Essays on the Origins of Western Music

by

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Essay Nr. 19: Music Defined as Mathematics

Music is the daughter of Arithmetic.

Anonymous, Scholia enchiriadis (c. 900 AD)

Reading the above quotation, perhaps it should be no surprise that much of our modern music notation system is characterized by simple arithmetic. This anonymous treatise, written at the very dawn of the creation of our modern music notation system, also epitomizes the central problem which all earlier philosophers had with music and which confuses music educators today. In a word, how do you make music, whose essence and values are non-rational, fit into a rational world? The answer for modern music educators is to ignore the inherent values in music and focus instead on teaching *about* music. For the early philosophers the answer was to simply ignore the characteristics of music they couldn't explain and take what they could understand and declare music Rational.

> **[Music is] the rational discipline of agreement and discrepancy of sounds according to numbers in their relation to those things which are found in sounds.... Because everything comprehended by these disciplines exists through reason formed of numbers and without numbers can be neither understood nor made known.**[1]

[1] Anonymous, "Of Symphonies," in Oliver Strunk, Source Readings in Music History (New York: Norton, 1950), 135.

On the other hand, for musicians of today, and the past, music is characterized more by pitches we hear, not numbers that we count. It follows that the symbols representing these pitches constantly remind us that music is for the ear. It is also very important to remember that for a very long time there was no notation of music at all. The ancient Egyptians had none and the ancient Greeks didn't even have names for the individual pitches.

For the ancient philosophers music was a special problem. They understood, through observing its impact on listeners, that it was important, but they could not <u>see</u> it and they had built a philosophical world in which the eye was the most important of the senses (we still say, "Seeing is believing!"). And then music obviously dwelt with the emotions, a topic which always made early philosophers uncomfortable because they could not generalize about them and because they seemed to them the antithesis of Reason.

Once the ancients discovered they could use numbers to describe musical sounds they were delighted, for now they could bring music into their world of Reason. The early Church was also delighted, for now it could admit music into the curriculum as a branch of mathematics and entirely avoid discussion of the emotions in music, a very sensitive topic since they had for so long preached that the emotions were "the first step toward sin!"

The origin of the idea of using numbers to describe music is usually attributed to Pythagoras, 580 – 500 BC. Like much of ancient Greek culture, however, perhaps some of his ideas came from the older society of Egypt. Iamblichus tells us, for example, that Pythagoras spent twelve years in Egypt where he studied "arithmetic, music and all the other sciences."[2] Since nothing by his own hand has survived, and because there is a lot of nonsense attributed to Pythagoras, it is difficult to determine exactly his role in music history. He certainly was not the first to discover the overtone series, but he may have been the first to use numbers as symbols to represent the intervals between the lower pitches of the series.[3] One can see how the relationship between mathematics and music followed.

[2] Iamblichus (c. 250-325 A.D.), "The Life of Pythagoras."

[3] Thus his greatest contribution to mankind, the idea that numbers could represent abstract thought was the beginning of all higher mathematics.

Aristotle found here a fundamental theory for looking at the world.

> **The so-called Pythagoreans, who were the first to take up mathematics, not only advanced this study, but also having been brought up in it they thought that its principles were the principles of all things. And since of these principles *numbers* are by nature the first, and in numbers they seemed to see many resemblances to the things that exist and come into being -- more than in fire and earth and water...since again, they saw that the modifications and the ratios of the musical scales were expressible in numbers -- since, then, all other things seemed in their whole nature modeled on numbers, and numbers seemed to be the first things in the whole of nature, they supposed the elements of numbers to be the elements of all things, and the whole heaven to be a musical scale and a number.[4]**

Following this line of thought, one can understand how medieval philosophers could come to the hypothesis that perhaps the organization of the planets was related to the intervals of the overtone series and that one could therefore study astronomy (in a time with no adequate telescopes) through studying music. But we are perhaps more surprised to discover that some early philosophers found a connection between music and *grammar*, as Sextus Empiricus (2nd c. AD) did,

> **For this is a feature of arts which are conjectural and subject to accidents such as navigation and medicine; but Grammar is not a conjectural art but akin to Music and Philosophy.[5]**

We find this association again 1,000 years later in Roger Bacon (b. c. 1214), who not only found a relationship between music and grammar but now added the field of logic!

