Semiconductor Devices and Circuits

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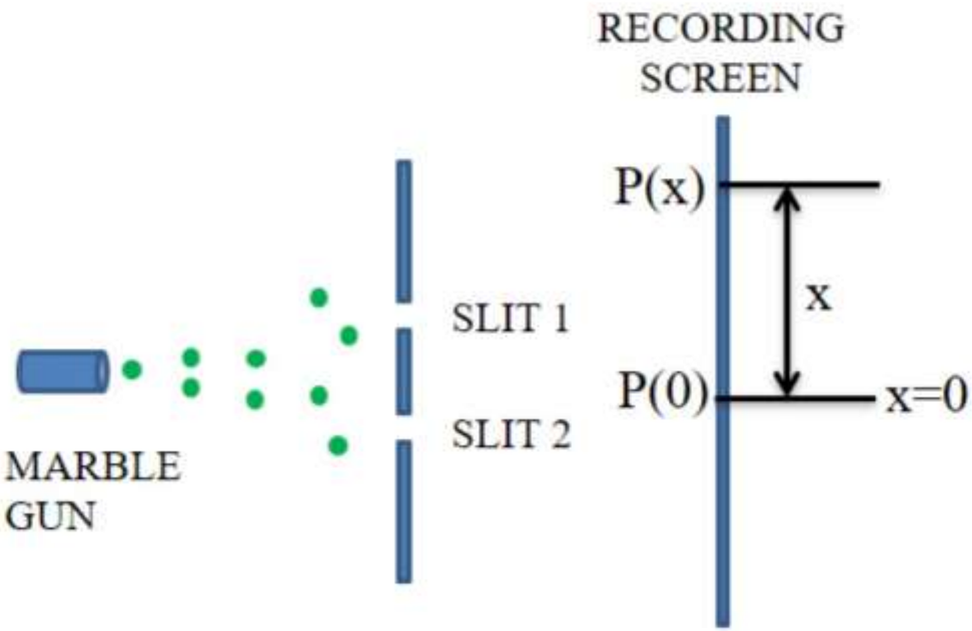
Quantum Mechanics

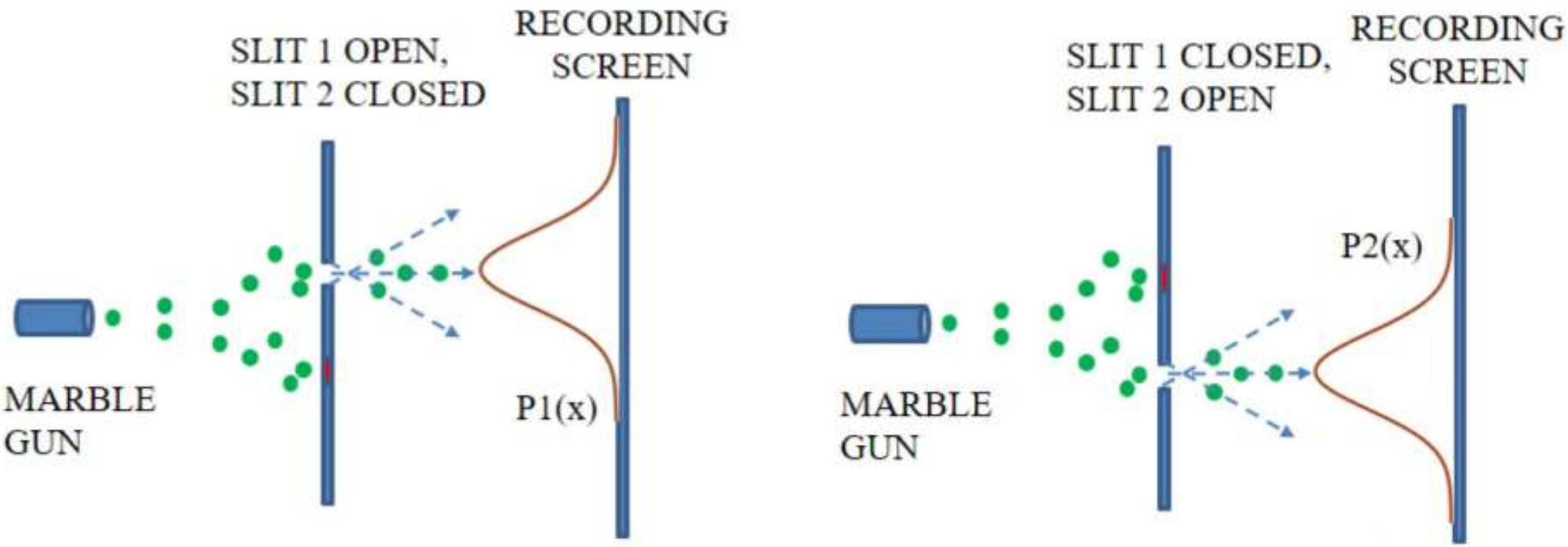
Feynman's Famous Thought Experiment



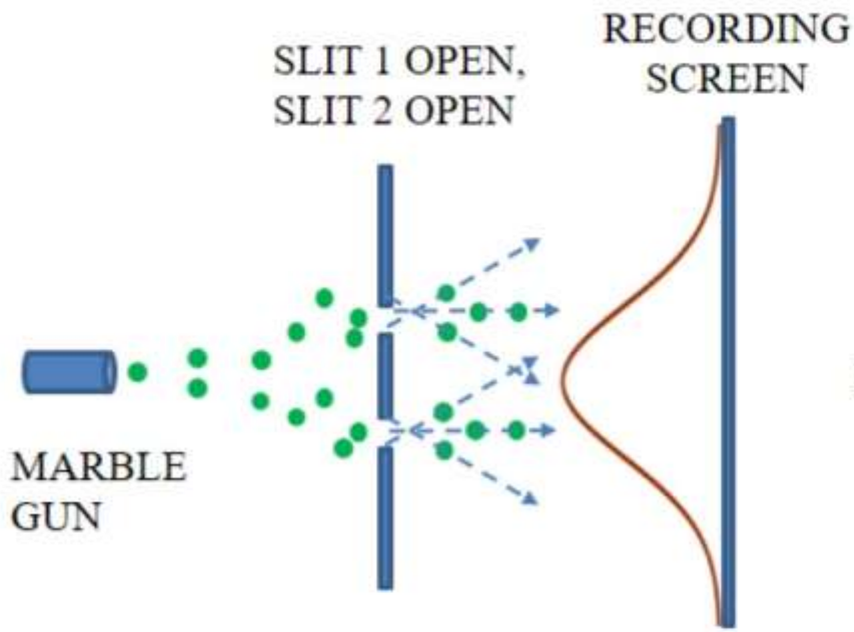
Probability=P(x)=?

$$P(x) = \frac{\text{number of marbles striking the screen at x}}{\text{Total number of marbles fired}}$$

