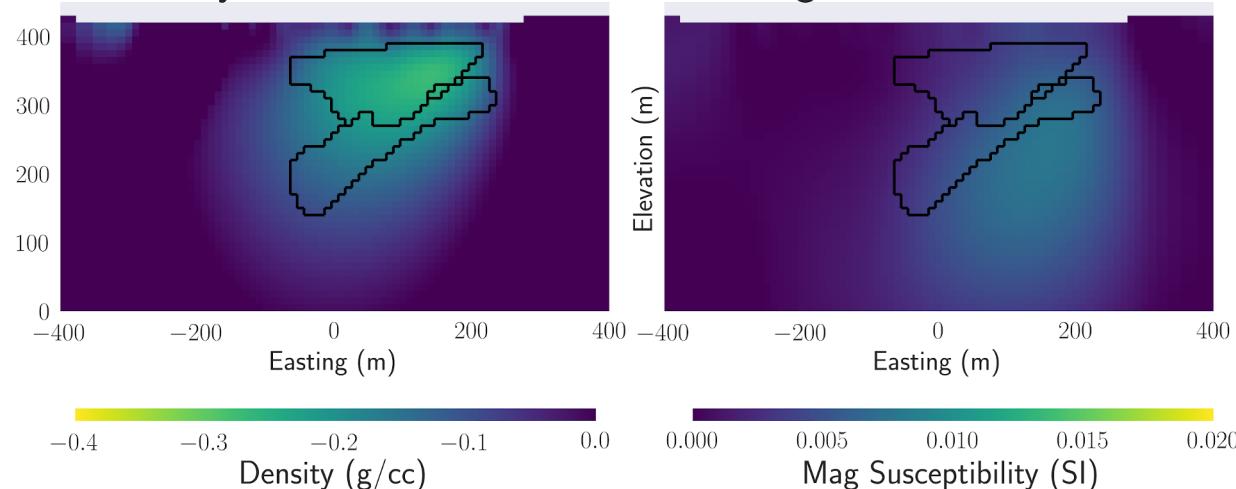
# Potential Field Inversion

Minimization of an objective-function:

$$\Phi(\mathbf{m}) = \Phi_d(\mathbf{m}) + \beta(\alpha_s \Phi_s(\mathbf{m}) + \alpha_{smooth} \Phi_{smooth}(\mathbf{m}))$$

With:  $\Phi_d(\mathbf{m}) = \frac{1}{2} \|\mathbf{W}_d(\mathcal{F}(\mathbf{m}) - \mathbf{d}_{obs})\|_2^2$  and  $\Phi_s(\mathbf{m}) = \frac{1}{2} \|\mathbf{W}_s(\mathbf{m} - \mathbf{m}_{ref})\|_2^2$

Gravity smooth inversion    Mag smooth inversion

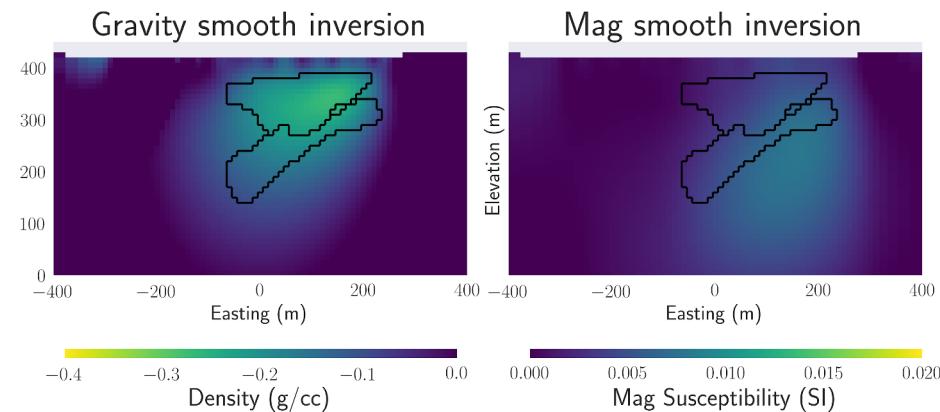


# Gaussian representation of physical properties

Minimization of an objective-function:

$$\Phi(\mathbf{m}) = \Phi_d(\mathbf{m}) + \beta(\alpha_s \Phi_s(\mathbf{m}) + \alpha_{smooth} \Phi_{smooth}(\mathbf{m}))$$

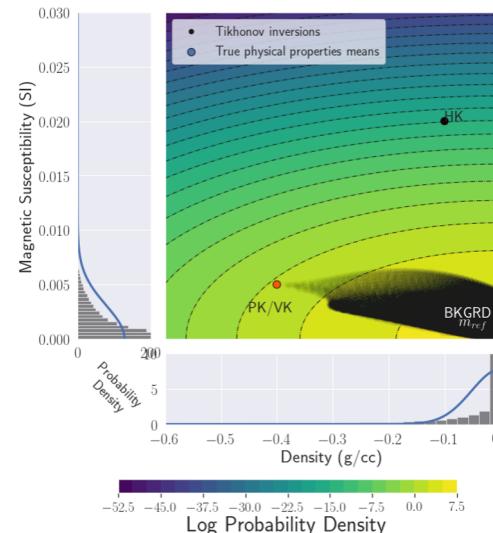
$$\Phi_s(\mathbf{m}) = \frac{1}{2} \|W_s(\mathbf{m} - \mathbf{m}_{ref})\|_2^2$$



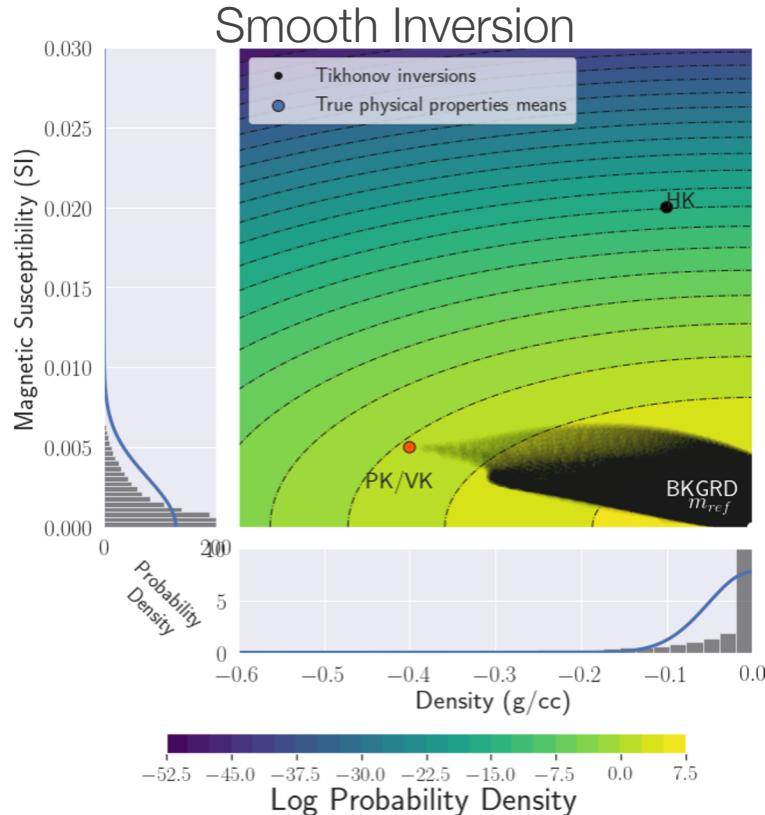
Maximization of a Posterior Distribution:

$$\mathcal{P}(\mathbf{m}|\mathbf{d}_{obs}) \propto \mathcal{P}(\mathbf{d}_{obs}|\mathbf{m}) P_{small}(\mathbf{m}) \mathcal{P}_{smooth}(\mathbf{m})$$

$$P_{small}(\mathbf{m}) \propto \mathcal{N}(\mathbf{m}|\mathbf{m}_{ref}, (W_s^T W_s)^{-1})^{\beta \alpha_s}$$