> **Now the accidental parts of philosophy are grammar and logic. Alpharabius makes it clear in his book on the sciences that grammar and logic cannot be known without mathematics. For although grammar furnishes children with the facts relating to speech and its properties in prose, meter, and rhythm, nevertheless it does so in a puerile way by means of statement and not through causes or reasons. For it is the function of another science to give the reasons for these things, namely, of that science, which must consider fully the nature of tones, and this alone is music, of which there are numerous varieties and parts. For one deals with prose, a second with meter, a third with rhythm, and a fourth with music in singing. And besides these it has more parts. The part dealing with prose teaches the**

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[4] Metaphysics.

[5] Sextus Empiricus, "Against the Professors," trans., R. G. Bury (Cambridge: Harvard University Press, 1949), I, 72.

> **reasons for all elevations of the voice in prose, as regards differences of accents and as regards colons, commas, periods, and the like. The metrical part teaches all the reasons and causes for feet and meters. The part on rhythm teaches about every modulation and sweet relation in rhythms, because all those are certain kinds of singing, although not so treated as in ordinary singing.... Therefore grammar depends causatively on music.**
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> **In the same way logic...Alpharabius especially teaches this in regard to the poetic argument, the statements of which should be sublime and beautiful, and therefore accompanied with notable adornment in prose, meter, and rhythm.... And therefore the end of logic depends upon music.[6]**

The reader will forgive us if we pause to reflect that much of what we call music theory functions like grammar. As a result some music theory teachers teach only the grammar of music and not music at all. Consequently we also have a tradition of analysis in which we analyze only the grammar and discover nothing of the great truths of music – a form of analysis that happens in no other discipline. One cannot imagine, for example, an English literature course where the study of Shakespeare's Hamlet ended with the analysis of the grammar of his language.

Following Aristotle's observation, quoted above, that numbers were to be thought of as the basis of nature, we can see that by the 6th century AD virtually all of science was now incorporated into mathematics. This is clearly expressed in a letter by Cassiodorus (480 – 573 AD) to the famous mathematician, Boethius (475 – 524 AD).

> **You have thoroughly imbued yourself with Greek philosophy. You have translated Pythagoras the musician, Ptolemy the astronomer, Nicomachus the arithmetician, Euclid the geometer, Plato the theologian, Aristotle the logician, and have given back the mechanician Archimedes to his own Sicilian countrymen. You know the whole science of Mathematics...[7]**

Boethius certainly considered that music was within his expertise as a mathematician, leaving his readers to choke on musical description such as this,

> **But since the nete synemmenon to the mese (3,456 to 4,608) holds a sesquitertian ratio -- that is, a diatessaron -- whereas the trite synemmenon to the nete synemmenon (4,374 to 3,456) holds the ratio of two tones....[8]**

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[6] Opus Majus, in The Opus Majus of Roger Bacon, trans., Robert Burke (New York: Russell & Russell, 1962), II. See also XVI for his views on the relationship of music to both mathematics and theology.
[7] Letter to Boethius, in The Letters of Cassiodorus (London: Frowde, 1886), 169.
[8] Boethius, Fundamentals of Music, trans., Calvin Bower (New Haven: Yale University Press), IV, ix.

Aurelian of Reome, in his Musica Disciplina of c. 843 AD, treats the relationship of mathematics and music as if the *numbers* of the intervals were subject to weighing and not hearing.

> **Music has the greatest correspondence to mathematics and encompasses that part of mathematics that compares one quantity with another....[9]**

In another place, he says if one wishes to become more versed in *music*,

> **let him turn his eyes to the harmony of proportions, to the contemplation of intervals, and to the exactitude of mathematics.[10]**

We have mentioned above one of the most important philosophers at the end of the Middle Ages, the Englishman, Roger Bacon (b. c. 1214), who studied at Oxford and at the University of Paris. Perhaps reflecting the power of the Church at this time, he was outspoken in his disrespect for the masses, the "unenlightened throng," the "ignorant multitude," whom he says can never rise to the perfection of wisdom.[11] For this reason, he maintains, the wise have always been an elite segment of society, separated from the masses. He found this true in religion ("as with Moses so with Christ the common throng does not ascend the mountain") and well as in the universities. He cites a book by A. Gellius in which the author maintained that the great Greek philosophers had discussions among themselves at night, so as to "avoid the multitude."

