



$$46 \times 4 = 184$$
$$(40 \times 4) + (6 \times 4) = 184$$

## Multiplication and Division

My name



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# Multiplication facts – factors and multiples

Factors are the numbers we multiply together to get to another number:

$$\text{factor} \times \text{factor} = \text{whole number}$$

How many factors does the number 12 have?  $4 \times 3 = 12$ ,  $6 \times 2 = 12$ ,  $1 \times 12 = 12$   
4, 3, 6, 2, 1 and 12 are all factors of 12.

## 1 List the factors of these numbers:

a 18

--	--	--	--	--	--	--	--	--	--

c 14

--	--	--	--	--	--	--	--	--	--

e 16

--	--	--	--	--	--	--	--	--	--

g 30

--	--	--	--	--	--	--	--	--	--

b 25

--	--	--	--	--	--	--	--	--	--

d 9

--	--	--	--	--	--	--	--	--	--

f 15

--	--	--	--	--	--	--	--	--	--

h 42

--	--	--	--	--	--	--	--	--	--

## 2 Fill the gaps in these sentences. The first one has been done for you.

a 1 or 16 or 2 or 8 or 4 people can share 16 sweets evenly.

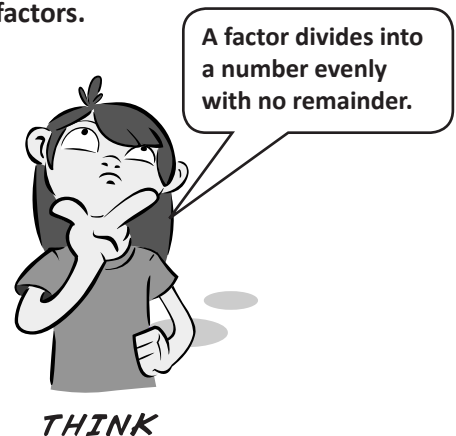
b \_\_\_\_\_ or \_\_\_\_\_ or \_\_\_\_\_ or \_\_\_\_\_ or \_\_\_\_\_ or \_\_\_\_\_ people can share 20 slices of pie evenly.

c \_\_\_\_\_ or \_\_\_\_\_ or \_\_\_\_\_ or \_\_\_\_\_ or \_\_\_\_\_ or \_\_\_\_\_ or \_\_\_\_\_ or \_\_\_\_\_ people can share 24 cherries.

d \_\_\_\_\_ or \_\_\_\_\_ or \_\_\_\_\_ or \_\_\_\_\_ or \_\_\_\_\_ or \_\_\_\_\_ or \_\_\_\_\_ or \_\_\_\_\_ people can share 30 pencils.

e \_\_\_\_\_ or \_\_\_\_\_ people can share 5 balls evenly.

## 3 Work out which number under 50 has the most factors. List the factors.



# Multiplication facts – factors and multiples

Multiples are the answers we get when we multiply 2 factors.

Think about the 3 times tables where 3 is always a factor.

What are the multiples of 3?

3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33 and 36 ...

3

×

factor

=

multiple

4 Fill in the gaps on these multiple boards:

a	4	b	5	c	9	d	7
	8		10				
	16						
							35
					63		

Numbers can be either factors or multiples depending on where they sit in the number sentence.

5 Choose 2 numbers between 2 and 5 and put them in the first frame as factors. Your answer is the multiple. Now take that multiple and make it a factor in another number sentence. Write in the other factor and solve the problem. Then make the answer a factor again. Can you fill the grid? The first one has been done for you.

a	$\boxed{3} \times \boxed{4} = \boxed{12}$	$\boxed{12} \times \boxed{2} = \boxed{24}$	$\boxed{24} \times \boxed{2} = \boxed{48}$
b	$\boxed{\phantom{00}} \times \boxed{\phantom{00}} = \boxed{\phantom{00}}$	$\boxed{\phantom{00}} \times \boxed{\phantom{00}} = \boxed{\phantom{00}}$	$\boxed{\phantom{00}} \times \boxed{\phantom{00}} = \boxed{\phantom{00}}$
c	$\boxed{\phantom{00}} \times \boxed{\phantom{00}} = \boxed{\phantom{00}}$	$\boxed{\phantom{00}} \times \boxed{\phantom{00}} = \boxed{\phantom{00}}$	$\boxed{\phantom{00}} \times \boxed{\phantom{00}} = \boxed{\phantom{00}}$
d	$\boxed{\phantom{00}} \times \boxed{\phantom{00}} = \boxed{\phantom{00}}$	$\boxed{\phantom{00}} \times \boxed{\phantom{00}} = \boxed{\phantom{00}}$	$\boxed{\phantom{00}} \times \boxed{\phantom{00}} = \boxed{\phantom{00}}$

# Multiplication facts – prime and composite numbers

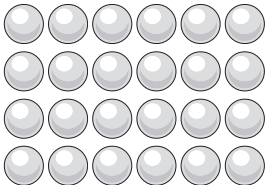
A factor is a number that divides equally into another number.

$$5 \times 4 = 20$$

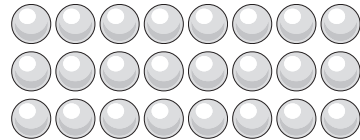
20 arranged in 5 rows means 4 in each row.

5 and 4 are factors of 20.

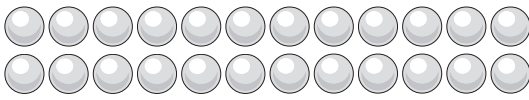
1 How many ways can 24 objects be arranged? Use the arrays below to complete the facts:

a 


×  = 24

b 

×  = 24

c 

×  = 24

d 

×  = 24

24 can be arranged in many different ways. The factors of 24 are 1, 2, 3, 4, 6, 8, 12 and 24.

**Composite numbers** are numbers with more than two factors.

4 (3 factors: 1, 2, 4) and 10 (4 factors: 1, 2, 5, 10) are examples of composite numbers.

If a number only has two factors (itself and 1) it is known as a **prime number**. For example, 7 is divisible only by itself and 1 so is a prime number.

One itself is a special case. It is not a composite or a prime number.

2 How many ways can 12 objects be arranged?



Draw all the combinations you can think of:

# Multiplication facts – prime and composite numbers

Eratosthenes (276 BC – 194 BC) was a Greek mathematician who developed a clever way to find prime numbers.

**3** Find all the prime numbers in the hundred grid below. (Do not shade the number itself as it is not a multiple.)

- a Cross out 1 since it is not prime.
- b Shade all the multiples of 2.
- c Shade all the multiples of 3.
- d Shade all the multiples of 5.
- e Shade all the multiples of 7.
- f The remaining numbers are prime numbers, apart from 1 which is a special case. List them:

---

---

## The Sieve of Eratosthenes

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100



The factors of a composite number may themselves be prime or composite numbers. For example, the factors of 6 are 1, 2, 3 and 6. Of these 2 and 3 are **prime factors**, and 6 is a composite number. (Remember that 1 is neither.)

**4** List the prime factors of these numbers:

- a 9 \_\_\_\_\_
- b 15 \_\_\_\_\_
- c 12 \_\_\_\_\_
- d 28 \_\_\_\_\_



# Multiplication facts – square numbers

A square number is a number multiplied by itself.

$1 \times 1 = 1$

$2 \times 2 = 4$

$3 \times 3 = 9$

$1^2 = 1$

$2^2 = 4$

$3^2 = 9$

1 Show these square numbers on the grid and write what they are equal to:

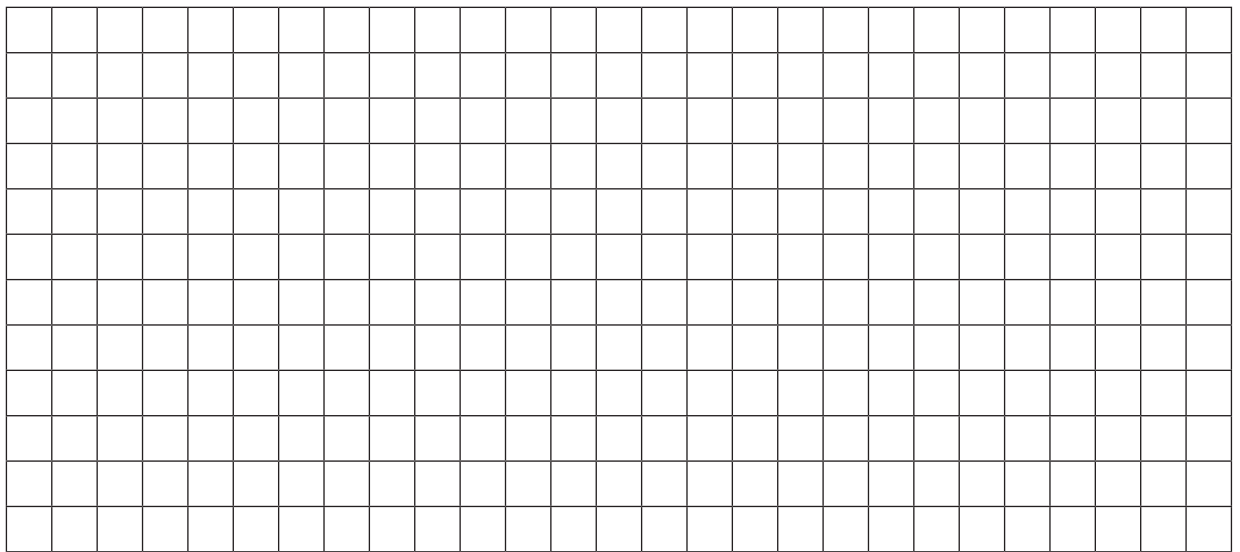
a  $4^2 =$

b  $6^2 =$

c  $5^2 =$

d  $3^2 =$

e  $7^2 =$



2 Shade the square numbers on this multiplication grid:

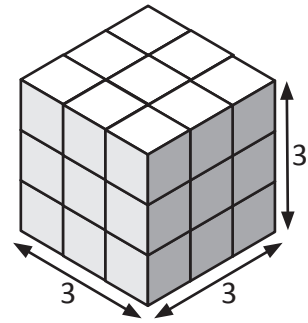
×	2	3	4	5	6	7	8	9	10
2	4	6	8	10	12	14	16	18	20
3	6	9	12	15	18	21	24	27	30
4	8	12	16	20	24	28	32	36	40
5	10	15	20	25	30	35	40	45	50
6	12	18	24	30	36	42	48	54	60
7	14	21	28	35	42	49	56	63	70
8	16	24	32	40	48	56	64	72	80
9	18	27	36	45	54	63	72	81	90
10	20	30	40	50	60	70	80	90	100

# Multiplication facts – cube numbers

A cube number is a number multiplied by itself three times.

For example, the cube of 3 is  $3 \times 3 \times 3$ , which equals 27.

We can write '3 cubed' as  $3^3$ .



**1** Write these cubed numbers out as full multiplications and find the answers:

a  $1^3 =$    $\times$    $\times$    $=$

b  $4^3 =$    $\times$    $\times$    $=$

c  $2^3 =$    $\times$    $\times$    $=$

d  $5^3 =$    $\times$    $\times$    $=$

e  $0^3 =$    $\times$    $\times$    $=$

**2** True or false?

a  $1^2 = 1^3$

b  $1^3 + 2^3 = 3^2$

c  $3^3 < 5^2$

d  $8^2 = 4^3$

e  $5^2 + 6^2 > 4^3$

f  $5^3 - 10^2 = 5^2$

# Mental multiplication strategies – doubling strategy

Doubling is a useful strategy to use when multiplying.

To multiply a number by four, double it twice.

$$15 \times 4 \text{ double once} = 30$$

$$\text{double twice} = 60$$

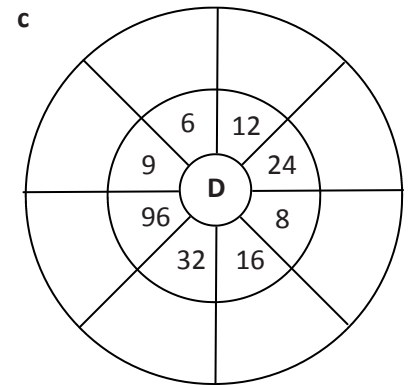
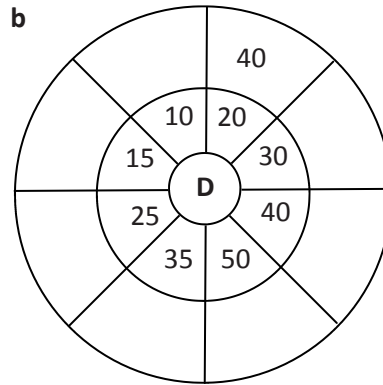
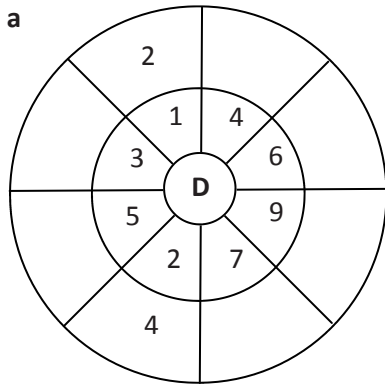
To multiply a number by eight, double it three times.

$$13 \times 8 \text{ double once} = 26$$

$$\text{double twice} = 52$$

$$\text{double three times} = 104$$

## 1 Warm up with some doubling practice:



## 2 Finish the doubling patterns:

a	4	<u>8</u>	<u>16</u>	<u>        </u>	<u>64</u>	<u>        </u>
b	3	<u>        </u>	<u>        </u>	<u>        </u>	<u>        </u>	<u>96</u>
c	5	<u>        </u>	<u>        </u>	<u>40</u>	<u>        </u>	<u>        </u>
d	25	<u>50</u>	<u>        </u>	<u>        </u>	<u>        </u>	<u>        </u>
e	7	<u>        </u>	<u>28</u>	<u>        </u>	<u>        </u>	<u>224</u>
f	75	<u>        </u>	<u>300</u>	<u>        </u>	<u>        </u>	<u>        </u>

## 3 Choose a number and create your own doubling pattern. How high can you go? What patterns can you see within your pattern?

## 4 Two sets of twins turn 12. They decide to have a joint birthday party with 1 giant cake but they all want their own candles. How many candles will they need?

# Mental multiplication strategies – doubling strategy

5 Use the doubling strategy to solve these:

	× 2	× 4
a $13 \times 4$	<u>26</u>	<u>52</u>
b $16 \times 4$	_____	_____
c $24 \times 4$	_____	_____
d $25 \times 4$	_____	_____
e $32 \times 4$	_____	_____
f $21 \times 4$	_____	_____
g $35 \times 4$	_____	_____

To multiply by 4, double twice. To multiply by 8, double three times.



**REMEMBER**

6 Use the doubling strategy to solve these:

	× 2	× 4	× 8
a $12 \times 8$	<u>24</u>	_____	<u>96</u>
b $14 \times 8$	_____	_____	<u>112</u>
c $25 \times 8$	_____	_____	_____
d $21 \times 8$	_____	<u>84</u>	_____
e $13 \times 8$	_____	_____	_____
f $16 \times 8$	<u>32</u>	_____	_____

7 Work out the answers in your head using the appropriate doubling strategy. Use a table like the one above if it helps.

a  $18 \times 4 =$

b  $16 \times 4 =$

c  $26 \times 4 =$

d  $24 \times 8 =$

e  $15 \times 8 =$

f  $22 \times 8 =$

8 Nick's dad offered him two methods of payment for helping with a 5 week landscaping project.

**Method 1:** £24 a week for 5 weeks.

**Method 2:** £8 for the first week, then double the payment each week.

Which method would earn Nick the most money? Why?