Quantum Mechanics



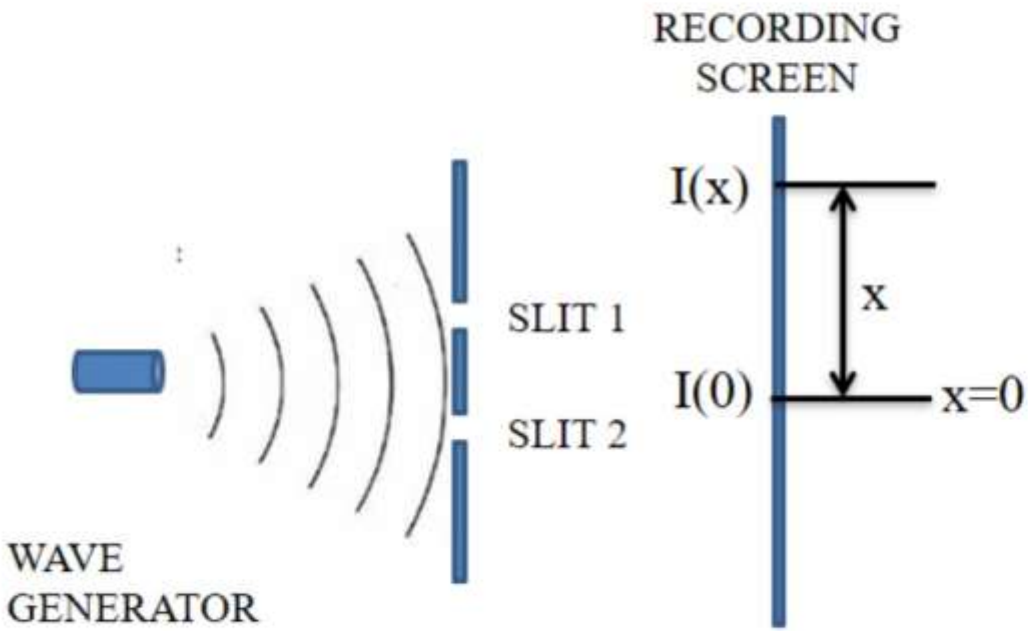
Quantum Mechanics



$P(x) = P1(x) + P2(x)$

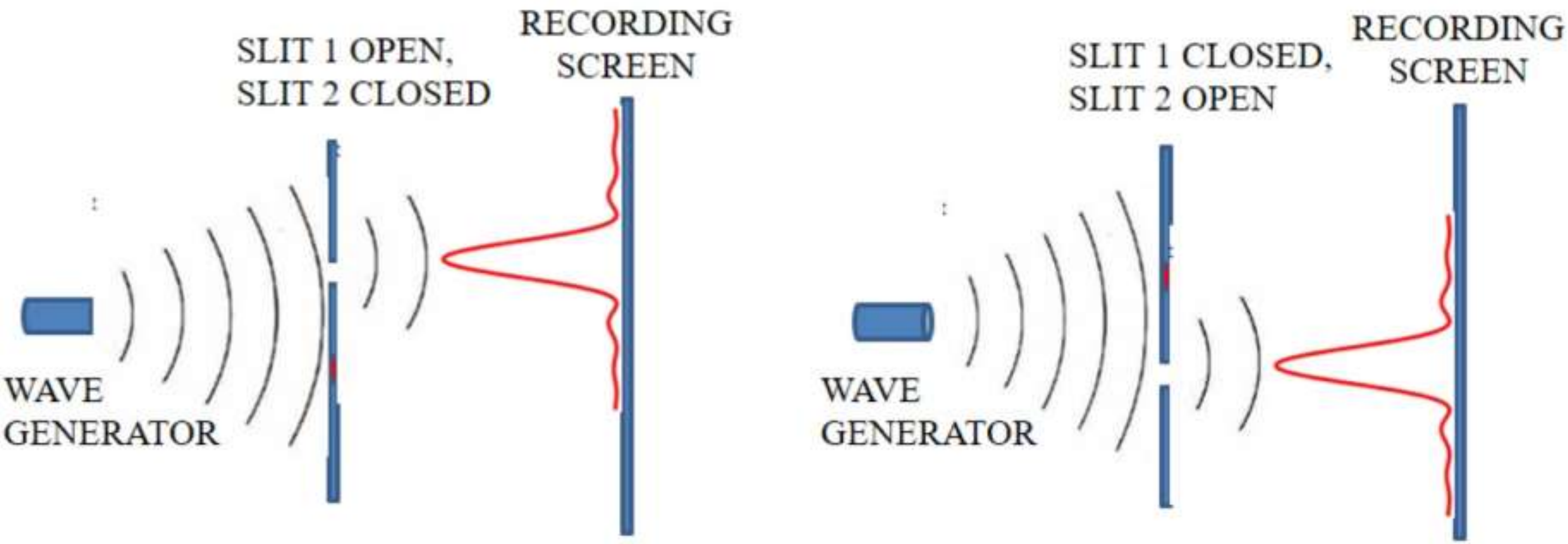
Classical Mechanic Intuition: A Marble can go through only one of the two slits (or so we think)

Quantum Mechanics

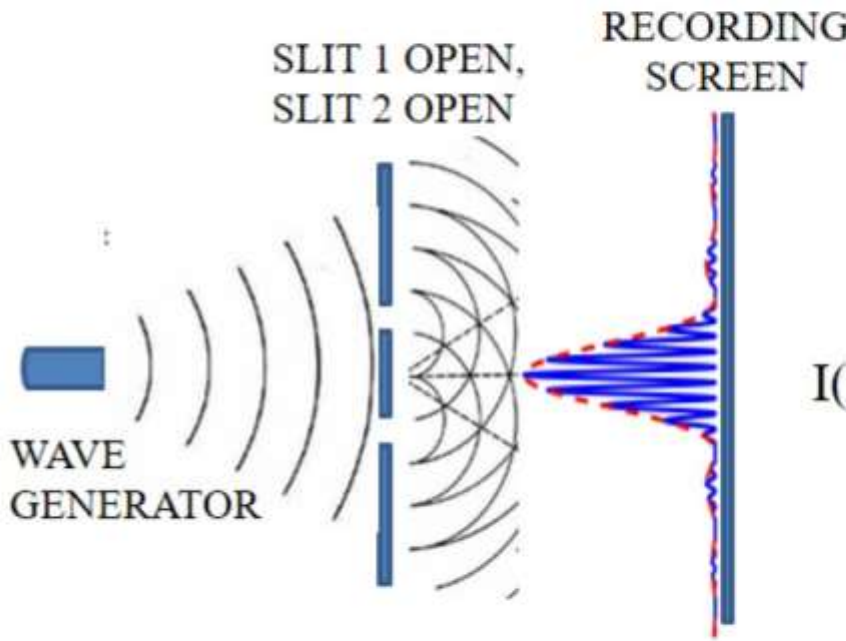


Intensity=I(x)=?

Quantum Mechanics



Quantum Mechanics

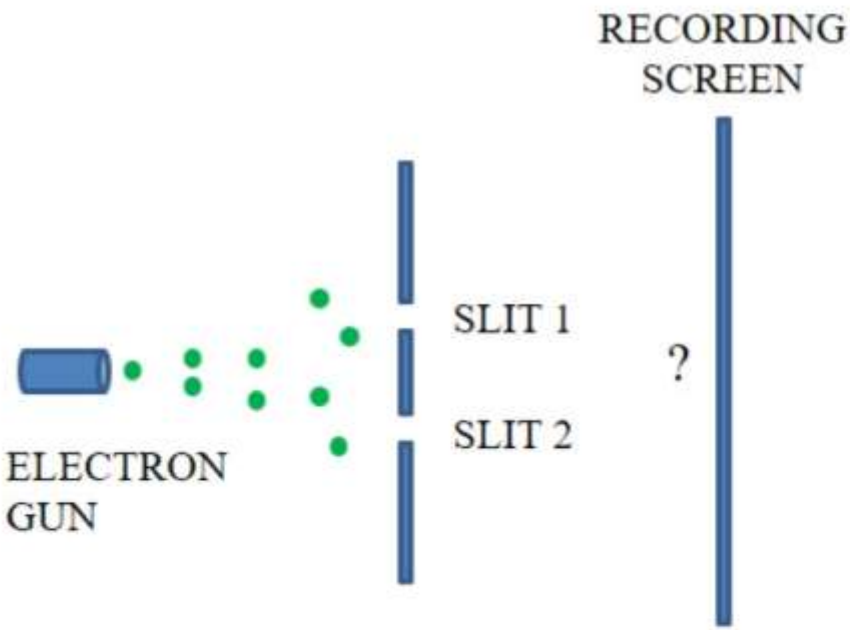


$$I(x)=I_1+I_2+2(I_1I_2)^{1/2}\cos(\phi)$$

Classical Mechanic Intuition: Wave fronts go through both slits simultaneously and we have interference