> **In this book he says that it is foolish to feed an ass lettuces when thistles suffice him. He is speaking of the multitude for whom rude, cheap, imperfect food of science is sufficient. Nor ought we to cast pearls before swine....**

Johannes de Grecheo, in his De Musica, c. 1300, makes it sound more like a matter of professional jealousy,

> **…many speculative thinkers make a secret of their calculations and their discoveries, not wishing to reveal them to others….[12]**

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[9] Aurelian of Reome, The Discipline of Music, trans., Joseph Ponte (Colorado Springs: Colorado College Music Press, 1968), VI.

[10] Ibid., X.

[11] Nicholas of Cusa (1401-1464), in "Compendium," XIV, trans., William Wertz, Jr., in Toward a New Council of Florence (Washington, D.C.: Schiller Institute, 1993), 539ff, says the uneducated man is nothing but an animal.

[12] Johannes de Grocheo, De Musica, trans., Albert Seay (Colorado Springs: Colorado College Music Press,

In his discussion of the Liberal Arts, Bacon first comments that while the ancients knew of the various sciences, they only actually used two: astronomy for the calendar, and music for worship.[13] **Mathematics, he calls the "gate and key" for the other Liberal Arts**[14] **and he specifically recommends that for children the study of mathematics should come before the study of music.**

> **The natural road for us is to begin with things which befit the state and nature of childhood, because children begin with facts that are better known by us and that must be acquired first. But of this nature is mathematics, since children are first taught to sing, and in the same way they can learn the method of making figures and of counting, and it would be far easier and more necessary for them to know about numbers before singing, because in the relations of numbers in music the whole theory of numbers is set forth by example, just as the authors on music teach, both in ecclesiastical music and in philosophy.**[15]

With the beginning of the Renaissance we find a representative treatise by one of the *ars antique*, Jacques de Liege. His treatise is called a "music treatise," Speculum Musicae (1313), but the first five of its seven books deal with mathematics. We see the importance of "numbers" in music when he complains that he is being attached by the younger generation, the *ars nova*, for his belief that music should be based on the number 3, a purely rational construction based on Church dogma. His contemporary, Jean de Muris (c. 1290 - 1350), takes the same position and offers some "proof" for the importance of the number 3. He includes not only the Trinity, but the 3 aspects of time of celestial bodies, the 3 attributes of the stars and sun, the 3 attributes of the elements, the 3 intellectual operations, the 3 terms in the syllogism and many more.[16] **De Muris also points to the relationship of music and geometry when he observes that "the wiser ancients long ago agreed and conceded that geometrical figures should be the symbols of musical sounds."**[17] **This he follows with an extraordinary omission, which, had he filled it, would be more**

1967), 2.
[13] Opus Majus, Op. cit., XIV.
[14] Ibid., "Mathematics," I.
[15] Ibid., III. See also XVI for more on the relationship of music to both mathematics and theology.
[16] Strunk, Op. cit.,.,173.
[17] Ibid., 175.

interesting to us today than the rest of his entire treatise. It also reminds us that theory and notation always follow the actual practice of music.

> **For reasons which we shall pass over, their symbols did not adequately represent what they sang.**

One of the more respected theorists of the Middle Renaissance was Johannes Tinctoris (1435 - 1511) who, once again, had actually made his reputation as a mathematician. In the Prologue to his own treatise Concerning the Nature and Propriety of Tones, Tinctoris identifies himself as one who professes "the mathematical sciences."[18] In this same work, in speaking of Church modes he says these were named,

> **according to arithmetic, without which it is obvious no famous musicians escapes.[19]**

During his tenure in Naples under Ferdinand I, Tinctoris must have been exposed not only to Italian Humanism, but to a wide variety of secular art music of high quality. Yet he assigns little space to these things in his treatises and, as an official of the Church, concluded Jesus Christ to have been the greatest singer. His main testimony of the new values in music is found in his complaints over composers who were breaking the old mathematical rules.

> **As a result of this tempest, the musical ability of our time has undergone such an increase that it seems to be a new art....**
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> **But alas! I wonder not only at these but even at many other famous composers, for while they compose so subtly and so ingeniously with incomprehensible smoothness, I have known them to ignore entirely musical proportions or to signify incorrectly those which they do know. I do not doubt that this results from a lack of arithmetic, without which no brilliant achievement in music escapes, for proportion is produced from its entrails.[20]**

What Tinctoris called the "new art" was a return to the ancient idea that music should communicate emotions rather than mathematical principles. In his treatise Tinctoris refers to an incident which must be regarded as a hallmark of this change, although we doubt that Tinctoris recognized it as such. He tells us that in response to his old-fashioned treatise on the importance of mathematics in music a