# Mental multiplication strategies – multiply by 10s, 100s and 1,000s

When we multiply by 10 we move the number one place value to the left.

When we multiply by 100 we move the number two place values to the left.

When we multiply by 1,000 we move the number three place values to the left.

Look at how this works with the number 45:

Ten Thousands	Thousands	Hundreds	Tens	Ones	
			4	5	
		4	5	0	× 10
	4	5	0	0	× 100
4	5	0	0	0	× 1,000

1 Multiply the following numbers by 10, 100 and 1,000:

a

T Th	Th	H	T	O	
			1	7	
					× 10
					× 100
					× 1,000

b

T Th	Th	H	T	O	
			4	3	
					× 10
					× 100
					× 1,000

c

T Th	Th	H	T	O	
			8	5	
					× 10
					× 100
					× 1,000

d

T Th	Th	H	T	O	
			9	9	
					× 10
					× 100
					× 1,000

2 Try these:

a  $14 \times 10 =$

b  $14 \times 100 =$

c  $14 \times 1,000 =$

d  $92 \times 10 =$

e  $92 \times 1,000 =$

f  $92 \times 100 =$

g  $0.1 \times 1,000 =$

h  $0.1 \times 100 =$

i  $0.1 \times 10 =$

3 You will need a partner for this activity. Take turns giving each other  $\times 10$ ,  $\times 100$  and  $\times 1,000$  problems, such as “What is  $678 \times 100$ ?” “What is  $0.92 \times 1,000$ ?” Both independently work out the answer. If you are correct you get 10 points. If you disagree, ask the teacher to adjudicate. The first person to 50 points wins.

# Multiplication facts – multiply by 10s, 100s and 1,000s

It is also handy to know how to multiply multiples of 10 such as 20 or 200 in our heads.

$4 \times 2$  helps us work out  $4 \times 20$ :       $4 \times 2 = 8$        $4 \times 20 = 80$

We can express this as  $4 \times 2 \times 10 = 80$       How would you work out  $4 \times 200$ ?

## 4 Use patterns to help you solve these:

- |   |                            |                             |                              |
|---|----------------------------|-----------------------------|------------------------------|
| a | $5 \times 2$ _____         | $5 \times 20$ _____         | $5 \times 200$ _____         |
| b | $2 \times 9$ _____         | $2 \times 90$ _____         | $2 \times 900$ _____         |
| c | $6 \times \text{£}4$ _____ | $6 \times \text{£}40$ _____ | $6 \times \text{£}400$ _____ |
| d | $8 \times 0.3$ _____       | $8 \times 3$ _____          | $8 \times 30$ _____          |
| e | $3 \times \text{£}7$ _____ | $3 \times \text{£}70$ _____ | $3 \times \text{£}700$ _____ |
| f | $0.02 \times 8$ _____      | $0.2 \times 8$ _____        | $2 \times 8$ _____           |
| g | $3 \times 9$ _____         | $30 \times 9$ _____         | $300 \times 9$ _____         |

## 5 Answer these problems:

- a Jock runs 7.5 km per week. How far does he run over 10 weeks?
- b Huy earns £20 pocket money per week. If he saves half of this, how much will he have saved at the end of 8 weeks?
- c The sum of two numbers is 28. When you multiply them together, the answer is 160. What are the numbers?

If you're struggling with your tables, get onto Live Mathematics and practise!



## 6 Finish these counting patterns:

- |   |     |     |           |           |           |             |
|---|-----|-----|-----------|-----------|-----------|-------------|
| a | 10  | 20  | _____ 30  | _____     | _____     | _____ 60    |
| b | 20  | 40  | _____     | _____ 80  | _____     | _____       |
| c | 30  | 60  | _____     | _____     | _____ 150 | _____       |
| d | 40  | 80  | _____     | _____     | _____ 200 | _____ 240   |
| e | 50  | 100 | _____ 150 | _____     | _____     | _____       |
| f | 100 | 200 | _____     | _____ 400 | _____     | _____       |
| g | 200 | 400 | _____     | _____     | _____     | _____ 1,200 |

# Mental multiplication strategies – split strategy

Sometimes it's easier to split a number into parts and work with the parts separately.

Look at  $64 \times 8$

Split the number into 60 and 4

Work out  $(60 \times 8)$  and then  $(4 \times 8)$

Add the answers together  $480 + 32 = 512$

## 1 Use the split strategy to answer the questions:

**a**  $46 \times 4$

$(40 \times 4) + (6 \times 4)$

\_\_\_\_\_ + \_\_\_\_\_  
=

**b**  $74 \times 5$

$(\_\_ \times \_\_) + (\_\_ \times \_\_)$

\_\_\_\_\_ + \_\_\_\_\_  
=

**c**  $48 \times 4$

$(\_\_ \times \_\_) + (\_\_ \times \_\_)$

\_\_\_\_\_ + \_\_\_\_\_  
=

**d**  $37 \times 7$

$(\_\_ \times \_\_) + (\_\_ \times \_\_)$

\_\_\_\_\_ + \_\_\_\_\_  
=

**e**  $62 \times 8$

$(\_\_ \times \_\_) + (\_\_ \times \_\_)$

\_\_\_\_\_ + \_\_\_\_\_  
=

**f**  $91 \times 5$

$(\_\_ \times \_\_) + (\_\_ \times \_\_)$

\_\_\_\_\_ + \_\_\_\_\_  
=

## 2 Use the split strategy to answer the questions. This time see if you can do the brackets in your head:

**a**  $48 \times 8 = \_\_\_\_\_\_ + \_\_\_\_\_\_ = \text{$

**b**  $52 \times 7 = \_\_\_\_\_\_ + \_\_\_\_\_\_ = \text{$

**c**  $9 \times 43 = \_\_\_\_\_\_ + \_\_\_\_\_\_ = \text{$

**d**  $8 \times 29 = \_\_\_\_\_\_ + \_\_\_\_\_\_ = \text{$

**e**  $86 \times 7 = \_\_\_\_\_\_ + \_\_\_\_\_\_ = \text{$



**THINK**

## 3 These problems have been worked out incorrectly. Circle where it all went wrong.

**a**  $37 \times 6$

$(30 \times 6) + (7 \times 6)$

$180 + 13$

$= 193$

**b**  $17 \times 5$

$(10 \times 5) + (7 \times 5)$

$70 + 35$

$= 105$

**c**  $32 \times 9$

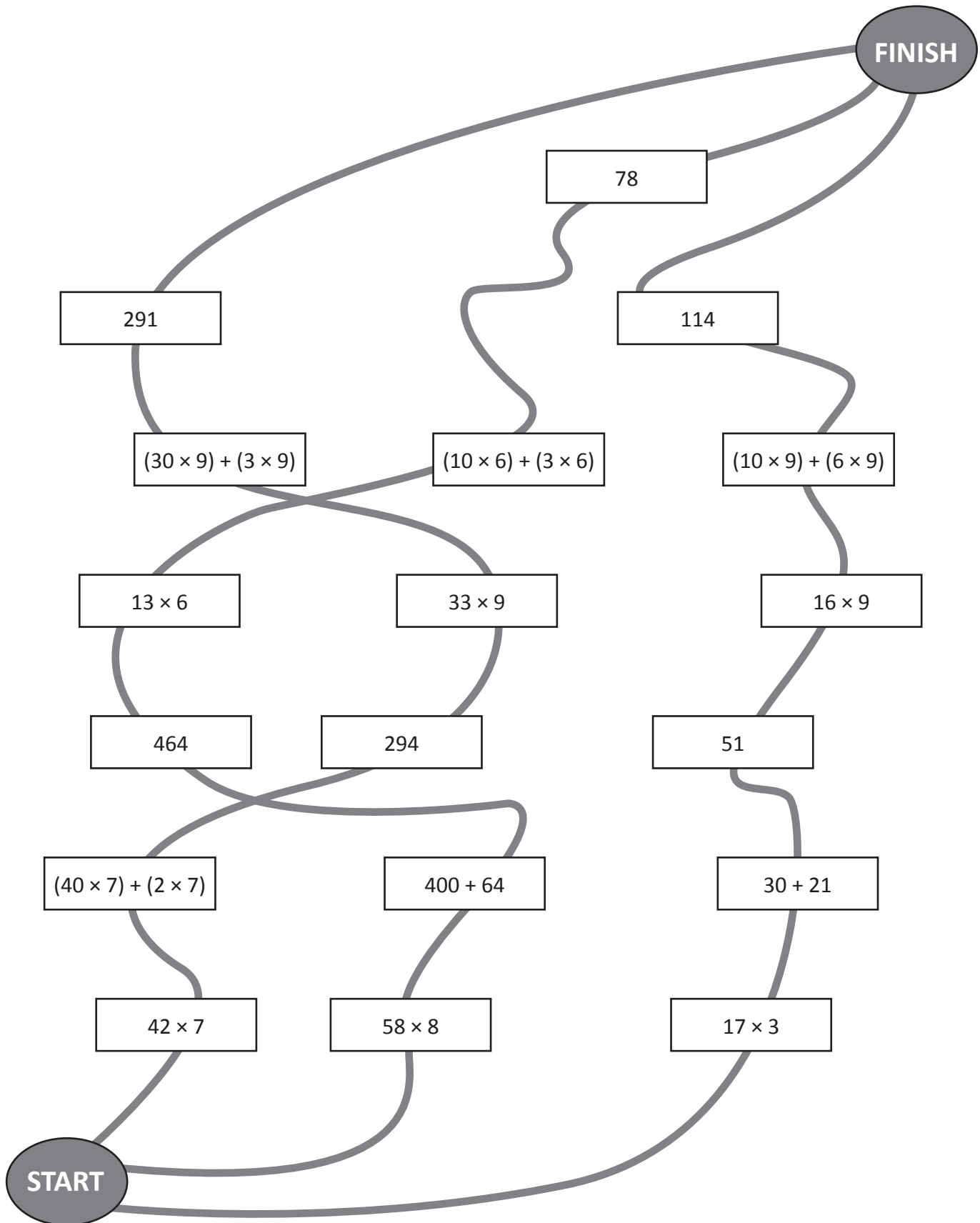
$(30 \times 9) + (2 \times 9)$

$27 + 18$

$= 45$

# Mental multiplication strategies – split strategy

- 4 Each trail contains 2 multiplication problems and steps to solve them. Only one trail has been solved correctly. There are errors in the other two. Find and colour the winning trail.





## Mental multiplication strategies – compensation strategy

When multiplying we can round to an easier number and then adjust.

Look how we do this with  $4 \times 29$ :

29 is close to 30. We can do  $4 \times 30$  in our heads because we know  $4 \times 3 = 12$

$$4 \times 30 = 120$$

We have to take off 4 because we used one group of 4 too many:  $120 - (1 \times 4) = 116$

$$4 \times 29 = 116$$

**1** Use the compensation strategy to answer the questions. The first one has been done for you.

a  $19 \times 3 = \underline{20} \times \underline{3} - \underline{3} = \boxed{57}$

b  $8 \times 29 = \underline{\quad} \times \underline{\quad} - \underline{\quad} = \boxed{\quad}$

c  $18 \times 6 = \underline{\quad} \times \underline{\quad} - \underline{\quad} = \boxed{\quad}$

d  $7 \times 39 = \underline{\quad} \times \underline{\quad} - \underline{\quad} = \boxed{\quad}$

e  $28 \times 5 = \underline{\quad} \times \underline{\quad} - \underline{\quad} = \boxed{\quad}$

We can also adjust up. Look how we do this with  $6 \times 62$ :

62 is close to 60. We can do  $6 \times 60$  in our heads because we know  $6 \times 6 = 36$

$$6 \times 60 = 360$$

We have to then add 2 more lots of 6:  $360 + 12 = 372$

$$6 \times 62 = 372$$

**2** Use the compensation strategy and adjust up for these. The first one has been done for you.

a  $41 \times 3 = \underline{40} \times \underline{3} + \underline{3} = \boxed{123}$

b  $81 \times 4 = \underline{\quad} \times \underline{\quad} + \underline{\quad} = \boxed{\quad}$

c  $22 \times 9 = \underline{\quad} \times \underline{\quad} + \underline{\quad} = \boxed{\quad}$

d  $32 \times 9 = \underline{\quad} \times \underline{\quad} + \underline{\quad} = \boxed{\quad}$

e  $7 \times 62 = \underline{\quad} \times \underline{\quad} + \underline{\quad} = \boxed{\quad}$

Would I use the compensation strategy with numbers such as 56 or 84? Why or why not?



**THINK**

# Mental multiplication strategies – compensation strategy

**3** In this activity you'll work alongside a partner. You'll each need two dice and your own copy of this page. For each line, roll the dice to find the tens digit and then roll it again to find the multiplier. Your partner will do the same. Use the compensation strategy to mentally work out the answers to the problems.



Tens	Ones		Multiplier		Answer
<input type="text"/>	1	×	<input type="text"/>	=	<input type="text"/>
<input type="text"/>	9	×	<input type="text"/>	=	<input type="text"/>
<input type="text"/>	2	×	<input type="text"/>	=	<input type="text"/>
<input type="text"/>	1	×	<input type="text"/>	=	<input type="text"/>
<input type="text"/>	8	×	<input type="text"/>	=	<input type="text"/>
<input type="text"/>	1	×	<input type="text"/>	=	<input type="text"/>
<input type="text"/>	9	×	<input type="text"/>	=	<input type="text"/>
<input type="text"/>	8	×	<input type="text"/>	=	<input type="text"/>
<input type="text"/>	2	×	<input type="text"/>	=	<input type="text"/>
<input type="text"/>	1	×	<input type="text"/>	=	<input type="text"/>

- a Check each other's calculations. You may want to use a calculator.
- b Now, use the calculator to add your answers. The person with the highest score wins.

