Assignment 5 - Theory of Computation

Feb 2024

Equivalence, Pumping Lemma

1. (2 points) Consider a regular language R and a context free language C. Let the PDA that recognizes C be called $P = (Q_P, \Sigma, \Gamma, \delta_P, q_P^0, F_P)$, and the DFA that reconginzes R be $(Q_R, \Sigma, \delta_R, q_R^0, F_R)$.

   Pick all the statements that are true.

   - ■ **The intersection of these two languages $C \cap R$ is also context free.**
   - ■ **The union of these languages $(C \cup R)$ is context free.**
   - ☐ Construct PDA P' such that its states are $Q_P \times Q_R$, start state q (q_p^0, q_R^0) and it's accept states are $F_P \times F_R$. Suppose the transition function of $P' = \delta'$ is such that $\delta'((q_1, q_2), a, \gamma) = \{Q_P \times \{\delta_R(q_2, a)\}, \epsilon)\}\ \forall \gamma \in \Gamma$. Then this PDA recognizes $C \cap R$
   - ■ **Construct PDA P' such that its states are $Q_P \times Q_R$, start state (q_p^0, q_R^0) and it's accept states are $F_P \times F_R$. Suppose the transition function of $P' = \delta'$ is such that $\delta'((q_1, q_2), a, \gamma) = \{(q_1', q_2'), \gamma' \mid \{q_1', \gamma'\} \in \delta_P(q_1, a, \gamma) \wedge q_2' = \delta_R(q_2, a)\}\ \forall \gamma \in \Gamma$. Then this PDA recognizes $C \cap R$**

2. (2 points) Using the result of Question 1, Consider the language $L = \{w|w$ has an equal number of $a, b, c\}$, $L_n = \{a^n b^n c^n | n \geq 0\}$, $L_2 = \{w | w \in a^* b^* c^*\}$

   Pick the **arguments** that are true. An option is correct, both the argument and the conclusion derived in the option must be correct.

   - ☐ The language L is regular, and hence it is also context free.
   - ☐ The language L is not regular because the language $L_n = \{a^n b^n c^n | n \geq 0\}$ is not regular, and this is a subset of L
   - ■ **The language $L_2 = \{w | w \in a^* b^* c^*\}$ is context free**
   - ☐ The language $L \cap L_2$ is L_n, and L_n is not regular, therefore L is not context-free.
   - ■ **The language $L \cap L_2$ is L_n, and L_n is not context free, therefore L is not context-free.**

   The below questions will be based on the following Context-Free Grammar G:

$$S \to aXb$$
$$X \to dSc \mid dc$$

3. (2 points) Consider the grammar G given above. Suppose a string s has a parse tree of height $|V| + 2$ where V is the number of variables in this grammar. Which of the following statements is/are true? Select all the correct statements.

   - ■ **The length of the string is at most $2^{|V|+2}$**
   - ■ **There is a path (from the root to a leaf) in the parse tree that has the variable S occurring twice.**
   - ☐ A path from the root can contain only unique variables.

☐ The length of the string is $|V| + 2$

4. (2 points) Consider the language $L(G)$ defined by the grammar G defined above. According to the pumping lemma, which of the following splits of the string $s = adadcbcb$ can be pumped (i.e., $uv^ixy^iz \in L(G)$, for all $i \geq 0$) so that they remain in the context-free language described by the above grammar. Select all the correct options.

   ☐ $u = ad, v = adc, x = b, y = c, z = b$

   ■ $u = a, v = da, x = dc, y = bc, z = b$

   ☐ $u = a, v = d, x = adcb, y = c, z = b$

   ■ $u = ad, v = ad, x = cb, y = cb, z = \varepsilon$

5. (1 point) Consider a context-free grammar, where the right hand side of every production rule has length at most 3. Which of the following is an upper bound on the maximum size of a string produced by this grammar, given that the parse tree for this string has height h?

   A. $h^2 + h + 1$

   B. $h + 1$

   C. 3^h

   D. $3h$

6. (2 points) Consider the grammar G'

$$S \rightarrow aSb \mid bY \mid Ya$$
$$Y \rightarrow bY \mid aY \mid \epsilon$$

   Pick the correct statements from the following about the language generated from this grammar $G' = L(G')$. Select all the correct options.

   ☐ The generated language can only generate strings with equal number of a's and $b's$

   ☐ The generated language is regular

   ■ **The complement of the generated language is not regular**

   ■ **The complement of the generated language is context free, and must contain an equal number of a's and b's**

   Solution: The generated language is $\bar{L}$, where $L = \{a^kb^k | k \geq 0\}$

7. (2 points) Which of the following is/are **not** a characteristic feature of Turing machines? Select all the options that are not a feature of TMs.

   ■ **Finite memory**

   ■ **Can only process inputs of a fixed length**

   ■ **Infinite number of states**

   ☐ Ability to read and write as well as move left or right on the infinite tape

8. (2 points) Which of the following class of languages can be recognized by a Turing machine but not by a finite automaton? Select all the correct options.

   ☐ Regular languages

   ■ **Context-free languages**

   ■ **Recursively enumerable (Turing-recognizable) languages**

- ■ **Recursive (decidable) languages**

9. (2 points) Which of the following is/are true about a Turing Machine M described by the 7-tuple,

$$M = (Q, \Sigma, \Gamma, \delta, q_0, q_{accept}, q_{reject})$$

Select all the correct options.

- ☐ The input alphabet Σ contains the blank symbol ␣
- ■ **The tape alphabet Γ contains the blank symbol ␣**
- ■ **The input alphabet Σ is a proper subset of the tape alphabet Γ.**
- ■ **The transition function δ takes the form:** $Q \times \Gamma \longrightarrow Q \times \Gamma \times \{L, R\}$

10. (1 point) In a deterministic Turing machine, the state transition is determined by

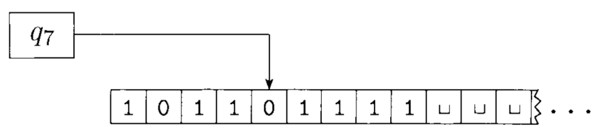
A. Whether the read-write head of the TM moves left or right

B. Current state and the symbol read currently

C. Symbols read until the previous transition

D. Random Selection

11. (1 point) From the diagram below, which of the following strings represent the configuration of the Turing Machine given below?



A. $q_7101101111$

B. $10110q_71111$

C. 101101111

D. $1011q_701111$

12. (1 point) In question 11 above, let $\delta(q_7, 0) = (q_6, 1, R)$. Then which of the following is the next configuration?

A. $q_6101111111$

B. $101111q_6111$

C. 101111111

D. $10111q_61111$