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[18] Concerning the Nature and Propriety of Tones, trans., Albert Seay (Colorado Springs, 1976),1.
[19] Ibid., 3.
[20] Proportionale Musices, trans., Albert Seay in Journal of Music Theory (1957), I, 1, 27.

singer, a representative of the new Humanistic focus on the emotions, wrote to Tinctoris telling him he was going to make him eat his treatise,

> **…has not been afraid to menace me with a violent meal of this little book if ever I should return to my native land.**

In 16th century Italy, in spite of the activity of the Humanists, we still find some curiously old-fashioned views. In Galilei's Fronimo, a book dealing with intabulating for the lute, the author comes upon his friend, Fronimo, a distinguished lutanist. His friend is sitting outdoors on a stump, playing for himself, and Galilei describes him not lost in the rapture of his performance, but absorbed in the mathematics of music.

> **He has not yet seen me, so intent is he on considering the proportions of the musical intervals.**[21]

It will seem curious to the modern reader that for a long time the interval of the sixth was considered a dissonance, long after its inversion, the third, had become considered a consonant. This judgment was made on the basis of the "proportions of the intervals," which is what Fronimo was listening to, rather than by the ear. Therefore, it is a harbinger of a new era when theorists begin to accept the ear's judgment over mathematics. We see an excellent illustration of this process in Girolamo Cardano (1501 - 1576), an important mathematician and writer on almost every subject.[22] **He personally recognizes that the sixth sounds consonant and he admits "why should we reject what the ear already approves," even if the mathematical ratios do not agree. He seems a bit frustrated that he cannot explain mathematically why it should sound good and finally concludes that it is just a matter of time before it is understood.**

> **So it is necessary to consider why a connection of tones which is pleasing to the ears does not have a rational explanation. Accordingly, the usefulness of the aural sense is clear, but its rationale is found in the discovery of many things which are not yet fully known through experience....**[23]

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[21] Vincenzo Galilei, Fronimo (1584), trans., Carol MacClintock (Neuhasen-Stuttgart: Hanssler-Verlag, 1985), 32.
[22] In his own catalog of his works, his music treatise is found among those on mathematics.
[23] Quoted in Clement Miller, Hieronymus Cardanus, Writings on Music (American Institute of Musicology, 1973), 104.

Another harbinger of our times, sad to say, is the fall of music from being a science. While maintaining that geometry and arithmetic are "vital instruments for the search of Truth," Vives in his important treatise, On Education, now finds the role for music to be, "for relaxation and recreation of the mind through the harmony of sounds."[24] It is noteworthy that Vives finds this the primary purpose of music in education.

> **In music we have deteriorated much from the older masters, on account of the dullness of the ear which has utterly lost all discrimination of subtle sounds, so that now we no longer distinguish even the long and short sounds in common speech; and for this reason we have lost some kinds of meters, and that primitive harmony of tones, the effects of which the ancient writers testify were vast and marvelous. Young men should receive theoretical instruction in music, and should also have some practical ability. Only let the pupil practice pure and good music which, after the Pythagorean mode, soothes, recreates, and restores to itself the wearied mind of the student; then let it lead back to tranquility and tractability all the wild and fierce parts of the student's nature….[25]**

As with France and Italy, it was not to the universities of the German-speaking countries that one could look for new ideas in music. Here also they remained locked in the old medieval Scholastic notion that music belonged to mathematics. Thus, in 1505, the University of Leipzig appointed Sebastianus Muchelon as "*lector musicae et aritmetice*,"[26] a document of the University of Koln in 1515 specifies the teaching of "the books on mathematics, that is geometry, arithmetic, music and astronomy" and in 1558 the University of Heidelberg employed a lecturer in mathematics who was expected to include music in his teaching. Johannes Cochlaeus, a professor at Koln, in his Tetrachordum Musices of 1511, still finds music firmly attached to mathematics.

> **…arithmetic is concerned with absolute numerals, music with numerals related to each other….[27]**

[24]Foster Watson, trans., Vives: On Education (Cambridge: University Press, 1913), I, v. He classifies all poetry under the heading of music.
[25] Ibid.
[26] Nan Cooke Carpenter, Music in the Medieval and Renaissance Universities (Norman: University of Oklahoma Press, 1958), 251. Carpenter documents the association with mathematics extensively.
[27] Johannes Cochlaeus, Tetrachordum Musices, trans., Clement Miller (American Institute of Musicology, 1970), 21.