# Mental division strategies – use multiplication facts

Knowing our multiplication facts helps us with division as they do the reverse of each other. They are inverse operations.

$$3 \times 5 = 15$$

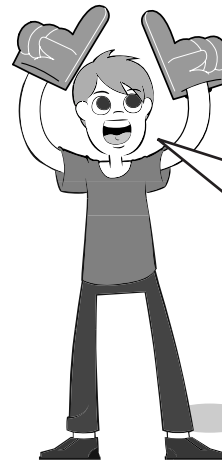
$$15 \div 5 = 3$$

1 Use your knowledge of multiplication facts to help answer these division questions:

- a  $56 \div 7 \rightarrow \underline{\quad 8 \quad} \times 7 = 56 \rightarrow 56 \div 7 = \boxed{\quad}$
- b  $121 \div 11 \rightarrow \underline{\quad} \times 11 = 121 \rightarrow 121 \div 11 = \boxed{\quad}$
- c  $72 \div 8 \rightarrow \underline{\quad} \times 8 = 72 \rightarrow 72 \div 8 = \boxed{\quad}$
- d  $49 \div 7 \rightarrow \underline{\quad} \times 7 = 49 \rightarrow 49 \div 7 = \boxed{\quad}$
- e  $36 \div 9 \rightarrow \underline{\quad} \times 9 = 36 \rightarrow 36 \div 9 = \boxed{\quad}$
- f  $64 \div 8 \rightarrow \underline{\quad} \times 8 = 64 \rightarrow 64 \div 8 = \boxed{\quad}$
- g  $108 \div 12 \rightarrow \underline{\quad} \times 12 = 108 \rightarrow 108 \div 12 = \boxed{\quad}$

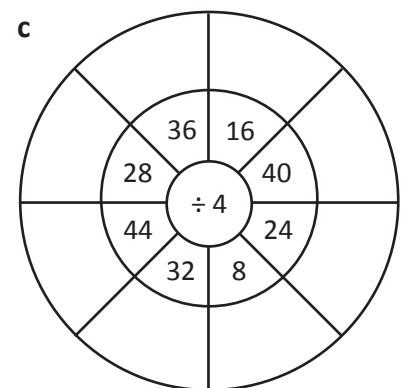
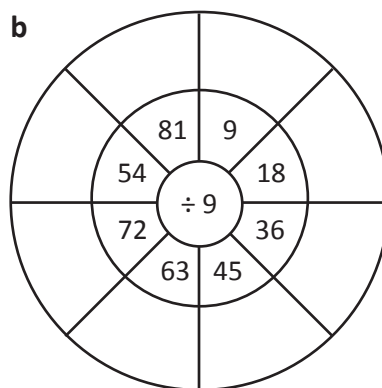
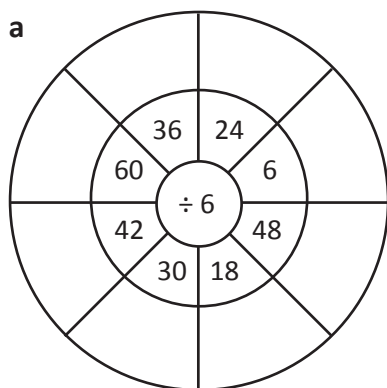
2 Now try these:

- a  $81 \div 9 = \boxed{\quad}$
- b  $40 \div 5 = \boxed{\quad}$
- c  $21 \div 3 = \boxed{\quad}$
- d  $54 \div 6 = \boxed{\quad}$
- e  $42 \div 7 = \boxed{\quad}$
- f  $63 \div 9 = \boxed{\quad}$
- g  $36 \div 4 = \boxed{\quad}$
- h  $45 \div 9 = \boxed{\quad}$
- i  $39 \div 3 = \boxed{\quad}$
- j  $24 \div 6 = \boxed{\quad}$



Doing maths without knowing your multiplication facts is hard. Learning them makes your life much easier. It's worth persevering to conquer them!

3 Fill in the division wheels. Use multiplication facts to help you.



## Mental division strategies – use multiplication facts

Knowing our families of facts is also helpful.

$3 \times 5 = 15$

$5 \times 3 = 15$

$15 \div 5 = 3$

$15 \div 3 = 5$

- 4 Complete the following patterns. How many more multiplication and division facts can you find, given the first fact?

a  $7 \times 8 = 56$

$8 \times 7 = \square$

$56 \div \square = 8$

$\square \div 8 = 7$

b  $8 \times 9 = 72$

$9 \times 8 = \square$

$72 \div \square = 9$

$\square \div 9 = 8$

c  $7 \times 9 = 63$

$9 \times 7 = \square$

$63 \div \square = 9$

$\square \div 9 = 7$

- 5 Write down another multiplication fact and two division facts for each question.

a  $6 \times 7 = 42$

b  $5 \times 9 = 45$

c  $9 \times 6 = 54$

d  $17 \times 8 = 136$

e  $12 \times 8 = 96$

f  $11 \times 21 = 231$

- 6 Look at these two division facts:  $20 \div 5 = 4$  and  $20 \div 4 = 5$

Imagine you're explaining to a younger child how they're related yet different. How would you do it? What would you say/write/draw?

# Mental division strategies – divide by 10s, 100s and 1,000s

When we divide by 10 we move the number one place value to the right.

When we divide by 100 we move the number two place values to the right.

When we divide by 1,000 we move the number three place values to the right.

Look what happens to 45,000 when we apply these rules:

Ten Thousands	Thousands	Hundreds	Tens	Ones	
4	5	0	0	0	
	4	5	0	0	÷ 10
		4	5	0	÷ 100
			4	5	÷ 1,000

1 Divide the following numbers by 10, 100 and 1,000:

a

T Th	Th	H	T	O	
4	5	0	0	0	
					÷ 10
					÷ 100
					÷ 1,000

b

T Th	Th	H	T	O	
4	3	0	0	0	
					÷ 10
					÷ 100
					÷ 1,000

c

T Th	Th	H	T	O	
8	5	0	0	0	
					÷ 10
					÷ 100
					÷ 1,000

d

T Th	Th	H	T	O	
8	8	0	0	0	
					÷ 10
					÷ 100
					÷ 1,000

2 Draw lines to match the answers with the questions:

a What number is one thousand times smaller than 32,000?

9,500

b What number is one hundred times smaller than 32,000?

88

c What number is one hundred times smaller than 95,000?

950

d What number is ten times smaller than 95,000?

880

e What number is one hundred times smaller than 8,800?

320

f What number is ten times smaller than 8,800?

32

# Mental division strategies – halving strategy

When the two numbers seem too large to work with in our heads, we can halve them till we get to a division fact we recognise. Both numbers must be even for this to work.

$$126 \div 14$$

$$(\text{half } 126) \div (\text{half } 14)$$

$$63 \div 7 = 9$$

1 Practise your halving. The first one has been done for you.

a

32	halve	16
56		
36		
84		
96		

b

24	halve	
48		
72		
144		
192		

c

50	halve	
500		
1,000		
250		
100		

2 Halve each number to get to a recognisable division fact. The first one has been done for you.

a  $112 \div 14$       $\frac{56}{\quad} \div \frac{7}{\quad} = \boxed{8}$

b  $144 \div 16$       $\frac{\quad}{\quad} \div \frac{\quad}{\quad} = \boxed{\quad}$

c  $96 \div 12$       $\frac{\quad}{\quad} \div \frac{\quad}{\quad} = \boxed{\quad}$

d  $220 \div 4$       $\frac{\quad}{\quad} \div \frac{\quad}{\quad} = \boxed{\quad}$

e  $162 \div 18$       $\frac{\quad}{\quad} \div \frac{\quad}{\quad} = \boxed{\quad}$

3 Match the problems with their halved equivalents. Then solve the problem. The first one has been done for you.

a $90 \div 18$	$60 \div 6$	= $\boxed{5}$
b $64 \div 16$	$24 \div 8$	= $\boxed{\quad}$
c $120 \div 12$	$35 \div 7$	= $\boxed{\quad}$
d $70 \div 14$	$45 \div 9$	= $\boxed{\quad}$
e $144 \div 24$	$72 \div 12$	= $\boxed{\quad}$
f $48 \div 16$	$32 \div 8$	= $\boxed{\quad}$

## Mental division strategies – halving strategy

Sometimes we need to keep halving until we reach an easy division fact.

$$144 \div 36 \rightarrow 72 \div 18 \rightarrow 36 \div 9 = 4$$

- 4 Keep halving until you get to a fact you can work with. If you can do it in your head, just fill in the last box. Otherwise, use the lines to help you.

a  $216 \div 36 =$  \_\_\_\_\_  $\div$  \_\_\_\_\_  $=$  \_\_\_\_\_  $\div$  \_\_\_\_\_  $=$

b  $196 \div 28 =$  \_\_\_\_\_  $\div$  \_\_\_\_\_  $=$  \_\_\_\_\_  $\div$  \_\_\_\_\_  $=$

c  $224 \div 32 =$  \_\_\_\_\_  $\div$  \_\_\_\_\_  $=$  \_\_\_\_\_  $\div$  \_\_\_\_\_  $=$

d  $168 \div 24 =$  \_\_\_\_\_  $\div$  \_\_\_\_\_  $=$  \_\_\_\_\_  $\div$  \_\_\_\_\_  $=$

e  $144 \div 36 =$  \_\_\_\_\_  $\div$  \_\_\_\_\_  $=$  \_\_\_\_\_  $\div$  \_\_\_\_\_  $=$

f  $288 \div 72 =$  \_\_\_\_\_  $\div$  \_\_\_\_\_  $=$  \_\_\_\_\_  $\div$  \_\_\_\_\_  $=$

- 5 Draw lines to connect numbers that could be doubled or halved to reach each other.

48	10	16	40
20	32	25	64
128	60	96	30
125	256	192	120
100	250	80	50

- 6 Work with a partner to solve this problem using halving:

You have an after school job at the local sweet shop, making up the mixed sweet bags. Today, you have to evenly share 288 sweets among 48 bags. How many sweets will you put in each bag? Show each halved sum.

# Mental division strategies – split strategy

Division problems also become easier if you split the number to be divided into recognisable facts.

Look at the problem  $144 \div 9$

Can we divide 144 into 2 multiples of 9?

We can divide it into 54 and 90. These are both easily divided by 9. Then we add the two answers together.

$$\begin{array}{r}
 144 \div 9 \\
 \swarrow \quad \searrow \\
 \begin{array}{r} 90 \\ \hline \div 9 \\ \hline 10 \end{array} \quad \begin{array}{r} 54 \\ \hline \div 9 \\ \hline 6 \end{array} \\
 10 + 6 = 16
 \end{array}$$

**1** Use the split strategy to divide these numbers. Use the clues to guide you:

**a**

$$\begin{array}{r}
 112 \div 8 \\
 \swarrow \quad \searrow \\
 \begin{array}{r} 80 \\ \hline \div 8 \\ \hline \end{array} \quad \begin{array}{r} 32 \\ \hline \div 8 \\ \hline \end{array} \\
 \underline{\quad} + \underline{\quad} = \boxed{\quad}
 \end{array}$$

**b**

$$\begin{array}{r}
 85 \div 5 \\
 \swarrow \quad \searrow \\
 \begin{array}{r} 50 \\ \hline \div 5 \\ \hline \end{array} \quad \begin{array}{r} \underline{\quad} \\ \hline \div 5 \\ \hline \end{array} \\
 \underline{\quad} + \underline{7} = \boxed{\quad}
 \end{array}$$

**c**

$$\begin{array}{r}
 78 \div 6 \\
 \swarrow \quad \searrow \\
 \begin{array}{r} \underline{\quad} \\ \hline \div 6 \\ \hline \end{array} \quad \begin{array}{r} 18 \\ \hline \div 6 \\ \hline \end{array} \\
 \underline{10} + \underline{\quad} = \boxed{\quad}
 \end{array}$$

**d**

$$\begin{array}{r}
 64 \div 4 \\
 \swarrow \quad \searrow \\
 \begin{array}{r} 24 \\ \hline \div 4 \\ \hline \end{array} \quad \begin{array}{r} \underline{\quad} \\ \hline \div 4 \\ \hline \end{array} \\
 \underline{\quad} + \underline{\quad} = \boxed{\quad}
 \end{array}$$

**e**

$$\begin{array}{r}
 91 \div 7 \\
 \swarrow \quad \searrow \\
 \begin{array}{r} 21 \\ \hline \div 7 \\ \hline \end{array} \quad \begin{array}{r} \underline{\quad} \\ \hline \div 7 \\ \hline \end{array} \\
 \underline{\quad} + \underline{\quad} = \boxed{\quad}
 \end{array}$$

**f**

$$\begin{array}{r}
 144 \div 8 \\
 \swarrow \quad \searrow \\
 \begin{array}{r} 80 \\ \hline \div 8 \\ \hline \end{array} \quad \begin{array}{r} 64 \\ \hline \div 8 \\ \hline \end{array} \\
 \underline{\quad} + \underline{\quad} = \boxed{\quad}
 \end{array}$$

**2** Now try these:

**a**

$$90 \div 6 \begin{cases} \rightarrow \frac{60}{\hline \div 6} \\ \rightarrow \frac{30}{\hline \div 6} \end{cases} = \boxed{\quad}$$

**b**

$$105 \div 7 \begin{cases} \rightarrow \frac{70}{\hline \div \quad} \\ \rightarrow \frac{\quad}{\hline \div \quad} \end{cases} = \boxed{\quad}$$

**c**

$$72 \div 4 \begin{cases} \rightarrow \frac{\quad}{\hline \div \quad} \\ \rightarrow \frac{24}{\hline \div \quad} \end{cases} = \boxed{\quad}$$

**d**

$$144 \div 8 \begin{cases} \rightarrow \frac{\quad}{\hline \div \quad} \\ \rightarrow \frac{96}{\hline \div \quad} \end{cases} = \boxed{\quad}$$

Hmmm ...  $91 \div 7$ .  
The unit digit helps me here. What multiple of 7 ends in 1? I know, 21.  
So that makes the other number 70!




*DISCOVER*



## Mental division strategies – split strategy

- 3 Play this game with a partner. Use one copy of this page between you. Cut out the problems on the left and stack them face up. Cut out and spread the other cards face up. Work together (or race) to find two numbers you could divide to solve the problem on the top card of the pile. One card in the pair will be grey, the other white. For example, if the problem was  $76 \div 4$ , you could locate 36 and 40.



$96 \div 4$	45	90
$75 \div 5$	25	21
$87 \div 3$	60	50
$98 \div 7$	80	70
$135 \div 9$	55	36
$78 \div 6$	30	60
$112 \div 8$	60	60
$51 \div 3$	27	32
$95 \div 5$	24	40
$84 \div 6$	28	18

# Mental division strategies – tests of divisibility

Divisibility tests tell us if a number can be divided evenly by another (that is with no remainders).

1 Use the rules to test out the numbers in the last column. The first two have been done for you:

Divisible by	Rule	Test
2	A number is divisible by 2 if it's even (ends in 0, 2, 4, 6 or 8).	Is 458 divisible by 2? <i>Yes, because it ends in an even number.</i>
3	A number is divisible by 3 if the sum of its digits is divisible by 3.	Is 7,281 divisible by 3? $7 + 2 + 8 + 1 = 18$ <i>Yes, because 18 is divisible by 3.</i>
4	A number is divisible by 4 if the number made by the last 2 digits is divisible by 4.	Is 3,912 divisible by 4?
5	A number is divisible by 5 if there's a 0 or 5 in the ones place.	Is 455 divisible by 5?
8	A number is divisible by 8 if the last 3 digits are divisible by 8.	Is 74,160 divisible by 8?
9	A number is divisible by 9 if the sum of its digits is divisible by 9.	Is 6,345 divisible by 9?
10	A number is divisible by 10 if there is a zero in the ones place.	Is 5,680 divisible by 10?

# Mental division strategies – tests of divisibility

- 2 These numbers can all be divided with no remainders. Work with a partner to find the rule/s that can be used to divide them. Fill in the tables.

36	90	84	99	50	72
456	330	888	120	981	548
1,025	3,486	6,993	1,256	9,050	10,072

$\div 4$

$\div 9$

$\div 5$

$\div 3$

$\div 8$

Numbers may go onto more than 1 table!



# Written methods – short multiplication

	H	T	O
	1	5	6
x			3
	4	6	8
	1	1	

Short multiplication is one way to solve a multiplication problem.

First we use our mental strategies to estimate an easier problem:  
 $3 \times 150 = 450$ . The answer will be around 450.

We start with the ones.  $3 \times 6$  is 18 ones. We rename this as 1 ten and 8 ones.

We put 8 in the ones column and carry the 1 to the tens column.

$3 \times 5$  plus the carried 1 is 16 tens. We rename this as 1 hundred and 6 tens.