# Gaussian representation of physical properties



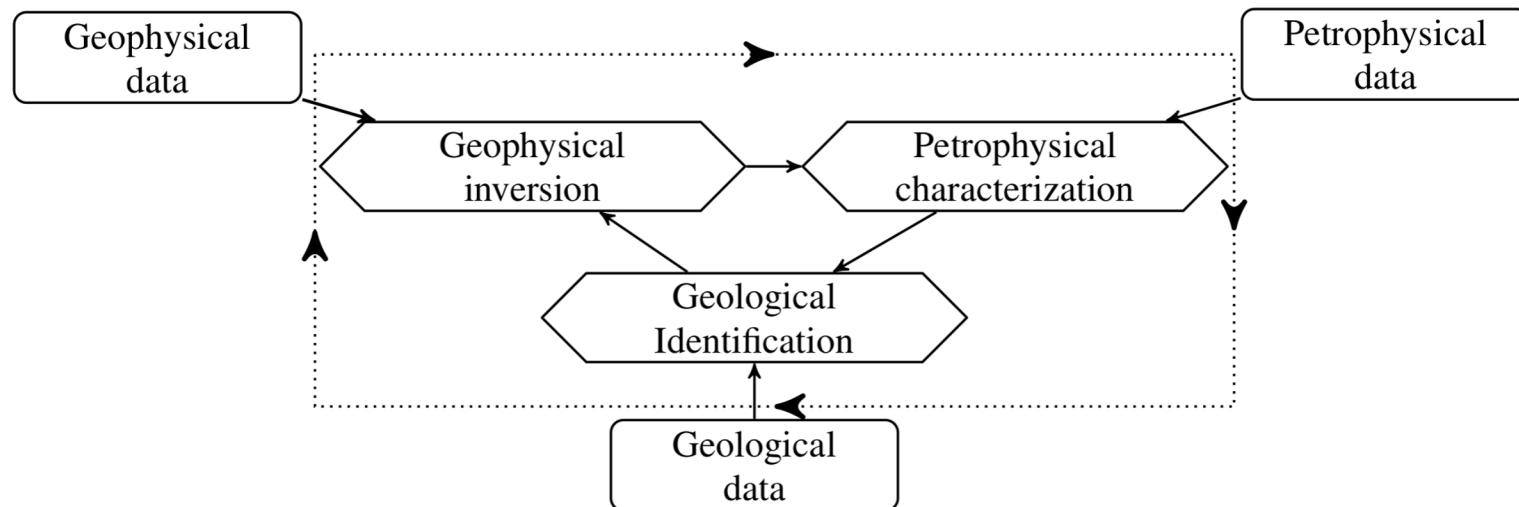
Tikhonov inversion assumes  
a Gaussian distribution  
around the reference model

$$\Phi_s(\mathbf{m}) = \frac{1}{2} \|W_s(\mathbf{m} - \mathbf{m}_{ref})\|_2^2$$

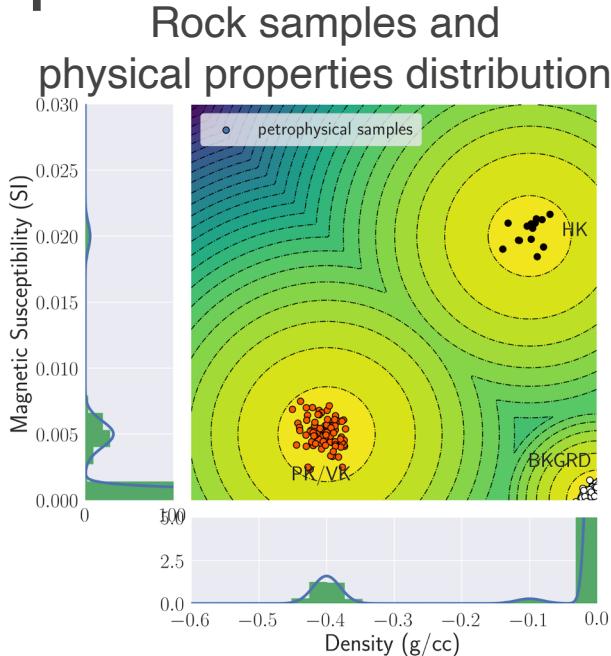
$\Leftrightarrow$

$$\mathcal{P}_{small}(\mathbf{m}) \propto \mathcal{N}(\mathbf{m} | \mathbf{m}_{ref}, (W_s^T W_s)^{-1})$$

# Linking Geophysics, Petrophysics and Geology



# Gaussian mixture representation of physical properties

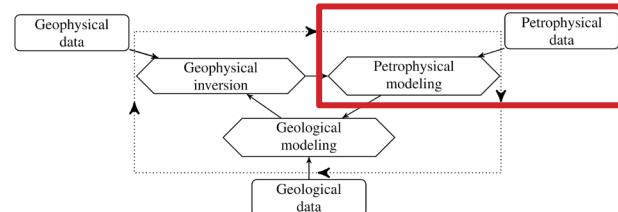


Can we recover the physical properties distribution in our inversions?

Gaussian mixture representation:

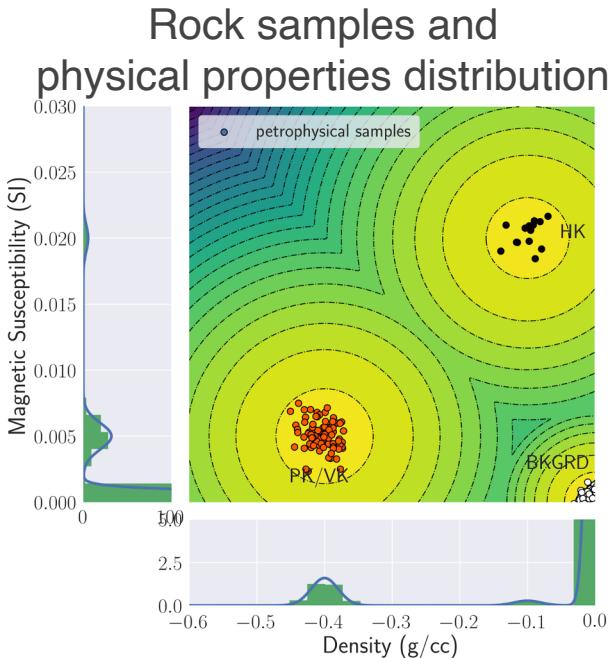
- Physical properties as probability distribution
- One Gaussian = One rock unit

$$\mathcal{M}(\mathbf{m}|\Theta) = \sum_{j=1}^c \mathcal{P}(\mathbf{z} = j) \mathcal{N}(\mathbf{m}|\mu_j, \Sigma_j)$$



# Gaussian mixture prior

- Include global and local parameters



Can we recover the physical properties distribution in our inversions?

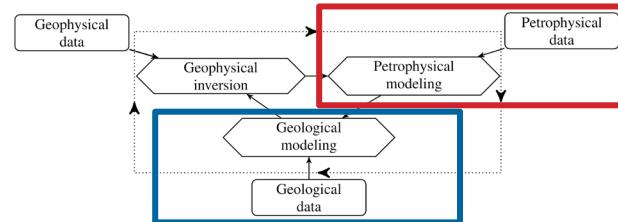
Number of expected units

$$\mathcal{P}_{small}(m|\Theta) = \prod_{i=1}^n \sum_{j=1}^c p(z_i = j) \mathcal{N}(\mathbf{m}_i | \mu_j, \text{diag}(\mathbf{w}_{ij}^{-2}) \Sigma_j)$$

Geological information

reference model

Depth/Sensitivity weighting



# Petrophysically guided inversion scheme

At each iteration:

- Update  $\mathbf{m}_{ref}$  and  $W_s$  according to  $\Theta$

$$\Phi_{small}(m) = \frac{1}{2} \sum_{i=1}^n \|W_s(\Theta, z_i)(\mathbf{m}_i - \mathbf{m}_{ref}(\Theta, z_i))\|_2^2$$