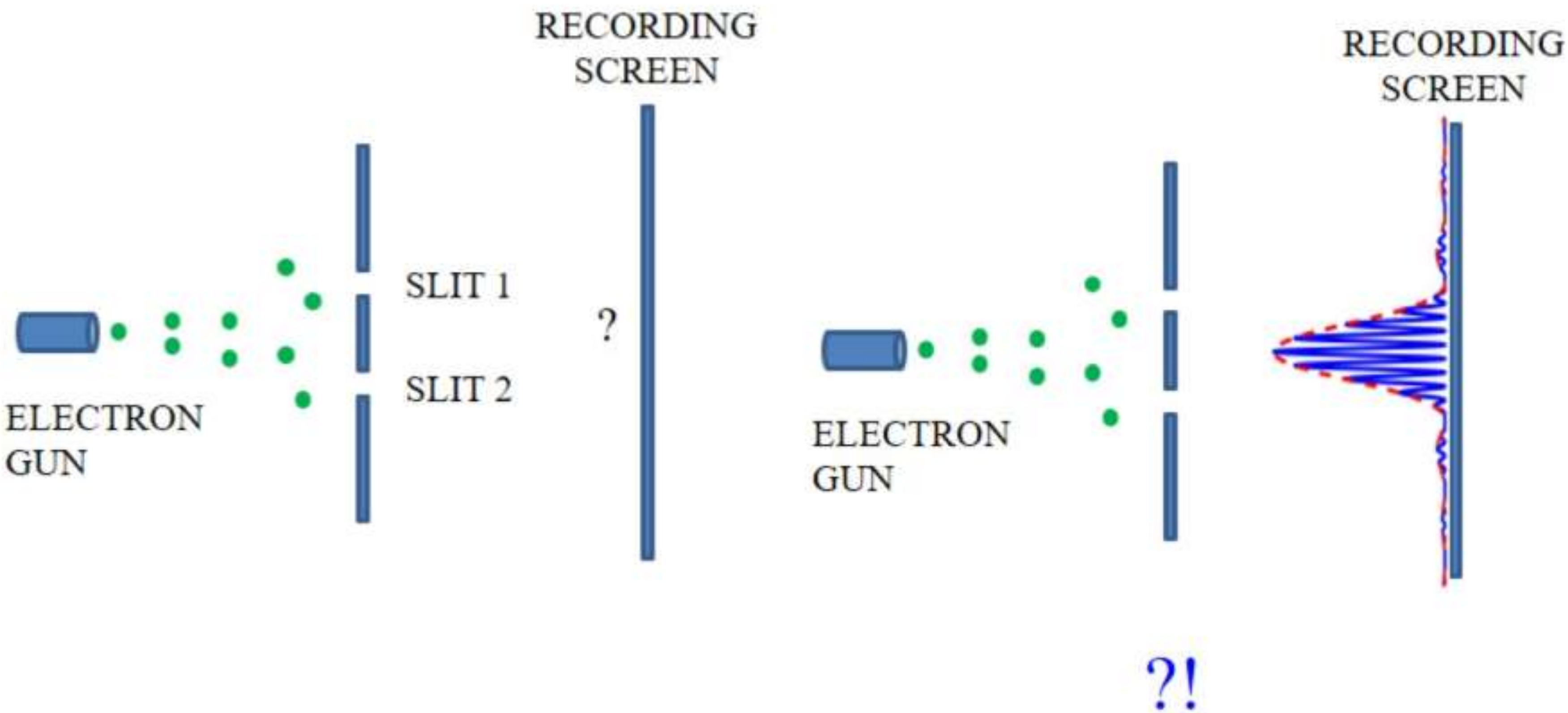
Quantum Mechanics

Feynman's Famous Thought Experiment



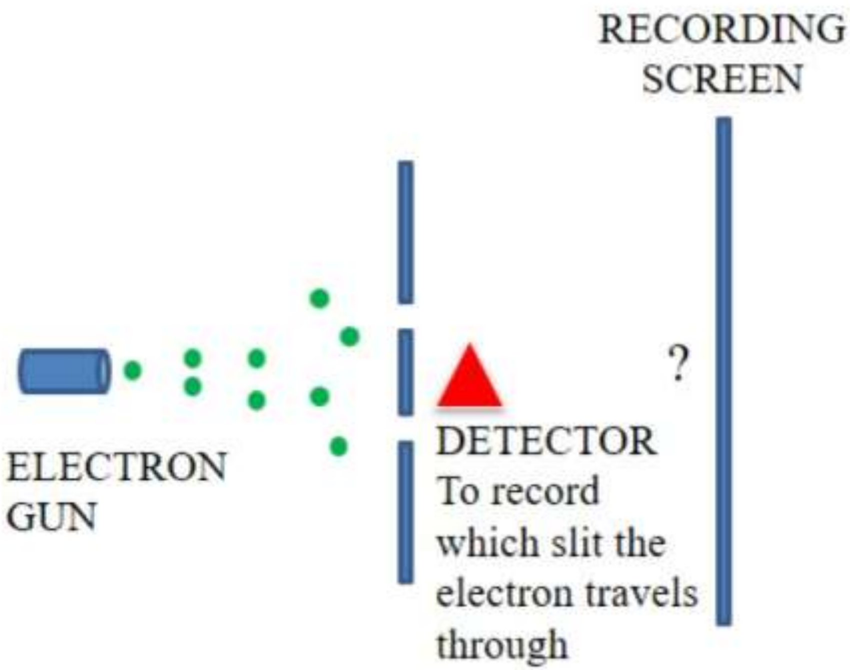
Quantum Mechanics

Feynman's Famous Thought Experiment



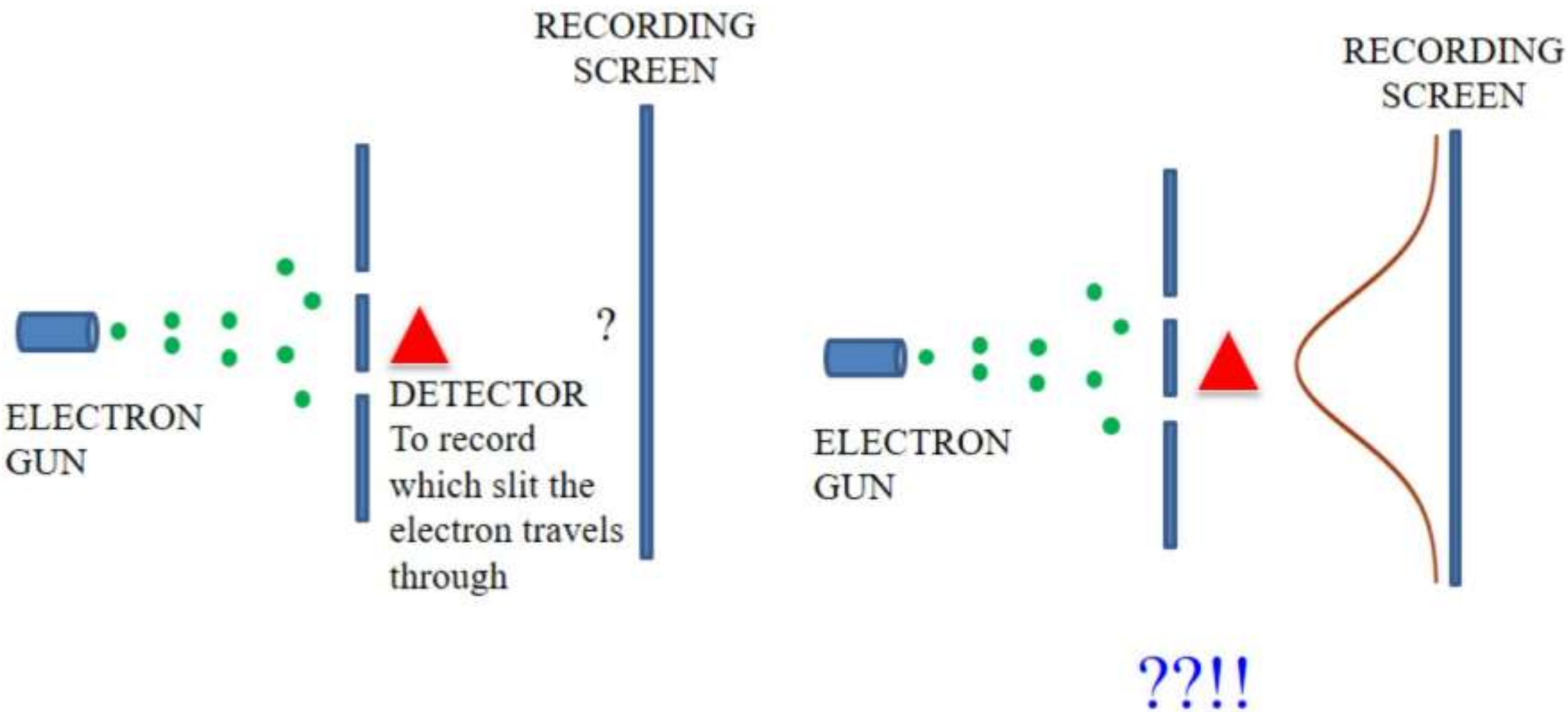
Quantum Mechanics

Feynman's Famous Thought Experiment



Quantum Mechanics

Feynman's Famous Thought Experiment



Wave-Particle Duality

Need both the Wave and Particle like treatment to explain Experiments
Photons, electrons etc behave as particle and wave
Wavelength of the Particle=λ=h/p (De Broglie's Equation)
$h=6.6\times10^{-34}$ Js is the Planck's constant

<u>Example: Electron</u>
Charge=-1.6×10^{-19} C
Mass = $m=9.1\times10^{-31}$ kg
Velocity = v = say 1×10^{5} m/s
Momentum = p = mv =9.1×10^{-26} kg.m/s
Kinetic Energy = $mv^2/2=p^2/2m=4.55\times10^{-21}$ J=$(4.55\times10^{-21})/(1.6\times10^{-19})$ eV= 28.4meV
Wavelength=$\lambda=h/p=6.6\times10^{-34}/9.1\times10^{-26}$ =7.2 nm

<u>Example: Tennis Ball</u>
Mass = m=0.058 kg
Velocity = v = say 50 m/s
Momentum = p = mv = 2.9 kg.m/s
Kinetic Energy = $mv^2/2=p^2/2m=72.5$ J
Wavelength=$\lambda=h/p=6.6\times10^{-34}/72.5=9.1\times10^{-32}$ m

Useful Relations to Note

k=wave vector=$2\pi/\lambda$
(k also used for the Boltzmann's coefficient later on, please note the context.)

$\lambda = h/p$
$p = h/\lambda = (h/2\pi)(2\pi/\lambda) = \hbar k$
