

TIME DEPENDENCE AND WIDTH CONSTRICTION IN ROCK FRACTURE – EXPERIMENTAL EVIDENCE AND MODELING RESPONSES

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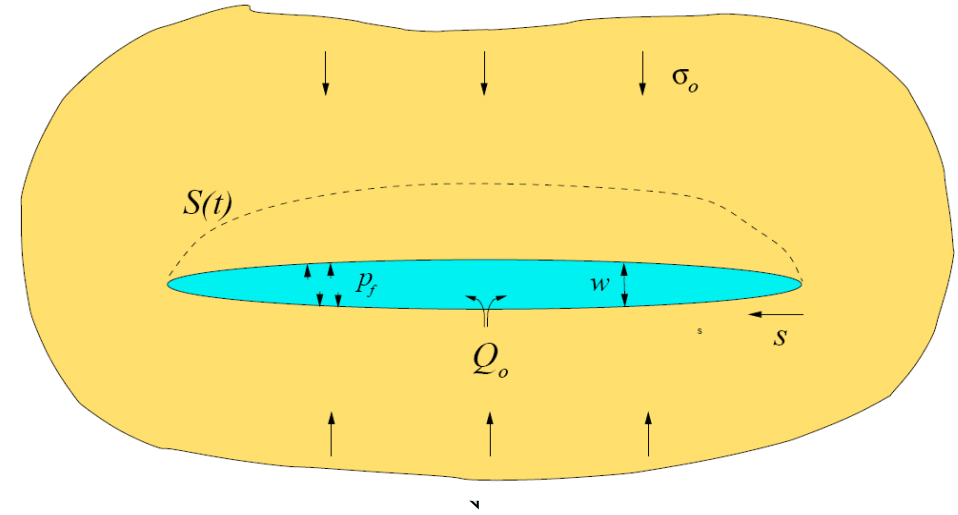
Modelling fluid-driven fracturing

Find

w = Crack opening

p_f = Fluid pressure p_f

$S(t)$ = Crack front location



Elasticity Equation:

$$p_f - \sigma_o = -\frac{E'}{8\pi} \int_{C(t)} \frac{w(x', y', t) dC(x', y')}{[(x' - x)^2 + (y' - y)^2]^{3/2}}$$

Reynold's Lubrication Equation: $\frac{\partial w}{\partial t} = \frac{1}{\mu'} \nabla \cdot (w^3 \nabla p_f) + Q_o \delta(x, y)$

Propagation Condition:

$$\lim_{s \rightarrow 0} \frac{w}{s^{1/2}} = \frac{K'}{E'}$$

Boundary Conditions:

$$\lim_{s \rightarrow 0} w^3 \frac{\partial p_f}{\partial s} = 0$$

Coupled, non-linear, non-local system of PDEs with a moving boundary

μ' = 12 × Dynamic fluid viscosity

E' = (Plane strain) Elastic modulus of the rock

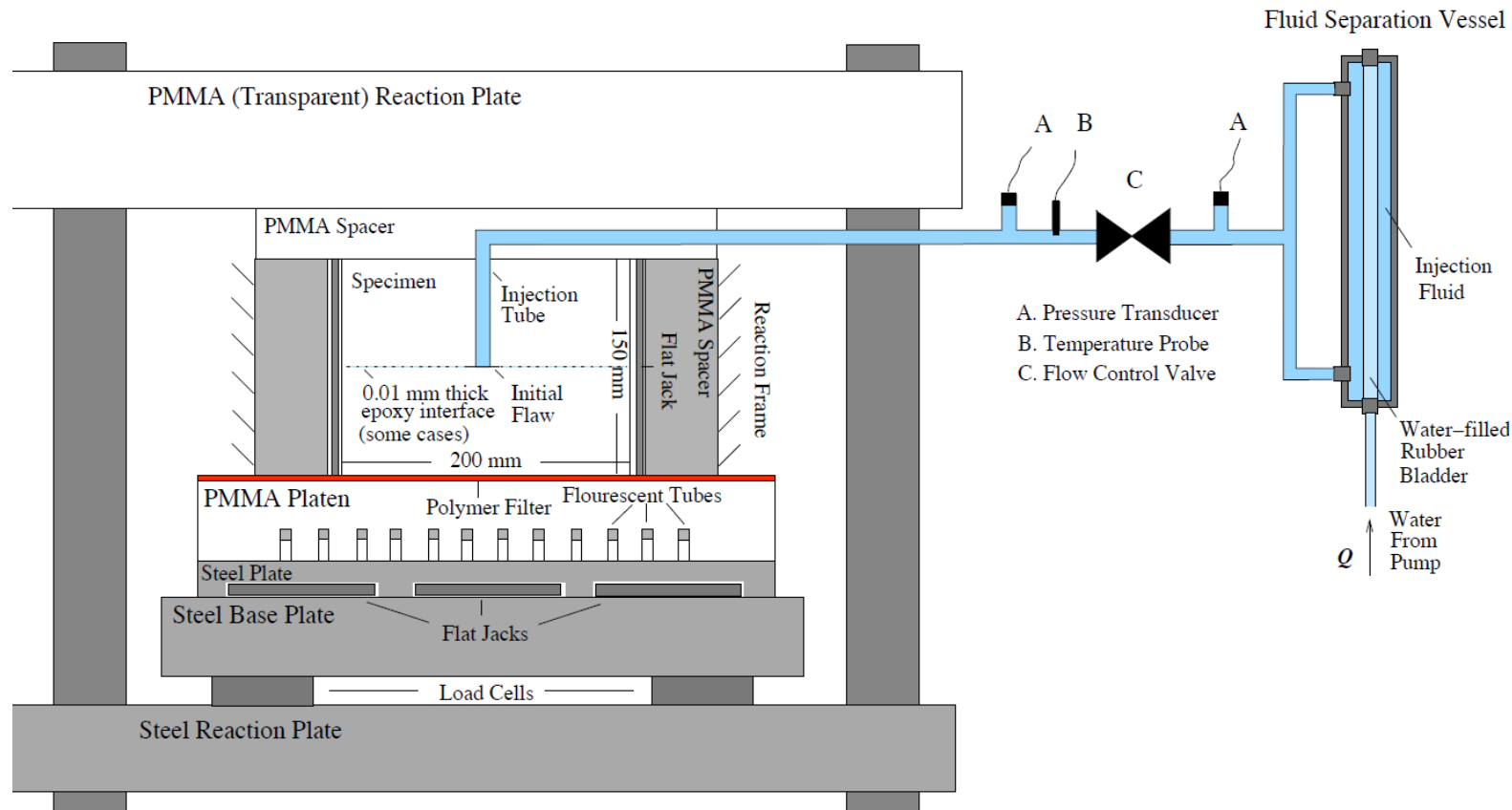
$K' = \left(\frac{32}{\pi}\right)^{1/2} \times$ Fracture toughness of the rock