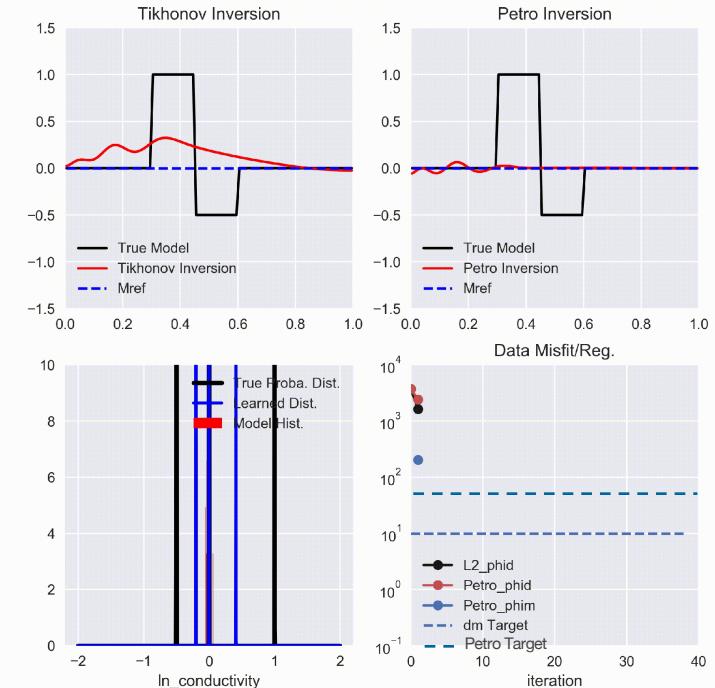
with:

$$\mathbf{m}_{ref}(\Theta, z_i) = \mu_{z_i}$$

$$W_s(\Theta, z_i) = \text{diag}(\mathbf{w}_{i,z_i}) \Sigma_{z_i}^{-1/2}$$

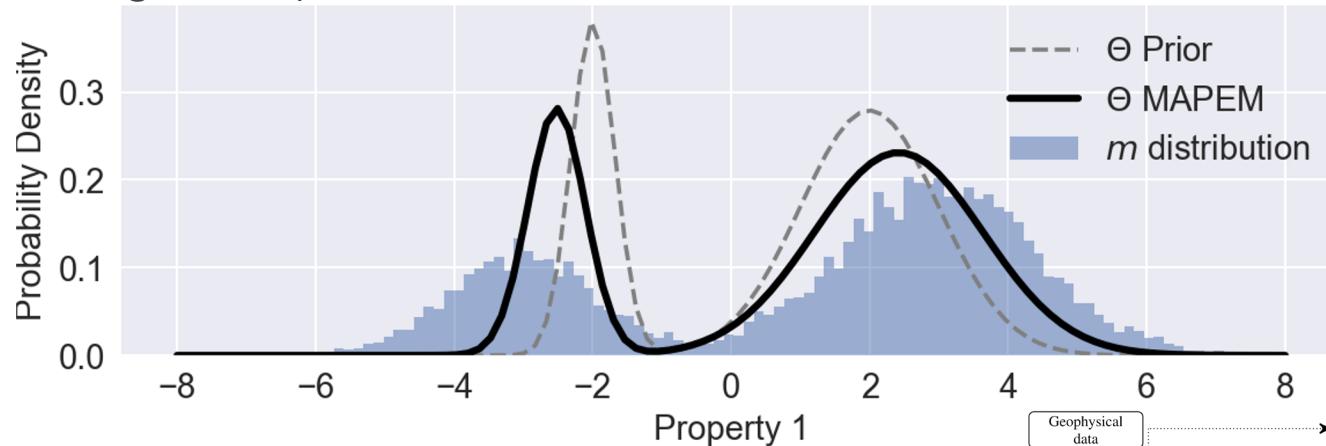
$$\text{and } z_i = \underset{z}{\operatorname{argmax}} p(z) \mathcal{N}(\mathbf{m}_i | \mu_z, \text{diag}(\mathbf{w}_{i,z}^{-2}) \Sigma_z)$$

Target misfits:  $\Phi_d^* = \frac{\#d}{2}$      $\Phi_{small}^* = \frac{n}{2}$

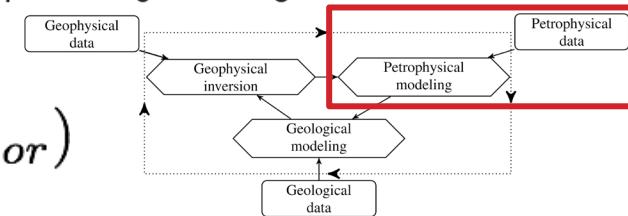


# Updating the petrophysical distribution

At each iteration, we learn a new physical properties distribution  $\Theta$ , “averaging” the input petrophysical distribution  $\Theta_{prior}$  and current inversion model  $m$  through the optimization of a Posterior Distribution on  $\Theta$ .



$$\underset{\Theta}{\operatorname{argmax}} \mathcal{P}(\Theta|m) \propto \mathcal{M}(m|\Theta)\mathcal{P}(\Theta|\Theta_{prior})$$



# Max. A Posteriori - Expectation Maximization

- E-step: Responsibilities

$$n_{ij}^{(k)} = \frac{p(z_i = j)^{(k-1)} \mathcal{N}((\mathbf{m}_i | \mu_j^{(k-1)}, \Sigma_j^{(k-1)})}{\sum_{t=1}^c p(z_i = t)^{(k-1)} \mathcal{N}(\mathbf{m}_i | \mu_t^{(k-1)}, \Sigma_t^{(k-1)})}$$

- M-step:

- proportions

- means

- covariances

$$\mathcal{P}(\pi) = \text{Dir}(\zeta n \pi_{prior} - 1)$$

$$\pi_j^{(k)} = \frac{n_j^{(k)} + \zeta_j n \pi_{j prior}}{n(1 + \sum_{t=1}^c \zeta_t \pi_{t prior})}$$

with:

$$n_j^{(k)} = \sum_{i=1}^n n_{ij}^{(k)}$$

$$\mathcal{P}(\mu | \Sigma) = \mathcal{N}(\mu | \mu_{prior}, (\kappa n \pi_{prior})^{-1})$$

$$\mu_j^{(k)} = \frac{n_j^{(k)} \bar{\mathbf{m}}_j^{(k)} + \kappa_j \pi_{j prior} n \mu_{j prior}}{n_j^{(k)} + \kappa_j \pi_{j prior} n}$$

with:

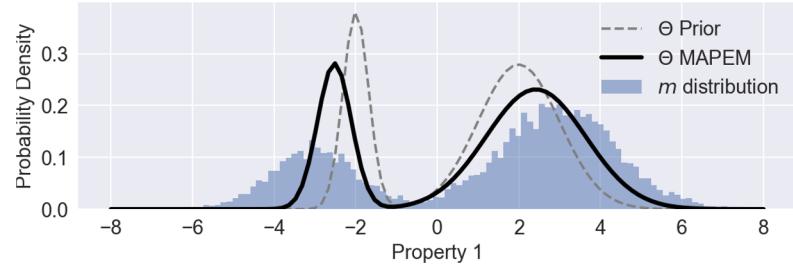
$$\bar{\mathbf{m}}_j^{(k)} = \frac{\sum_{i=1}^n n_{ij}^{(k)} \mathbf{m}_i}{n_j^{(k)}}$$

$$\mathcal{P}(\Sigma | \mu) = \text{IW}(\Sigma | \mathbf{V}, \mathbf{V} \Sigma_{prior})$$

$$\Sigma_j^{(k)} = \frac{n_j^{(k)} \Sigma_{\bar{\mathbf{m}}_j^{(k)}} + \nu_j \pi_{j prior} n \Sigma_{j prior}}{n_j^{(k)} + \nu_j \pi_{j prior} n}$$