We put 6 in the tens column and carry the 1 to the hundreds column.

$3 \times 1$  plus the carried 1 is 4 hundreds. We put 4 in the hundreds column.

## 1 Solve these problems using short multiplication. Estimate first:

a

	H	T	O
	3	2	7
x			3

b

	H	T	O
	2	4	7
x			4

c

	H	T	O
	1	5	4
x			5

d

	H	T	O
	3	1	5
x			3

e

	H	T	O
	2	8	6
x			2

f

	H	T	O
	1	9	4
x			5

## 2 Solve these word problems. Show how you worked them out:

- a Dan's dad has resorted to bribery to counteract Dan's PlayStation addiction. For every evening, Dan spends away from the PlayStation, his dad pays him £3. So far, Dan has racked up an impressive 27 nights (though he looks like breaking any day now). How much money does this equate to?

- b Dan's mum thinks she might get in on the action too and pays Dan £4 for every week that he puts his dishes in the dishwasher and his dirty clothes in the basket. Dan is less keen on this plan but does manage 33 weeks in 1 year. How much has he made out of this scheme?

# Written methods – short multiplication

3 Below are Jess and Harry's tests. Check them and give them a mark out of 5. If they made mistakes, give them some feedback as to where they went wrong.

**Jess**

$$\begin{array}{r} 387 \\ \times \quad 2 \\ \hline 774 \\ \hline \end{array}$$

□ □ □

$$\begin{array}{r} 119 \\ \times \quad 7 \\ \hline 773 \\ \hline \end{array}$$

□ □ □

$$\begin{array}{r} 2203 \\ \times \quad 3 \\ \hline 6609 \\ \hline \end{array}$$

□ □ □ □

$$\begin{array}{r} 436 \\ \times \quad 3 \\ \hline 1208 \\ \hline \end{array}$$

□ □ □ □

$$\begin{array}{r} 401 \\ \times \quad 7 \\ \hline 2807 \\ \hline \end{array}$$

□ □ □ □

**Harry**

$$\begin{array}{r} 387 \\ \times \quad 2 \\ \hline 774 \\ \hline \end{array}$$

□ □ □

$$\begin{array}{r} 119 \\ \times \quad 7 \\ \hline 833 \\ \hline \end{array}$$

□ □ □

$$\begin{array}{r} 3203 \\ \times \quad 3 \\ \hline 969 \\ \hline \end{array}$$

□ □ □ □

$$\begin{array}{r} 436 \\ \times \quad 3 \\ \hline 1308 \\ \hline \end{array}$$

□ □ □ □

$$\begin{array}{r} 1401 \\ \times \quad 7 \\ \hline 7287 \\ \hline \end{array}$$

□ □ □ □

# Written methods – short division

In short division, we use our knowledge of multiplication to help us. We can split 936 into  $900 + 30 + 6$ .

$$\begin{array}{r} 3 \quad 1 \quad 2 \\ 3 \overline{) 936} \end{array}$$

900 divided by 3 is 300, so we put a 3 in the hundreds place.

30 divided by 3 is 10, so we put a 1 in the tens place.

6 divided by 3 is 2, so we put a 2 in the ones place.

$$936 \div 3 = 312$$

## 1 Divide these numbers:

a  $4 \overline{) 84}$

b  $5 \overline{) 55}$

c  $3 \overline{) 93}$

d  $9 \overline{) 990}$

e  $4 \overline{) 484}$

f  $6 \overline{) 666}$

g  $3 \overline{) 999}$

h  $2 \overline{) 462}$

i  $3 \overline{) 693}$

Sometimes it's easier to split the numbers differently. We can also split 936 into  $900 + 36$ .

$$\begin{array}{r} 3 \quad 1 \quad 2 \\ 3 \overline{) 936} \end{array}$$

900 divided by 3 is 300 so we put a 3 in the hundreds place

36 divided by 3 is 12. We put the 1 in the tens place and the 2 in the ones place.

$$936 \div 3 = 312$$

In these problems, if there are no tens in a number we put a 0 in to show this and also to hold the place of the other numbers!



## 2 Decide how you'll split these numbers and then divide. Remember to put in zeros as needed.

a  $5 \overline{) 515}$

b  $3 \overline{) 669}$

c  $9 \overline{) 927}$

d  $4 \overline{) 804}$

e  $4 \overline{) 812}$

## Written methods – short division

Sometimes a number doesn't easily split and we have to use a different method to solve a division.

$$\begin{array}{r} 166 \\ 5 \overline{) 830} \\ \underline{5} \phantom{0} \\ 33 \phantom{0} \\ \underline{30} \phantom{0} \\ 30 \\ \underline{30} \\ 0 \end{array}$$

Look at 830 divided by 5. We start with the largest place value. 8 hundreds divided by 5 is 100. There is 300 left over, which we rename and carry over to the tens column. 33 tens divided by 5 is 6 with 3 left over. We rename and carry these 3 tens to the ones. 30 divided by 5 is 6 exactly.

So  $830 \div 5 = 166$

### 3 Solve these divisions:

a  $6 \overline{) 738}$

b  $5 \overline{) 865}$

c  $3 \overline{) 702}$

d  $4 \overline{) 944}$

e  $8 \overline{) 744}$

f  $9 \overline{) 954}$

g  $7 \overline{) 4788}$

h  $4 \overline{) 9468}$

i  $3 \overline{) 2607}$

j  $6 \overline{) 6216}$

k  $8 \overline{) 9872}$

l  $5 \overline{) 2945}$

### 4 Solve these problems:

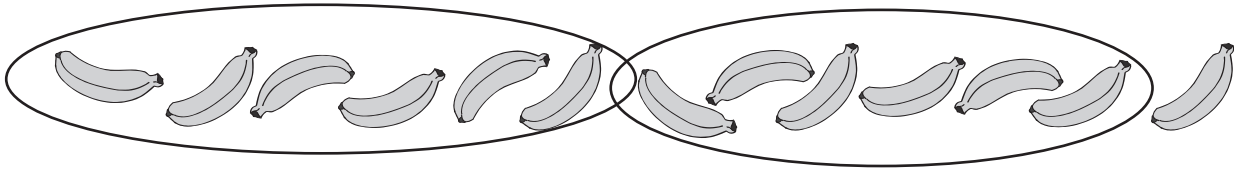
a Four friends find £936 in the street. They take it to the police but no-one claims it and the money is eventually returned to them to keep. If they share it fairly between themselves, how much do they get each?

b A carpenter has a piece of timber 1,878 mm long. He needs to split it into 6 equal pieces. How long should each piece be?

# Written methods – division with remainders

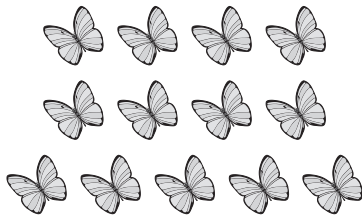
Sometimes when we make equal groups there are some left over.

Here are 13 bananas. If we make 2 equal groups of 6, there is 1 banana left over.



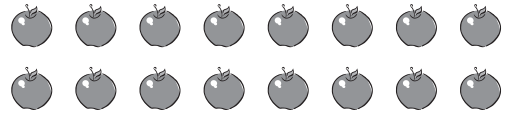
## 1 Make groups of each of the following items and show the left overs:

a Here are 13 butterflies:



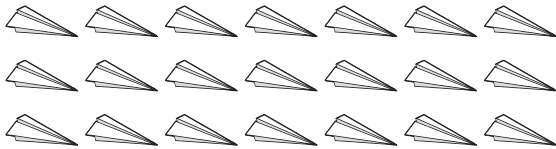
If we make \_\_\_\_\_ equal groups  
of 3 there is \_\_\_\_\_ left over.

b Here are 16 apples:



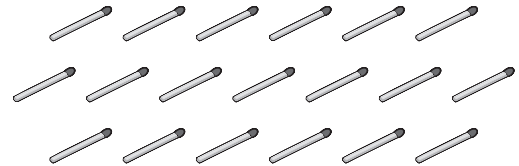
If we make \_\_\_\_\_ equal groups  
of 7 there are \_\_\_\_\_ left over.

c Here are 21 paper planes:



If we make \_\_\_\_\_ equal groups  
of 6 there are \_\_\_\_\_ left over.

d Here are 19 match sticks:



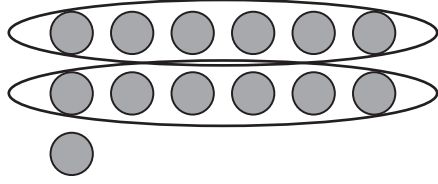
If we make \_\_\_\_\_ equal groups  
of 5 there are \_\_\_\_\_ left over.

## 2 Draw a picture to show 12 groups of 2 with 1 left over.

How many are there in total?



## Written methods – division with remainders



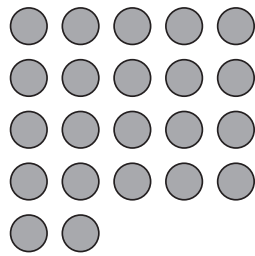
Sometimes division is not exact.

From 13, we can make 2 fair shares of 6 with 1 left over.  
We call the left over the remainder.

$$13 \div 6 = 2 \text{ remainder } 1$$

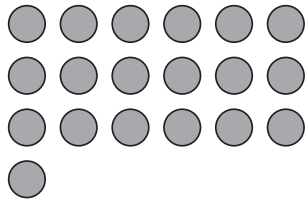
**3** In each array, ring the fair shares to see the remainder:

a



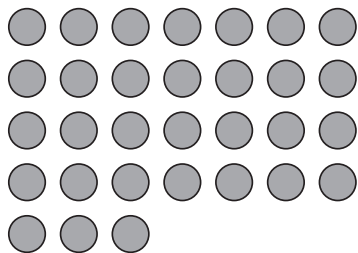
$$22 \div 5 = \square \text{ remainder } \square$$

b



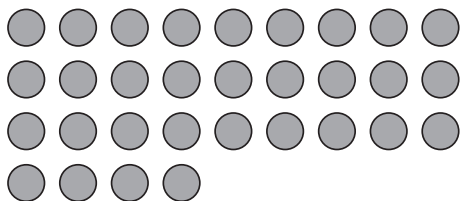
$$19 \div 6 = \square \text{ remainder } \square$$

c



$$31 \div 7 = \square \text{ remainder } \square$$

d



$$31 \div 9 = \square \text{ remainder } \square$$

## Written methods – division with remainders

Now use your multiplication facts.

$$25 \div 6 = \boxed{?}$$

Think  $4 \times 6 = 24 + 1$  is 25

So,  $25 \div 6 = 4$  remainder 1

### 4 Use your multiplication facts to write the division facts and the remainder:

a  $32 \div 10 = \boxed{?}$

Think  $\boxed{\phantom{00}} \times \boxed{\phantom{00}} = \boxed{\phantom{00}} + \boxed{\phantom{00}}$  is  $\boxed{\phantom{00}}$

So,  $\boxed{\phantom{00}} \div \boxed{\phantom{00}} = \boxed{\phantom{00}}$  remainder  $\boxed{\phantom{00}}$

b  $30 \div 4 = \boxed{?}$

Think  $\boxed{\phantom{00}} \times \boxed{\phantom{00}} = \boxed{\phantom{00}} + \boxed{\phantom{00}}$  is  $\boxed{\phantom{00}}$

So,  $\boxed{\phantom{00}} \div \boxed{\phantom{00}} = \boxed{\phantom{00}}$  remainder  $\boxed{\phantom{00}}$

c  $37 \div 9 = \boxed{?}$

Think  $\boxed{\phantom{00}} \times \boxed{\phantom{00}} = \boxed{\phantom{00}} + \boxed{\phantom{00}}$  is  $\boxed{\phantom{00}}$

So,  $\boxed{\phantom{00}} \div \boxed{\phantom{00}} = \boxed{\phantom{00}}$  remainder  $\boxed{\phantom{00}}$

### 5 Complete each word problem:

a 39 pencils were shared between 6 kids. How many did each kid get?

$$\boxed{\phantom{00}} \div \boxed{\phantom{00}} = \boxed{\phantom{00}} \text{ remainder } \boxed{\phantom{00}}$$

b 43 fish were divided between 6 tanks. How many fish are in each tank?

$$\boxed{\phantom{00}} \div \boxed{\phantom{00}} = \boxed{\phantom{00}} \text{ remainder } \boxed{\phantom{00}}$$

c From 17 flowers, 5 flowers were arranged in each vase. How many vases were used?

$$\boxed{\phantom{00}} \div \boxed{\phantom{00}} = \boxed{\phantom{00}} \text{ remainder } \boxed{\phantom{00}}$$

### 6 Write in the missing digit to make this statement true:

$$\boxed{\phantom{00}} \div 6 = 8 \text{ remainder } 2$$

## Written methods – division with remainders

This is the way we write remainders when using the division symbol.

$$\begin{array}{r} 2 \text{ r } 3 \\ 6 \overline{) 15} \\ \underline{12} \\ 3 \end{array}$$

This is the same as  $15 \div 6 = 2$  remainder 3.

Check your work with the closest multiplication fact:

$$6 \times 2 = 12$$

$$\text{Then add on the remainder: } 12 + 3 = 15$$

**7** Solve these division problems and then check them.

**a**

$$\begin{array}{r} \square \text{ r } \square \\ 8 \overline{) 27} \end{array}$$

Check with the multiplication fact and add the remainder:

$$\square \times \square = \square + \square$$

**b**

$$\begin{array}{r} \square \text{ r } \square \\ 9 \overline{) 38} \end{array}$$

Check with the multiplication fact and add the remainder:

$$\square \times \square = \square + \square$$

**c**

$$\begin{array}{r} \square \text{ r } \square \\ 6 \overline{) 45} \end{array}$$

Check with the multiplication fact and add the remainder:

$$\square \times \square = \square + \square$$

**d**

$$\begin{array}{r} \square \text{ r } \square \\ 5 \overline{) 48} \end{array}$$

Check with the multiplication fact and add the remainder:

$$\square \times \square = \square + \square$$

**8** What is the question if I am checking with this multiplication fact?

$$\begin{array}{r} \square \text{ r } \square \\ \overline{) \quad \quad} \end{array}$$

$$5 \times 6 = 30 + 3$$

# Written methods – division with remainders

Sometimes numbers don't divide evenly. The amount left over is called the **remainder**.

Look at 527 divided by 5.

500 divided by 5 is 100.

27 divided by 5 is 5 with 2 left over (this is the remainder).

This can be written as r 2.

$$527 \div 5 = 105 \text{ r } 2.$$

$$\begin{array}{r} 105 \text{ r } 2 \\ 5 \overline{) 527} \\ \underline{5} \phantom{00} \\ 2 \phantom{0} \\ \underline{25} \\ 7 \end{array}$$

9 Divide these 2-digit numbers. Each problem will have a remainder.

a  $9 \overline{) 75} \text{ r } \square$

b  $4 \overline{) 47} \text{ r } \square$

c  $6 \overline{) 38} \text{ r } \square$

d  $5 \overline{) 63} \text{ r } \square$

e  $4 \overline{) 49} \text{ r } \square$

f  $6 \overline{) 62} \text{ r } \square$

10 Divide these 3-digit numbers. Each problem will have a remainder.

a  $5 \overline{) 557} \text{ r } \square$

b  $3 \overline{) 661} \text{ r } \square$

c  $4 \overline{) 481} \text{ r } \square$

d  $9 \overline{) 994} \text{ r } \square$

e  $6 \overline{) 6945} \text{ r } \square$

f  $4 \overline{) 8963} \text{ r } \square$

11 Solve these problems:

- a Giovanni's Nan has given him a bag of gold coins to share among him and his two sisters. There are 47 gold coins altogether. How many does each child get if they're shared evenly? How would you suggest they deal with the remainder?

