INVESTIGATION

Farmer's Field

MathSphere

Farmer's Field Investigation

A farmer had two sons.
One of the sons, Harry, was hard working and made the best use of all the land he had.
The other son, Larry, was lazy and never did more work than he had to.

One day the farmer decided to give his sons 24 kilometres of fencing each. He told them they could fence in as much land as they wanted and that land would then be theirs for ever.

The hard working son, Harry, wanted to fence in the largest area of land he could so he could grow as many crops as possible.

The lazy son, Larry, wanted to fence in the smallest area so he would not have to do much work.

The Problem

The problem is to find the largest area for Harry and the smallest area for Larry you can make with 24 kilometres of fence.

The first thing you need to decide is the scale you are going to use.

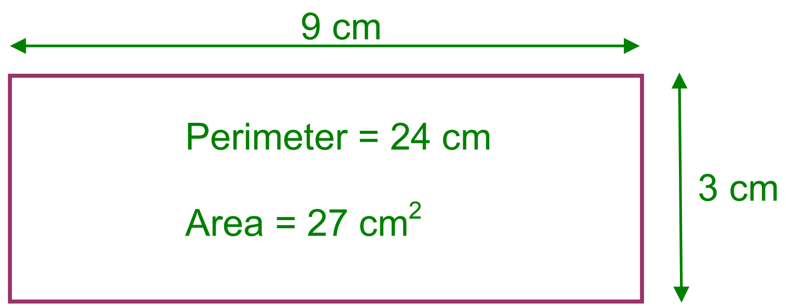
One centimetre to represent each kilometre might be a good starting point.

Some Ideas

Work in a methodical way, recording your results carefully as you go.

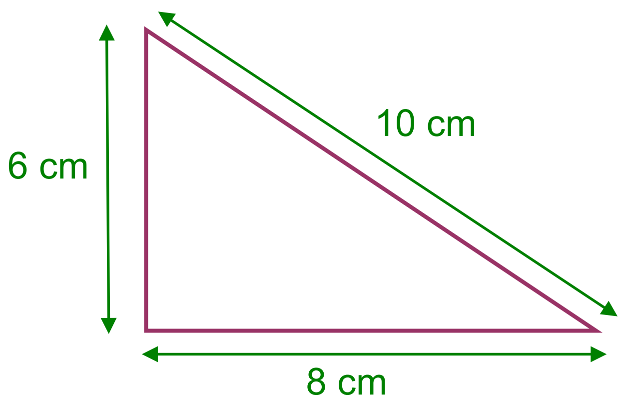
Make sure every shape has a perimeter of 24 units.

Begin with rectangles. Try to find the largest and smallest areas you can using rectangles.



Then move on to other shapes you know how to find the areas of. Triangles might be a good move!

Make sure you know how to find the area of a triangle!



Then you can explore other shapes - perhaps shapes made of several triangles or triangles and rectangles together.

Do not jump from shape to shape - do not do one or two rectangles and then one or two triangles and then go back to rectangles - you will get into a real muddle.

Try to find as many rules and patterns as you can.

Answer Guide

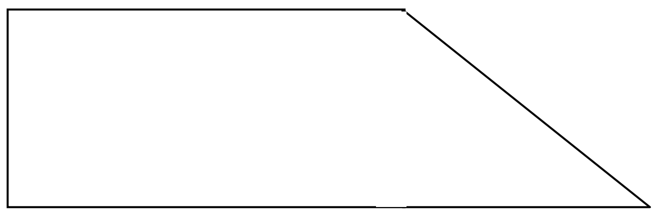
Here are some possible answers and notes for guidance.

The reason for starting with rectangles is that this is the easiest shape for which the area may be found. Some children will not progress beyond rectangles and some not beyond using whole numbers with rectangles.

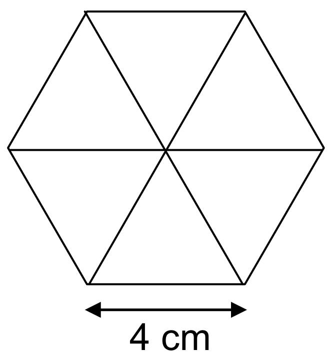
More confident children may work with decimals (eg 8.3 cm × 3.7cm) and be able to find the area of these rectangles (with or without a calculator).

Before moving to triangles, make sure children can draw a triangle with a perimeter of 24 cm using a pair of compasses or some other suitable method and then find its area.

Children can then combine shapes, eg.



Regular polygons provide a good source as they are made up of a number of congruent triangles and children need only find the area of one and then multiply by the number of triangles, eg.



Older and more proficient children may be encouraged to look at circles.

General rules:

The regular shapes have the largest areas for a given number of sides.
The more sides a regular shape has, the greater its area.

Therefore a circle gives the largest area of all and a rectangle 12 × 0 has the smallest area.
Some children will only get as far as discovering that a square of 6 × 6 has the largest area of all rectangles and may think that a rectangle of 11 × 1 cm has the smallest area.
Use your judgement about the children's age and ability when assessing the investigation.

Answer Guide (Contd)

Other ideas:

Encourage the children to record their results in a table, eg.

Area of rectangles	
Length of one side (cm)	Area (cm^2)
0	0
1	11
2	20
3	27
4	32
5	35
6	36

Encourage them to draw graphs. In the above case, the length of the side may be plotted on the horizontal axis and the area on the vertical axis.

Children that are not able to construct shapes with a perimeter of 24 cm could be given a piece of string 24 cm long and stretch it out to make the shape. They then only need to measure the shape to calculate its area.

eg. for triangles.