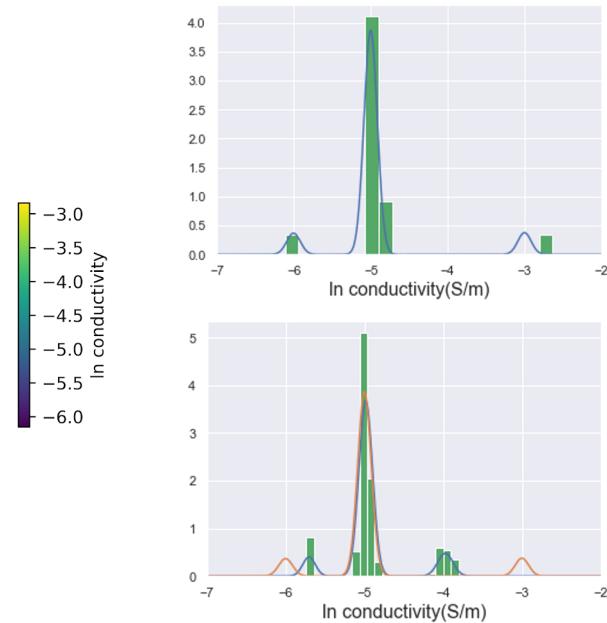
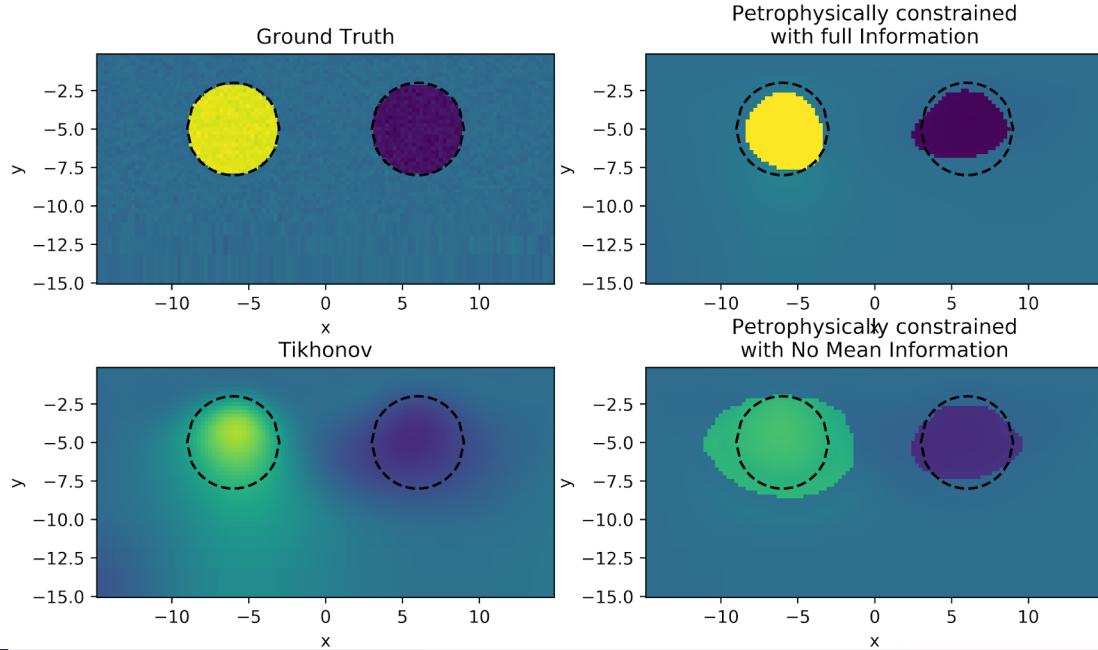
with:

$$\Sigma_{\bar{\mathbf{m}}_j^{(k)}} = \frac{1}{n_j^{(k)}} \sum_{i=1}^n n_{ij}^{(k)} (\mathbf{m}_i - \bar{\mathbf{m}}_j^{(k)}) (\mathbf{m}_i - \bar{\mathbf{m}}_j^{(k)})^T$$

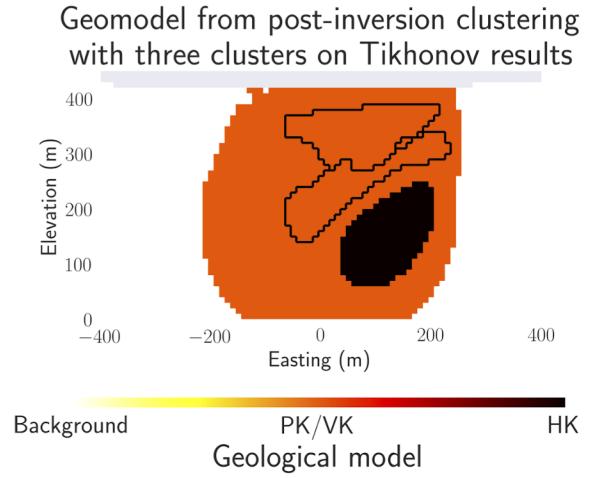
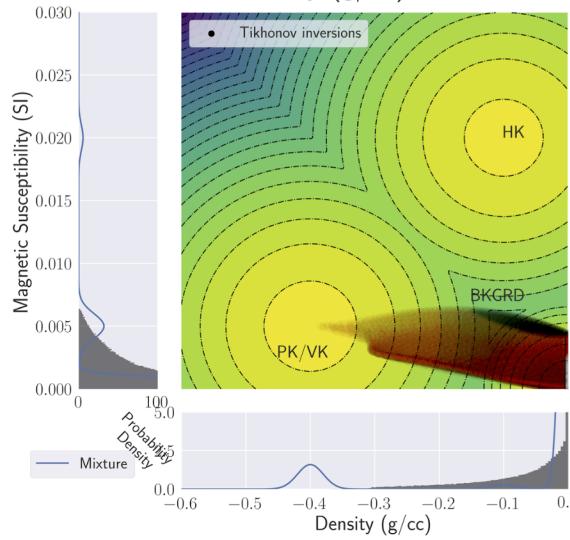
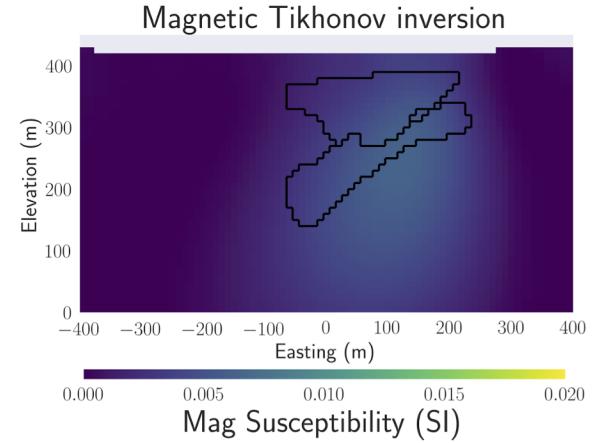
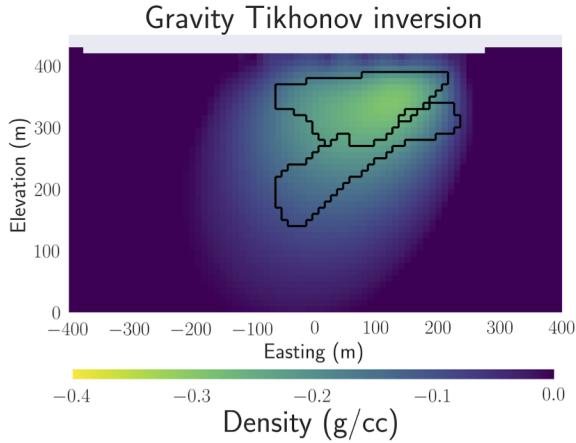
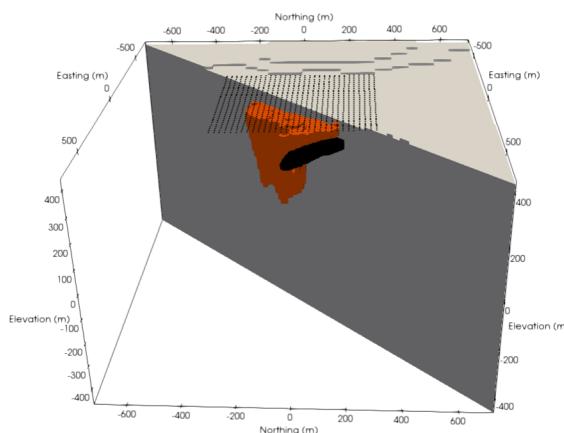


