

Week 10 - Theory of Computation

March 2024

1. (1 point) $P \subseteq NPC \subseteq NP$, where NPC is the set of NP-Complete languages. True or False?

A. True

B. False

Solution: P is not known to be contained in NP complete.

2. (2 points) Given languages A, B and C , which of the following statements is/are true?

■ $A \leq_P B$ and $B \leq_P C \implies A \leq_P C$

■ $A \leq_P B \implies \overline{A} \leq_P \overline{B}$

■ $A \leq_P B$ and $B \in NP \implies A \in NP$

■ $A \leq_P B$ and $A \notin P \implies B \notin P$

3. (2 points) Given two languages A and B , which of the following statements is/are true?

If $B \subseteq A$ and $A \in NP$ then $B \in NP$

If $B \subseteq A$ and $B \in NP$ then $A \in NP$

■ If $B \in NP$ and $A \in NP$ then $A \cup B \in NP$

■ If $B \in NP$ and $A \in NP$ then $A \cap B \in NP$

4. (2 points) Let $k - SAT$, for $k \in \mathbb{N}, k \geq 2$ be the subset of $CNF - SAT$, where each of the disjunctive clauses contain exactly k literals.

For example: $(x_1 \vee \overline{x_2} \vee x_3 \vee \overline{x_4}) \wedge (x_2 \vee x_3 \vee x_4 \vee x_5)$ is in $4 - SAT$ since it has a satisfiable assignment. Also in all of the disjunctive clauses, exactly 4 literals are present.

Assuming $P \neq NP$, which of the following options is/are correct? Select all that are correct.

■ $4 - SAT \leq_P 5 - SAT$

■ $6 - SAT \leq_P 3 - SAT$

$7 - SAT \leq_P 2 - SAT$

■ $2 - SAT \leq_P 8 - SAT$

Solution: k -SAT, for $k \geq 3$ is NP complete and reducible to each other. But 2-SAT is in P and hence we cannot reduce k -SAT to 2-SAT when $k \geq 3$.

5. (2 points) Let $B \in NP$ and we show that $B \in P$ then $P = NP$

A. False

B. True

6. (2 points) Which of the following is/are true?

- Checking if a graph is 2-colorable $\in P$
- Checking if a graph is 3-colorable is NP-Complete
- Checking if a graph is 3-colorable $\in NP$
- Checking if a graph is 3-colorable is reducible to SAT

7. (2 points) Suppose the following statements regarding the languages L_1, L_2, L_3 are true.

- $3 - SAT \leq_P L_1$
- $L_2 \in P$
- L_3 is NP-Complete

Which of the following options is/are true? Select all that are correct.

- $L_1 \leq_P L_3$
- L_1 is NP-Complete
- $L_3 \leq_P L_1$
- $L_2 \leq_P L_1$

Solution: L_1 need not be in NP. So, it need not be NP-Complete, just NP-HARD

8. (2 points) The boolean formula $(a \vee b \vee \bar{c}) \wedge (b \vee c \vee d) \wedge (\bar{a} \vee \bar{b} \vee d)$ over the Boolean variables $\{a, b, c, d\}$ has 10 satisfying assignments. (answer in numerical form)

9. (2 points) Which of the following language(s) is/are in P ? (Assuming $P \neq NP$) Select all the correct options.

- 2-SAT
- 3-SAT
- $A_{CFG} = \{(G, w) \mid G \text{ is a CFG and accepts the string } w\}$
- A_{TM}

10. (2 points) Which of the following statements, if true, can help us answer whether or not $P = NP$?

- 3-SAT is NP-complete
- 4-SAT is not in P
- CNF-SAT is in P
- 2-SAT is NP-complete

Solution: The second option implies that P is not equal to NP, since 4-SAT is NP complete. The third option implies that P equal to NP since CNF-SAT is NP complete.

The last option is true only when P equal to NP since 2-SAT is known to be in P.