Q_o = Fluid injection rate

LEFM and non-LEFM Tip Behavior in Experiments



Video
Camera



σ_o = "Overburden" stress

v_{tip} = Tip velocity

μ' = Dynamic fluid viscosity * 12

E' = Plane strain modulus

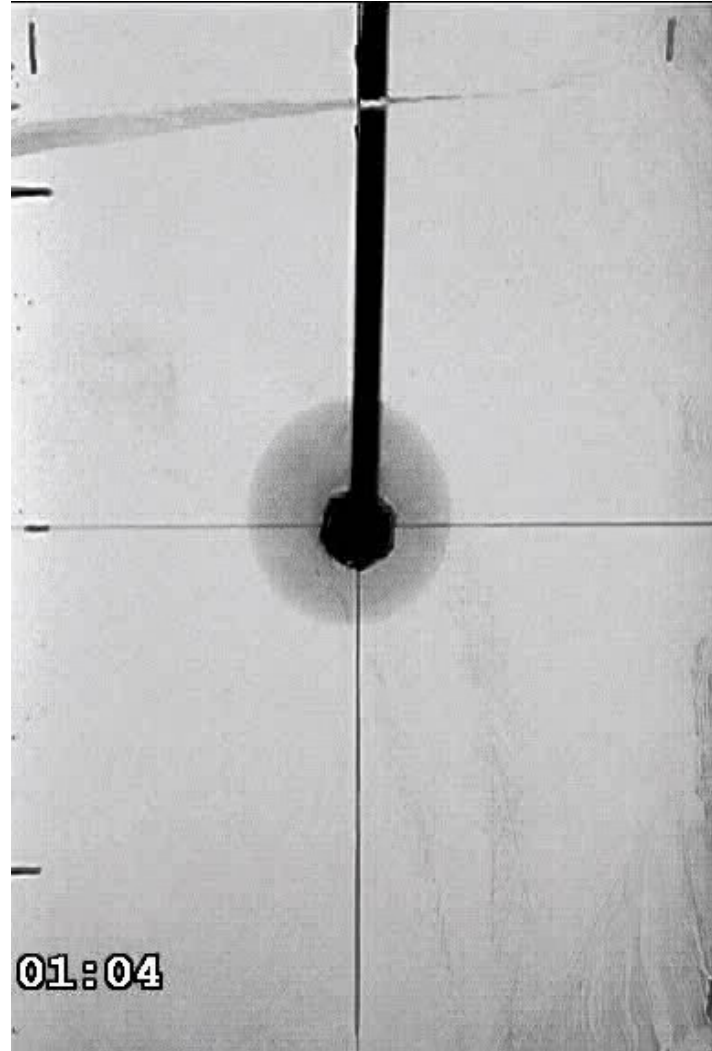
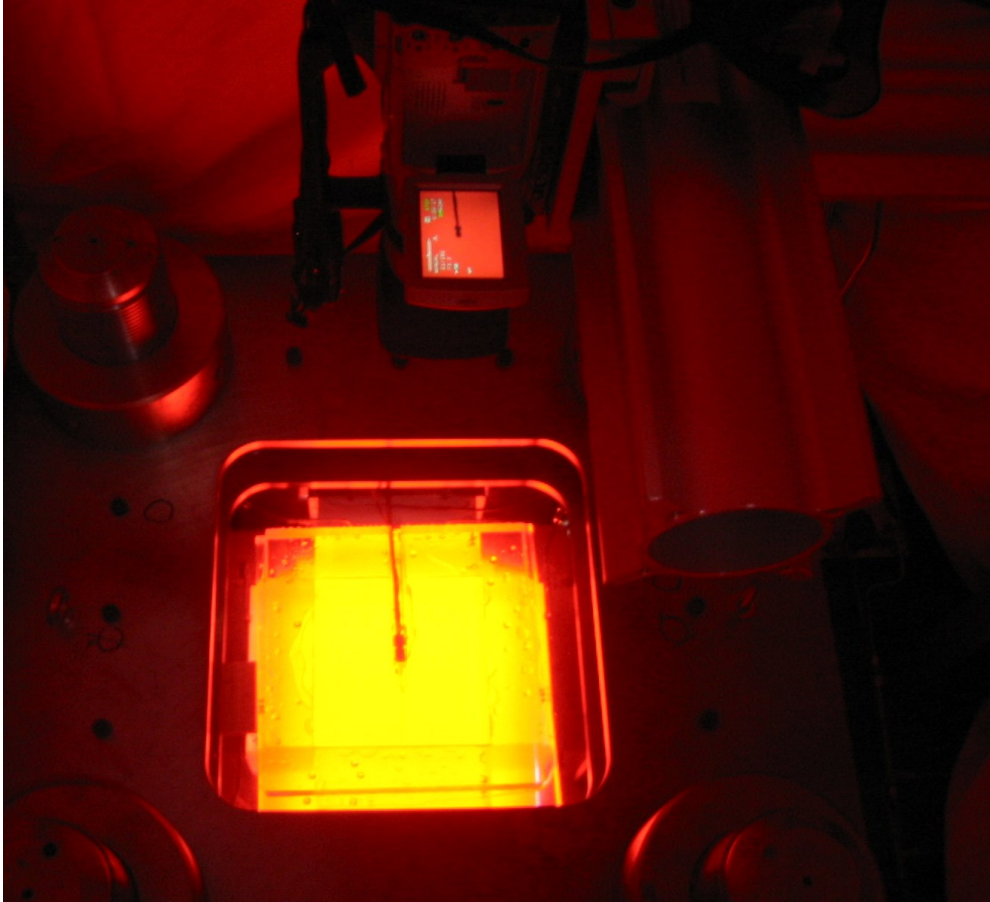
K' = Fracture Toughness * $(32/\pi)^{1/2}$

Particular Case: Zero Lag

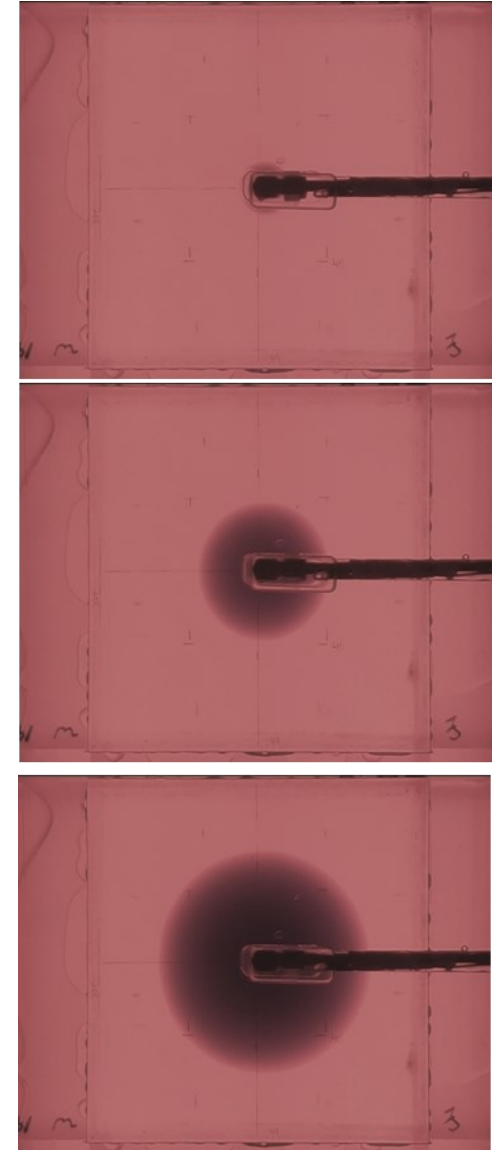
$$\kappa = \left(\frac{\sigma_o K'^2}{\mu' v_{tip} E'^2} \right)^{1/2} \geq 1$$

Experiment in Progress

Toughness-dominated
Example

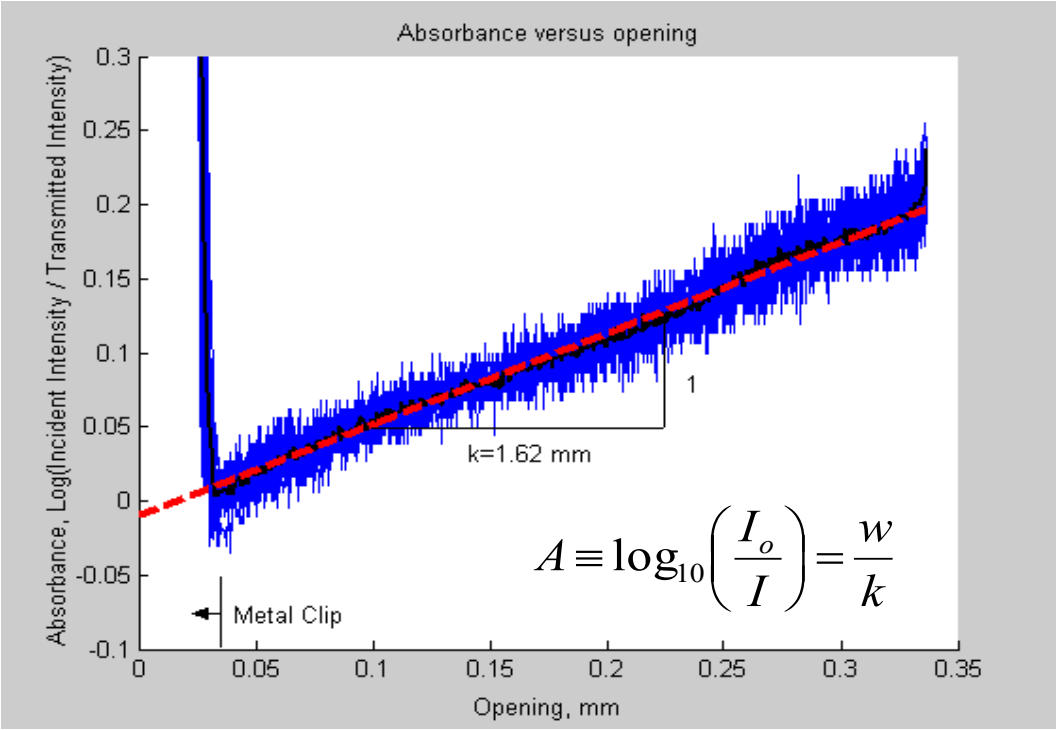
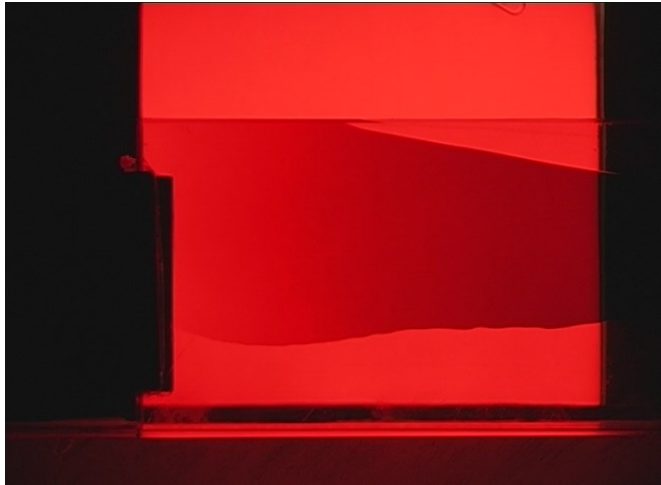
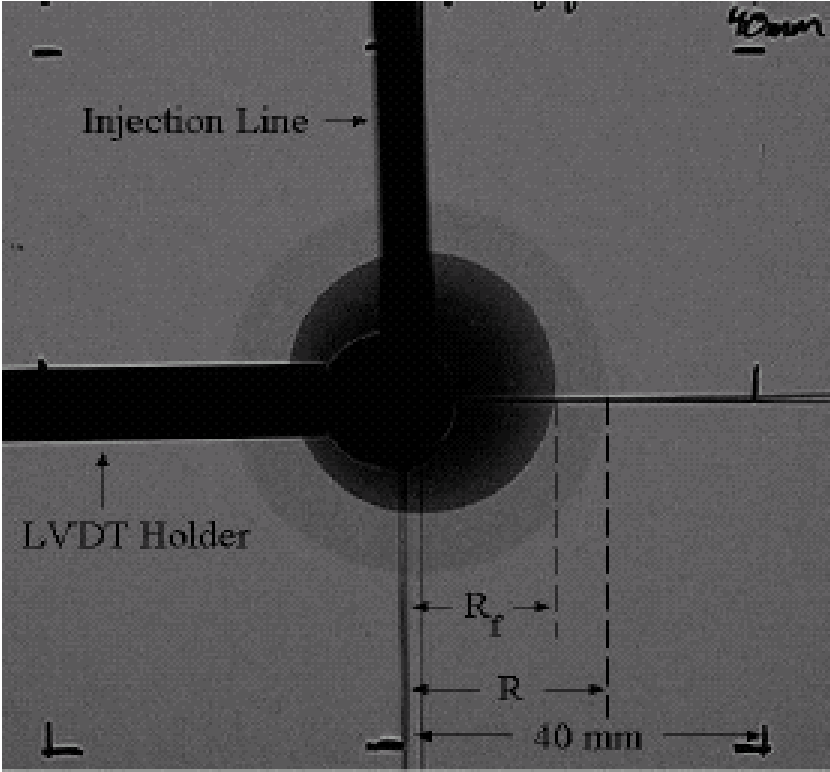


Viscosity-dominated
Example



For predictions of symmetry breaking see: Gao, H., & Rice, J. R. (1987). Somewhat circular tensile cracks. *International Journal of Fracture*, 33(3), 155-174.