'h bar' is the reduced Planck's constant $\hbar = h/2\pi \approx 1.05\times10^{-34}$ Js

Energy of a photon
$h\nu = hc/\lambda$
$h\nu = (h/2\pi)(2\pi\nu) = \hbar\omega$

Heisenberg's Uncertainty Principle

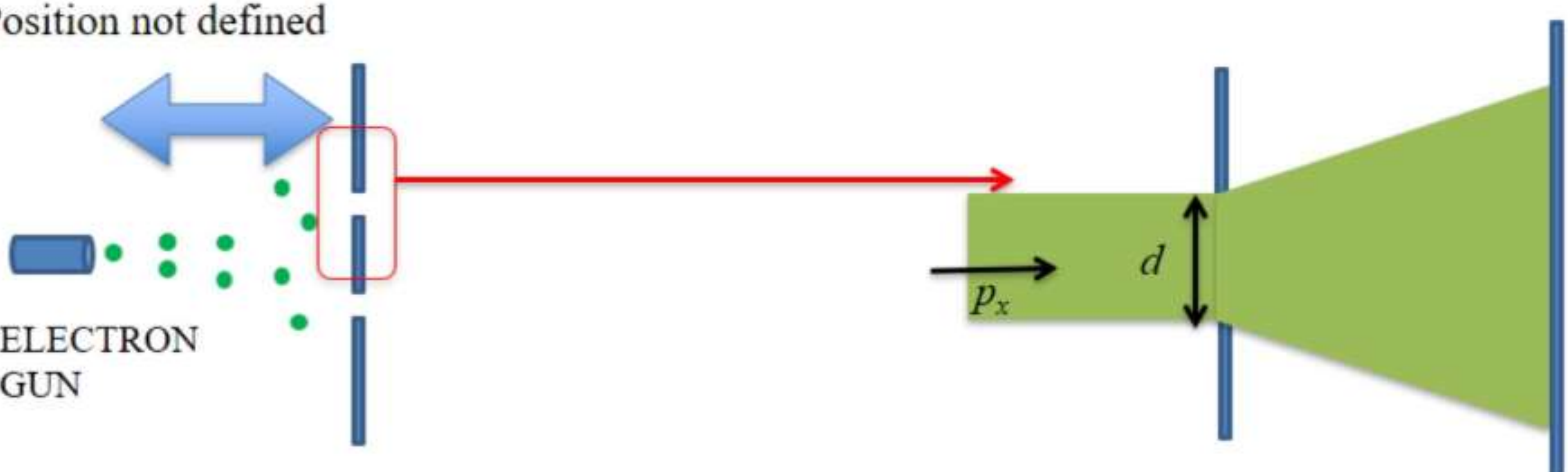
Cannot measure position and momentum – both – absolutely accurately.
(Equivalently, error in energy and error in time of measurement)
If Δx is the error in determining position and Δp the error in determining momentum, $\Delta p \Delta x \geq \hbar / 2$

In general a signal in time/space ←→ frequency will have this restriction

Momentum well defined
(eg. energy of the electron gun is known)
Position not defined

$p_x \sim h / \lambda; \; d \sin(\Delta\theta) = \lambda \Rightarrow \Delta\theta \sim \lambda / d$

$\Delta y \sim d; \; \Delta p_y \sim p_x \Delta\theta \sim h / d \sim h / \Delta y; \; \Delta p_y \Delta y \sim h$



Quantum Mechanics

The Wave function

$$\psi$$

Contains information on all measurable parameters of the particle

If we consider 1D (say x direction), the wavefunction is $\psi(x,t)$

$\psi\psi^*$= Probability density of finding the particle between x and x+dx

If $\psi(x,t)$ is real,
Probability of finding the particle between x and x+dx $=|\psi(x,t)|^2 dx$

Probability of finding the particle between x=a and x=b is $\int_a^b |\psi|^2 dx$

Quantum Mechanics

Operators

Contains information on all measurable parameters of the particle.

