Week 3 Questions - Theory of Computation

February 2, 2024

1. (1 point) The language L_1 is given by $L_1 = \{0^n 1^n | n \geq 0\}$. Then $\overline{L_1}$ is a regular language

   A. True

   B. False

2. (1 point) Let L_2 be the language of finite length strings that contain equal number of occurrences of the substrings 01 and 10. Then L_2 is regular.

   A. True

   B. False

3. (1 point) The language L_3 is given by $L_3 = \{a^n b^m c^n | m, n \geq 0\}$. Then L_3 is a regular language

   A. True

   B. False

4. (1 point) Consider the following CFG.

$$S \rightarrow A$$
$$A \rightarrow aAa \mid B$$
$$B \rightarrow bB \mid b$$

   The Context Free Language generated by this CFG is

   A. $\{a^m b^n a^\ell \; | m, n, \ell \geq 0\}$

   B. $\{a^m b^n a^m \; | n > m \geq 0\}$

   C. $\{a^m b^n a^m \; | m, n \geq 0\}$

   D. $\{a^m b^n a^m \; | m \geq 0, n > 0\}$

5. (2 points) The index of the regular language $L_4 = \{01^m | m \geq 0\}$ is ____**3**____

   Solution: The set of strings $\{\varepsilon, 0, 00\}$ is a largest pairwise distinguishable set.

6. (2 points) Consider the following CFG.

$$S \rightarrow X$$
$$X \rightarrow BA$$
$$A \rightarrow aA \mid \epsilon$$
$$B \rightarrow bB \mid b$$

   The Language generated by this CFG is

   A. $\{(ab)^m \; | m \geq 0\}$ and it is regular

B. $\{(ab)^m \ |m \geq 1\}$ and it is not regular

C. $\{b^n a^m \ |n \geq 1, m \geq 0\}$ and it is not regular

D. $\{b^m a^n \ |m \geq 1, n \geq 0\}$ and it is regular

7. (2 points) The language L_5 is given by $L_5 = \{a^m b^n | m, n \geq 0\}$. Which of the following pairs of strings is/are indistinguishable with respect to L_5? Select all the pairs that are indistinguishable.

■ **aa and a**

☐ aaa and aab

■ **ba and baa**

☐ aa and ba

8. (2 points) The language L_6 is given by $L_6 = \{a^i b^j c^k | i, j, k \geq 0$ and if $i = 1$ then $j = k\}$. Then which of the following is/are correct? Select all the correct options.

☐ L_6 is regular

■ **L_6 satisfies pumping lemma**

■ **L_6 is not regular**

☐ L_6 does not satisfy pumping lemma

Solution: The laguage satisfies the pumping lemma because you can always pump the first character. Let the string be $w = xyz$ then let $x = \epsilon$ and y be the first character. This will satisfy the pumping lemma. There are two cases,

1. $y = a$: In this case pumping a will make $i > 1$
2. $y = b$: In this case we have i=0

9. (2 points) Out of the given options, which of the languages is/are regular? (The alphabet $\Sigma = \{0, 1\}$) Select all the regular languages.

■ **{All finite length strings that contain exactly four 0's}**

■ **{All finite length strings that contain at most four 0's}**

■ **{All finite length strings that contain at least four 0's}**

☐ {All finite length strings that contain equal number of 0's and 1's}

10. (2 points) Let L_7 be the language generated by the grammar

$$S \to aSa|bSb|\epsilon$$

Which of the following statements is true?

A. L_7 is regular and $\overline{L_7}$ is not context free

B. L_7 is regular and $\overline{L_7}$ is context free

C. L_7 is context free and $\overline{L_7}$ is not regular

D. L_7 is context free and $\overline{L_7}$ is regular

11. (2 points) Which of the following languages is/are regular? Select all that are regular.

☐ $\{1^p | \ p$ is a prime $\}$

☐ $\{1^{p^2} | \ p$ is a prime $\}$

☐ $\{1^{n^2} | \ n$ is a natural number $\}$

■ $\{1^n |$ n **is a natural number** $\}$

12. (2 points) Notice that DFA and NFA are memoryless i.e., they do not store the symbols that have been encountered. Consider the below set of languages over $\Sigma = \{0, 1\}$. The number of regular languages from the following set is ____**0**____

$$
\begin{aligned}
A = \{&\{ww|\ w \in \Sigma^*\},\\
&\{www|\ w \in \Sigma^*\},\\
&\{w0w|\ w \in \Sigma^*\},\\
&\{ww^R|\ w \in \Sigma^*\},\\
&\{0^n1^m0^n|\ m, n \geq 0\},\\
&\{0^n1^n|\ n \geq 0\}\}
\end{aligned}
$$