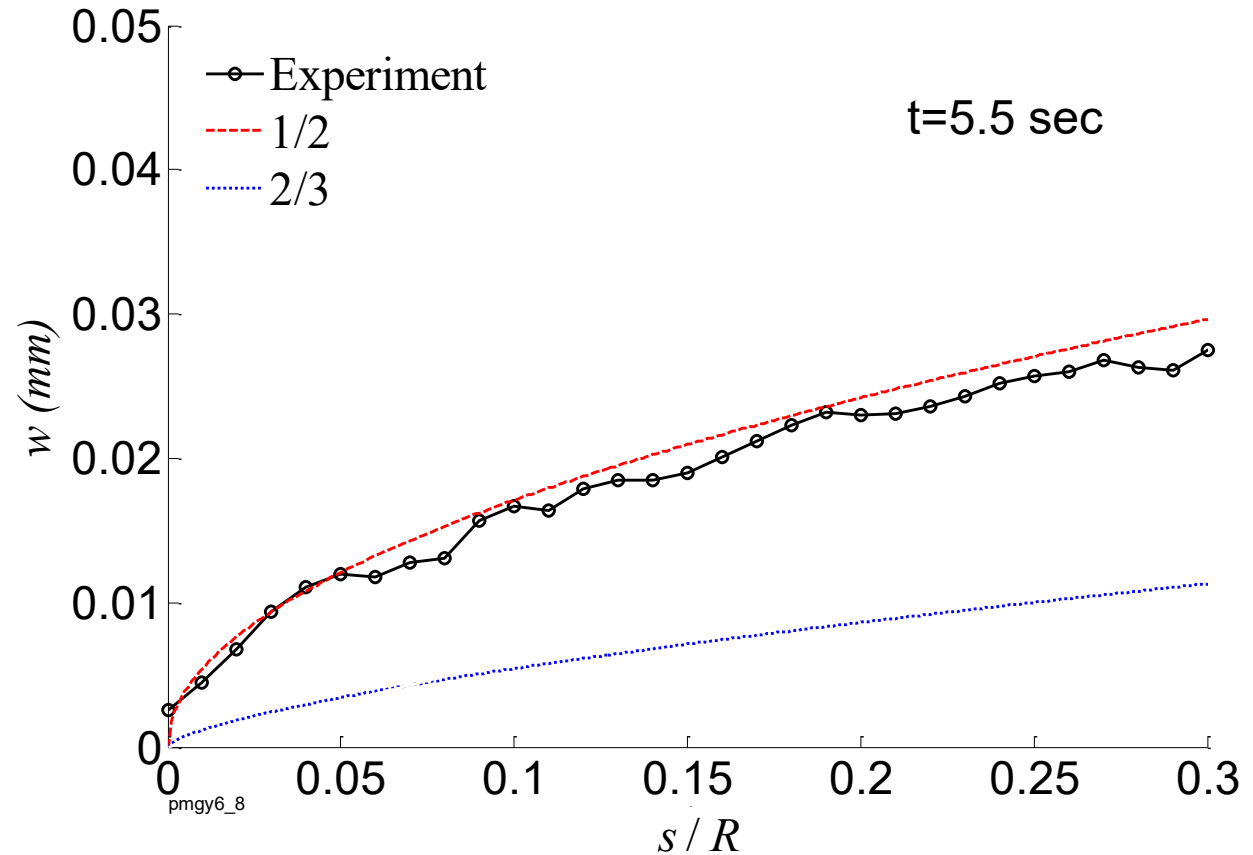
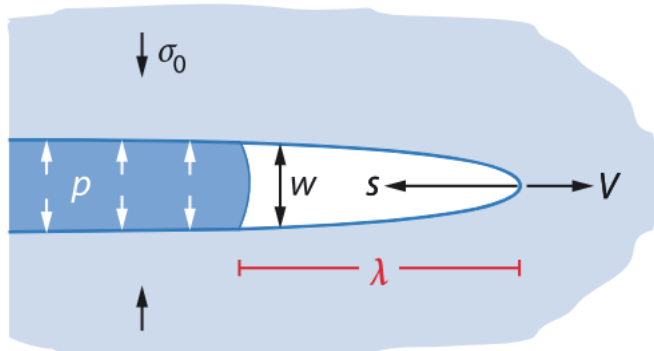
Photometric Analysis



Near-Tip Behavior Following LEFM

Linear Elastic
Fracture
Mechanics

$$w \sim 4 \left(\frac{2}{\pi} \right)^{1/2} \frac{K_{Ic}}{E'} s^{1/2}$$

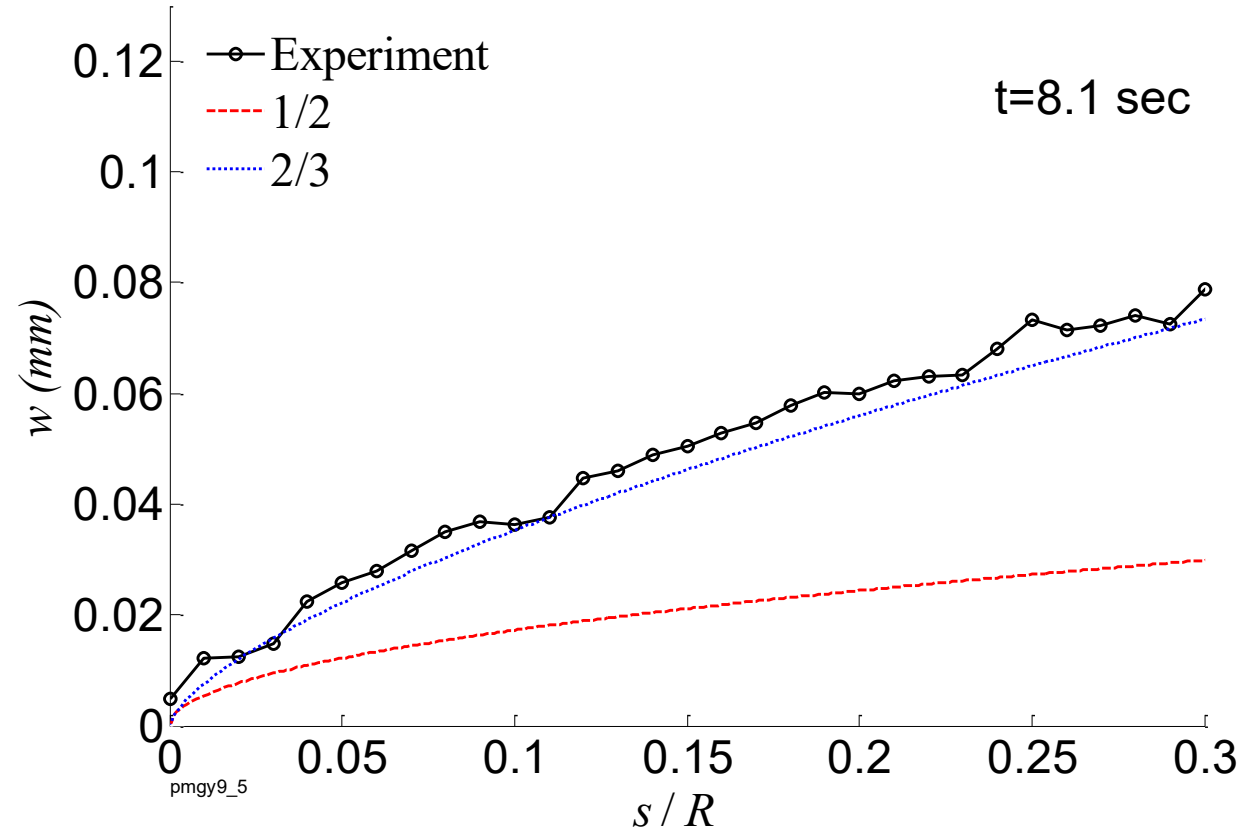
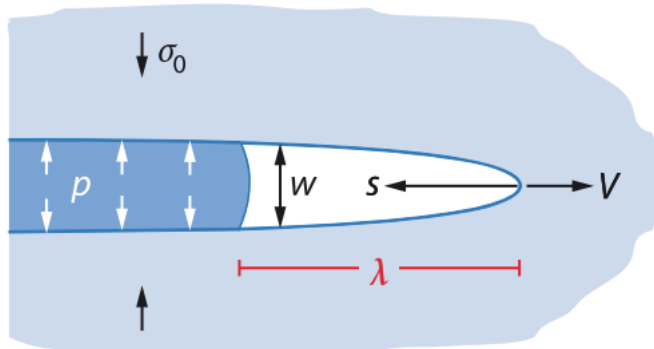