In one of the works of Erasmus (1469 - 1536) we again find the harbinger of the modern age when the ear begins to receive equal billing, so to speak, with the eye. It is an interesting discussion and concludes with a memorable phrase. Erasmus defines the common usage of an old Greek proverb, "Double diapason," to mean any two things very far apart. In the course of his musical discussion he seeks to make the principal point that the range of two octaves is a kind of natural furthermost limit, with respect to the ear hearing the mathematical proportions in music. Clearly concerned that he was writing on a subject which he had limited experience, he tells us that as he was writing, a famous philosopher, Ambrogio Leone of Nola, just happened to walk in and thus he attributes to this man the remainder of the discussion. Leone finds two reasons for calling the double octave the natural limit. First, he has observed that the [male] voice can not reach beyond the fifteenth without becoming forced and artificial. The second argument is because Reason and the senses must work together. While Reason can comprehend numbers of any size, hence, for example, the possibility of a distance of a thousand octaves, the senses do not distinguish relationships beyond two octaves.

> **But the physical senses have had their own limits prescribed for them by nature, and if they transgress these, they gradually become misty and wandering, and can no longer judge with certainty as they used to do, but through a cloud, as they say, or in a dream. It was not fitting that principles of art should be drawn from an uncertainty of judgment. But since the ancients understood that beyond the fifteenth note of the scale the judgment of the ears began to fail, they decided to fix the bounds of harmony there, so that no one could have any reason to bring up that adage of yours, "unheard music is useless."**[28]

There was one theorist who was far ahead of his time and attacked the whole idea of the inclusion of mathematics in the teaching of music. He was a Flemish man teaching at Wittenberg in 1545 when he arrived at the thoughts he expressed in his <u>Compendium Musices</u> of 1552, a treatise intended as a manual for the teaching of singing to choir boys. Tine and time again he advises the reader to forget the

[28] "Adages," in <u>The Collected Works of Erasmus</u> (Toronto: University of Toronto Press, 1992)., XXXI, 202ff. Erasmus discusses the last phrase in a discussion of the proverb, "Hidden music has no listeners [and is thus worthless]" in <u>Ibid</u>., XXXII, 117ff.

"books of the musician-mathematicians" and comes very strongly to the aesthetic basis of music – the listener.

> **As a singer, [the boy] will study especially how to please the ears of men and how to inspire pleasure in them, as well as admiration and favor for himself. He will also be continually guided by the judgment of his ears. The ears easily understand what is done correctly or badly and are truly the masters of the art of singing.**[29]

It is a treatise of practical music, not the theory of music, and he attributes his viewpoints to his own teacher, the "most noble musician, Josquin," from whom he learned "incidentally, from no book."

During the 16th century the English remained married to the old scholastic, mathematical world of music. Thus even Shakespeare, always a mirror of real (aristocratic) life in London, found it necessary to introduce a music teacher as a man "Cunning in music and the mathematics."[30]

With the dawn of the Baroque these views continued, as for example we see in the definition of music by William Wooten (1666-1727) found in his Reflections upon Ancient and Modern Learning (1694),

> **Musick is a Physico-Mathematical Science, built upon fixed rules, and stated proportions.**

Wooten finds it particularly objectionable that musicians do not respect, and do not even read, the great treatises of the past.

> **Whereas all modern mathematicians have paid a mighty deference to the ancients; and have not only used the names of *Archimedes*, *Apolonius* and *Diophantus*, and the other ancient mathematicians with great respect; but have also acknowledged, that what further advancements have since been made, are, in a manner, wholly owing to the first rudiments, formerly taught. Modern musicians have rarely made use of the writings of *Aristoxenus*, *Ptolemee*, and the rest of the ancient musicians; and, of those that have studied them, very few, unless their editors have confessed that they could understand them. Others have laid them so far aside, as useless for their purpose; that it is very probable, that many excellent composers have scarce ever heard of their names.**

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[29] Adrian Coclico, Musical Compendium, trans., Albert Seay (Colorado Springs: Colorado College Music Press, 1973), 6.
[30] The Taming of the Shrew, II, ii, 57.