Some useful operators:

Momentum Operator : $\hat{p}_x = \frac{\hbar}{i}\frac{\partial}{\partial x}$ Thus, $\frac{\hbar}{i}\frac{\partial \psi}{\partial x} = p_x \psi$

Energy Operator : $\hat{E} = -\frac{\hbar}{i}\frac{\partial}{\partial t}$ Thus, $-\frac{\hbar}{i}\frac{\partial \psi}{\partial t} = E\psi$

Kinetic Energy : $\frac{p_x^2}{2m} = \frac{1}{2m}\frac{\hbar}{i}\frac{\partial}{\partial x}\left(\frac{\hbar}{i}\frac{\partial \psi}{\partial x}\right) = -\frac{\hbar^2}{2m}\frac{\partial^2 \psi}{\partial x^2}$

Quantum Mechanics

Properties of The Wave function

Contains information on all measurable parameters of the particle

ψ must be a solution of Schrodinger's Equation (Energy Balance)
ψ must be continuous
dψ/dx must be continuous
ψ must be normalizable – i.e. it cannot for eg. blow up.
This condition is realized by noting that the particle must exist somewhere in all allowed regions and that the Probability of finding the particle somewhere is 1

$$\int_{-\infty}^{\infty} |\psi|^2 dx = 1$$

Quantum Mechanics

The Schrodinger's Equation

Kinetic Energy + Potential Energy = Total Energy

Potential Energy: Depends on the potential terrain the particle is placed in=Vψ

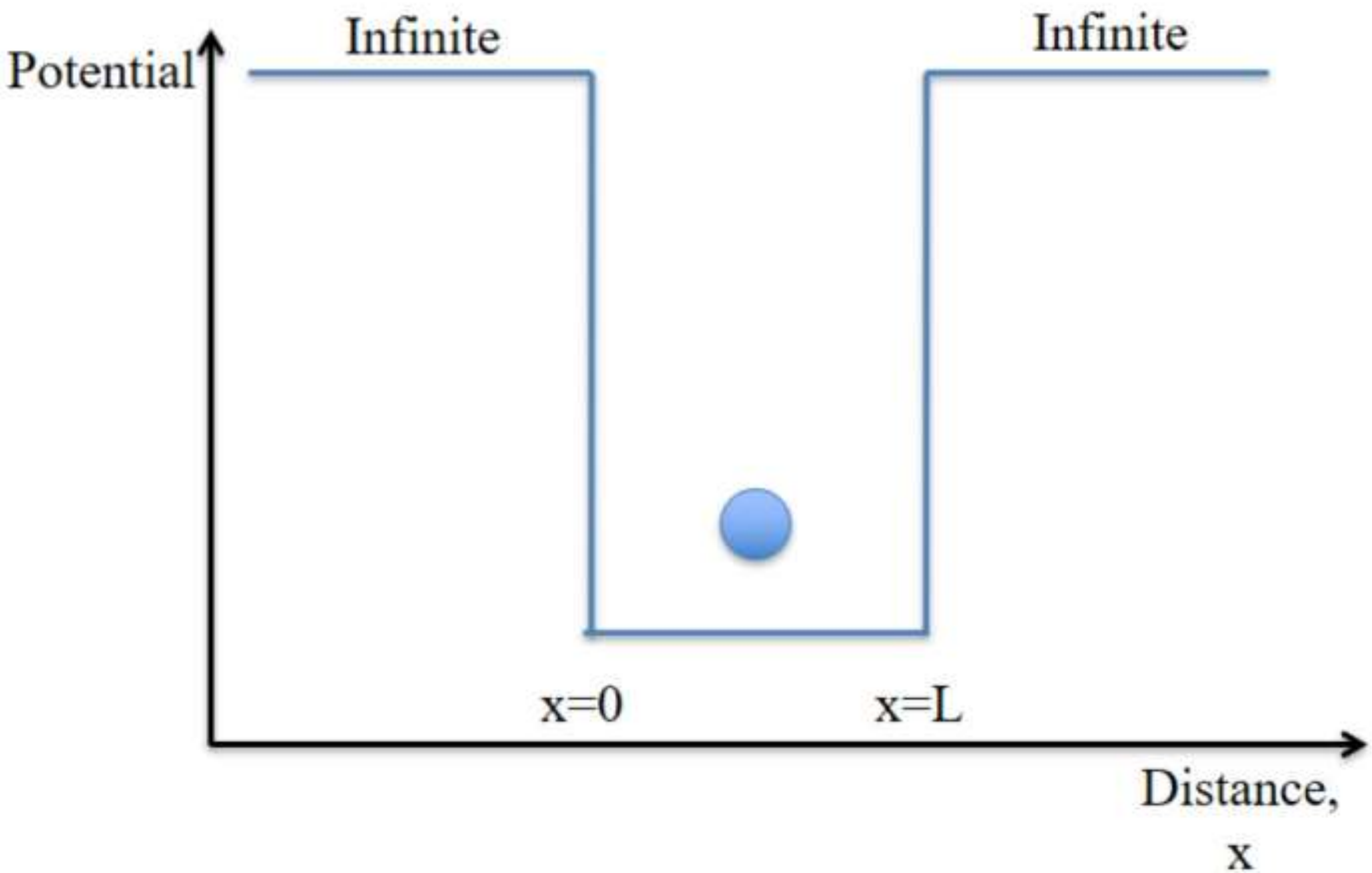
$$-\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2}+V\psi=-\frac{\hbar}{i}\frac{\partial\psi}{\partial t}$$

