DEEP LEARNING WEEK 5

1. Given below are the possible number of linearly independent eigenvectors for a 5x5 matrix. Choose the incorrect option.
a)1
b)4
c)5
d)6

Answer: d)

Solution: Maximum number of the independent eigenvectors(or vectors) is always less than the dimension of the matrix

2. Which of the following is an eigenvalue of matrix A = $\begin{bmatrix} 0.3 & 0.8 \\ 0.7 & 0.2 \end{bmatrix}$.
a) 1
b)3
c)-1
d)2

Answer: 1

Solution: The Following matrix is a stochastic matrix, hence its dominant eigenvalue is 1. One can also compute eigenvalues directly by using the characteristic equation.

3. What is/are the limitations of PCA?
a) It is less interpretable than neural networks
b) It can only identify linear relationships in the data.
c) It can be sensitive to outliers in the data.
d) It can only reduce the dimensionality of a dataset by a fixed amount.

Answer: b), c)

Solution: PCA can be sensitive to outliers in the data since the principal components are calculated based on the covariance matrix of the data. Outliers can have a large impact on the covariance matrix and can skew the results of the PCA. Also, it can only capture linear relationships in the data.

4. What is the determinant of a 2x2 matrix with eigenvalues $\lambda 1$ and $\lambda 2$?
a) $\lambda 1 + \lambda 2$
b) $\lambda 1$ - $\lambda 2$
c) $\lambda 1$ * $\lambda 2$
d) $\lambda 1$ / $\lambda 2$

Answer: c)$\lambda 1$ * $\lambda 2$

Solution: The determinant of a matrix is defined as the product of its eigenvalues. Therefore, if a matrix has eigenvalues $\lambda 1$ and $\lambda 2$, its determinant is given by det(A) = $\lambda 1$ * $\lambda 2$. This implies that option c is correct.

Questions 5-9 are based on common data.

Consider the following data points $x1, x2, x3$ to answer following questions: $x1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, $x2 = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$, $x3 = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$

5. What is the mean of the given data points $x1, x2, x3$?

a) $\begin{bmatrix} 3 \\ 3 \end{bmatrix}$

b) $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$

c) $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$

d) $\begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix}$

Answer: c)

Solution: Mean of $x1, x2, x3 = \dfrac{x1 + x2 + x3}{3} = \dfrac{1}{3}\begin{bmatrix} 3 \\ 3 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

6. The covariance matrix $C = \dfrac{1}{n}\sum_{i=1}^{n}(x - \bar{x})(x - \bar{x})^T$ is given by: ($\bar{x}$ is mean of the data points)

a) $\begin{bmatrix} 0.33 & -0.33 \\ -0.33 & 0.33 \end{bmatrix}$

b) $\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$

c) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

d) $\begin{bmatrix} 0.67 & -0.67 \\ -0.67 & 0.67 \end{bmatrix}$

Answer: d)

textbfSolution: $\dfrac{1}{3}(x_1' x_1'^T + x_2' x_2'^T + x_3' x_3'^T) = \dfrac{1}{3}\begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix}$

7. The maximum eigenvalue of the covariance matrix C is:

a) $\dfrac{1}{3}$

b) $\dfrac{4}{3}$

c) $\dfrac{1}{6}$

d) $\dfrac{1}{2}$

Answer: b)

Solution: If v is the eigenvector of A we get

$A = \dfrac{1}{3}\begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix} Av = \lambda v \implies \dfrac{1}{3}(A - \lambda I)v = 0 \implies \dfrac{1}{3}|A - \lambda I| = 0 \implies \dfrac{1}{3}(\lambda^2 - 4\lambda) = 0 \implies \dfrac{1}{3}\lambda(\lambda - 4) = 0$

Hence, $\lambda = 4/3, 0$

8. The eigenvector corresponding to the maximum eigenvalue of the given matrix C is:

a) $\begin{bmatrix} 0.7 \\ 0.7 \end{bmatrix}$

b) $\begin{bmatrix} -0.7 \\ 0.7 \end{bmatrix}$

c) $\begin{bmatrix} -1 \\ 0 \end{bmatrix}$

d) $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$

Answer: b)

Solution: Using the λ value found above we get the equation $\frac{1}{3}(A - 4I)v = 0$. The unit vector in the null space i.e v is the solution to this equation given by $\begin{bmatrix} -0.7 \\ 0.7 \end{bmatrix}$

9. The data points $x1, x2, x3$ are projected on the eigenvector calculated above. After projection what will be the new coordinate of $x2$? (Hint: $\begin{bmatrix} -0.7 \\ 0.7 \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} -1 \\ 1 \end{bmatrix}$)

a) $\begin{bmatrix} -1 \\ 1 \end{bmatrix}$

b) $\begin{bmatrix} 0 \\ 2 \end{bmatrix}$

c) $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$

d) $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$

Answer: a)

Solution: The required projection is given by $(x^T w)w = (\begin{bmatrix} 0 & 2 \end{bmatrix} \frac{1}{\sqrt{2}} \begin{bmatrix} -1 \\ 1 \end{bmatrix}) \frac{1}{\sqrt{2}} \begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$

10. If the dominant eigenvalue of the matrix A is less than 1 and all the eigenvalues are distinct and non-zero, which of the following is true for the sequence S given by $|Ax|, |A^2x|, |A^3x|, ...?$ ($|v|$ represents the magnitude of vector v)

a) S converges to 1
b) S converges to 0
c) S converges to maximum eigenvalue of A
d) S grows bigger and never converges to any point.

Answer: b)
Solution: Any vector x can be written as linear combination of eigenvectors say if A is 2x2 matrix then $x = a_1 v_1 + a_2 v_2$. When we apply A on x we get $Ax = a_1 \lambda_1 v_1 + a_2 \lambda_2 v_2$. Since $\lambda_1, \lambda_2 < 1$, $(\lambda)^n = 0$. Hence the sequence converges to 0.