Unit P3: Pure Mathematics 3

Compulsory unit for IAL Mathematics and Pure Mathematics

Externally assessed

P3.1 Unit description

Algebra and functions; trigonometry; exponentials and logarithms; differentiation; integration; numerical methods.

P3.2 Assessment information

1. Prerequisites

A knowledge of the specifications for P1 and P2, their prerequisites and associated formulae, is assumed and may be tested.

2. Examination

- First assessment: January 2020.
- The assessment is 1 hour and 30 minutes.
- The assessment is out of 75 marks.
- Students must answer all questions.
- Calculators may be used in the examination. Please see *Appendix 6: Use of calculators*.
- The booklet *Mathematical Formulae and Statistical Tables* will be provided for use in the assessments.

3. Notation and formulae

Students will be expected to understand the symbols outlined in *Appendix 7: Notation*.

Formulae that students are expected to know are given below and will **not** appear in the booklet *Mathematical Formulae and Statistical Tables*, which will be provided for use with the paper. Questions will be set in SI units and other units in common usage.

Trigonometry

$\cos^2 A + \sin^2 A \equiv 1$

$\sec^2 A \equiv 1 + \tan^2 A$

$\operatorname{cosec}^2 A \equiv 1 + \cot^2 A$

$\sin 2A \equiv 2 \sin A \cos A$

$\cos 2A \equiv \cos^2 A - \sin^2 A$

$$\tan 2A \equiv \frac{2 \tan A}{1 - \tan^2 A}$$

3. Notation and formulae *continued*

Differentiation

$\mathbf{f}(x)$	$\mathbf{f}'(x)$
$\sin kx$	$k\cos kx$
$\cos kx$	$-k\sin kx$
e^{kx}	$k\mathrm{e}^{kx}$
$\ln x$	$\frac{1}{x}$
$\mathrm{f}(x) + \mathrm{g}(x)$	$\mathrm{f}'(x) + \mathrm{g}'(x)$
$\mathrm{f}(x)\,\mathrm{g}(x)$	$\mathrm{f}'(x)\,\mathrm{g}(x) + \mathrm{f}(x)\,\mathrm{g}'(x)$
$\mathrm{f}(\mathrm{g}(x))$	$\mathrm{f}'(\mathrm{g}(x))\,\mathrm{g}'(x)$
a^x	$a^x \ln a$

Integration

$\mathbf{f}(x)$	$\int \mathbf{f}(x)\,\mathbf{d}x$
$\cos kx$	$\frac{1}{k}\sin kx + c$
$\sin kx$	$-\frac{1}{k}\cos kx + c$
e^{kx}	$\frac{1}{k}\mathrm{e}^{kx} + c$
$\frac{1}{x}$	$\ln\lvert x\rvert + c,\ x \neq 0$
$\mathrm{f}'(x) + \mathrm{g}'(x)$	$\mathrm{f}(x) + \mathrm{g}(x) + c$
$\mathrm{f}'(\mathrm{g}(x))\,\mathrm{g}'(x)$	$\mathrm{f}(\mathrm{g}(x)) + c$
a^x	$\frac{a^x}{\ln a} + c$

P3.3 Unit content

What students need to learn:		Guidance
1. Algebra and functions		
1.1	Simplification of rational expressions including factorising and cancelling, and algebraic division.	Denominators of rational expressions will be linear or quadratic, e.g. $\frac{1}{ax+b}$, $\frac{ax+b}{px^2+qx+r}$, $\frac{x^3+1}{x^2-1}$.
1.2	Definition of a function. Domain and range of functions. Composition of functions. Inverse functions and their graphs.	The concept of a function as a one-one or many-one mapping from $\mathbb{R}$ (or a subset of $\mathbb{R}$) to $\mathbb{R}$. The notation $f: x \mapsto$ and $f(x)$ will be used. Students should know that fg will mean 'do g first, then f'. Students should know that if f^{-1} exists, then $f^{-1}f(x) = ff^{-1}(x) = x$.
1.3	The modulus function.	Students should be able to sketch the graphs of $y = \lvert ax + b \rvert$ and the graphs of $y = \lvert f(x) \rvert$ and $y = f(\lvert x \rvert)$, given the graph of $y = f(x)$. For example, sketch the graph with equation $y = \lvert 2x - 1 \rvert$ and use the graph to solve the equation $\lvert 2x - 1 \rvert = x + 5$ or the inequality $\lvert 2x - 1 \rvert > x + 5$.
1.4	Combinations of the transformations $y = f(x)$ as represented by $y = af(x)$, $y = f(x) + a$, $y = f(x + a)$, $y = f(ax)$.	Students should be able to sketch the graph of, for example, $y = 2f(3x)$, $y = f(-x) + 1$, given the graph of $y = f(x)$ or the graph of, for example, $y = 3 + \sin 2x$, $y = -\cos\left(x + \frac{\pi}{4}\right)$. The graph of $y = f(ax + b)$ will *not* be required.
2. Trigonometry		
2.1	Knowledge of secant, cosecant and cotangent and of arcsin, arccos and arctan. Their relationships to sine, cosine and tangent. Understanding of their graphs and appropriate restricted domains.	Angles measured in both degrees and radians.
2.2	Knowledge and use of $\sec^2\theta = 1 + \tan^2\theta$ and $\operatorname{cosec}^2\theta = 1 + \cot^2\theta$.	

