Week 6 Questions - Theory of Computation

February 25, 2024

1. (1 point) The minimum number of tapes needed by a deterministic TM to simulate an NTM is **1**.
2. (1 point) Any Python program can be run on a Turing machine.
   - **A. True**
   - B. False
3. (2 points) An NTM decider for a language runs in linear time for a given input. If a deterministic Turing machine (or DTM) for the same language is constructed using the BFS algorithm from the lecture, then its runtime on the same input would be
   - A. Logarithmic
   - B. Linear
   - C. Polynomial
   - D. **Exponential**
4. (1 point) Let N be an NTM that recognizes language L. If we construct a DTM M that traverses the configuration graph using DFS instead of BFS, then M is recognizer for language L
   - A. **True**
   - B. False
5. (1 point) Define $L = \{\langle P\rangle \mid P$ is a polynomial with integer roots$\}$, where $\langle P\rangle$ is a representation of a polynomial over the variables $x_1, \ldots, x_k$. That is, if $\langle P\rangle \in L$, then there exist an integer combination of $x_1, \ldots, x_k$ such that P evaluates to zero. Which of the following is true?
   - A. L is Turing recognizable, but not decidable
   - B. L is decidable
6. (1 point) Consider L as defined in the previous problem. Consider the following description of a DTM M to recognize L:

   1. Try all possible settings of $x_1, \ldots, x_k$ to integer values.
   2. Evaluate P on all of these settings.
   3. If any of these settings evaluates to 0, accept ; otherwise, reject."

   Which of the following are true?
   - A. The description of M is legitimate.
   - **B. The description of M is not legitimate.**

> **Solution:** There are infinitely many choices possible for k integers. It is not possible for a TM to try out "all possible settings" of $x_1, \ldots, x_k$ in a finite time, and then decide accept/reject. Hence this is not a valid description of a TM.
>
> However, note that the following alternative machine works. The TM could consider an infinite order of each integer combination of $x_1, \ldots, x_k$. The TM can go through all the integer combinations. After considering each combination, the TM has to check if that combination is a root of P, and accept if it is the case. Else, the TM will continue with the next combination. This yields a recognizer for L as there are infinitely many combinations.

7. (2 points) Turing-recognizable languages are closed under which of the following operations? Select all the correct choices.

   - ■ **union**
   - ■ **intersection**
   - ■ **concatenation**
   - ■ **star**

8. (2 points) Define a *One-Way Turing Machine*, or OWTM as a deterministic Turing machine where the head can only stay in place or move right, but cannot go left. Which of the following is/are true?

   - ■ **Any language that can be recognised by an OWTM is regular.**
   - ■ **Any regular language can be recognized by an OWTM.**
   - ☐ Any context-free language can be recognized by an OWTM.
   - ■ **Any language that can be recognised by an OWTM is context-free.**

> **Solution:** A deterministic OWTM is equivalent to a DFA, and hence the class of languages recognized by the OWTM is same as the class of regular languages. Notice that once the OWTM moves right, it cannot go back and read what is written on the left cells. Hence anything that it writes on the tapes is not going to be used again.

9. (1 point) Suppose M_1 is a deterministic single tape Turing machine. Let C be a configuration of M_1, which is neither an accepting nor a rejecting configuration. Out of the below choices, which of the following can be the number of valid successor configurations that can immediately follow C? Select all the possible options.

   - ☐ 0
   - ■ **1**
   - ☐ 3
   - ☐ 10

10. (1 point) Suppose M_2 is a deterministic multi tape Turing machine with 3 tapes. Let C be a configuration of M_2, which is neither an accepting nor a rejecting configuration. Out of the below choices, which of the following can be the number of valid successor configurations that can immediately follow C? Select all the possible options.

    - ☐ 0
    - ■ **1**
    - ☐ 3
    - ☐ 10

11. (2 points) Suppose N is a nondeterministic single tape Turing machine. Let C be a configuration of n, which is neither an accepting nor a rejecting configuration. Out of the below choices, which of the following can be the number of valid successor configurations that can immediately follow C? Select all the possible options.

   - ■ **0**
   - ■ **1**
   - ■ **3**
   - ■ **10**

Paragraph for Q12 - Q13

Recall the definitions of PTAs (PDAs with queues instead of stacks) and multi-stack PDAs from Week 4.

12. (2 points) If a k-stack PDA can emulate any Turing machine, then the minimum value of k is ____**2**____.

Solution: Two stacks can emulate a tape, since we can store everything left of the head in one stack and everything right of the head in the second stack. Now to move the head left, we can push the current head value to the right stack, and pop the new head value from the left stack. Same logic works to move the head right as well.

13. (2 points) If a k-queue PTA can emulate any Turing machine, then the minimum value of k is ____**1**____.

Solution: This can be done similar to the construction used to convert a multi-tape TM to a single tape TM.

We store the entire tape in the queue, with one marked value where the head currently is. To do any operation, we pop all elements out and push them back into the queue, cycling through it entirely. When we find the marked element, we perform the required operation and continue cycling.