Even so remarkable a mind as Isaac Newton, in formulating his ideal university curriculum, has the "mathematics lecturer" teaching music.[31] Newton, by the way, labored for years in an attempt to correlate the numbers representing the vibrations of musical pitches with the numbers representing the light waves of the colors of the visual realm. When he died he left a drawing, a kind of chart, in which he has created his hypothetical correspondence of tones and colors, but he left no text to describe how he arrived at these findings and no one has ever been able to make sense of them.[32]

The great French treatise of the Baroque, the Harmonie universelle of 1636, is again the work of a mathematician, Marin Mersenne (1588-1648). There is much fascinating reading here which we will discuss in future essays, but with regard to the topic at hand, music and mathematics, there is one new proposal of note. He invented a new notation system consisting entirely of numbers, which he regarded as useful primarily in correspondence.

> **This manner of composing can be used by learned theoreticians, who wish to compare and send their compositions to each other, or who wish to have their compositions printed without using the [normal musical] notes of practice, which not every printer has.[33]**

The reader must remember that the 17th century was also the beginning of The Enlightenment and was a period of fervent activity in inventions of all kinds. This climate produced another work related to new notation and it is found in the ten books on music of the Musurgia Universalis (1650) by the German born Athanasius Kircher (1601-1680). In Book Three, "Arithmetical," Kircher presents a system of "musical arithmetic," through which the rules of addition, subtraction, multiplication and division of intervals are represented by special characters.

And perhaps another experiment of this sort is found by Gottfried Leibniz (1646 - 1716), a system which might be of profit to student composers!

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[31] We do, however, support his position that the faculty should be given lifetime supervision of the alumni!

> All Graduates without exception found by the Proctors in Taverns or other drinking houses, unless with travelers at their Inns, shall at least have their names given in to the Vice-Chancellor who shall summon them to answer for it before the next Consistory.

[32] The reader can find this chart reproduced in The Correspondence of Isaac Newton (Cambridge: University Press, 1959), I, 377.

[33] Treatise IV, book iv, 17.

> **Music is subordinate to Arithmetic and when we know a few fundamental experiments with harmonies and dissonances, all the remaining general precepts depend on numbers; I recall once drawing a harmonic line divided in such a fashion that one could determine with the compass the different compositions and properties of all musical intervals. Besides, we can show a man who does not know anything about music, the way to compose without mistakes.**[34]

On the other hand, it is among the German writers of the Baroque that we find the first clear documentation of a new era of philosophy in music, a philosophy *not* based on mathematics. The most important discussion on this view has come down to us is by Johann Mattheson (1681 - 1764). In his Neu-Eroffnete Orchestre Mattheson attacks the old notion of mathematics-based theory in music by going directly to the elements upon which the older theorists had based their reasoning, in particular the nature of the intervals. In his discussion of whether the interval of the fourth should regarded as a consonance or dissonance, Mattheson concludes it is not a matter of mathematics, but rather a matter of the ear, that is how the fourth is used. The reader should particularly notice, as a hallmark of the Baroque's movement away from music based on concepts to music based on feeling, that Mattheson specifies here that music communicates with "the inner soul."

> **Numbers in music do not govern but merely instruct. The Hearing is the only channel through which their force is communicated to the inner soul of the attentive listener.... The true aim of music is not its appeal to the eye, nor yet altogether to the so-called "Reason," but only to the Hearing, which communicates pleasure, as it is experienced, to the Soul and the "Reason." Hence, if the testimony of the ear is followed, it will be discovered that in its relation to the surrounding sounds and harmony, the fourth will be either consonant or dissonant.**[35]

Such views, which would seem obvious to most modern readers, were nevertheless a direct attack on the old mathematics-based theories of music and resulted in letters and books attacking Mattheson for his views. Johann Buttstedt,

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[34] Leibniz, untitled manuscript, known as "Precepts for Advancing the Sciences and Arts" (1680), in Leibniz Selections, ed., Philip Wiener (New York: Scribner's, 1951), 42ff.
[35] Johann Mattheson, Das Neu-Eroffnete Orchestre (Hamburg, 1713), 126ff. Mattheson also writes at length in opposition to the old dogma that mathematics is the basis of music in his book, Das Forschende Orchestre of 1721.

an organist in Erfurt, attacked Mattheson in a book, Ut, Mi, Sol, Re, Fa, La, Tota Musica et harmonia Aeterna...entgegen gesetzt Dem neu-eršffneten Orchestre... in which he contends that since German music is now practiced only by craftsmen [*Spielmanns-Wesen*] the current musicians are not even educated in the older rules.