What students need to learn:		Guidance
2. Trigonometry *continued*		
2.3	Knowledge and use of double angle formulae; use of formulae for $\sin(A \pm B)$, $\cos(A \pm B)$ and $\tan(A \pm B)$ and of expressions for $a\cos\theta + b\sin\theta$ in the equivalent forms of $r\cos(\theta \pm a)$ or $r\sin(\theta \pm a)$.	To include application to half angles. Knowledge of the $t\left(\tan\frac{1}{2}\theta\right)$ formulae will *not* be required. Students should be able to solve equations such as $a\cos\theta + b\sin\theta = c$ in a given interval, and to prove identities such as $\cos x\cos 2x + \sin x\sin 2x \equiv \cos x$.
3. Exponential and logarithms		
3.1	The function e^x and its graph.	To include the graph of $y = e^{ax+b} + c$
3.2	The function $\ln x$ and its graph; $\ln x$ as the inverse function of e^x.	Solution of equations of the form $e^{ax+b} = p$ and $\ln(ax + b) = q$ is expected.
3.3	Use logarithmic graphs to estimate parameters in relationships of the form $y = ax^n$ and $y = kb^x$.	Plot $\log y$ against $\log x$ and obtain a straight line where the intercept is $\log a$ and the gradient is n. Plot $\log y$ against x and obtain a straight line where the intercept is $\log k$ and the gradient is $\log b$.
4. Differentiation		
4.1	Differentiation of e^{kx}, $\ln kx$, $\sin kx$, $\cos kx$, $\tan kx$ and their sums and differences.	
4.2	Differentiation using the product rule, the quotient rule and the chain rule.	Differentiation of $\operatorname{cosec} x$, $\cot x$ and $\sec x$ are required. Skill will be expected in the differentiation of functions generated from standard functions using products, quotients and composition, such as $2x^4\sin x$, $\frac{e^{3x}}{x}$, $\cos x^2$ and $\tan^2 2x$.
4.3	The use of $\frac{dy}{dx} = \frac{1}{\left(\frac{dx}{dy}\right)}$	For example, finding $\frac{dy}{dx}$ for $x = \sin 3y$
4.4	Understand and use exponential growth and decay.	Students should be familiar with terms such as 'initial', 'meaning when' $t = 0$. Students may need to explore the behaviour for large values of t or to consider whether the range of values predicted is appropriate. Consideration of a second improved model may be required. Knowledge and use of the result $\frac{d}{dx}(a^x) = a^x\ln a$ is expected.

What students need to learn:		Guidance
5. Integration		
5.1	Integration of e^{kx}, $\frac{1}{x^n}$, $\sin kx$, $\cos kx$ and their sums and differences.	To include integration of standard functions such as $\sin 3x$, e^{5x}, $\frac{1}{2x}$
5.2	Integration by recognition of known derivatives to include integrals of the form $\int \frac{f'(x)}{f(x)} dx = \ln(f(x)) + c$ and $\int f'(x)[f(x)]^n dx = \frac{[f(x)]^{n+1}}{n+1} + c$	For example, to include integration of $\tan x$, $\sec^2 2x$. Students are expected to be able to use trigonometric identities to integrate, for example, $\sin^2 x$, $\tan^2 x$, $\cos^2 3x$.
6. Numerical methods		
6.1	Location of roots of $f(x) = 0$ by considering changes of sign of $f(x)$ in an interval of x in which $f(x)$ is continuous.	
6.2	Approximate solution of equations using simple iterative methods, including recurrence relations of the form $x_{n+1} = f(x_n)$	Solution of equations by use of iterative procedures, for which leads will be given.