Material: PMMA
Viscosity=0.1 Pa s
“Toughness Regime”

Near-Tip Behavior Deviating from LEFM

Fluid/Solid
Coupling in Tip
Region

$$w \sim 2.3^{7/6} \left(\frac{\mu v_{tip}}{E'} \right)^{1/3} s^{2/3}$$



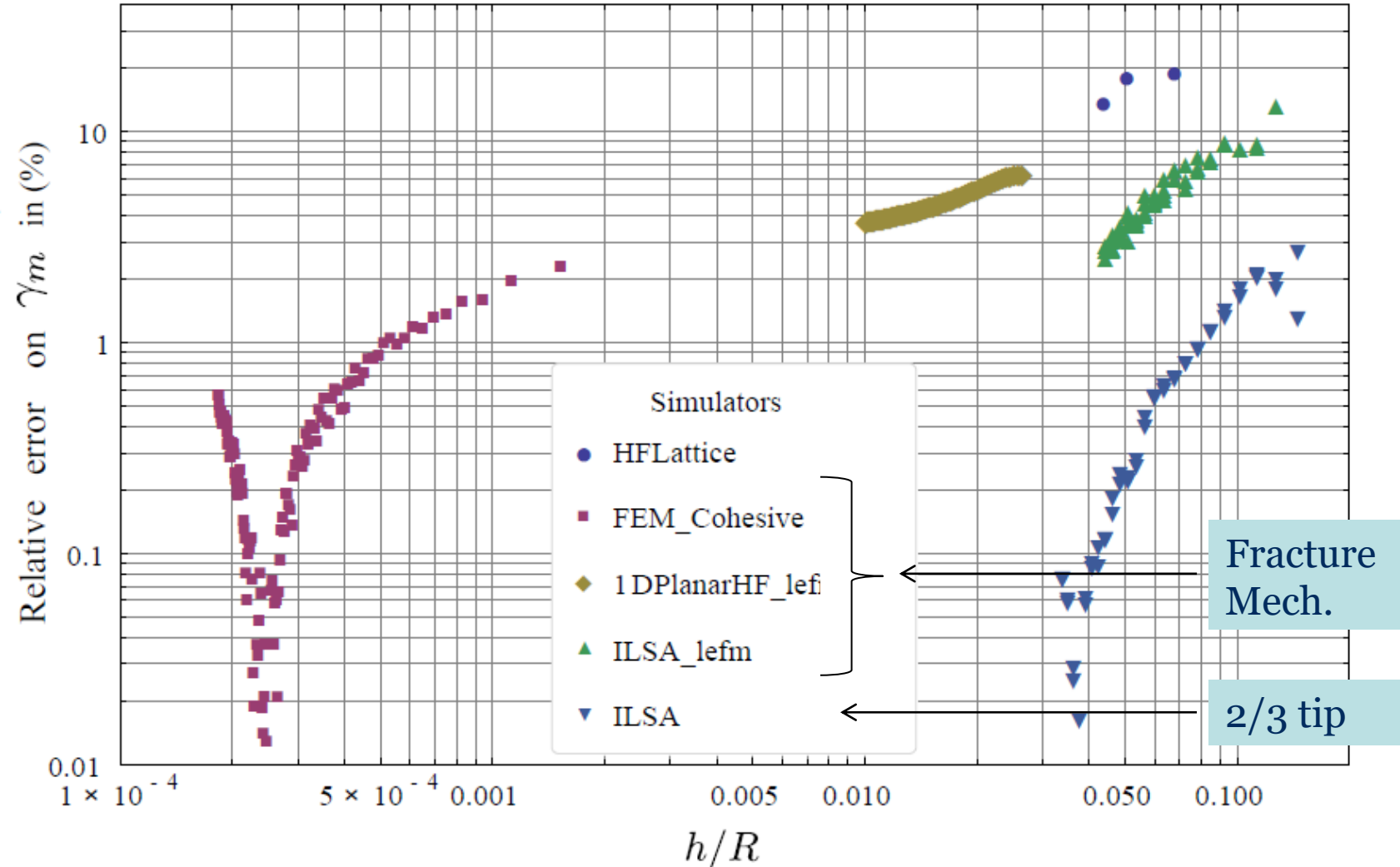
Material: PMMA
Viscosity=29 Pa s
“Viscosity Regime”

Implications of Fluid-Solid Coupling for Simulations

- Propagation implies LEFM width profile near tip
- But, elasticity and fluid flow near the tip brings in a different tip profile
- Each physical process brings its own characteristic length scale

Multiphysics problems are multiscale problems

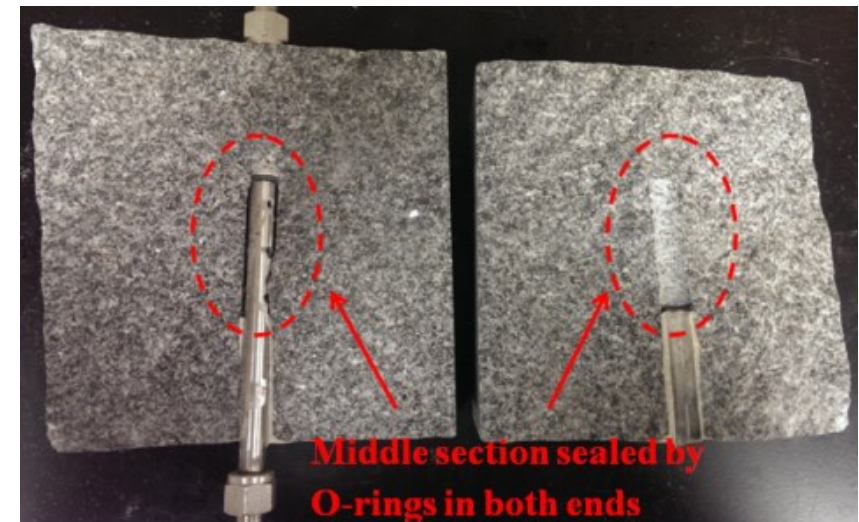
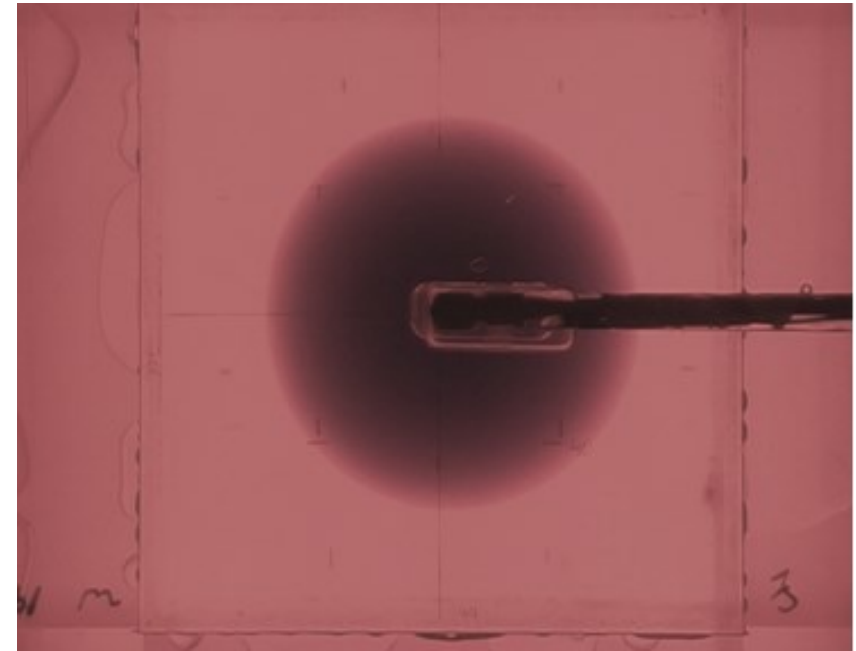
- **In simulations leads to mesh dependence and very slow convergence with mesh refinement**



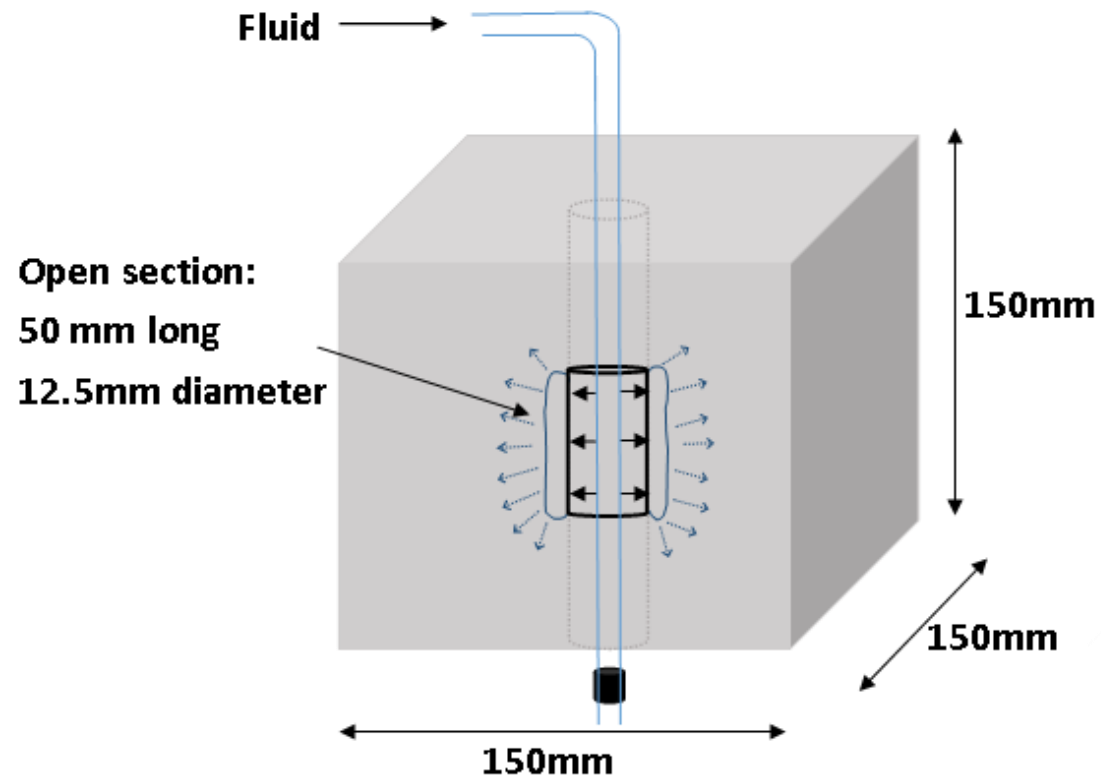
Homogeneous, transparent, brittle materials (PMMA, glass)

...to...