- b You have 59 jelly beans to add to party bags. Each bag gets five jelly beans. How many full party bags can you make?

# Written methods – division with remainders

There are 3 ways of expressing remainders. How we do it depends on how we'd deal with the problem in the real world. Look at:

$$\begin{array}{r}
 105r2 \\
 5 \overline{) 527}
 \end{array}$$

**12** One way is to write  $r2$  as in the example above. We use this when we don't care about being absolutely precise and when the remainder can't be easily broken up. An example would be sharing 527 jelly beans among 5 people. Solve these problems expressing the remainders as  $r$ .

a Share 126 blue pencils among 4 people.

b Share 215 paper clips among 7 people.

**13** We can also express a remainder as a fraction. We do this when we can easily share the remainder. For example, 19 cakes shared among 3 people is 6 and one third each. Solve these problems expressing the remainder as a fraction:

a Share 13 pizzas among 4 people.

b Share 50 sandwiches among 3 people.

$$\begin{array}{r}
 6\frac{1}{3} \\
 3 \overline{) 19}
 \end{array}$$



**REMEMBER**

**14** We express remainders as decimals when we must be absolutely precise. Sharing pound amounts is a good example of this. We add the pence after the decimal point to help us. Try these:

a Share 12 pounds among 4 people.

b Share 27 pounds between 2 people.

$$\begin{array}{r}
 3.00 \\
 4 \overline{) 12.00}
 \end{array}$$

$$\begin{array}{r}
 13.50 \\
 2 \overline{) 27.00}
 \end{array}$$

27 divided by 2 is 13. Now we have one pound left. How many pence is half of one pound?



**THINK**

# Written methods – solving problems

We regularly come across multiplication and division problems in our everyday life. It doesn't matter which strategy we use to solve them, we can choose the one that suits us or the problem best.

**1** One real-life problem is comparing prices to find the best deal. It's easy if the prices and amounts are the same but what if the amounts are different? Use a strategy to help you find the best deal on these:

**a**

100 g      £1.95

300 g      £5.43

Best deal is \_\_\_\_\_

**b**

500 g      £3.95

1 kg      £8.50

Best deal is \_\_\_\_\_

**c**

10 pack CD      £22.90

Single CD      £2.75

Best deal is \_\_\_\_\_

**d**

500 ml      £1.40

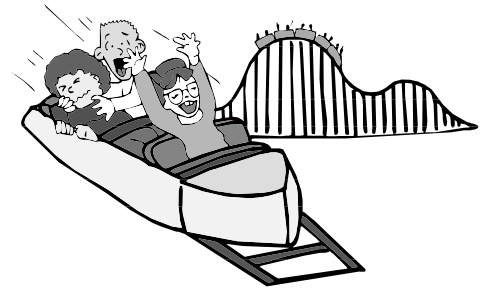
2 litres      £2.80

Best deal is \_\_\_\_\_

**2** You go to the service station with your weekly pocket money of £5. When you take a £1.75 chocolate bar to the counter, they offer you the special of 3 bars for £4.50. Which is a better deal? Show why.

## Written methods – solving problems

- 3 You're planning a trip to the Wet and Wild theme park and there are many ticket options. Use a strategy of your choice and the price list below to answer the following questions:



### Entry

1-day pass **£16**

2-day pass **£24**

Annual pass **£45**

Individual rides **£6**

10-ride pass **£50**

Order online **£2 discount**

### Extras

5-minute helicopter ride **£21**

10-minute helicopter ride **£35**

30-minute helicopter ride **£105**

Sunset cruise **£6**

Lunch cruise **£11**

Swim with the dolphins **£35**

- a If you buy a 2-day pass, what is the cost per day?
- b How much cheaper is this option than buying two 1-day passes?
- c If you bought an annual pass, how many times would you need to visit to make it a better option than buying either a 1-day or 2-day pass?
- d What if you choose just the rides? How much would you save if you bought the 10-ride pass instead of the individual rides?
- e If you took a 5-minute helicopter ride, what would be the cost per minute?
- f What about if you chose the 10-minute flight option? What would be the cost per minute?
- g Plan a day's itinerary for you and a partner. How much will this cost?

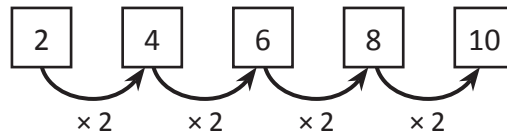
# Patterns and algebra – patterns and functions

Look around you, can you see a pattern? A pattern is an arrangement of shapes, numbers or objects formed according to a rule. Patterns are everywhere, you can find them in nature, art, music and even in dance!

In this topic, we are looking at number patterns. A number pattern is a sequence or list of numbers that is formed according to a rule.

Number patterns can use any of the four operations (+, −, ×, ÷) or even a combination.

In the example below, if we follow this instruction: “starting at 1 add 5 each time” we get this number pattern:



**1** Write the next 3 numbers in each sequence by following the rule:

- a Rule: multiply by 3    3 → 9 → 27 →  →  →
- b Rule: divide by 2    64 → 32 → 16 →  →  →
- c Rule: multiply by 4    1 → 4 → 16 →  →  →

**2** Identify the missing numbers in each pattern and write the rule.

- a    1    5        125    625
- Rule \_\_\_\_\_
- b    2,187    729    243        27
- Rule \_\_\_\_\_
- c    4,096        256    64        4
- Rule \_\_\_\_\_
- d    3        27    81        729
- Rule \_\_\_\_\_

**3** Complete these grid patterns. Look closely at the numbers in the grid and follow the patterns.

a

	→ × 3			
	5	15	45	135
↓ × 2	10			

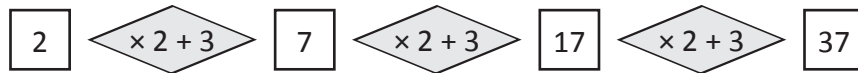
b

	→ × 10		
			1,000
↓ × 4	4		
			1,600



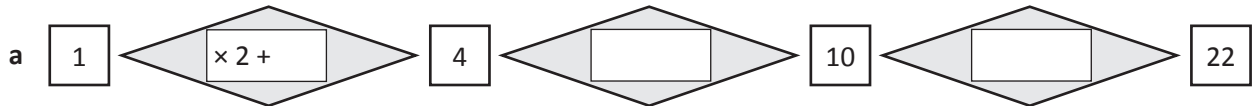
# Patterns and algebra – recursive number patterns

Some number patterns can be formed with 2 operations each time. For example:

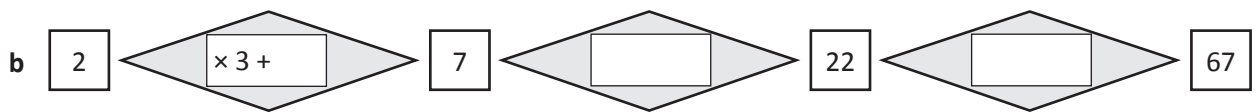


The rule is multiply by 2 and add 3 each time.

**4** With these number patterns, write the rule as 2 operations in the diamond shapes and describe it underneath.



The rule is \_\_\_\_\_



The rule is \_\_\_\_\_

**5** Lena and Max were asked to show a number pattern for different rules. Check each sequence and put a circle around any errors. You may use a calculator.

a Start at 2, add 1 and multiply by 2

Lena	2	6	14	30	62	126	254	510	1,022
------	---	---	----	----	----	-----	-----	-----	-------

b Start at 3, add 1 and multiply by 2

Max	3	8	18	38	78	158	320	640	1,280
-----	---	---	----	----	----	-----	-----	-----	-------

**6** Look at each pattern of shapes and see if you can predict the following:



What will shape number 20 look like? Draw it here:

What will shape number 33 look like? Draw it here:



What will shape number 15 look like? Draw it here:

What will shape number 26 look like? Draw it here:

# Patterns and algebra – function number patterns

There are 2 different types of rules that a number pattern can be based upon:

- 1 A recursive rule – used to continue the sequence by doing something to the **number** before it.
- 2 A function rule – used to predict any number by applying the rule to the **position** of the number.

A function rule is a rule based on the position of a number.

Consider this. Lucia was given this number pattern: 

5	10	15	20	25
---	----	----	----	----

Her teacher asked her to work out what the 20th number would be without continuing the sequence. Lucia used a table to work out the rule between the position of a number and the number in the pattern. She worked out the rule to be  $\times 5$ .

Position of number	1	2	3	4	5	20
Function rule	$\times 5$	$\times 5$	$\times 5$	$\times 5$	$\times 5$	$\times 5$
Number pattern	5	10	15	20	25	100

So, following the rule based on the position of a number, the 20th number is 100.

This is a function rule.

## 1 Use the function rule and then apply the rule to position 20.

**a**

Position of number	1	2	3	4	5	20
Function rule						
Number pattern	6	12	18	24	30	

**b**

Position of number	1	2	3	4	5	20
Function rule						
Number pattern	4	8	12	16	20	

**c**

Position of number	1	2	3	4	5	20
Function rule						
Number pattern	8	16	24	32	40	

**d**

Position of number	1	2	3	4	5	20
Function rule	$\times 4 +$					
Number pattern	7	11	15	19	23	

**HINT:** In the last pattern, the rule has 2 operations.



**THINK**

## Patterns and algebra – function number patterns

Function rules with 2 operations are easy to work out when we look at how they are linked to the multiplication tables.

Position of number	1	2	3	4	5
<b>2 times table + 3</b>	<b>2 + 3</b>	<b>4 + 3</b>	<b>6 + 3</b>	<b>8 + 3</b>	<b>10 + 3</b>
Number pattern	5	7	9	11	13
Function rule	Multiply by 2 and then add 3				

This table shows that the number pattern is the same as the 2 times table with 3 added to each answer.

**2** Complete each table to show how function rules with 2 operations can be linked to multiplication tables.

a

Position of number	1	2	3	4	5
<b>3 times table + _____</b>	<b>3 + _____</b>	<b>6 + _____</b>	<b>9 + _____</b>	<b>12 + _____</b>	<b>15 + _____</b>
Number pattern	7	10	13	16	19
Function rule	Multiply by 3 and then add _____				

b

Position of number	1	2	3	4	5
<b>6 times table + _____</b>	<b>6 + _____</b>	<b>12 + _____</b>	<b>18 + _____</b>	<b>24 + _____</b>	<b>30 + _____</b>
Number pattern	8	14	20	26	32
Function rule	Multiply by 6 and then add _____				

c

Position of number	1	2	3	4	5
<b>_____ times table + _____</b>					
Number pattern	11	19	27	35	43
Function rule					

**3** Complete this table to show the 4 times tables with 2 added.

a

Position of number	1	2	3	4	5
<b>4 times table + 2</b>					
Number pattern					
Function rule					

b What would the number in the 20th position be? \_\_\_\_\_

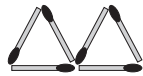
# Patterns and algebra – matchstick patterns

Use the function rule to predict geometric patterns with matchsticks. Here is an example.  
Mia made this sequence of shapes with matchsticks:

Shape 1



Shape 2



Shape 3



Shape 4



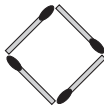
If Mia followed this sequence, how many matchsticks will she need for shape 20?

Shape number	1	2	3	4	5	<b>20</b>
Number of matchsticks	3	6	9	12	15	60
Function rule	Number of matchsticks = Shape number $\times$ <u>3</u>					

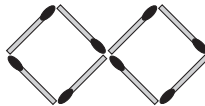
**1** Complete the table for each sequence of matchstick shapes. Use the function rule for finding the number of matchsticks needed for the shape in the 20th position.

a

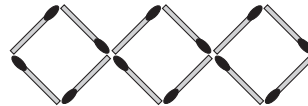
Shape 1



Shape 2



Shape 3



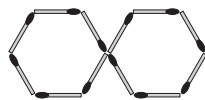
Shape number	1	2	3	4	5	<b>20</b>
Number of matchsticks	4	8	12			
Function rule	Number of matchsticks = Shape number $\times$ _____					

b

Shape 1



Shape 2



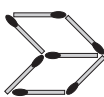
Shape 3



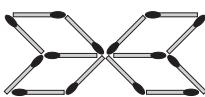
Shape number	1	2	3	4	5	<b>20</b>
Number of matchsticks	6	12	18			
Function rule	Number of matchsticks = Shape number $\times$ _____					

c

Shape 1



Shape 2



Shape 3



Shape number	1	2	3	4	5	<b>20</b>
Number of matchsticks	7	14	21			
Function rule	Number of matchsticks = Shape number $\times$ _____					

# Patterns and algebra – matchstick patterns

This time the rule for this matchstick pattern has 2 operations. Can you see why? Look for a multiplication pattern and how many extra there are in each shape.

Look for a repeating element.

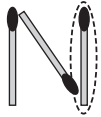


Then look to see what is added.

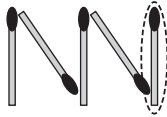
These are circled in the sequence below.