Unit P4: Pure Mathematics 4

Compulsory unit for IAL Mathematics and Pure Mathematics

Externally assessed

P4.1 Unit description

Proof; algebra and functions; coordinate geometry in the (x, y) plane; binomial expansion; differentiation; integration; vectors.

P4.2 Assessment information

1. Prerequisites

A knowledge of the specifications for P1, P2 and P3, their prerequisites and associated formulae, is assumed and may be tested.

2. Examination

- First assessment: June 2020.
- The assessment is 1 hour and 30 minutes.
- The assessment is out of 75 marks.
- Students must answer all questions.
- Calculators may be used in the examination. Please see *Appendix 6: Use of calculators*.
- The booklet *Mathematical Formulae and Statistical Tables* will be provided for use in the assessments.

3. Notation and formulae

Students will be expected to understand the symbols outlined in *Appendix 7: Notation*.

Formulae that students are expected to know are given below and will **not** appear in the booklet *Mathematical Formulae and Statistical Tables*, which will be provided for use with the paper. Questions will be set in SI units and other units in common usage.

This is a list of formulae that students are expected to remember and which will not be included in formulae booklets.

Vectors

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} \bullet \begin{pmatrix} a \\ b \\ c \end{pmatrix} = xa + yb + zc$$

P4.3 Unit content

What students need to learn:		Guidance
1. Proof		
1.1	Proof by contradiction	Including proof of the irrationality of $\sqrt{2}$ and the infinity of primes, and application to unfamiliar proofs.
2. Algebra and functions		
2.1	Decompose rational functions into partial fractions (denominators not more complicated than repeated linear terms).	Partial fractions to include denominators such as $(ax + b)(cx + d)(ex + f)$ and $(ax + b)(cx + d)^2$. The degree of the numerator may equal or exceed the degree of the denominator. Applications to integration, differentiation and series expansions. Quadratic factors in the denominator such as $(x^2 + a)$, $a > 0$, are *not* required.
3. Coordinate geometry in the (x, y) plane		
3.1	Parametric equations of curves and conversion between cartesian and parametric forms.	
4. Binomial expansion		
4.1	Binomial Series for any rational n.	For $\lvert x \rvert < \frac{b}{a}$, students should be able to obtain the expansion of $(ax + b)^n$, and the expansion of rational functions by decomposition into partial fractions.
5. Differentiation		
5.1	Differentiation of simple functions defined implicitly or parametrically.	The finding of equations of tangents and normals to curves given parametrically or implicitly is required.
5.2	Formation of simple differential equations.	Questions involving connected rates of change may be set.

What students need to learn:		Guidance
6. Integration		
6.1	Evaluation of volume of revolution.	$\pi\int y^2\,\mathrm{d}x$ is required, but *not* $\pi\int x^2\,\mathrm{d}y$. Students should be able to find a volume of revolution, given parametric equations.
6.2	Simple cases of integration by substitution and integration by parts. Understand these methods as the reverse processes of the chain and product rules respectively.	Students will be expected to use a substitution to find, e.g. $\int x\sqrt{x-2}\,\mathrm{d}x$ The substitution will be given in more complicated integrals. The integral $\int \ln x\,\mathrm{d}x$ is required. More than one application of integration by parts may be required, for example, $\int x^2\mathrm{e}^x\,\mathrm{d}x$, $\int \mathrm{e}^x \sin x\,\mathrm{d}x$.
6.3	Simple cases of integration using partial fractions.	Integration of rational expressions such as those arising from partial fractions, e.g. $\frac{2}{3x+5}$, $\frac{3}{(x-1)^2}$. Note that the integration of other rational expressions, such as $\frac{x}{x^2+5}$ and $\frac{2}{(2x-1)^4}$ is also required (*see P3 section 5.2*).
6.4	Analytical solution of simple first order differential equations with separable variables.	General and particular solutions will be required.
6.5	Use integration to find the area under a curve given its parametric equations.	Students should be able to find the area under a curve given its parametric equations. Students will not be expected to sketch a curve from its parametric equations.
7. Vectors		
7.1	Vectors in two and three dimensions.	
7.2	Magnitude of a vector.	Students should be able to find a unit vector in the direction of $\mathbf{a}$, and be familiar with $\lvert\mathbf{a}\rvert$.
7.3	Algebraic operations of vector addition and multiplication by scalars, and their geometrical interpretations.	
7.4	Position vectors.	$\overrightarrow{OB} - \overrightarrow{OA} = \overrightarrow{AB} = \mathbf{b} - \mathbf{a}$

