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ERROR ESTIMATES FOR NONCONFORMING AND DISCONTINUOUS DISCRETIZATIONS OF NONSMOOTH PROBLEMS VIA CONVEX DUALITY

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ABSTRACT

Various modern applications including image processing and fracture modeling require the use of nondifferentiable functionals. Their numerical solution by standard finite element methods leads to suboptimal convergence rates. The talk discusses the use of nonconforming and discontinuous finite element methods and provides quasi optimal error estimates based on [1, 2]. These are obtained by using appropriate discrete convex duality relations and identifying suitable regularity conditions. The techniques apply to a large class of convex minimization problems and lead to a postprocessing formula that provides the solution of the discrete dual problem via the nonconforming solution of the discrete primal problem. Following [3, 4], the discrete approximations give rise to the definition of a primal-dual gap error estimator which provides an exact representation of a meaningful error quanitity. Using the particular structure of the discrete dual variable we identify a monotonicity formula that allows us to establish refined efficiency properties of the estimator for a class of nonlinear Dirichlet problems.

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