# Why learning a new petrophysical model?

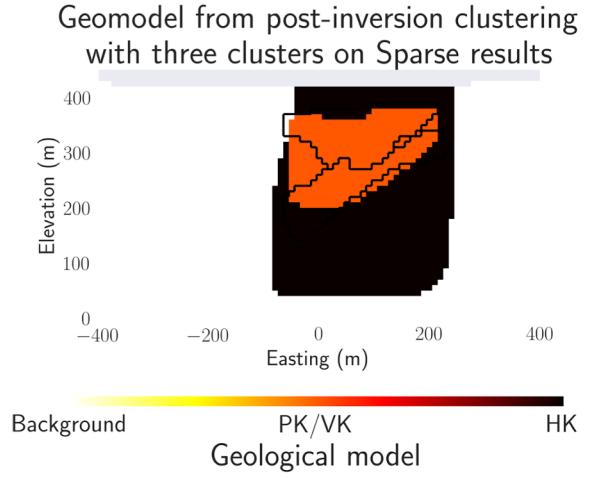
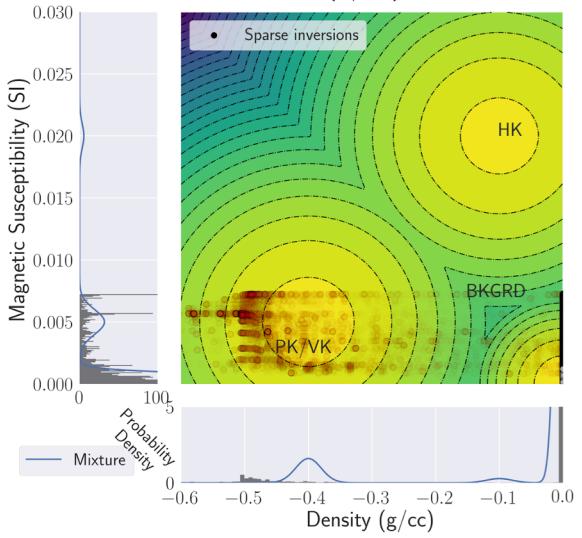
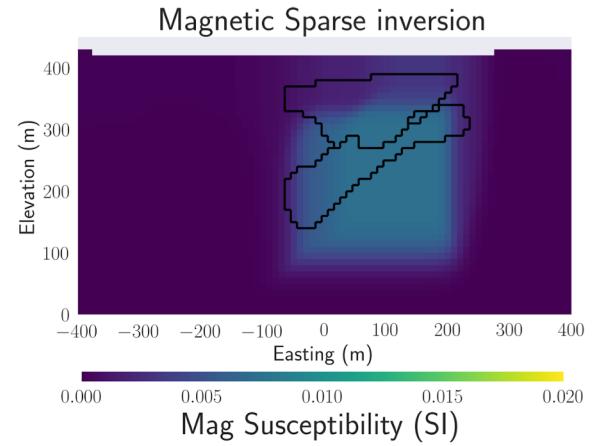
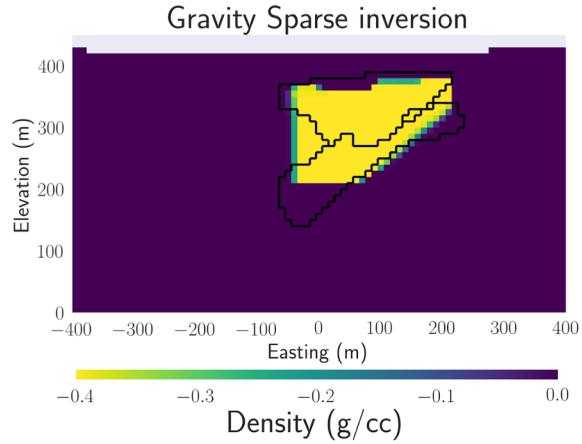
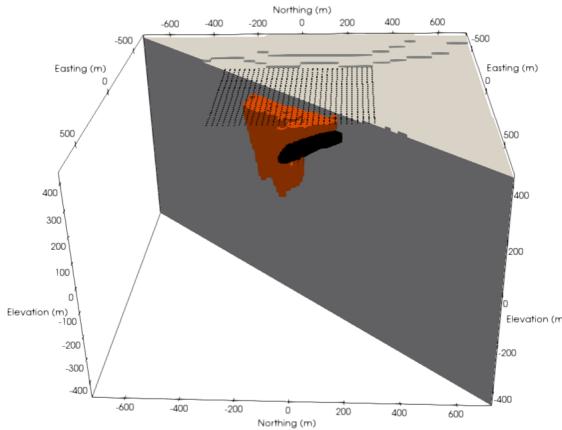
- We can work with partial, incomplete or biais information
  - DC2D example: no mean value information