Time independent Schrodinger Equation

$$-\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2}+V\psi=E\psi$$

Quantum Mechanics

Particle in a 1D Box: Infinite Potential Barrier



Quantum Mechanics

Particle in a 1D Box: Infinite Potential Barrier

$$-\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2}+V\psi=E\psi$$

Inside the Box

$$-\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2}=E\psi$$

$$\psi=A\sin(kx)+B\cos(kx)$$

Quantum Mechanics

Particle in a 1D Box: Infinite Potential Barrier

$$\psi = A\sin(kx) + B\cos(kx)$$

ψ must be continuous

Quantum Mechanics

Particle in a 1D Box: Infinite Potential Barrier

$$\psi = A\sin(kx)$$

ψ must be continuous

Quantum Mechanics

Particle in a 1D Box: Infinite Potential Barrier

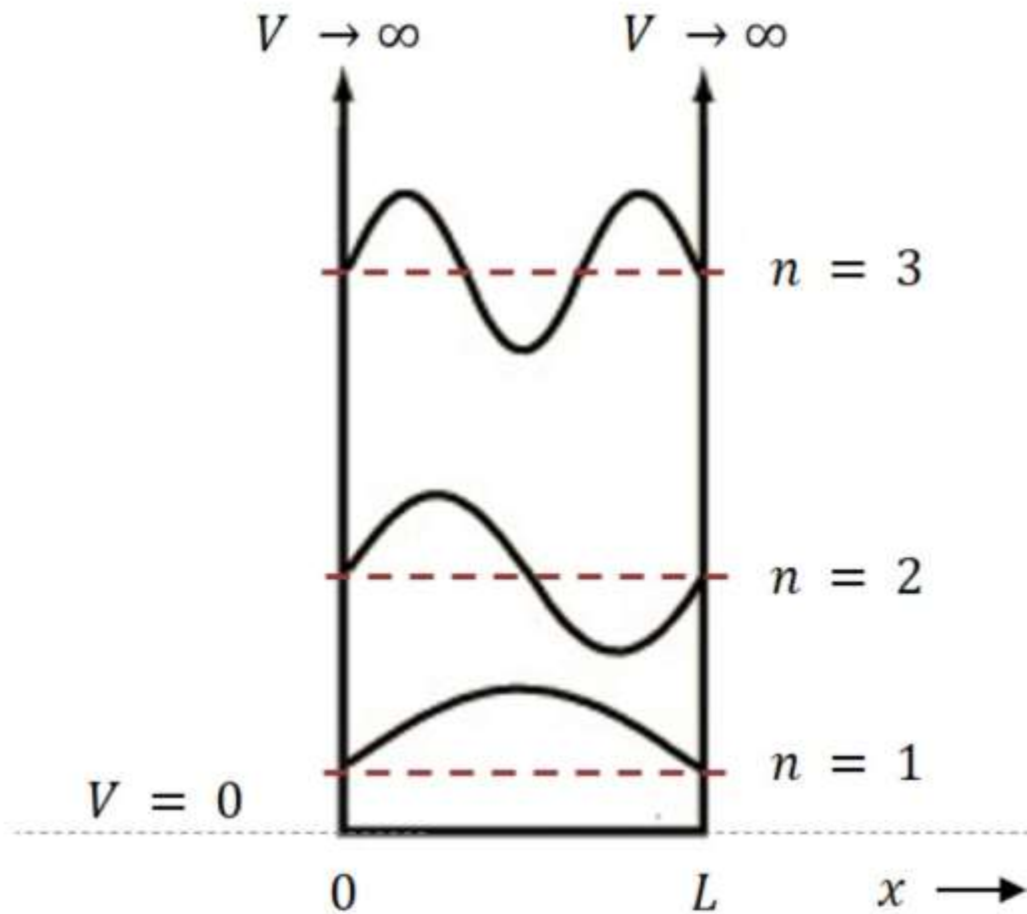
$$\psi = A\sin(\frac{n\pi}{L}x)$$

$$\int_0^L |\psi|^2 dx = 1$$

Quantum Mechanics

Particle in a 1D Box: Infinite Potential Barrier

$$E_n = \frac{n^2\pi^2\hbar^2}{2mL^2}$$



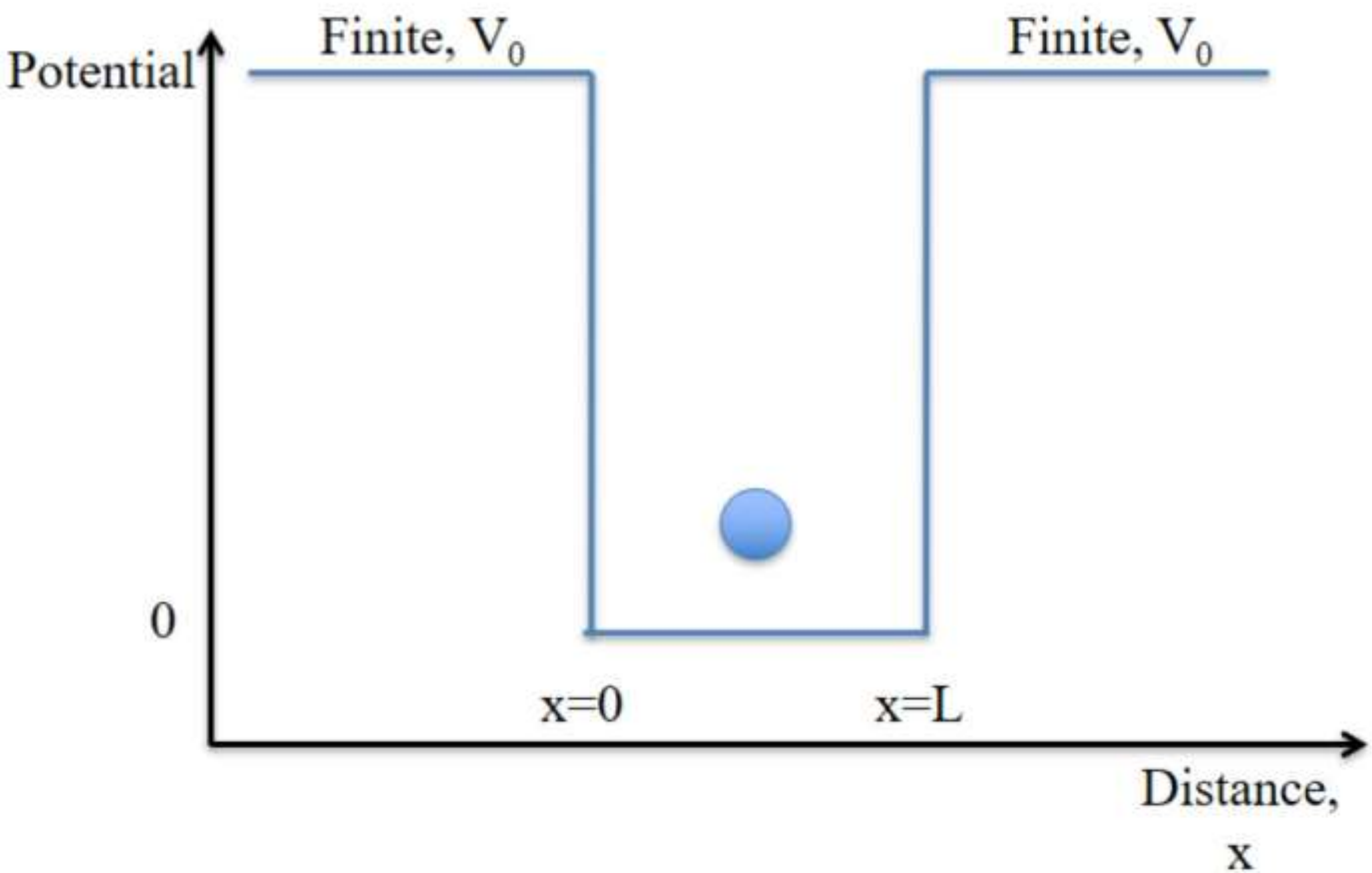
Quantum Mechanics

Particle in a 2D Box: Infinite Potential Barrier

$$-\frac{\hbar^2}{2m}\left(\frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial y^2}\right)=E\psi$$