> **How many musicians will one find today who have real knowledge? Most of them do not even know how many styles and modes there are and what music is suitable for ecclesiastical or motet styles. The knowledge of such styles is almost entirely lost.... Why? [Modern music] is hard to understand and not well paid for. And so, instead of correct knowledge mere *Galanterie* suffices, just as the finery of ladies once consisted of pearls and golden chains but now of mere ribbons and laces....**[36]

To defend himself, Mattheson published a new book, Das Beschutzte Orchestre, in which he appealed to a number of distinguished German musicians to join in the debate over mathematics versus feelings, and some distinguished musicians came to the defense of Mattheson. Handel wrote Mattheson at this time, taking a very practical approach to the debate.

> **The question seems to me to reduce itself to this: whether one should prefer an easy & most perfect Method to another that is accompanied by great difficulties capable not only of disgusting pupils with Music, but also making them waste much precious time that could better be employed in plunging deeper into this art & in the cultivation of one's genius?**[37]

Johann Heinichen, in language much stronger than Mattheson's, ridiculed the old-fashioned theorists as having wasted their entire life in pursuit of *rudera antiquitatis*.

> **All will be sheer Greek to those steeped in prejudices when nowadays they hear that a moving music composed for the ears requires even more subtle and skillful rules -- to say nothing of lengthy practice -- than the heavily oppressive music composed for the eyes which the cantors of even the tiniest towns maltreat on innocent paper according to all the venerable rules of counterpoint.... And we Germans alone are such fools as to jog on in the old groove and, absurdly and ridiculously, to make the appearance of the composition on paper, rather than the hearing of it, the aim of music.**[38]

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[36] Quoted in Beekman Cannon, Johann Mattheson, Spectator in Music (Archon Books, 1968), 135ff.
[37] George Friedrich Handel, letter to Johann Mattheson, February 24, 1719, quoted in Piero Weiss, Letters of Composers Through Six Centuries (Philadelphia: Chilton, 1967), 63.
[38] Quoted in Cannon, Op. cit., 141ff.

Johann Kuhnau also was strong in his support of Mattheson.

> **As regards the great controversy that the gentleman of Erfurt has brought upon you, I do not believe that, save for him, anyone will disapprove of your *Orchestre*. This is especially true of your point of view in matters of the solmisation and the old ecclesiastical modes; for you wrote your *Orchestre* for a *galant-homme* who, being no professional musician, has not the least interest in amusing himself with innumerable old freaks which are usually outmoded at best and worth -- virtually nothing.**[39]

In his Der vollkommene Capellmeister of 1739 Mattheson returns to this question.[40] **Here he begins with the basic point that mathematics is an aid to music, as it is to most disciplines. However, "they are wrong who believe or want to teach others that mathematics is the heart and soul of music" or that it is responsible for changes in emotion in the music. He begins his argument with the concept of proportions in general, which he finds in natural, moral, rhetorical and mathematical relationships. For the first three of these, natural, moral and rhetorical relationships, Mattheson maintains no precise mathematical measure is possible. One cannot, for example measure the distance from the earth to the sun precisely because the flames leaping out from the sun render no fixed edge. His comment regarding precision in language is quite perceptive. Everyone would agree, he supposes, that "life" is a positive, happy word. But if one says "life is denied," the meaning is changed. Thus, "the heart's emotion no longer has its basis in mere sounds and words."**

Turning to music, he proposes two rhetorical questions:

> **1. If someone wants to be a sound musician, must he not attain this through mathematics?**
>
> **2. Cannot one become an admirable composer and musician without thorough knowledge of the arts of measuring?**
>
> **Now if someone says yes to the first question, and no to the second, then he contradicts ancient and modern experience, indeed, his own eyes, ears, hands, the combined senses of all mankind, and shuts the only door through which his intelligence gives him what he has. Whereas if he answers**

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[39] Quoted in Ibid.,142.

[40] Johann Mattheson, Der vollkommene Capellmeister (1739), trans., Ernest Harriss (Ann Arbor: UMI Research Press, 1981), Foreword, VI.

> **no to the first question and yes to the second, then mathematics cannot possibly be the heart and soul of music.**

From this he concludes mathematics can measure, but not determine the essence of a thing. "Everything that goes on in music is based on mathematical relationships of intervals just about as much as seamanship is based on anchors and cables."

> **However one defines the mathematical relationships of sounds and their quantities, no real connection with the passions of the soul can ever be drawn from this alone.**

Mathematics is only the "science, theory and scholarship" of music. To introduce what exists beyond this he quotes Andreas Papius.