Actual rock



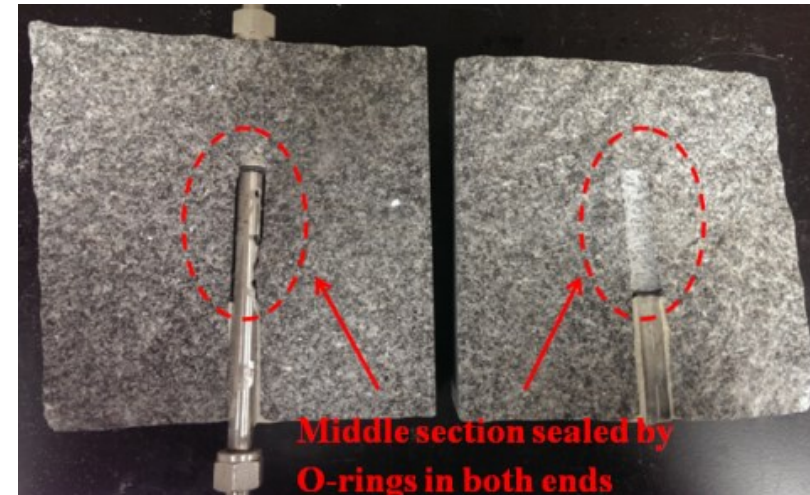
Lab Experiments – Time-Dependent Initiation



- Hold constant pressure
- Measure time to HF initiation

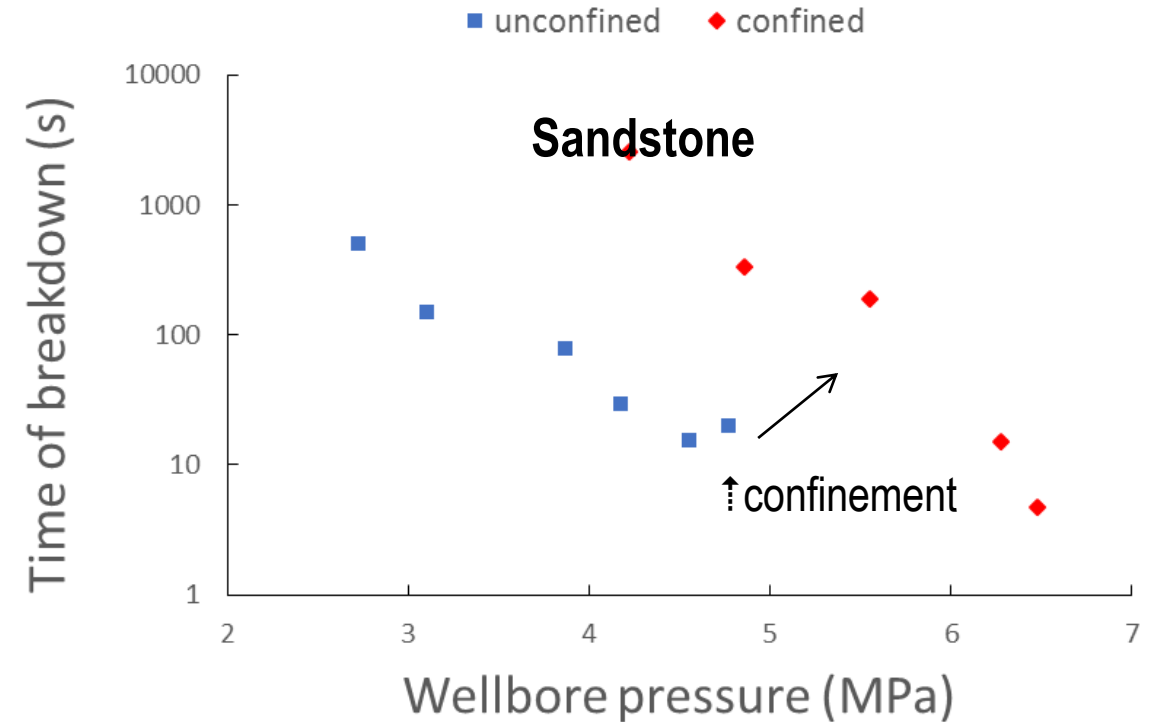
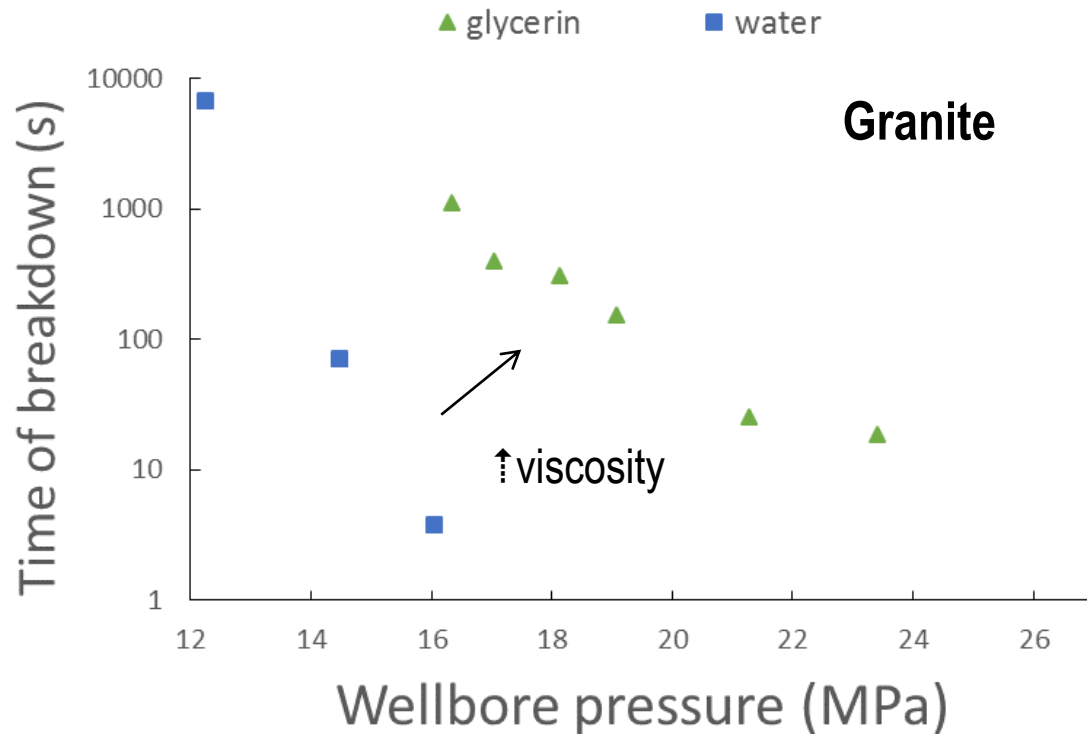


Before



After

Lab Experiments – Time-Dependent Initiation



- Similar observation in both rock types
- Delay time increases with viscosity and confining stress