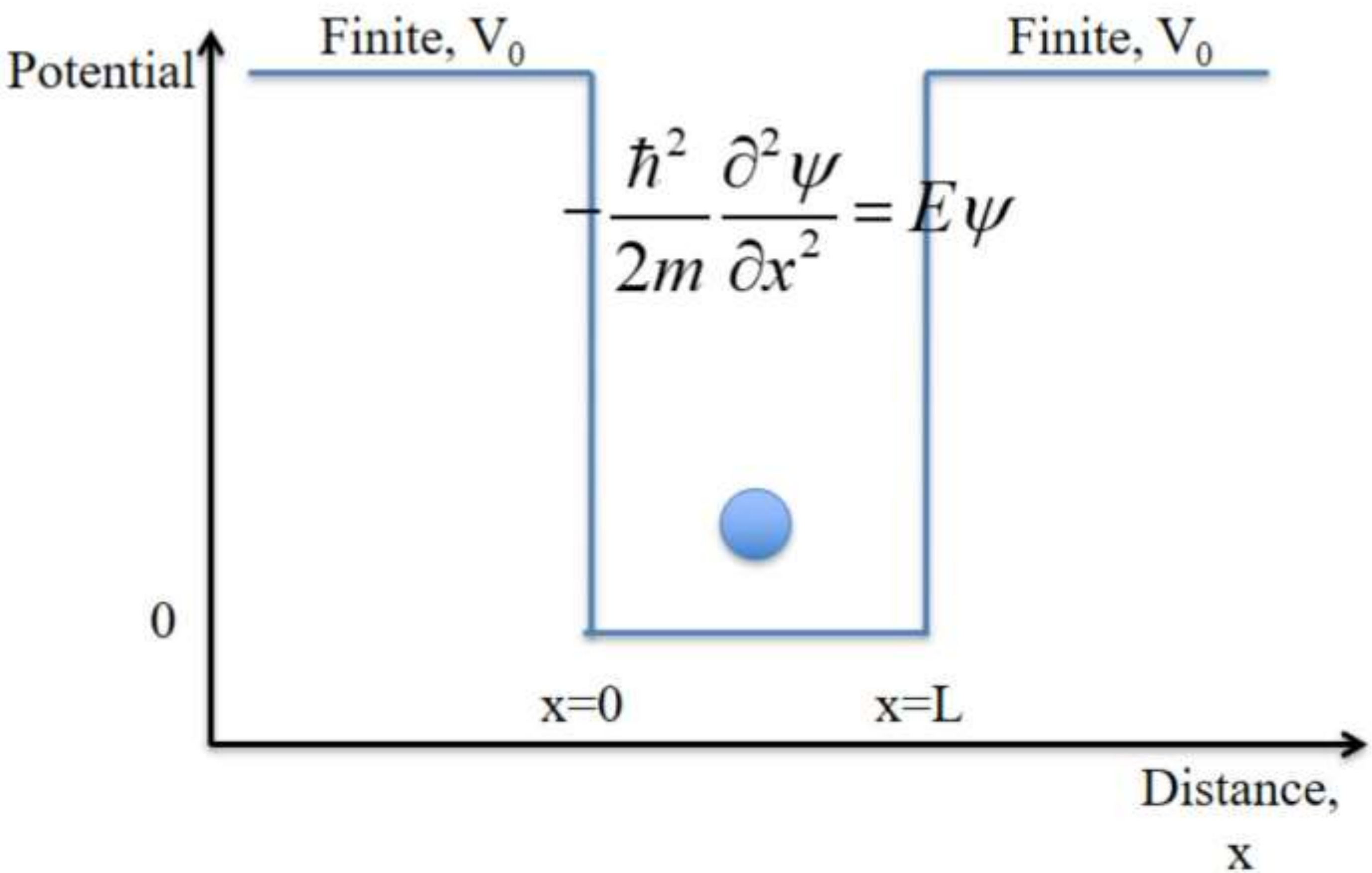
Quantum Mechanics

Particle in a 1D Box: Finite Potential Barrier



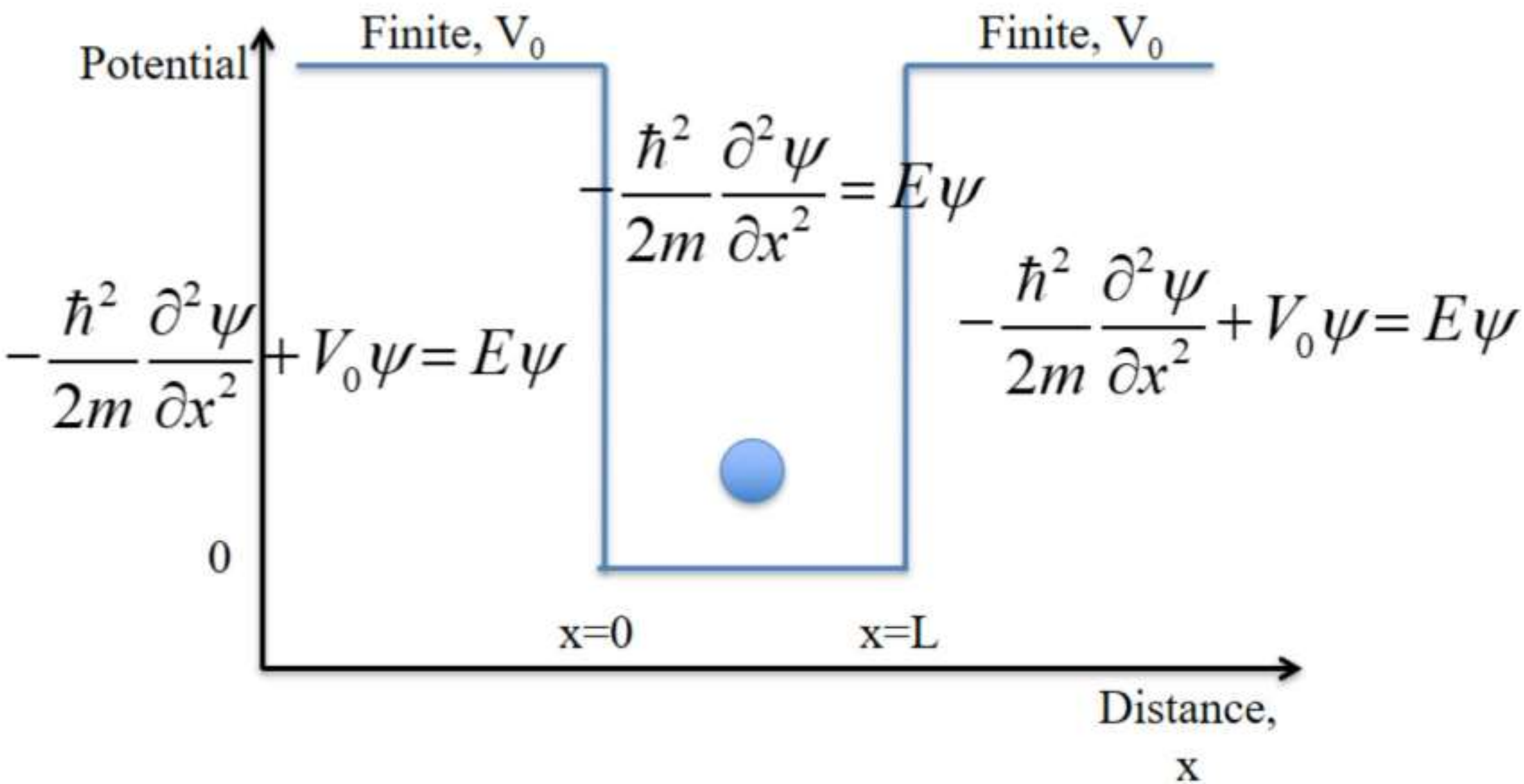
Quantum Mechanics

Particle in a 1D Box: Finite Potential Barrier



Quantum Mechanics

Particle in a 1D Box: Finite Potential Barrier



Quantum Mechanics

Particle in a 1D Box: Finite Potential Barrier

$$-\frac{\hbar^2}{2m}\frac{\partial^2 \psi_1}{\partial x^2}+V_0\psi_1=E\psi_1$$

$$-\frac{\hbar^2}{2m}\frac{\partial^2 \psi_2}{\partial x^2}=E\psi_2$$

$$-\frac{\hbar^2}{2m}\frac{\partial^2 \psi_3}{\partial x^2}+V_0\psi_3=E\psi_3$$

$$E<V_0$$

Quantum Mechanics

Particle in a 1D Box: Finite Potential Barrier

$$-\frac{\hbar^2}{2m}\frac{\partial^2\psi_1}{\partial x^2}+V_0\psi_1=E\psi_1 => \frac{\partial^2\psi_1}{\partial x^2}=\frac{2m(V_0-E)}{\hbar^2}\psi_1 => \frac{\partial^2\psi_1}{\partial x^2}=\alpha^2\psi_1$$

$\alpha > 0$

$$-\frac{\hbar^2}{2m}\frac{\partial^2\psi_2}{\partial x^2}=E\psi_2 => \frac{\partial^2\psi_2}{\partial x^2}=-\frac{2mE}{\hbar^2}\psi_2 => \frac{\partial^2\psi_2}{\partial x^2}=-k^2\psi_2$$

$k > 0$

$$-\frac{\hbar^2}{2m}\frac{\partial^2\psi_3}{\partial x^2}+V_0\psi_3=E\psi_3 => \frac{\partial^2\psi_3}{\partial x^2}=\frac{2m(V_0-E)}{\hbar^2}\psi_3 => \frac{\partial^2\psi_3}{\partial x^2}=\alpha^2\psi_3$$