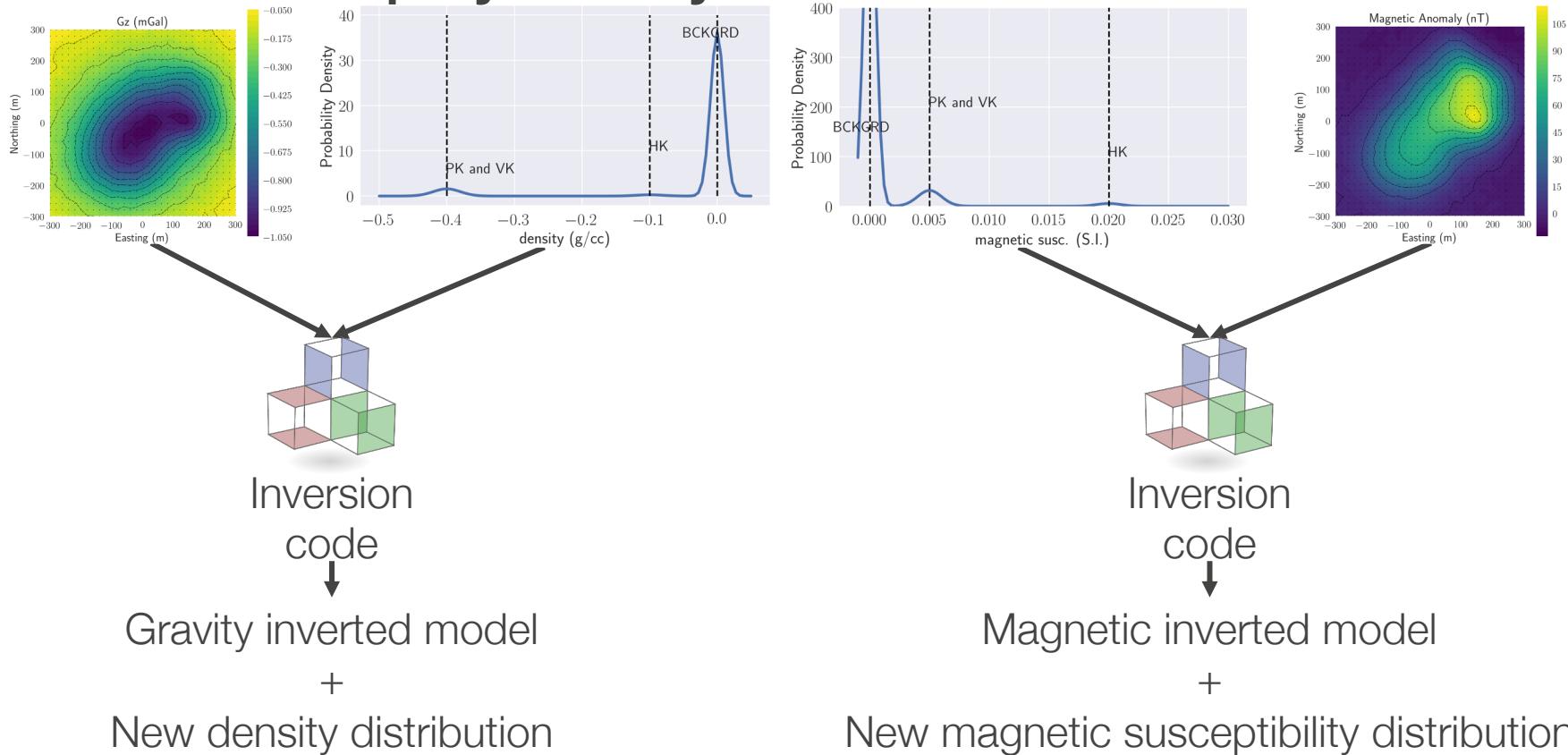
# Back to TKC: Tikhonov Inversion



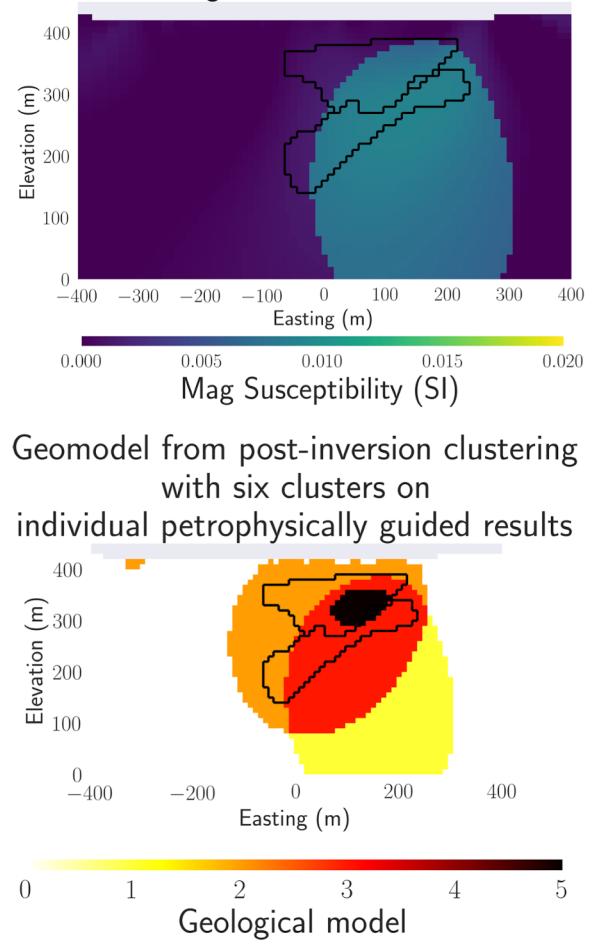
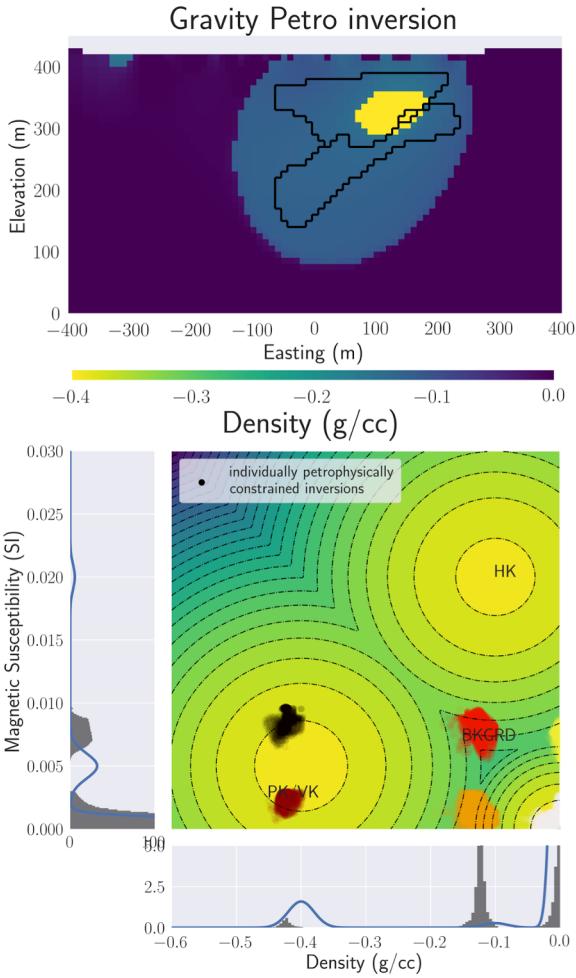
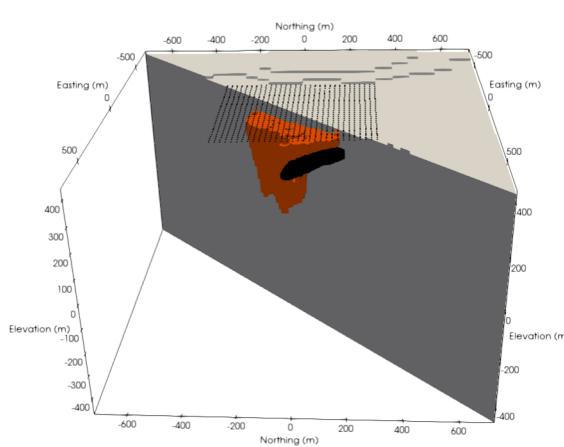
# TKC: Sparse Inversion



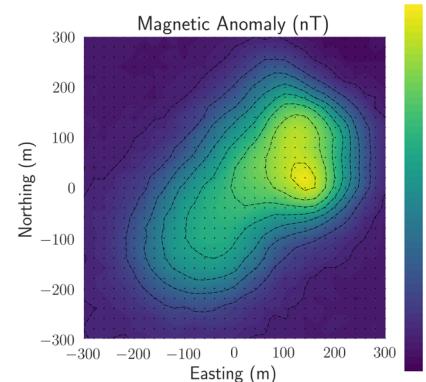
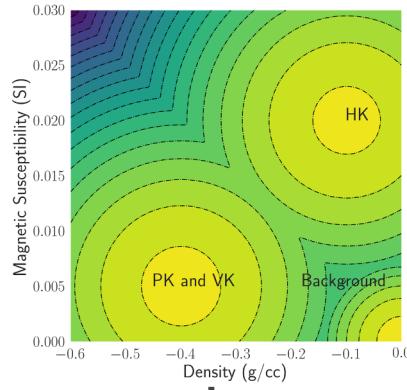
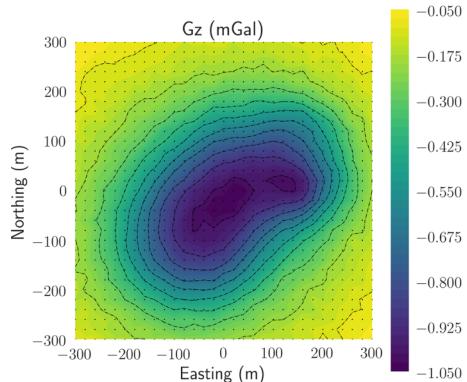
# TKC: Petrophysically constrained inversion