Modeling Time-Dependent Initiation

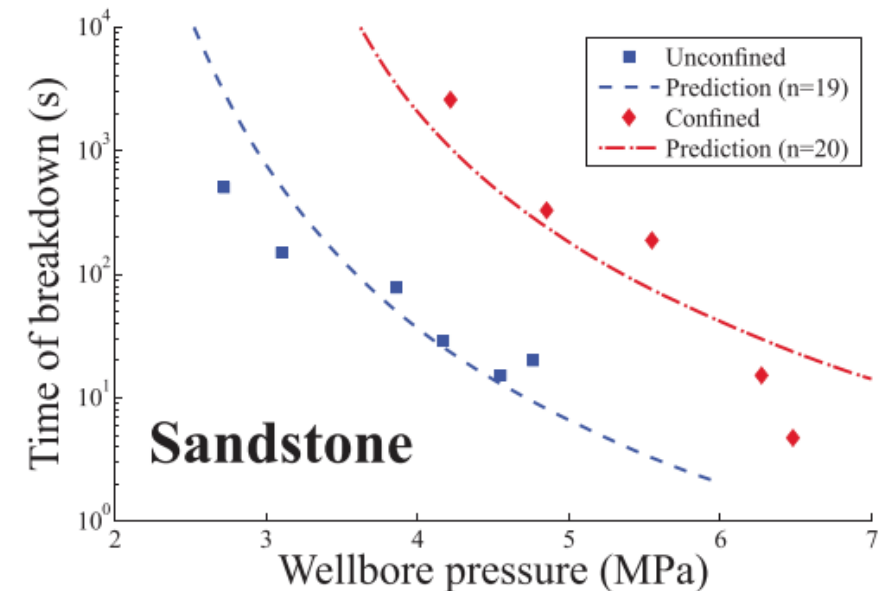
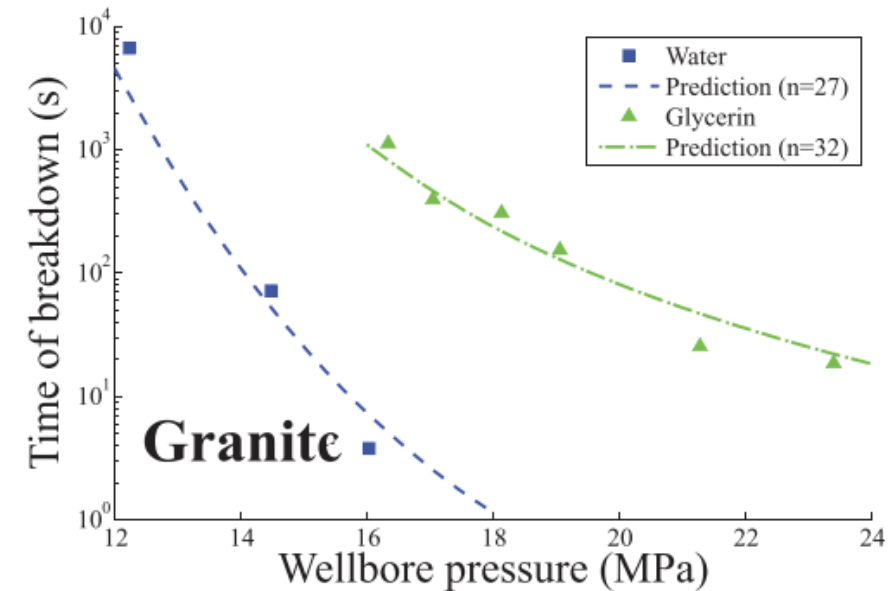
- Classical hydraulic fracture model but changing propagation velocity to Charles' law for subcritical crack growth

$$V = A \left(\frac{K_I}{K_{IC}} \right)^n,$$

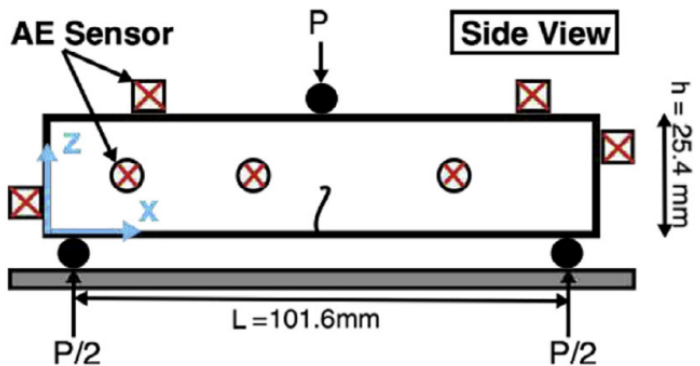
- With some algebra, tip condition becomes

$$w \sim \sqrt{\frac{32}{\pi} \frac{K_{IC}}{E'}} (x_{tip} - x)^{1/2} \cdot \left(\frac{V}{A} \right)^{1/n}, \quad x \rightarrow x_{tip}$$

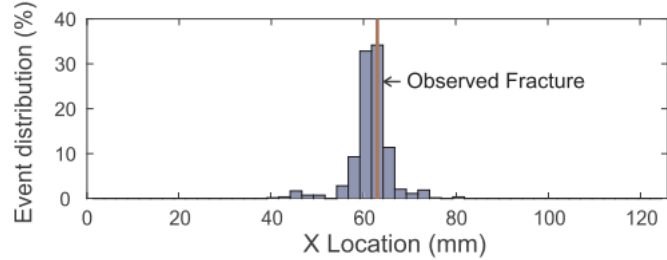
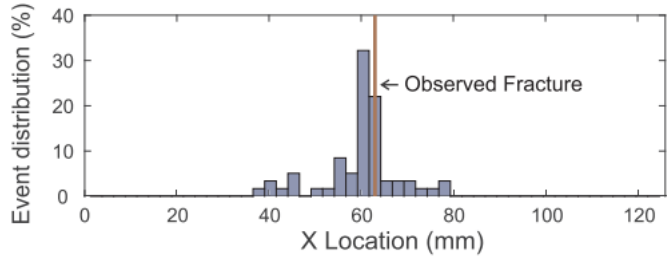
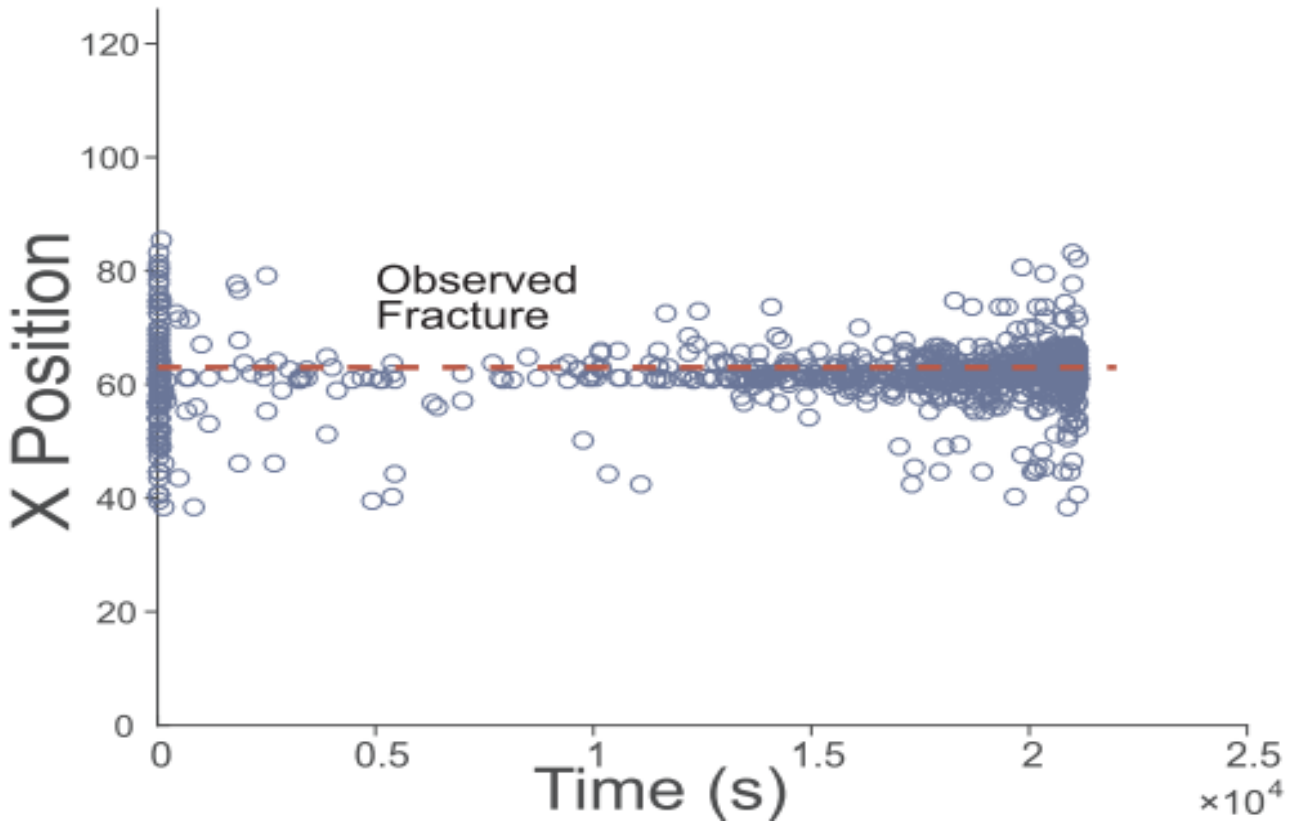
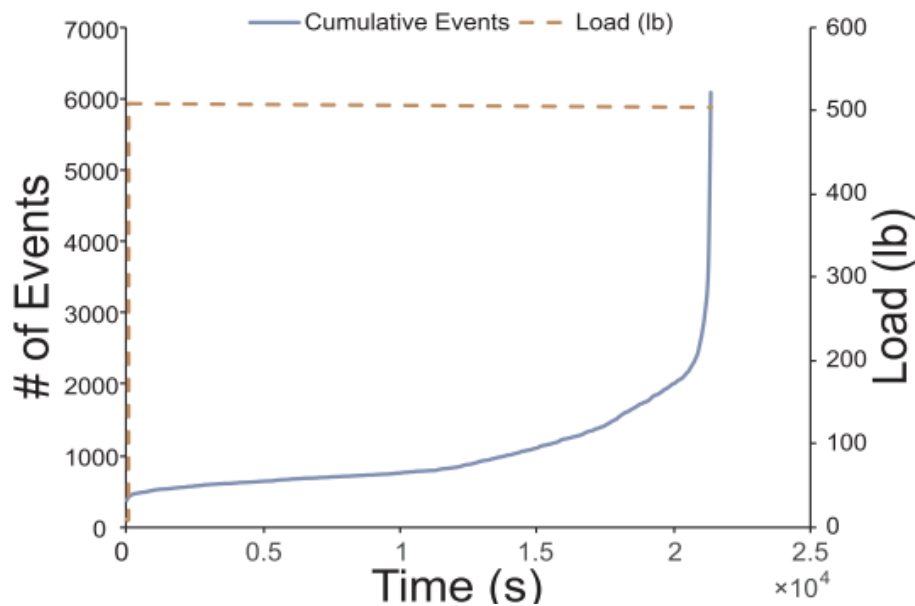
- Good match to experiments, using exponent n as a fitting parameter (noting n is in reasonable ranges for these two rocks)