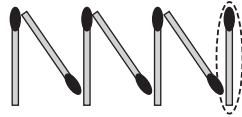
Shape 1



Shape 2



Shape 3



Shape 1 has 3 matchsticks  $1 \times 2 + 1 = 3$

Shape 2 has 5 matchsticks  $2 \times 2 + 1 = 5$

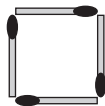
Shape 3 has 7 matchsticks  $3 \times 2 + 1 = 7$

Shape number	1	2	3	4	5	20
Number of matchsticks	3	5	7	9	11	41
Function rule	Number of matchsticks = Shape number $\times$ 2 + 1					

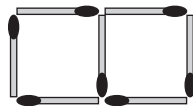
2 In each of these patterns, look for the repeating element and then what is added each time:

a

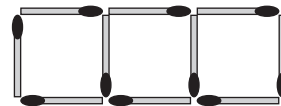
Shape 1



Shape 2



Shape 3



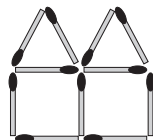
Shape number	1	2	3	4	5	20
Number of matchsticks	4	7	10			
Function rule	Number of matchsticks = Shape number $\times$ ____ + ____					

b

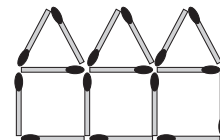
Shape 1



Shape 2



Shape 3



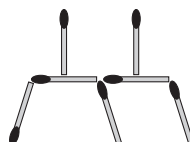
Shape number	1	2	3	4	5	20
Number of matchsticks						
Function rule	Number of matchsticks = Shape number $\times$ ____ + ____					

c

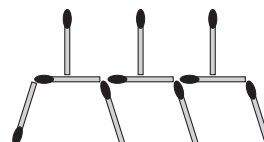
Shape 1



Shape 2



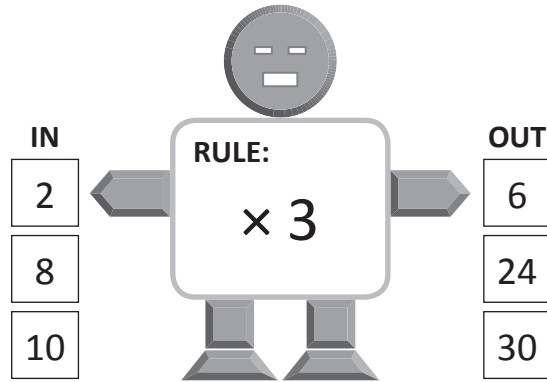
Shape 3



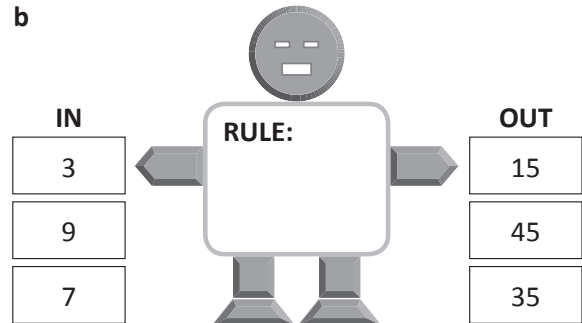
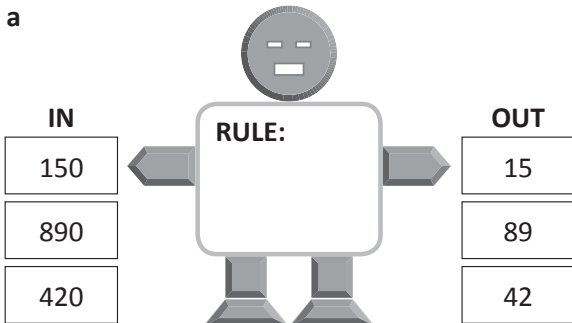
Shape number	1	2	3	4	5	20
Number of matchsticks						
Function rule	Number of matchsticks = Shape number $\times$ ____ + ____					

# Patterns and algebra – function machines

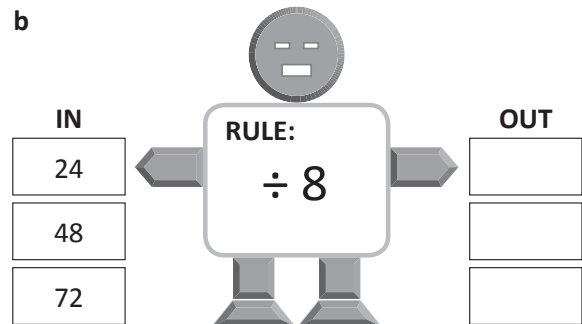
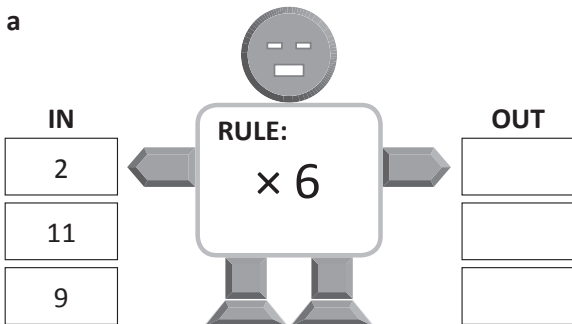
This is a function machine.  
Numbers go in, have the rule applied,  
and come out again.



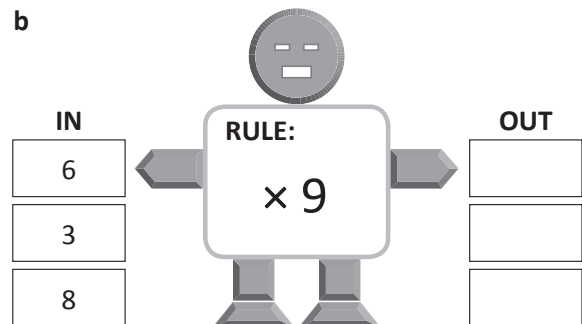
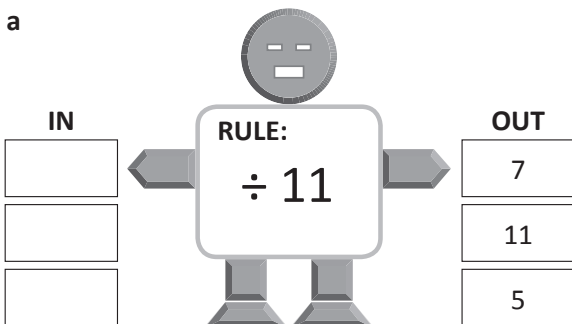
- 1 Look carefully at the numbers going *in* these function machines and the numbers coming *out*.  
What rule are they following each time?



- 2 What numbers will come *out* of these function machines?



- 3 What numbers go *in* to these number function machines?



# Patterns and algebra – function machines

4 Write the rule in each double function machine. Each rule is made up of 2 operations ( $\times$  then  $+$ ).

a

IN	RULE:	OUT
2		10
8		34
10		42

b

IN	RULE:	OUT
6		31
10		51
7		36

c

IN	RULE:	OUT
3		20
20		122
11		68

d

IN	RULE:	OUT
5		50
4		41
3		32

5 Which function machine will win this game of bingo? Write the numbers that come out and colour each machine's numbers in a different colour. Check which machine has 3 numbers in a line in any direction.

<b>MATHS</b> <b><math>\times \div</math> BINGO <math>+-</math></b>				
27	16	45	12	17
42	32	22	18	23
47	68	★ FREE SPACE	18	29
15	20	37	15	32
14	30	43	16	35

IN	RULE:	OUT
1	$\times 2 + 12$	
2		
3		
4		
5		

IN	RULE:	OUT
1	$\times 5 + 22$	
2		
3		
4		
5		

# Patterns and algebra – function tables with multiplication

Let's look at more real life function tables, this time based on multiplication.

By working out the function, you can extend the pattern to find out the unknowns.

For example:

A bakery makes 10 cupcakes an hour.

The rule to work out the number of cupcakes this bakery produces within a certain amount of time is:

$$\text{Number of hours} \times 10 = \text{Number of cupcakes}$$

Hours	1	2	3	4	5	6	7	8
Cupcakes	10	20	30	40	50	60	70	80

How many cupcakes will it make in 1 day?

*This table only goes up to 8 hours but we can use the function to answer this question:*

$$24 \text{ hours} \times 10 \text{ cupcakes} = 240 \text{ cupcakes}$$



## 1 Complete the function tables, write the rule and answer the question.

a A dry cleaner charges £2 to iron a shirt.

Number of shirts	1	2	3	4	5	6	7	8
Cost	£2	£4	£6					

Write the rule for finding out the cost of ironing shirts when you know how many shirts:

How much does it cost to have 12 shirts ironed?

b Monica and Anna have a lemonade stand outside their house. For every litre of lemonade they make 4 cups to sell.

Litres	1	2	3	4	5	6	7	8
Cups	4	8						

Write the rule for finding out how many cups are needed when you know how many litres have been made:

How many cups will be needed if they have enough to make 12 litres of lemonade?

c At a cinema, the ice cream is sold by weight. 1 scoop costs 50p.

Scoops of ice cream	1	2	3	4	5	6	7	8
Cost	50p	£1						

Write the rule to find out the cost of the ice cream when you know how many scoops:

How many scoops of ice cream can I get for £10?



# Patterns and algebra – understanding equivalence

An equation is like a set of balanced scales. Both sides are equal.  
Look at the scale on the right.

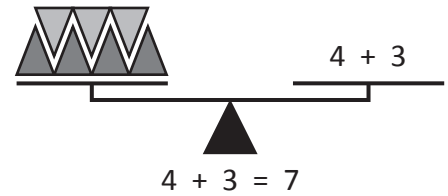
On one side are 4 black triangles and 3 grey triangles.

On the other side is the problem  $4 + 3$ .

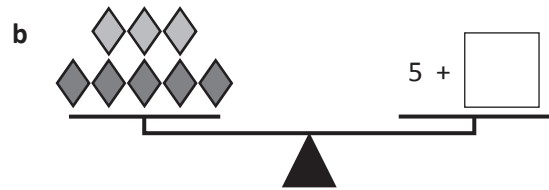
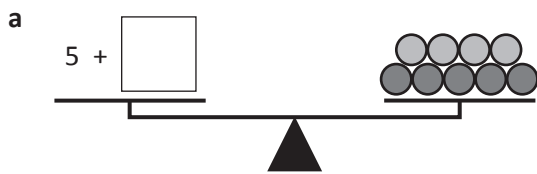
Is this a balanced equation?

Yes, because they both represent 7.

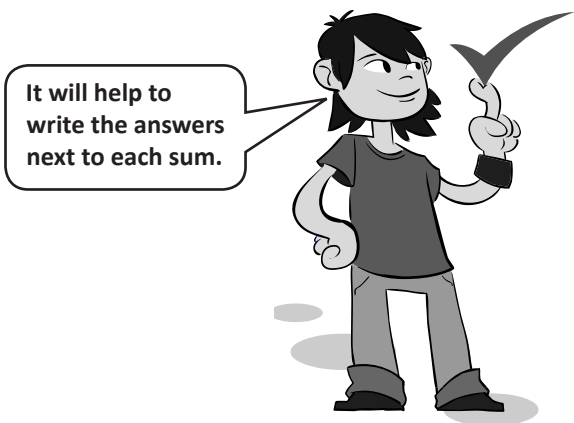
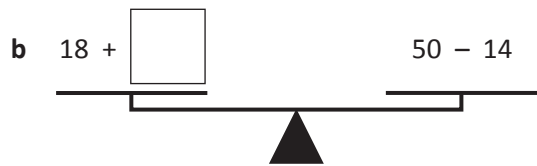
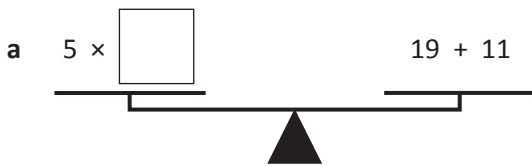
Sometimes, we haven't been given all the information and we have to work it out. This is what algebra is – solving missing number puzzles.



## 1 Make these scales balance by adding the missing value:



## 2 These scales have number problems on each side. One side has a complete problem. On the other side, you need to work out the missing value. Write the value in the box so that the scales balance:



**CHECK**

# Patterns and algebra – understanding equivalence

If the sides are not balanced, we say the equation is unequal.

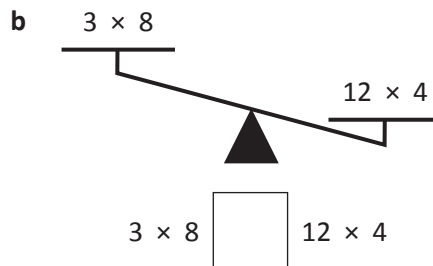
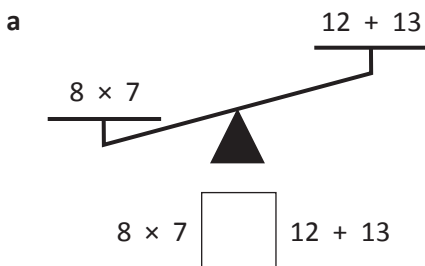
Look at these scales:

$5 \times 4$  is greater than  $5 + 4$

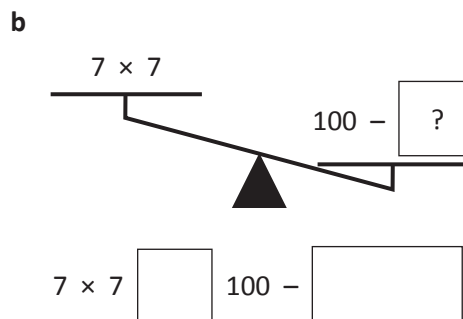
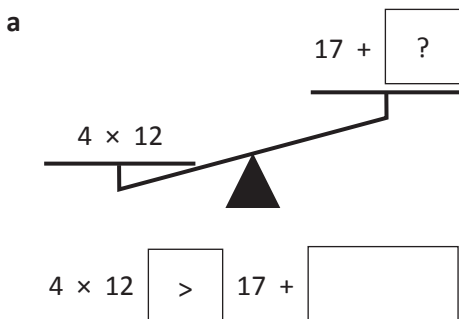
So instead of an equals sign, we use the greater than sign:

$$5 \times 4 > 5 + 4$$

3 Complete the following scales and inequalities by adding greater than (>) or less than (<):



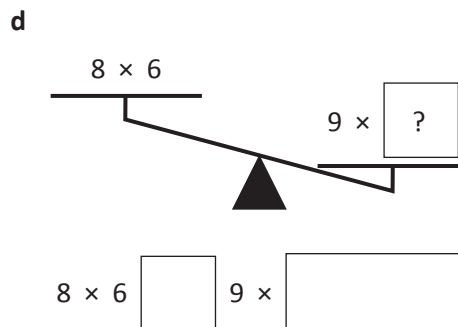
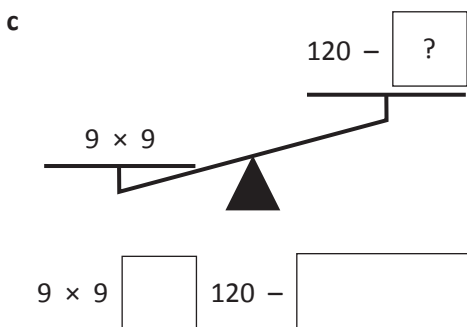
4 In these problems, you have to add both the symbol *and* a value that would make the equation true. Remember, just like with ordinary scales, the bigger value will be lower down.



HINT: there are many values that would work in the boxes!




**THINK**





# Patterns and algebra – using symbols

Symbols help us when we have more than one number to find.  
A symbol can be any shape and stands for any unknown numbers.

- 1 Work out the value of the diamond in each question. Notice the same symbol is added 3 times. Your 3 times tables will help here.

a   $\square + \square + \square = 12$

b   $\square + \square + \square = 36$

c   $\square + \square + \square = 45$

- 2 Find the value of the symbols. Remember that if a symbol is used more than once, it means it is the same value again.

a  $\star + \star + \star = 9$        $\star = \square$

b  $\heartsuit \times \heartsuit = 36$        $\heartsuit = \square$

c  $\smiley \times \smiley = 49$        $\smiley = \square$

Guess, check and improve strategy will help here.



DISCOVER

- 3 Find the value of the symbols and then check if you are right by using the same value in the question alongside it.

a  $\diamond \times \diamond = 81$        $\diamond \times \triangle = 36$   
 $\diamond = \square$        $\triangle = \square$

b  $\circ + \star + \star = 29$        $\circ \times \star = 60$   
 $\circ = \square$        $\star = \square$

# Patterns and algebra – using symbols

Known values can help us work out the values of the secret symbols.