# TKC: Single PGI

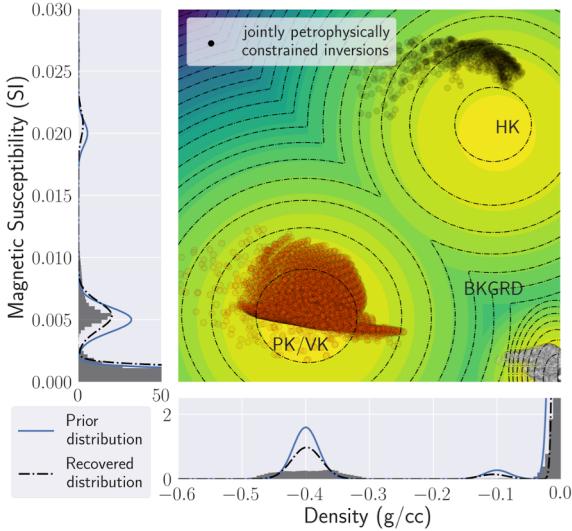
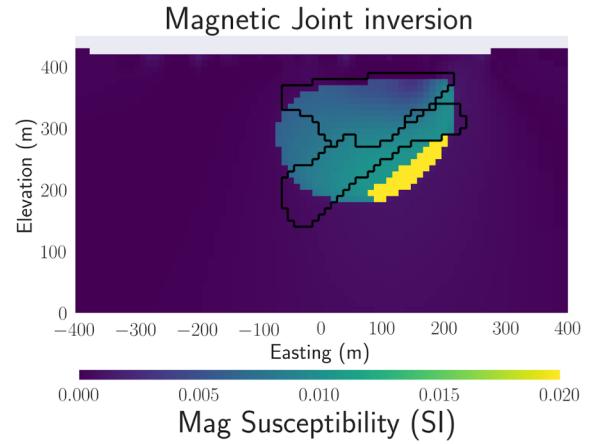
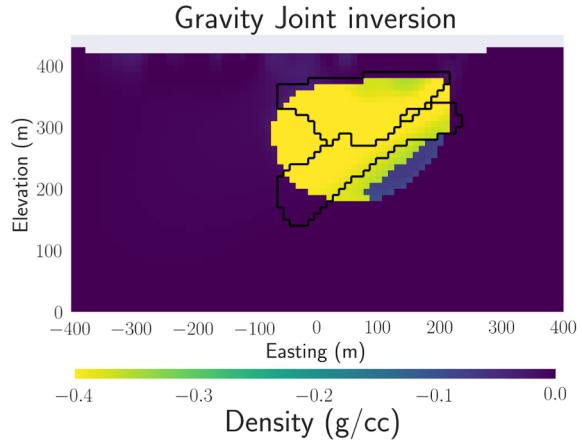
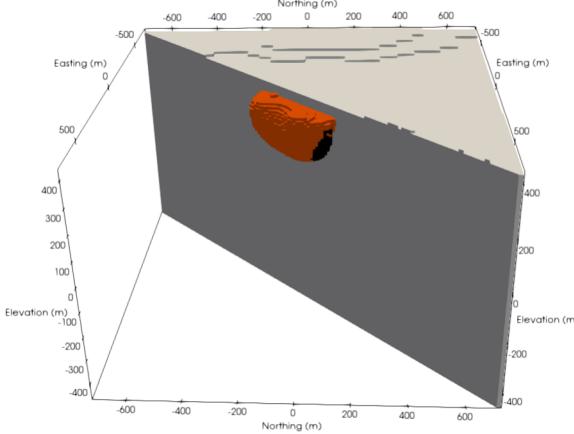


# TKC: Joint PGI

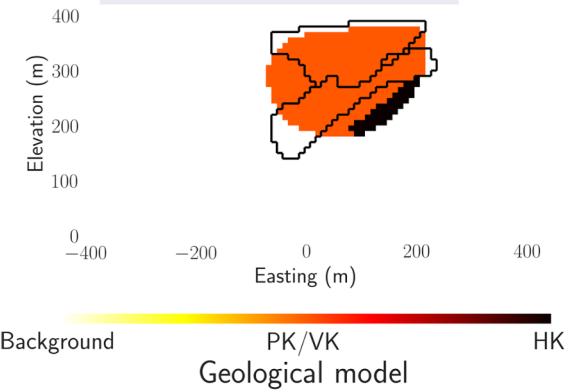


- Gravity and magnetic inverted model
- New physical properties distribution model

# TKC: Joint PGI



Geomodel from  
petrophysically guided inversion

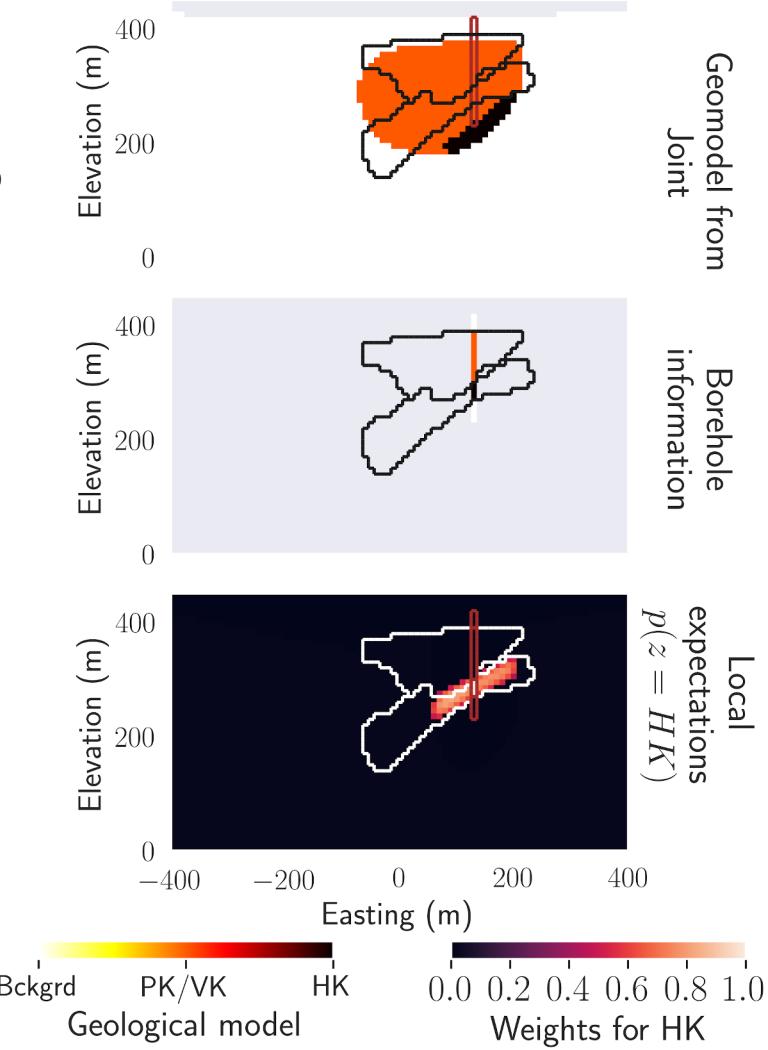


# TKC: geological constraints

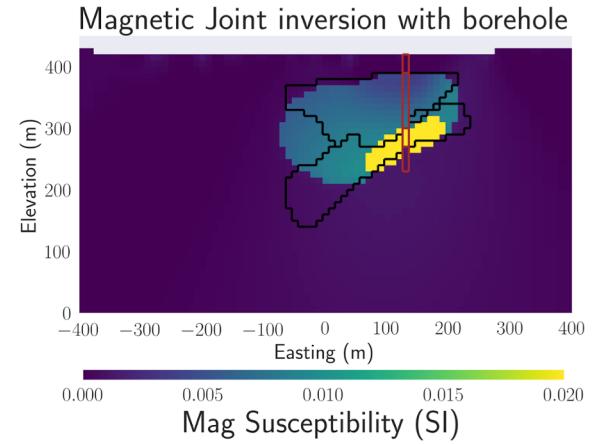
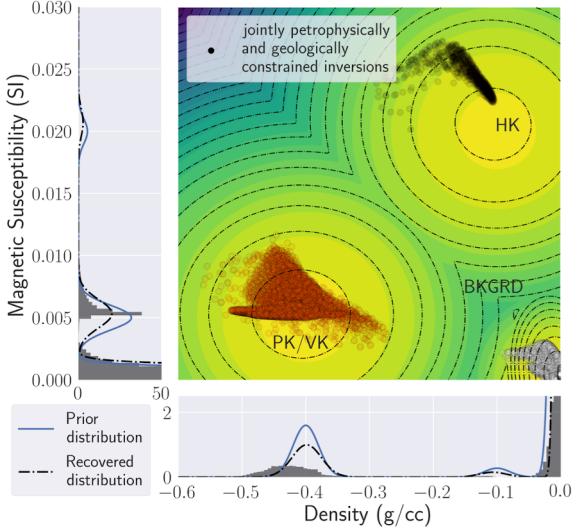
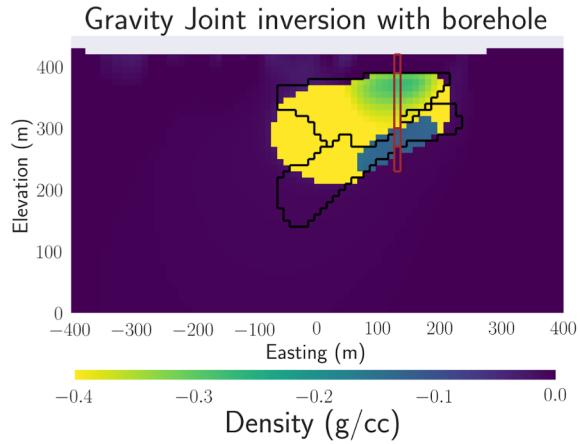
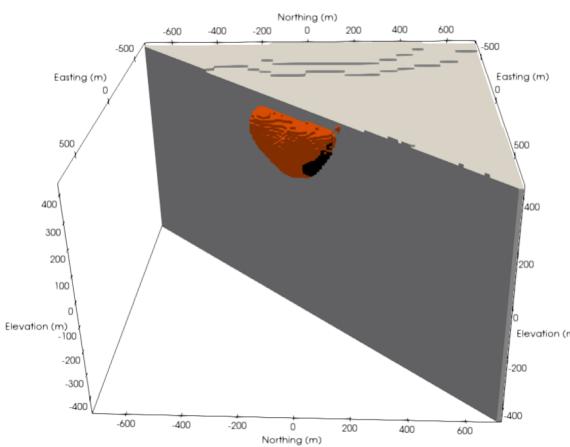
- add one geologic borehole log to constrain the classification
  - Increase the expectations of the observed units at and around the borehole (ellipses)