$\alpha > 0$

Quantum Mechanics

Particle in a 1D Box: Finite Potential Barrier

$$\psi_1 = A_1 e^{\alpha x} + B_1 e^{-\alpha x}$$

$$\psi_2 = A_2 e^{ikx} + B_2 e^{-ikx}$$

$$\psi_3 = A_3 e^{\alpha x} + B_3 e^{-\alpha x}$$

Quantum Mechanics

Particle in a 1D Box: Finite Potential Barrier

$$\psi_1 = A_1 e^{\alpha x} + B_1 e^{-\alpha x}$$

as x→ -inf, the second term → inf

Wave function cannot blow up => B_1=0

$$\psi_1 = A_1 e^{\alpha x}$$

$$\psi_3 = A_3 e^{\alpha x} + B_3 e^{-\alpha x}$$

as x→ inf, the second term → inf

Wave function cannot blow up => A_3=0

$$\psi_3 = B_3 e^{-\alpha x}$$

Wavefunction symmetric about x=0

Odd Sym or Even Sym

$$\psi_2 = C_2 \sin(kx)$$

$$\psi_2 = C_2 \cos(kx)$$

Quantum Mechanics

Particle in a 1D Box: Finite Potential Barrier

$$\psi_1 = A_1 e^{\alpha x}$$

$$\psi_2 = C_2 \cos(kx)$$

$$\psi_3 = B_3 e^{\alpha x}$$

$$\psi_2(x = L/2) = \psi_3(x = L/2)$$

$$C_2 \cos(-kL/2) = B_3 e^{-\alpha L/2}$$

Quantum Mechanics

Particle in a 1D Box: Finite Potential Barrier

$$\psi_1 = A_1 e^{\alpha x}$$

$$\psi_2 = C_2 \cos(kx)$$

$$\psi_3 = B_3 e^{\alpha x}$$

$$\psi_2(x = L/2) = \psi_3(x = L/2)$$

$$C_2 \cos(kL/2) = B_3 e^{-\alpha L/2}$$

$$\frac{d\psi_2}{dx}(x = L/2) = \frac{d\psi_1}{dx}(x = L/2)$$

$$-\frac{C_2 k}{2}\sin(kL/2) = -\frac{B_3 \alpha}{2} e^{-\alpha L/2}$$

Quantum Mechanics

Particle in a 1D Box: Finite Potential Barrier

$$\psi_1 = A_1 e^{\alpha x}$$

$$\psi_2 = C_2 \cos(kx)$$

$$\psi_3 = B_3 e^{\alpha x}$$

$$\psi_2(x = L/2) = \psi_3(x = L/2)$$

$$C_2 \cos(kL/2) = B_3 e^{-\alpha L/2}$$

$$\frac{d\psi_2}{dx}(x = L/2) = \frac{d\psi_1}{dx}(x = L/2)$$

$$-\frac{C_2 k}{2}\sin(kL/2) = -\frac{B_3 \alpha}{2} e^{-\alpha L/2}$$

$$\tan(kL/2) = \frac{\alpha}{k}$$

Quantum Mechanics

Particle in a 1D Box: Finite Potential Barrier

$$\psi_1 = A_1 e^{\alpha x}$$

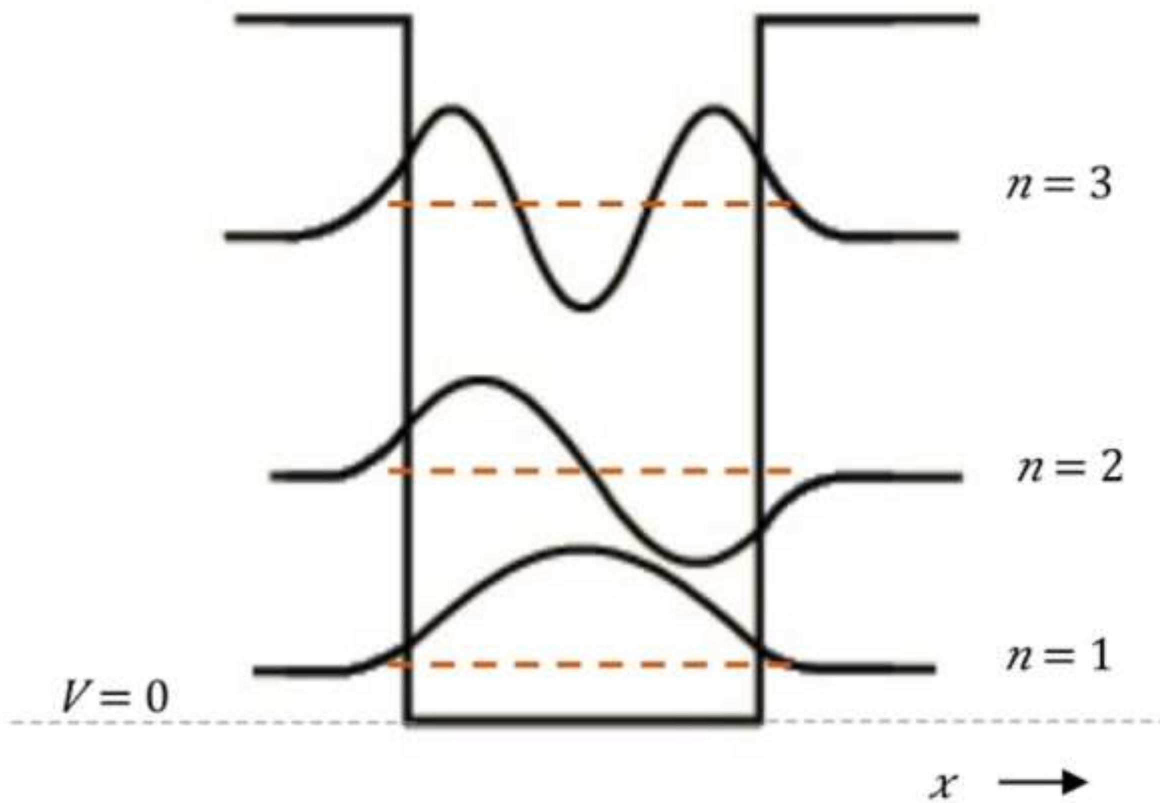
$$\psi_2 = C_2 \sin(kx)$$

$$\psi_3 = B_3 e^{\alpha x}$$

$$\cot(kL/2) = -\frac{\alpha}{k}$$

Quantum Mechanics

Particle in a 1D Box: Finite Potential Barrier



BIG MESSAGE

Particle can exist outside the box!

Quantum Mechanics

Harmonic Oscillator

$$-\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2}+V\psi=E\psi$$

$$V = fx^2/2$$

$$\omega=(f/m)^{1/2}$$

$$V = m\omega^2x^2/2$$

Quantum Mechanics

Harmonic Oscillator

$$-\frac{\hbar^2}{2m}\frac{\partial^2 \psi}{\partial x^2}+(m\omega^2 x^2/2)\psi = E\psi$$

$$\psi \sim e^{-\alpha x^2/2}$$

Quantum Mechanics

Harmonic Oscillator

$$-\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2}+\frac{m\omega^2x^2}{2}\psi\psi=E\psi$$

$$\psi\sim e^{-\alpha x^2/2}$$

$$-\frac{\hbar^2}{2m}(\alpha^2x^2-\alpha)\psi+\frac{m\omega^2x^2}{2}\psi=E\psi$$

Quantum Mechanics

Harmonic Oscillator

$$-\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2}+(m\omega^2x^2/2)\psi=E\psi$$

$$\psi\sim e^{-\alpha x^2/2}$$

$$-\frac{\hbar^2}{2m}(\alpha^2x^2-\alpha)\psi+\frac{m\omega^2x^2}{2}\psi=E\psi$$

$$-\frac{\hbar^2}{2m}\alpha^2x^2=\frac{m\omega^2x^2}{2}$$

Quantum Mechanics

Harmonic Oscillator

$$-\frac{\hbar^2}{2m}(\alpha^2 x^2 - \alpha)\psi + \frac{m\omega^2 x^2}{2}\psi = E\psi$$

$$\alpha = \frac{m\omega}{\hbar}$$

$$E = \frac{\hbar^2}{2m}\alpha = \frac{\hbar^2}{2m}\frac{m\omega}{\hbar} = \frac{\hbar\omega}{2}$$

Quantum Mechanics

Harmonic Oscillator

$$-\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2}+V\psi=E\psi$$

$$V = fx^2/2$$

$$V = m\omega^2 x^2/2$$

$$\omega=(f/m)^{1/2}$$

$$-\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2}+(m\omega^2x^2/2)\psi=E\psi$$

$$\psi \sim e^{-\alpha x^2/2}$$

$$-\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2}+(m\omega^2x^2/2)\psi=E\psi$$