Your knowledge of inverse operations will also come in handy.

$$\begin{aligned} \odot &= 3 \\ \odot \times \bigcirc &= 24 \\ \triangle + \bigcirc &= 80 \\ \bigcirc &= \underline{\hspace{2cm}} \\ \triangle &= \underline{\hspace{2cm}} \end{aligned}$$

By knowing the value of  $\odot$  we can work out  $\bigcirc$

$$3 \times \bigcirc = 24, \text{ so } \bigcirc = 24 \div 3, \text{ so } \bigcirc = 8$$

By knowing the value of  $\bigcirc$ , we can work out  $\triangle$

$$\triangle \times 8 = 80, \text{ so } \triangle = 80 \div 8, \text{ so } \triangle = 10$$

4 Look carefully at the example above and follow the steps to find out the values of these secret symbols:

a  $\ast = 5$

$$\ast \times \bigcirc = 45$$

$$\triangle \times \bigcirc = 63$$

$$\bigcirc = \underline{\hspace{2cm}}$$

$$\triangle = \underline{\hspace{2cm}}$$

b  $\diamond = 54$

$$\diamond \div \bigcirc = 9$$

$$\triangle \div \bigcirc = 3$$

$$\bigcirc = \underline{\hspace{2cm}}$$

$$\triangle = \underline{\hspace{2cm}}$$

5 This time you must find the value of 3 different symbols  $\triangle$   $\star$   $\bigcirc$  using the clues in each step:

a  $\star \times \star = 16$   
 $\bigcirc + \star = 100$   
 $\bigcirc - \star = \triangle$

$$\begin{aligned} \star &= \square \\ \bigcirc &= \square \\ \triangle &= \square \end{aligned}$$

b  $\triangle + \triangle = 50$   
 $\triangle \div \bigcirc = 5$   
 $\bigcirc + \triangle = \star$

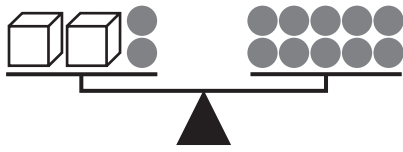
$$\begin{aligned} \star &= \square \\ \bigcirc &= \square \\ \triangle &= \square \end{aligned}$$

c  $\star + \bigcirc = 20$   
 $\star \times \triangle = 72$   
 $13 - \triangle = 5$

$$\begin{aligned} \star &= \square \\ \bigcirc &= \square \\ \triangle &= \square \end{aligned}$$

# Patterns and algebra – keeping balance

We can work out how many counters are in each box by keeping balance.



Here is our equation. How do we work out how many counters are in each box? We use a symbol to represent the unknown.

$$2 \times \square + 2 = 10$$

If we take away 2 from each side, we maintain the balance and make the problem easier. We now have to work out the value of  $\square$

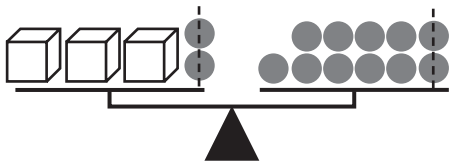
$$2 \times \square = 8$$

$$2 \times 4 = 8$$

This works because  $2 \times 4 + 2 = 10$

- 1 Find out how many counters are in each of the boxes. Remember to take away the same amount on both sides so the balance is kept.

a



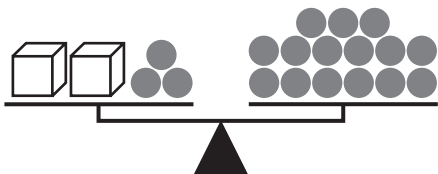
I will take away  $\square$  from each side. This leaves me with:

$$3 \times \square = \square$$

$$\square = \square$$

This works because  $3 \times \square + 2 = 11$

b



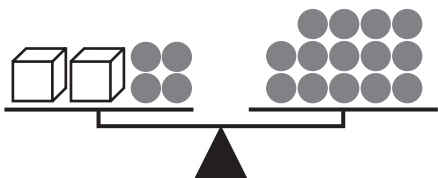
I will take away  $\square$  from each side. This leaves me with:

$$2 \times \square = \square$$

$$\square = \square$$

This works because  $2 \times \square + \square = \square$

c



I will take away  $\square$  from each side. This leaves me with:

$$\square \times \square = \square$$

$$\square = \square$$

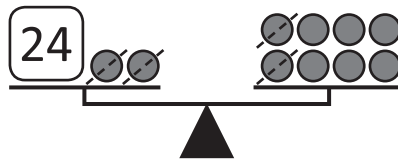
This works because  $2 \times \square + \square = \square$

# Patterns and algebra – keeping balance

In this activity you need to find out what each counter is worth.

**Step 1** Make the number stand alone by keeping balance.

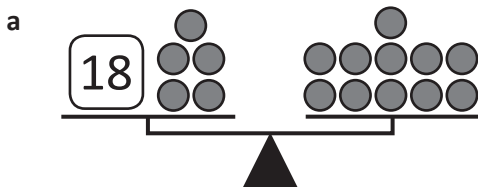
**Step 2** Write an equation to solve.



$$24 = \text{●} \times 6$$

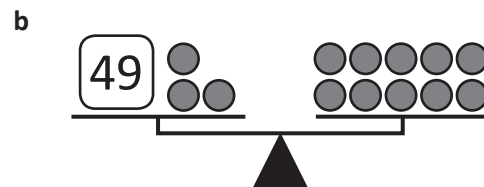
$$\text{●} = 4$$

**2** Look carefully at each balanced scale and work out what the symbols equal:



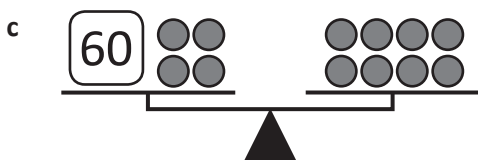
$$18 = \text{●} \times \square$$

$$\text{●} = \square$$



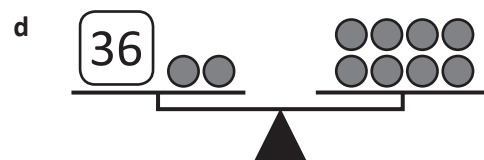
$$49 = \text{●} \times \square$$

$$\text{●} = \square$$



$$60 = \text{●} \times \square$$

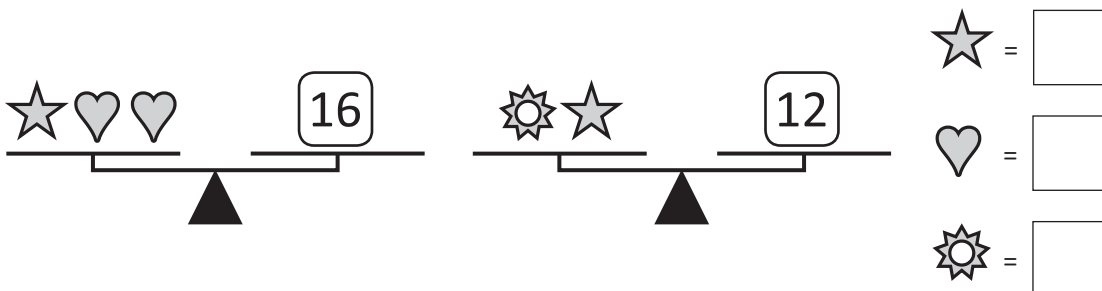
$$\text{●} = \square$$



$$36 = \text{●} \times \square$$

$$\text{●} = \square$$

**3** This time use guess, check and improve to work out what the value of the symbols could be. The symbols have the same value on both scales.



## Patterns and algebra – word problems

If you can solve equations with one unknown number using the balance strategy, you will be able to solve word problems with ease!

**A large group of friends signed up to participate in a fun run. 56 of them got food poisoning the day before so had to pull out. How many people signed up if a total of 84 people ran the race?**

$$\star - 56 = 84$$

$$\star - 56 = 84 + 56$$

$$\star = 140$$

To get the star on its own we use the inverse operation and do the same to the other side.



**REMEMBER**

- 1 Solve the following word problems using inverse operations. Start by choosing the matching equation from the box below.

$$£50 + \triangle = £130$$

$$\triangle - 70 \text{ m} = 38 \text{ m}$$

$$£83 + £100 + \triangle = £300$$

- a Jack had a piece of rope and cut off 70 metres. He was left with 38 metres. How long was the rope?
- b Tom found £50 on the bus on Monday and was given birthday money by his Gran on Wednesday. How much did his Gran give him if he ended up with £130?
- c Matilda saved £83 towards a trip to the snow and her parents gave her £100. How much more money does she need if the trip costs £300?

# Patterns and algebra – word problems

Kate saved each week for 5 weeks and then spent £25.

How much was she saving each week if she had £100 left at the end of 5 weeks and after spending £25?

**Step 1** Set up the equation. The triangle stands for the amount Kate was saving each week.

$$\blacktriangle \times 5 - 25 = \pounds 100$$

**Step 2** Cancel out the  $- 25$  with the inverse operation:  $+ 25$

$$\blacktriangle \times 5 = 100 + 25$$

$$\blacktriangle \times 5 = 125$$

**Step 3** Cancel out  $\times 5$  with the inverse operation:  $\div 5$

$$\blacktriangle = 125 \div 5$$

$$\blacktriangle = \pounds 25$$

Kate was saving £25 each week.



Make the unknown number stand on its own while keeping the equation balanced. We do this with inverse operations.

**REMEMBER**

**2** Solve the following word problems using inverse operations. The equations are partially set up. You may like to use a calculator.

- a For my school fete I baked 3 batches of cookies, realised that wasn't enough and so I bought a dozen more. How many were in one batch if I had 84 cookies altogether?

$$3 \times \blacktriangle + 12 = 84$$

$$3 \times \blacktriangle = 84 - \square$$

$$\blacktriangle = \square \div \square$$

$$\blacktriangle = \square$$

There were  cookies in each batch.

- b 8 same sized Year 5 classes assembled in the playground for photo day. There were 11 pupils absent. How many pupils are there in each class if there were 213 there on the day?

$$\square \times \blacktriangle - 11 = 213$$

$$\square \times \blacktriangle = 213 + 11$$

$$\blacktriangle = \square \div \square$$

$$\blacktriangle = \square$$

There were  pupils in each class.

- c Trin went on a holiday for 15 days. She collected 3 postcards a day for the first 10 days. By the end of her holiday she had 73 postcards. How many did she collect over the last 5 days?

$$3 \times \square + \blacktriangle = 73$$

Trin collected  postcards over the last 5 days.



# Patterns and algebra – think of a number

Lim thinks of a number, adds 3 to it and then multiplies it by 4.

The answer is 20. What is Lim's number?

To answer this, first we need to write an equation with the unknown:

**Step 1** Set up the equation. The heart shape stands for the unknown number.

$$\heartsuit + 3 \times 4 = 20$$

**Step 2** Cancel out the  $\times 4$  with the inverse operation:  $\div 4$

$$\heartsuit + 3 = 20 \div 4$$

**Step 3** Cancel out the  $+ 3$  with the inverse operation:  $- 3$

$$\heartsuit + 3 = 5$$

$$\heartsuit = 5 - 3$$

$$\heartsuit = 2$$

## 1 Work out the numbers these children are thinking of:

a Jamila says: "I'm thinking of a number. I divide it by 7 and then add 6. My answer is 13."

$$\heartsuit \div 7 + 6 = 13$$

$$\heartsuit \div 7 = 13 - 6$$

$$\heartsuit \div 7 = \square$$

$$\heartsuit = \square \times \square$$

$$\heartsuit = \square$$

b Pablo says: "I'm thinking of a number. I multiply it by 6 and then add 7. My answer is 55."

$$\heartsuit \times 6 + 7 = 55$$

$$\heartsuit \times 6 = 55 - 7$$

$$\heartsuit \times 6 = \square$$

$$\heartsuit = \square \div \square$$

$$\heartsuit = \square$$

c Mikaela says: "I'm thinking of a number. I multiply it by 4 then subtract 12. My answer is 20."

$$\heartsuit \square \square \square \square = \square$$

$$\heartsuit \square \square = \square \square \square$$

$$\heartsuit \square \square = \square$$

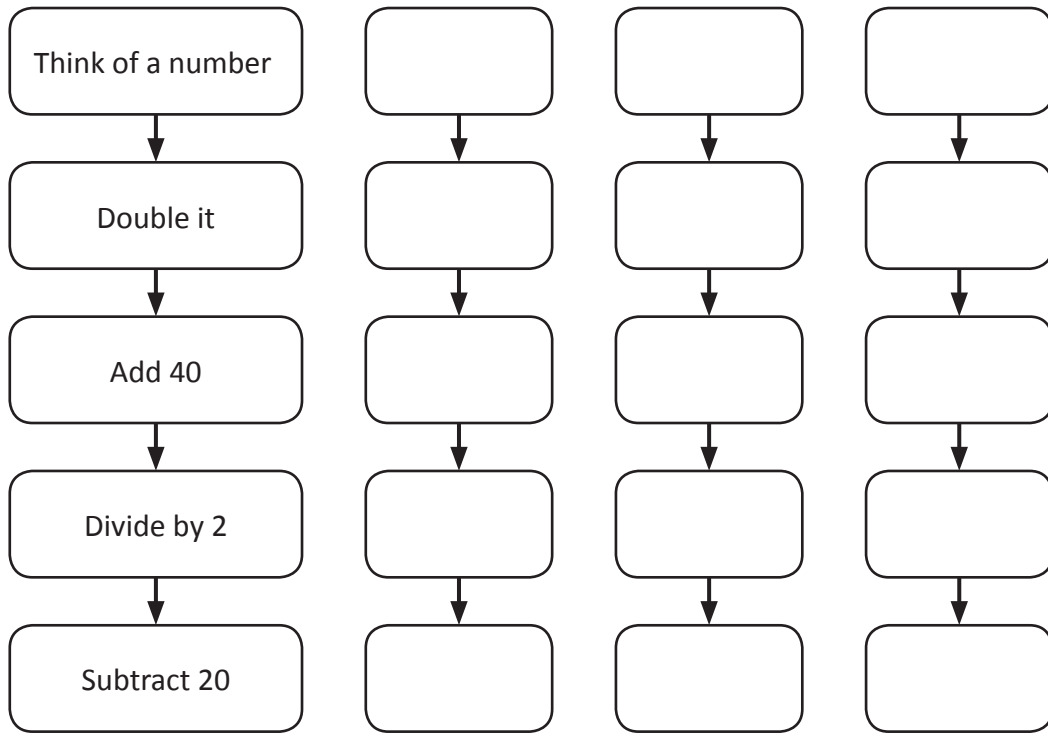
$$\heartsuit = \square \square \square$$

$$\heartsuit = \square$$

d Linh says: "I'm thinking of a number. I divide it by 8 and then add 11. My answer is 19."

# Patterns and algebra – think of a number

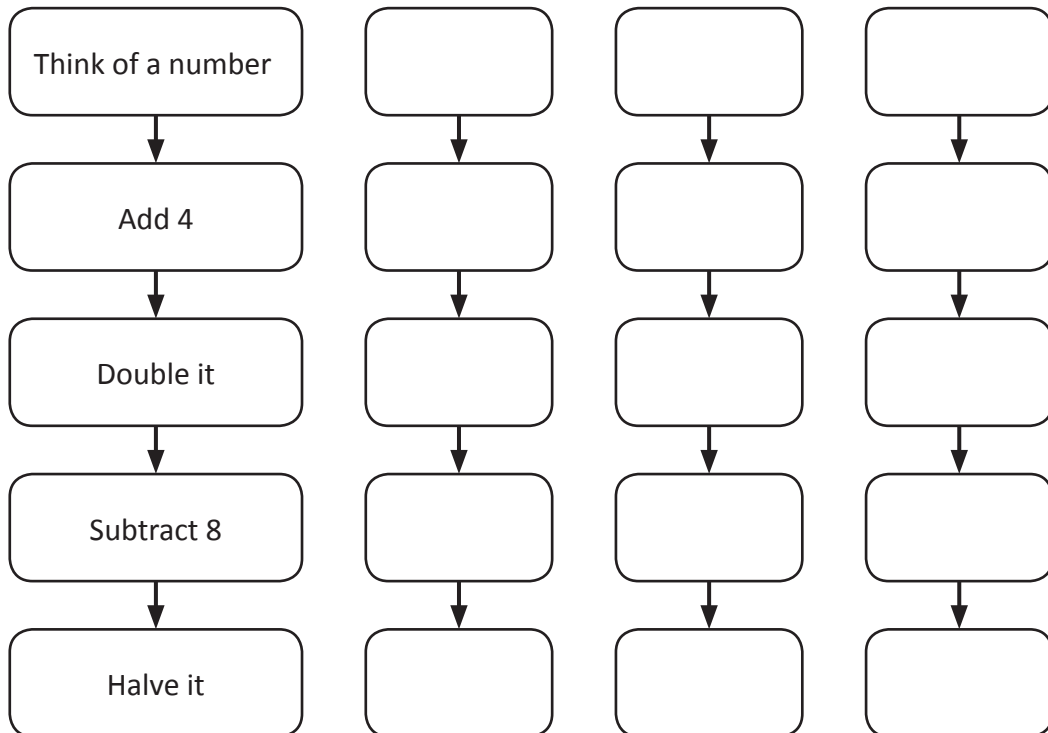
**2** Follow the steps for 3 different numbers.