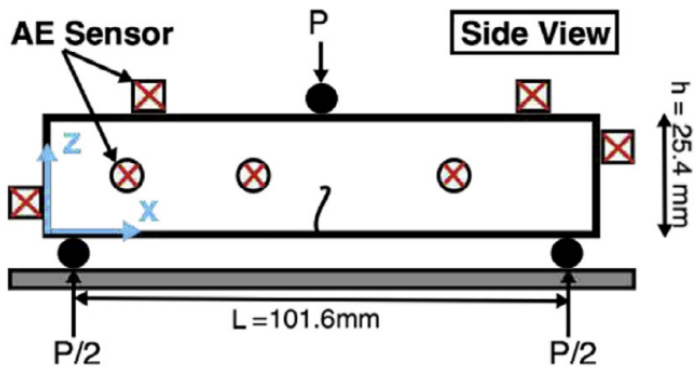
Acoustic Emission during Time Dependent Breakage of Rocks



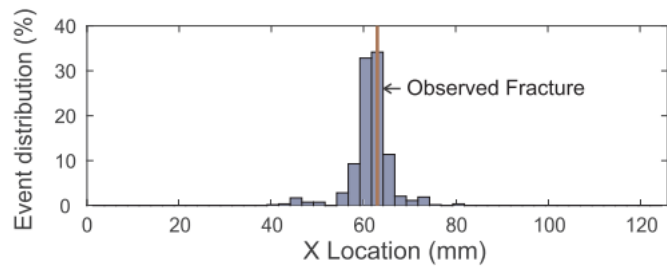
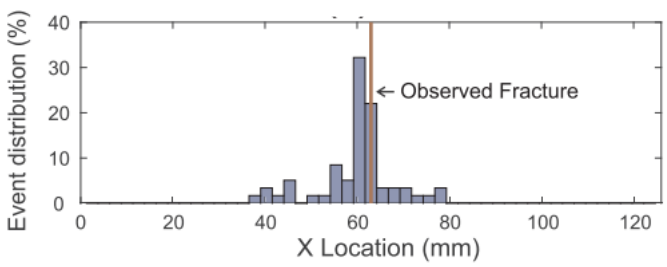
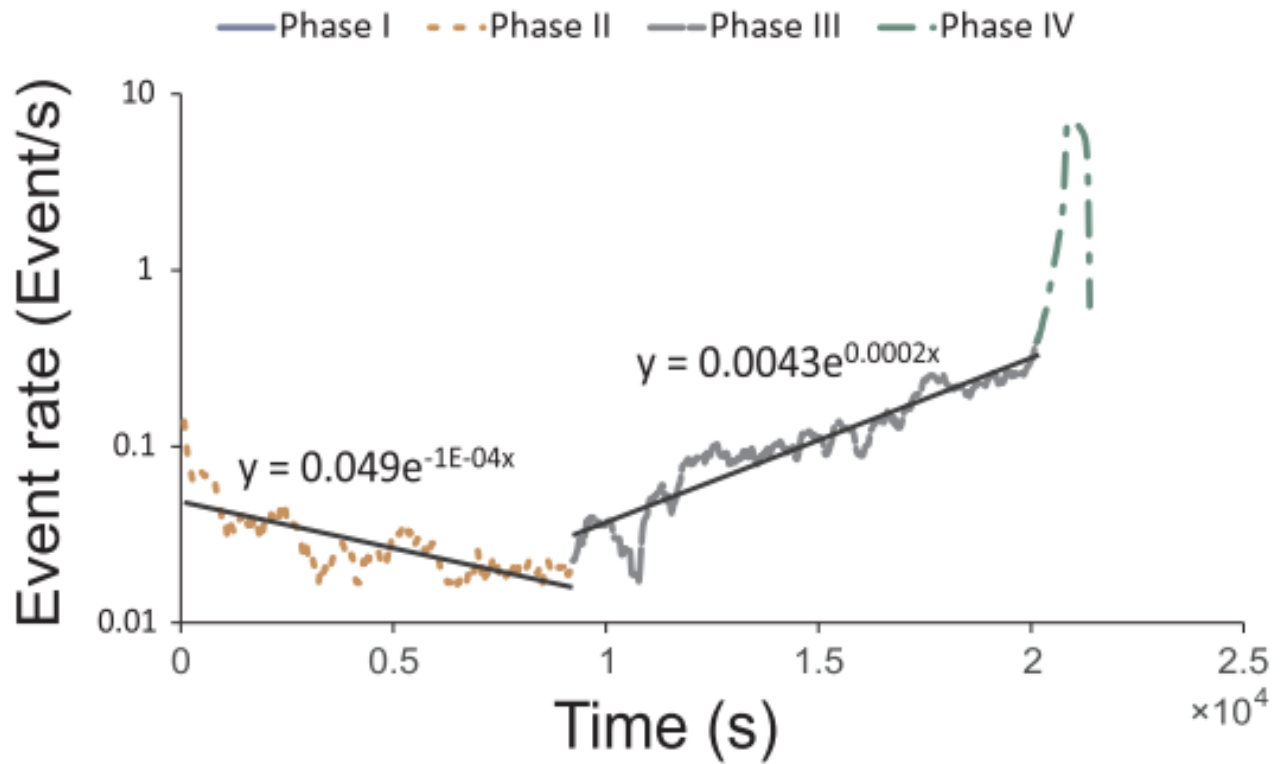
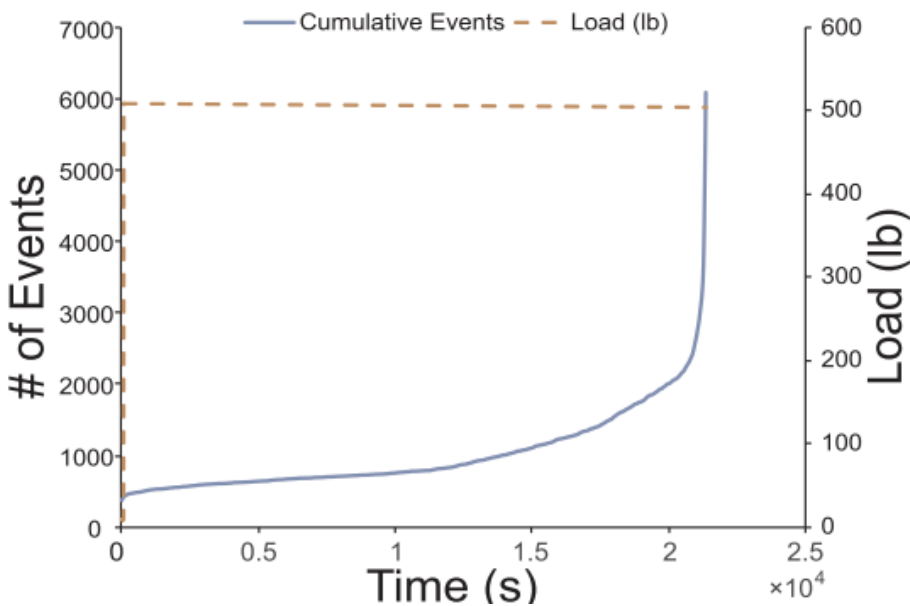
- Hold constant load
- Measure acoustic emission until time of failure