$$\mathcal{P}_{small}(m|\Theta) = \prod_{i=1}^n \sum_{j=1}^c p(z_i = j) \mathcal{N}(\mathbf{m}_i | \mu_j, \text{diag}(\mathbf{w}_{ij}^{-2}) \Sigma_j)$$

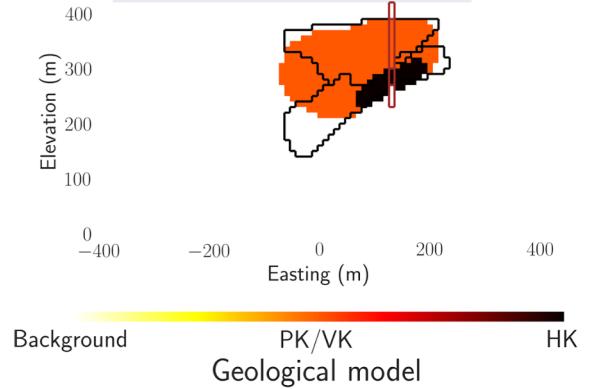
Prior expectation of finding  
rock unit  $j$   
at location  $i$



# TKC: Petrophysical and geological constraints



Geomodel from petrophysically  
and geologically guided inversion



# TKC: Summary

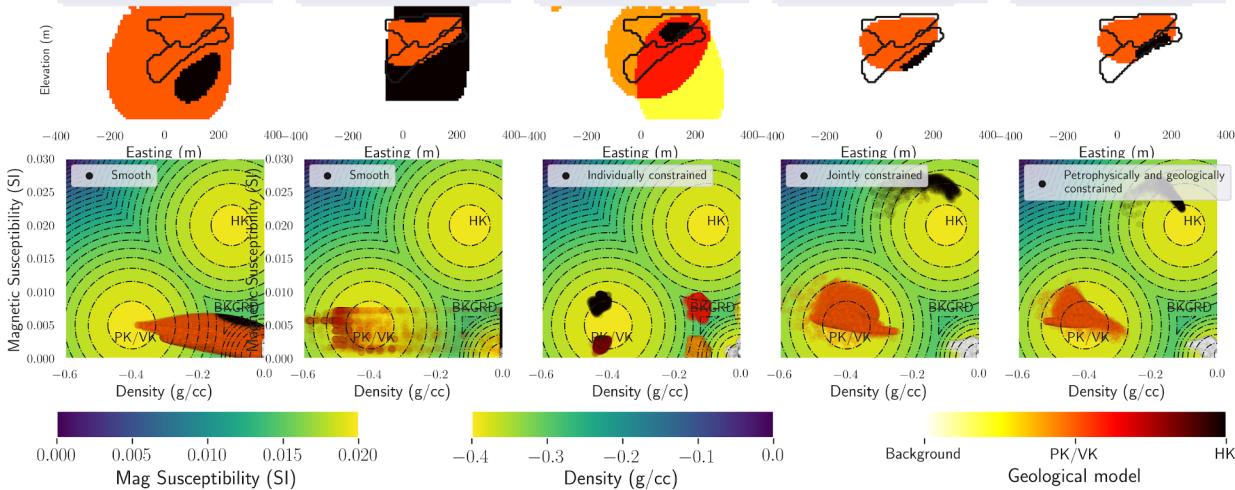
Gravity model



Magnetic model



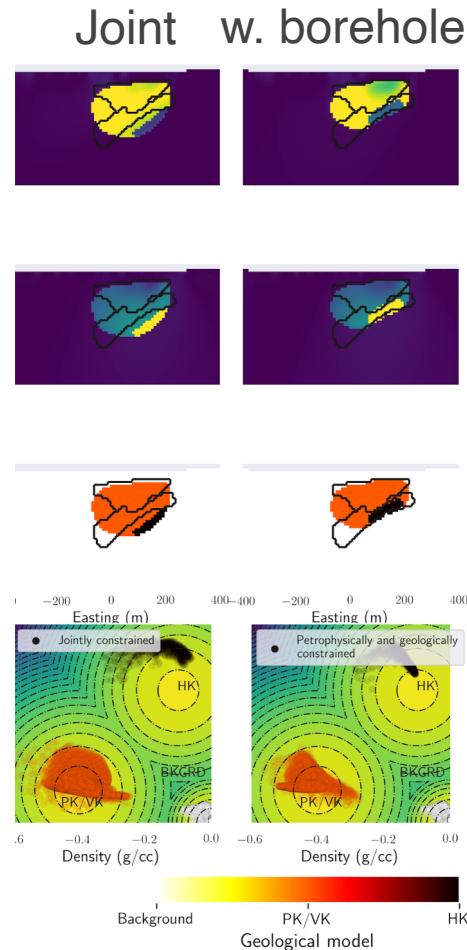
Geological model  
from inversion



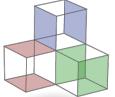
Petrophysical  
distribution

# Conclusions

- General framework applicable to any geophysical survey without extra-terms
- Can be used for joint inversion
- Works with partial or biais petrophysical information
- Can include geological information

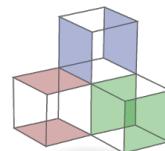
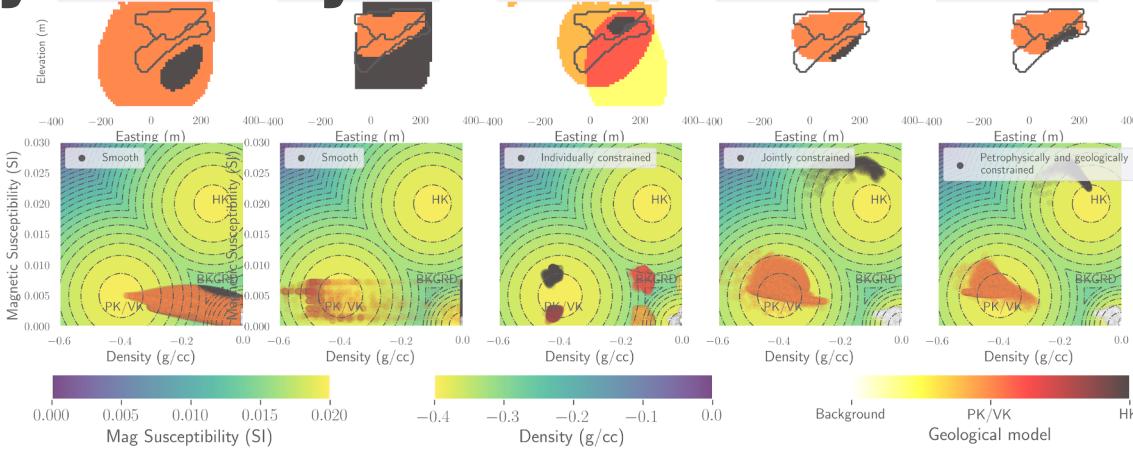


# Acknowledgments

- Condor Geophysics, Peregrine Diamonds and Kennecott for providing the TKC data sets
- The  simpeg open-source community for all the hard-work that made this research possible.



# Thank you for your attention. Questions?



# simpeg