What happens each time? \_\_\_\_\_

---

**3** Follow the steps for 3 different numbers.



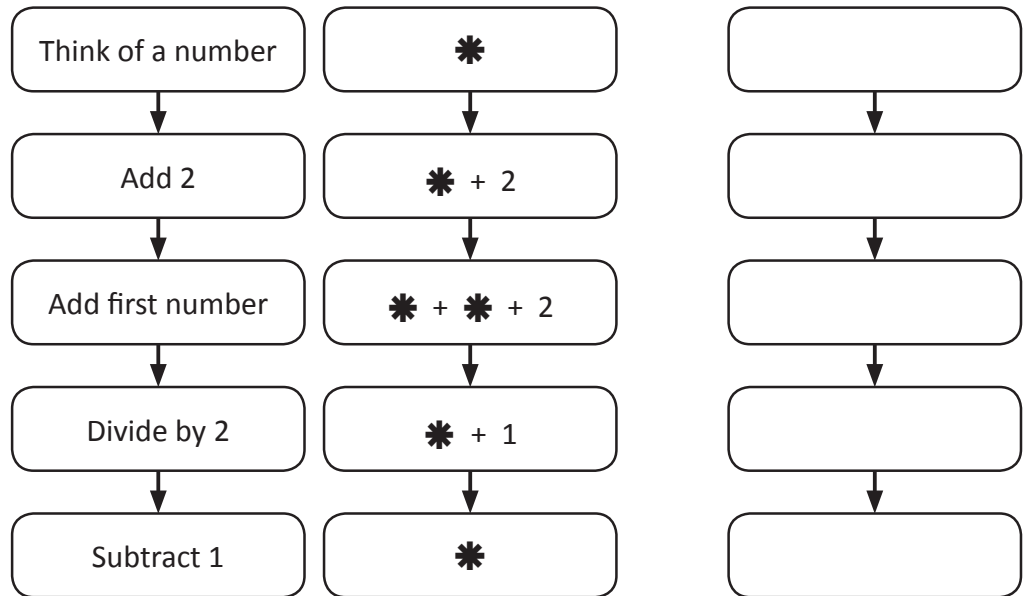
What happens each time? \_\_\_\_\_



What to do



Try this number puzzle by testing it out in the blank boxes.



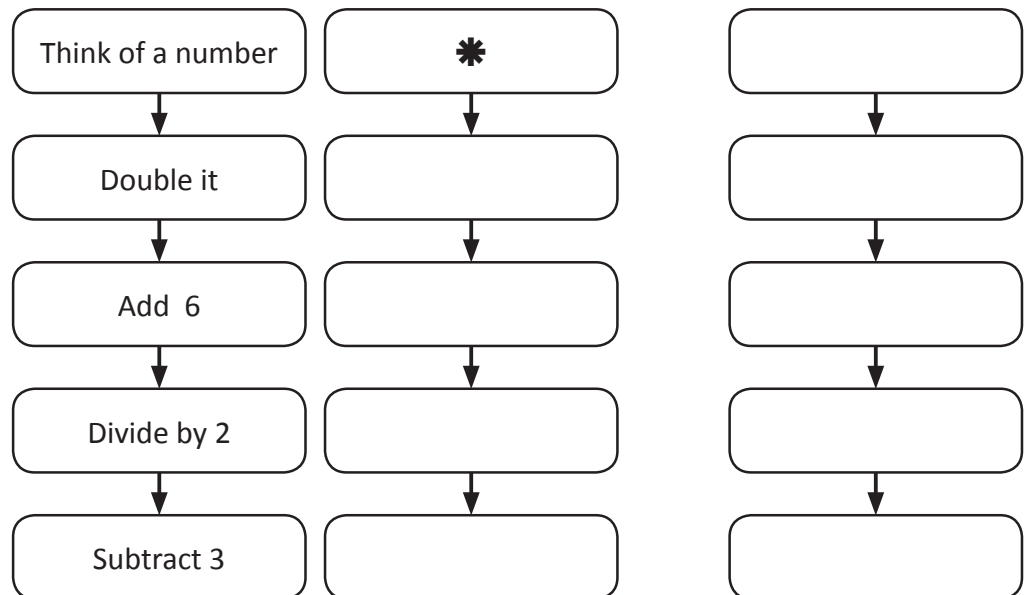
What do you notice? \_\_\_\_\_



What to do next



This number puzzle uses the same trick. This time complete the column of boxes with the number sentences using symbols. Then test it in the last column.



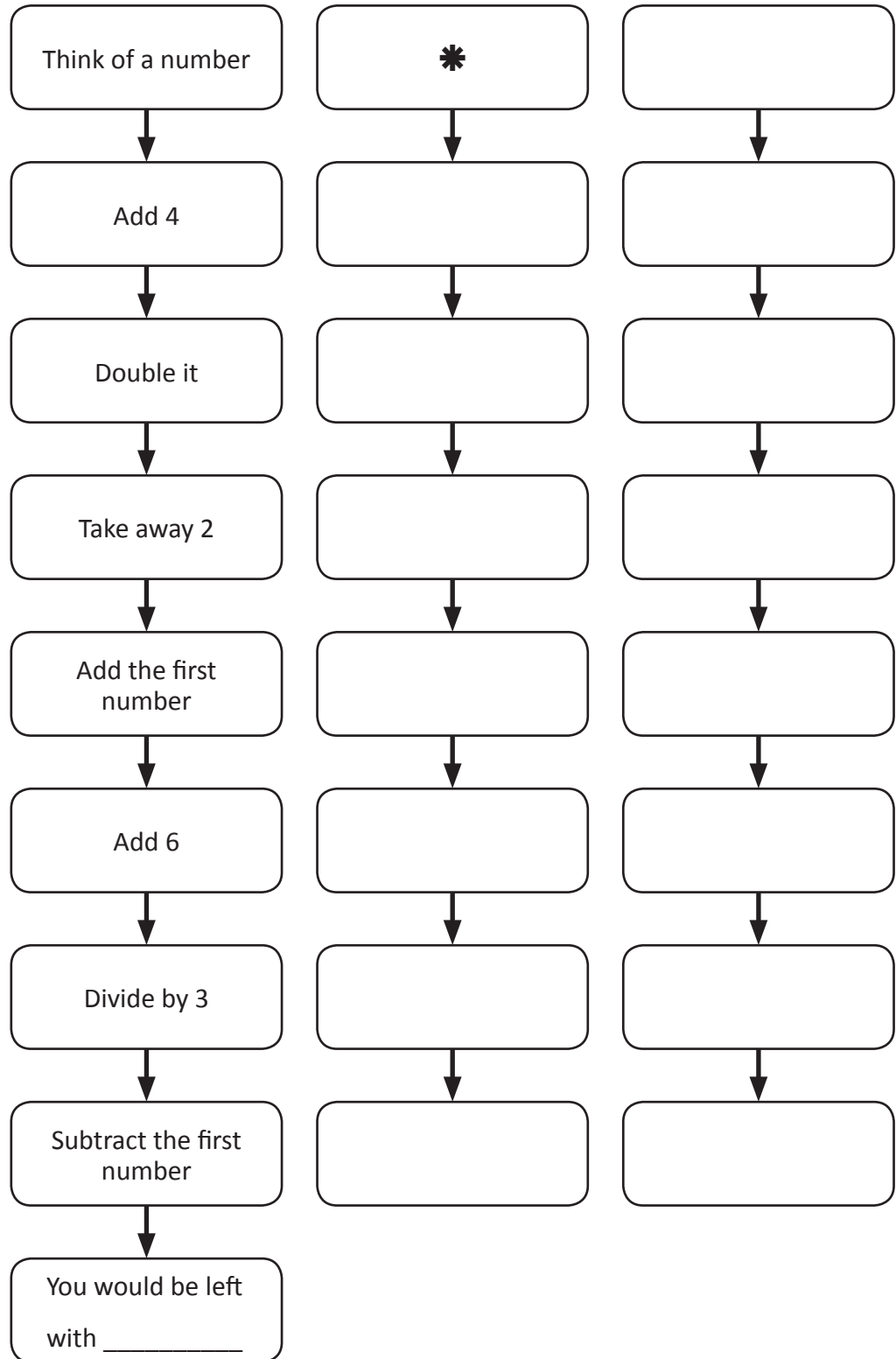
Why does this work for any number?



What to do

Write the symbols for this puzzle in column 2 and test it out.

What number is left?





What to do

Use the code below to work out the hidden message.

2 1 3 6 4 5 3 8 7 9 8 9 10 12 11

$A \times A = A$       **A is** \_\_\_\_\_

$M \times M = M + M$       **M is** \_\_\_\_\_

$T - M = A$       **T is** \_\_\_\_\_

$T + T = H$       **H is** \_\_\_\_\_

$H - M = L$       **L is** \_\_\_\_\_

$3 \times L = U$       **U is** \_\_\_\_\_

$F = H + L$       **F is** \_\_\_\_\_

$E = F \div 2$       **E is** \_\_\_\_\_

$2 \times L = I$       **I is** \_\_\_\_\_

$(2 \times L) - A = C$       **C is** \_\_\_\_\_

$F + A = N$       **N is** \_\_\_\_\_

$3 \times T = S$       **S is** \_\_\_\_\_

Once I work out the first couple, the rest come easily!



DISCOVER



What to do

Try this one:

2 9 4 12 13 8 2 7 4 9 2 12 3

4 2 6 6 3 12 0 8 9 1 2 5 3



If two letters are together, we read them as a tens digit and a ones digit.

$A \times A = A + A$       **A is** \_\_\_\_\_

$A + A = T$       **T is** \_\_\_\_\_

$T \times 2 = N$       **N is** \_\_\_\_\_

$AT \div N = E$       **E is** \_\_\_\_\_

$2 \times E = L$       **L is** \_\_\_\_\_

$E + T = U$       **U is** \_\_\_\_\_

$L + E = S$       **S is** \_\_\_\_\_

$N - N = I$       **I is** \_\_\_\_\_

$U - A = C$       **C is** \_\_\_\_\_

$S - (2 \times T) = P$       **P is** \_\_\_\_\_

$2 \times U - P = O$       **O is** \_\_\_\_\_

$S + E = R$       **R is** \_\_\_\_\_



Getting ready

In this activity, you'll use your knowledge of multiplication, division, subtraction and addition to find as many number statements you can to create one number.



What to do

Using ONLY the number 2, +, ×, − and ÷ keys on your calculator, find as many ways as you can to create the number 13.

For example, you could make:

$$22 + 2 + 2 = 26 \div 2 = 13$$

Record your statements on a piece of paper.

Now, compare your answers with a partner's. How many did they find?

Can you supplement each other's lists?

What's the longest statement? What's the shortest?



What to do next

Choose another number to make and see how many statements you can find *or* challenge a partner to a competition. Set a time limit and see who can find the most ways to make the number 15 within the time span.

## Bugs

## investigate



Getting ready

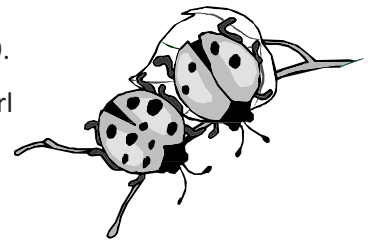
Use your knowledge of multiples to help you work out how many boy bugs and girl bugs there are in the problem below. Listing all the multiples is a strategy that would help.



What to do

Girl bugs have 4 splodges on their backs, boy bugs have 9.

Altogether there are 48 splodges. Work out how many girl bugs and how many boy bugs there are.



What to do next

What if girl bugs have 8 splodges and boy bugs have 6 and there are 120 splodges altogether? How many different answers can you find?



What to do

Use your knowledge of multiplication to work out the missing values:

a

$$\begin{array}{r} 28 \\ \times 3 \\ \hline 8 \square \end{array}$$

b

$$\begin{array}{r} 7 \square \\ \times 4 \\ \hline 288 \end{array}$$

c

$$\begin{array}{r} \square 7 \\ \times 5 \\ \hline 235 \end{array}$$

d

$$\begin{array}{r} 8 \square \\ \times 9 \\ \hline 729 \end{array}$$

e

$$\begin{array}{r} 68 \\ \times \square \\ \hline 204 \end{array}$$

f

$$\begin{array}{r} \square 23 \\ \times \square \\ \hline 6584 \end{array}$$

g

$$\begin{array}{r} 261 \\ \times \square \\ \hline \square \square 44 \end{array}$$

h

$$\begin{array}{r} 42 \\ \times \square 3 \\ \hline 126 \\ \square 680 \end{array}$$

i

$$\begin{array}{r} 56 \\ \times 27 \\ \hline 392 \\ \square \square \square \square \\ \square \square \square \square \end{array}$$



What to do

Fill in the multiplication and division tables by working out the missing digits. The arrows show you some good starting places.

			↓	
×			7	6
→		20	16	14
	5		40	
→				36
	3	30		

			↓		↓
×		8	9		
12	24				
3				12	
	14				
			54		

×			3	
4				32
		14		
	45		27	
12		24		

×			9	
	6			
11	33	44		
			63	
8				64



What to do

Complete this crossnumber puzzle:

1			2		
		3			
		4			5
	6			7	
			8		
9			10		

Across

1  $60 \div 5$

2  $25 \times 5$

3  $7 \times 6$

4  $15 \times 6$

7  $7 \times 3$

9  $9 \times 6$

10  $6 \times 50$

Down

1  $11 \times 11$

2  $12 \times 10$

3  $7 \times 7$

5  $66 \div 6$

6  $12 \times 12$

8  $39 \div 3$



What to do

Test your speed and accuracy. Race against a partner or the clock to complete each table:

÷ 8	
56	
16	
64	
80	
32	
72	
24	
8	

Time:

÷ 3	
9	
6	
18	
12	
24	
30	
27	
33	

Time:

÷ 7	
21	
7	
14	
70	
49	
28	
42	
35	

Time:



What to do

Use the “guess, check and improve” strategy to solve this problem. You could use a calculator to help if you wish.

Tracey paid £3.10 for 7 jelly snakes and 4 sherbets. Madison paid £2.95 for 4 jelly snakes and 7 sherbets. How much does one jelly snake cost? How much does one sherbet cost?

If the decimals are confusing me, I can change the amounts to 310 pence and 295 pence.



THINK