> **The mere *cognition of the ratio* of a step, a half step, a comma, the consonances, etc., will bring the name virtuoso or artistic prince to no one, but rather the minute examination *according to the laws of nature* of the various works which are produced by great artists: from this we can understand the composer's *soul*, in regard to how and to what extent, in his particular work, one thing more than another masters the *human mind and emotions*, which is the *highest pinnacle of the discipline of music*.**

Again, his point here is that mathematics can measure the elements of music, but not how these elements are used. It is the latter, not the former, which concern feelings in music.

> **A perfect understanding of the human emotions, which certainly are not to be measured by the mathematical yardstick, is of much greater importance to melody and its composition than the understanding of tones.... This is certain: it is not so much good *proportion*, but rather the apt *usage* of the intervals and keys, which establishes the beautiful, moving and natural quality in melody and harmony. Sounds, in themselves, are neither good nor bad; but they become good and bad according to the way in which they are used. No measuring or calculating art teaches this.**

How then does one describe the role mathematics plays in music, together with its other elements? Mattheson offers following metaphor:

> **The human mind is the paper. Mathematics is the pen. Sounds are the ink; but Nature must be the writer.**

Mattheson points out that sculptors know and can measure the proportions of the human body, but "heart and soul...and beauty is not on this account to be found in such mathematical measuring; but only in that force which God put in Nature."

Similarly, in painting, when "mathematics ceases entirely, true beauty really first begins." And so with music,

> **A composer can succeed quite well without special mathematical skills. Many who virtually climbed to the pinnacle of music can hardly name or interpret all parts of mathematics; not to mention anything more.... However, the best mathematician, as such, if he were to want to compose something, could not possibly achieve this with mere logic.**
>
> **Let it be said once in fact for all: Good mathematical proportions cannot constitute everything: this is an old, stubborn misconception.**

The point, he says, is this: "music draws its water from the spring of Nature; and not from the puddles of arithmetic." The composer expresses something understood from Nature. Only then can this be mathematically expressed, but not the other way around. When Mattheson speaks here of Nature, he is also thinking of God.

> **Mathematics is a human skill; nature, however, is a divine force.... Now the goal of music is to praise God in the highest, with word and deed, through singing and playing. All other arts besides theology and its daughter, music, are only mute priests. They do not move hearts and minds nearly so strongly, nor in so many ways....**
>
> **Music is *above*, not in *opposition* to mathematics.**

In conclusion, Mattheson cannot resist taking a shot at those remaining exponents of the old mathematics-based polyphony.

> **I have occupied myself with music, practical as well as theoretical, with great earnestness and ardor for over half a century already: I have also met many very learned *Mathematici* in this not insubstantial time who thought they made new musical wonders out of their old, logical writings; but they have, God knows! always failed miserably. On the other hand, I have quite certainly and very often experienced that not a single famous actor, musician, nor composer, not only in my time but as far as I can remember having read or heard about, has been able to construct even a simple melody which was of any value on the feeble foundations of mathematics or geometry.... What will happen in the future is yet to be seen.**

We might also add that in his biographical work, Ehrenpforte (Hamburg, 1740), in reference to a person who had claimed both a goal of making "music a scientific or scholarly pursuit" and an association with Bach, Johann Mattheson

adds that Bach certainly did not teach this man "the supposed mathematical basis of composition." "This," Mattheson testifies, "I can guarantee."[41]

And so the stage was set for a century and a half of the greatest, most heart-felt musical compositions ever written, the innumerable great Classic Period and Romantic Era compositions by the German and Austrian composers. They are a powerful rebuttal to 3000 years or so of philosophical arguments on the mathematical basis of music.

Nothing more was heard of mathematics until the 20th century when the serial composers appeared. The Twelve-Tone Era lasted about 50 years (the same length as the Classic Period) and is now dead.[42] It produced music the public did not want to hear and does not want to hear today. It was a failed experiment by composers who were ignorant of history's demonstration that great music is based neither on concepts nor numbers.

"But," these composers say, "we are not of this world; it is in the future when our compositions will be understood and appreciated." We can see nothing to suggest this will be the case. Actually, as we survey concert programs here and abroad, we do not find a single serial composition which can be said to be "in the repertoire."

[41]Quoted in Hans T. David and Arthur Mendel, The Bach Reader (New York: Norton, 1966), 440.
[42] A musician we admire told us, "Thank God I lived long enough to see the death of Communism and Twelve-Tone music!"