What students need to learn:		Guidance
7. Vectors *continued*		
7.5	The distance between two points.	The distance d between two points (x_1, y_1, z_1) and (x_2, y_2, z_2) is given by $d^2 = (x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2$
7.6	Vector equations of lines.	To include the forms $\mathbf{r} = \mathbf{a} + t\mathbf{b}$ and $\mathbf{r} = \mathbf{c} + t(\mathbf{d} - \mathbf{c})$ Conditions for two lines to be parallel, intersecting or skew.
7.7	The scalar product. Its use for calculating the angle between two lines.	Students should know that for $\overrightarrow{OA} = \mathbf{a} = a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}$ and $\overrightarrow{OB} = \mathbf{b} = b_1\mathbf{i} + b_2\mathbf{j} + b_3\mathbf{k}$ then $\mathbf{a.b} = a_1b_1 + a_2b_2 + a_3b_3$ and $\cos \angle AOB = \dfrac{\mathbf{a.b}}{\lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert}$ Students should know that if $\mathbf{a.b} = 0$, and $\mathbf{a}$ and $\mathbf{b}$ are non-zero vectors, then $\mathbf{a}$ and $\mathbf{b}$ are perpendicular.

Unit S1: Statistics 1

Optional unit for IAS Mathematics and Further Mathematics
Optional unit for IAL Mathematics and Further Mathematics

Externally assessed

S1.1 Unit description

Mathematical models in probability and statistics; representation and summary of data; probability; correlation and regression; discrete random variables; discrete distributions; the Normal distribution.

S1.2 Assessment information

1. Examination

- First assessment: June 2019.
- The assessment is 1 hour and 30 minutes.
- The assessment is out of 75 marks.
- Students must answer all questions.
- Calculators may be used in the examination. Please see *Appendix 6: Use of calculators*.
- The booklet *Mathematical Formulae and Statistical Tables* will be provided for use in the assessments.

2. Notation and formulae

Students will be expected to understand the symbols outlined in *Appendix 7: Notation*.

Formulae that students are expected to know are given below and will **not** appear in the booklet *Mathematical Formulae and Statistical Tables*, which will be provided for use with the paper. Questions will be set in SI units and other units in common usage.

Mean = $\bar{x} = \frac{\sum x}{n}$ or $\frac{\sum fx}{\sum f}$

Standard deviation = $\sqrt{(\text{variance})}$

Interquartile range = IQR = $Q_3 - Q_1$

$P(A') = 1 - P(A)$

For independent events A and B,

$P(B \mid A) = P(B)$, $P(A \mid B) = P(A)$,

$P(A \cap B) = P(A)\,P(B)$

$E(aX + b) = aE(X) + b$

$Var(aX + b) = a^2\,Var(X)$

2. Notation and formulae *continued*

Cumulative distribution function for a discrete random variable:

$$F(x_0) = P(X \leqslant x_0) = \sum_{x \leqslant x_0} p(x)$$

Standardised Normal Random Variable $Z = \dfrac{X - \mu}{\sigma}$

where $X \sim N(\mu, \sigma^2)$

S1.3 Unit content

What students need to learn:		Guidance
1. Mathematical models in probability and statistics		
1.1	The basic ideas of mathematical modelling as applied in probability and statistics.	
2. Representation and summary of data		
2.1	Histograms, stem and leaf diagrams, box plots.	Using histograms, stem and leaf diagrams and box plots to compare distributions. Back-to-back stem and leaf diagrams may be required. Drawing of histograms, stem and leaf diagrams or box plots will not be the direct focus of examination questions.
2.2	Measures of location – mean, median, mode.	Calculation of mean, mode and median, range and interquartile range will not be the direct focus of examination questions. Students will be expected to draw simple inferences and give interpretations to measures of location and dispersion. Significance tests will not be expected. Data may be discrete, continuous, grouped or ungrouped. Understanding and use of coding.
2.3	Measures of dispersion – variance, standard deviation, range and interpercentile ranges.	Simple interpolation may be required. Interpretation of measures of location and dispersion.
2.4	Skewness. Concepts of outliers.	Students may be asked to illustrate the location of outliers on a box plot. Any rule to identify outliers will be specified in the question.
3. Probability		
3.1	Elementary probability.	
3.2	Sample space. Exclusive and complementary events. Conditional probability.	Understanding and use of $P(A') = 1 - P(A)$, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$, $P(A \cap B) = P(A)\,P(B \mid A)$.
3.3	Independence of two events.	$P(B \mid A) = P(B)$, $P(A \mid B) = P(A)$, $P(A \cap B) = P(A)\,P(B)$.
3.4	Sum and product laws.	Use of tree diagrams and Venn diagrams. Sampling with and without replacement.

What students need to learn:		Guidance
4. Correlation and regression		
4.1	Scatter diagrams. Linear regression.	Calculation of the equation of a linear regression line using the method of least squares. Students may be required to draw this regression line on a scatter diagram.
4.2	Explanatory (independent) and response (dependent) variables. Applications and interpretations.	Use to make predictions within the range of values of the explanatory variable and the dangers of extrapolation. Derivations will not be required. Variables other than x and y may be used. Linear change of variable may be required.
4.3	The product moment correlation coefficient, its use, interpretation and limitations.	Derivations and tests of significance will not be required.
5. Discrete random variables		
5.1	The concept of a discrete random variable.	
5.2	The probability function and the cumulative distribution function for a discrete random variable.	Simple uses of the probability function $\mathrm{p}(x)$ where $\mathrm{p}(x) = \mathrm{P}(X = x)$. Use of the cumulative distribution function: $\mathrm{F}(x_0) = \mathrm{P}(X \leqslant x_0) = \sum_{x \leqslant x_0} \mathrm{p}(x)$.
5.3	Mean and variance of a discrete random variable.	Use of $\mathrm{E}(X)$, $\mathrm{E}(X^2)$ for calculating the variance of X. Knowledge and use of $\mathrm{E}(aX + b) = a\mathrm{E}(X) + b$, $\mathrm{Var}(aX + b) = a^2\,\mathrm{Var}(X)$.
5.4	The discrete uniform distribution.	The mean and variance of this distribution.
6. The Normal distribution		
5.1	The Normal distribution including the mean, variance and use of tables of the cumulative distribution function.	Knowledge of the shape and the symmetry of the distribution is required. Knowledge of the probability density function is not required. Derivation of the mean, variance and cumulative distribution function is not required. Interpolation is not necessary. Questions may involve the solution of simultaneous equations.

Appendix 1: Codes

Type of code	Use of code	Code
Unit codes	Each unit is assigned a unit code. This unit code is used as an entry code to indicate that a student wishes to take the assessment for that unit. Centres will need to use the entry codes only when entering students for their examination.	Unit P1: WMA11/01 Unit P2: WMA12/01 Unit P3: WMA13/01 Unit P4: WMA14/01 Unit FP1: WFM01/01 Unit FP2: WFM02/01 Unit FP3: WFM03/01 Unit M1: WME01/01 Unit M2: WME02/01 Unit M3: WME03/01 Unit S1: WST01/01 Unit S2: WST02/01 Unit S3: WST03/01 Unit D1: WDM11/01
Cash in codes	The cash-in code is used as an entry code to aggregate the student's unit scores to obtain the overall grade for the qualification. Centres will need to use the entry codes only when entering students for their qualification.	**International Advanced Subsidiary:** Mathematics – XMA01 Further Mathematics – XFM01 Pure Mathematics – XPM01 **International Advanced Level:** Mathematics – YMA01 Further Mathematics – YFM01 Pure Mathematics – YPM01
Entry codes	The entry codes are used to: • enter a student for the assessment of a unit • aggregate the student's unit scores to obtain the overall grade for the qualification.	Please refer to the *Pearson Information Manual*, available on our website.