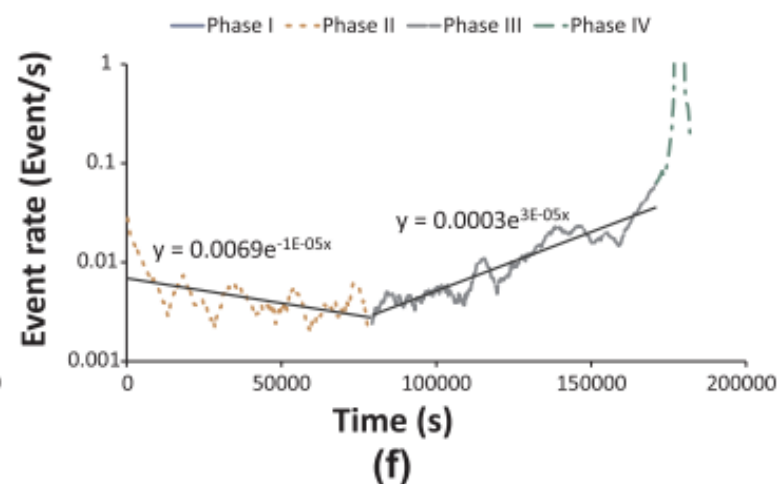
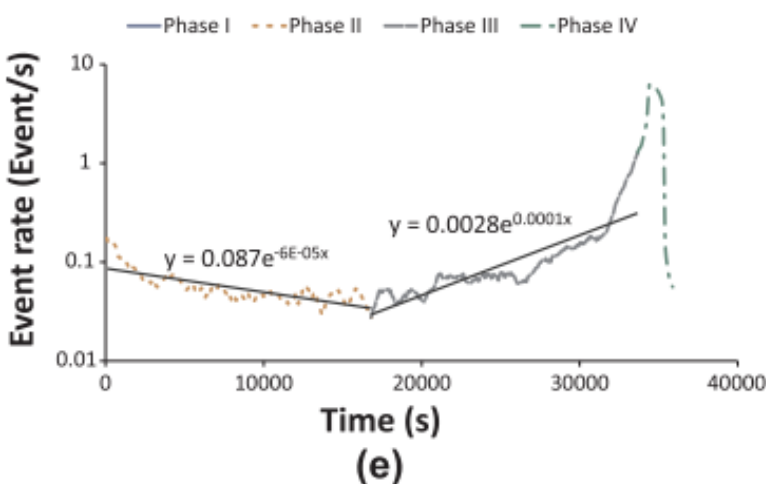
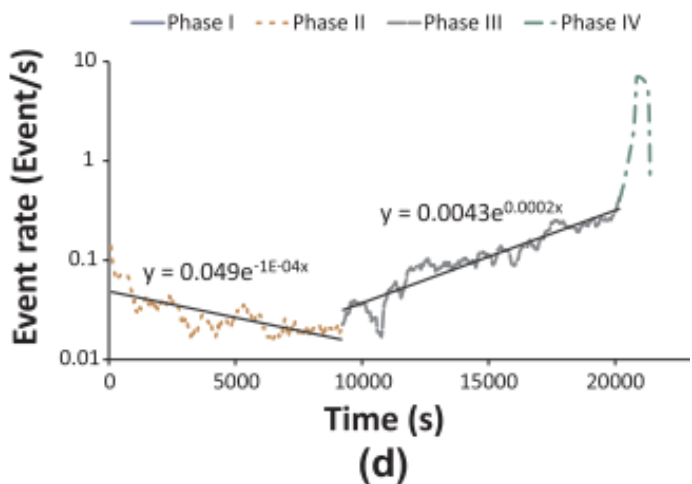
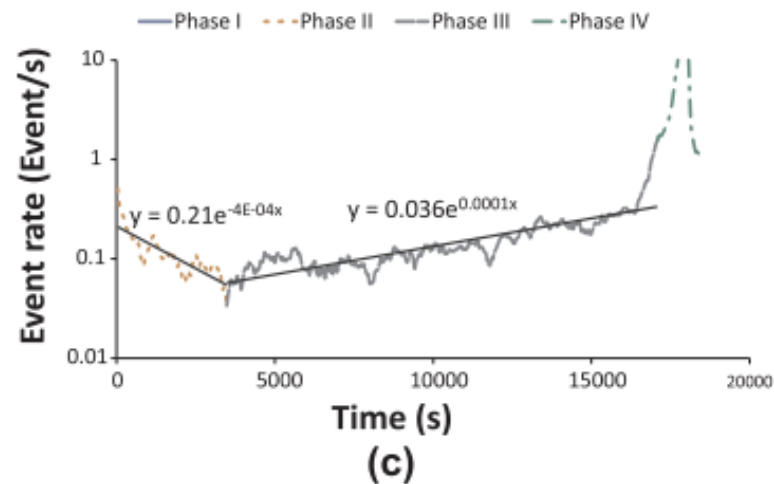
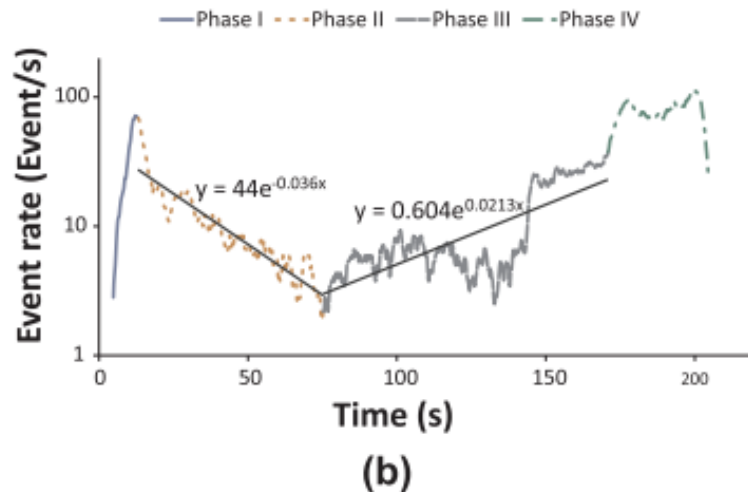
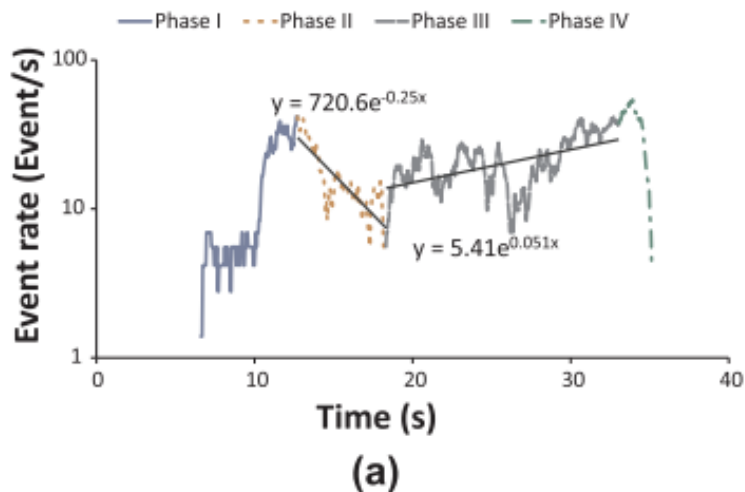
Acoustic Emission during Time Dependent Breakage of Rocks



- Hold constant load
- Measure acoustic emission until time of failure

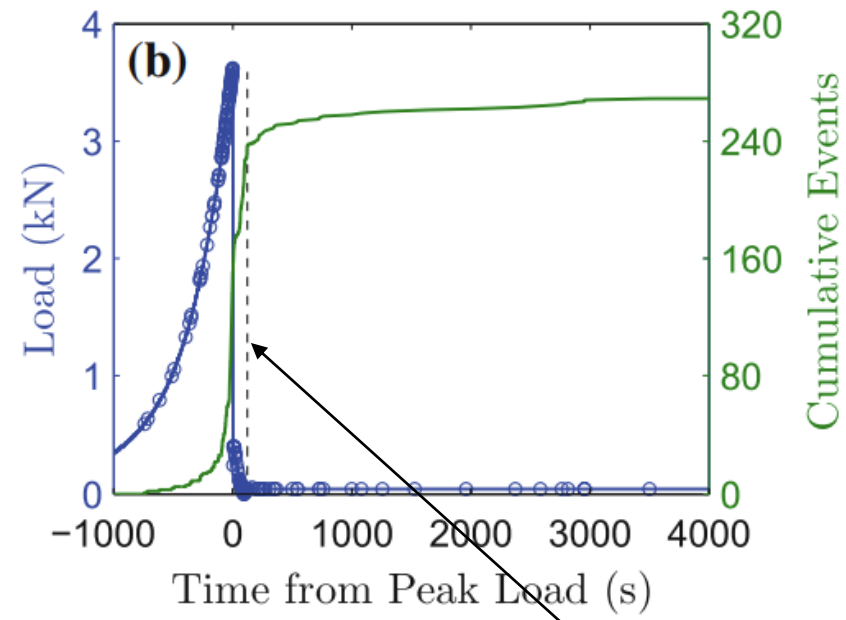
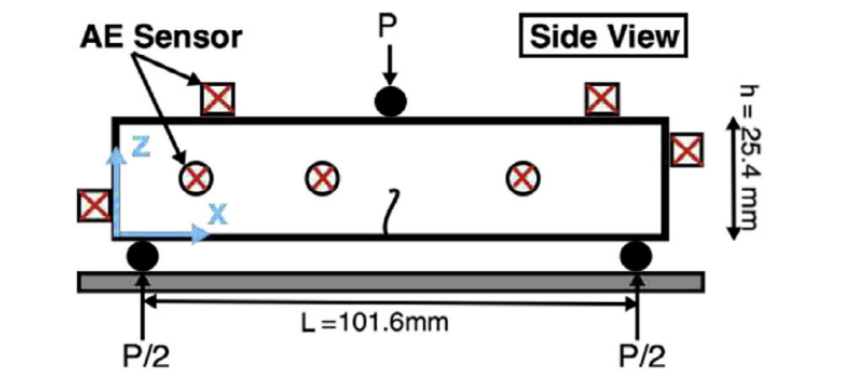


Acoustic Emission during Time Dependent Breakage of Rocks

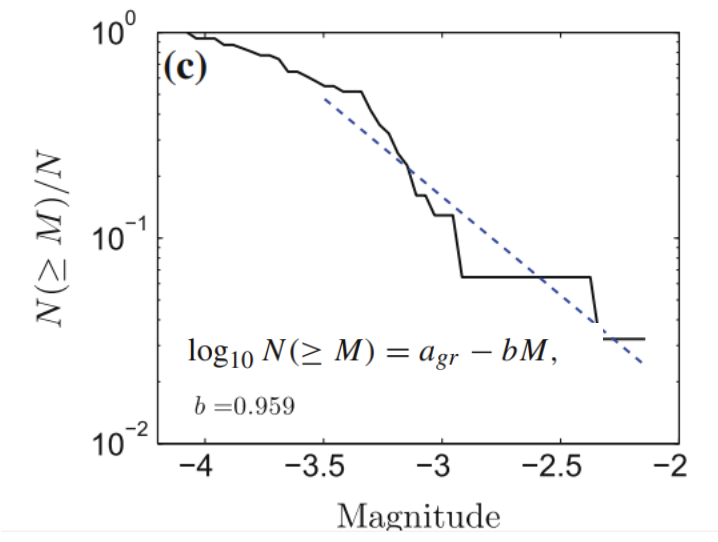
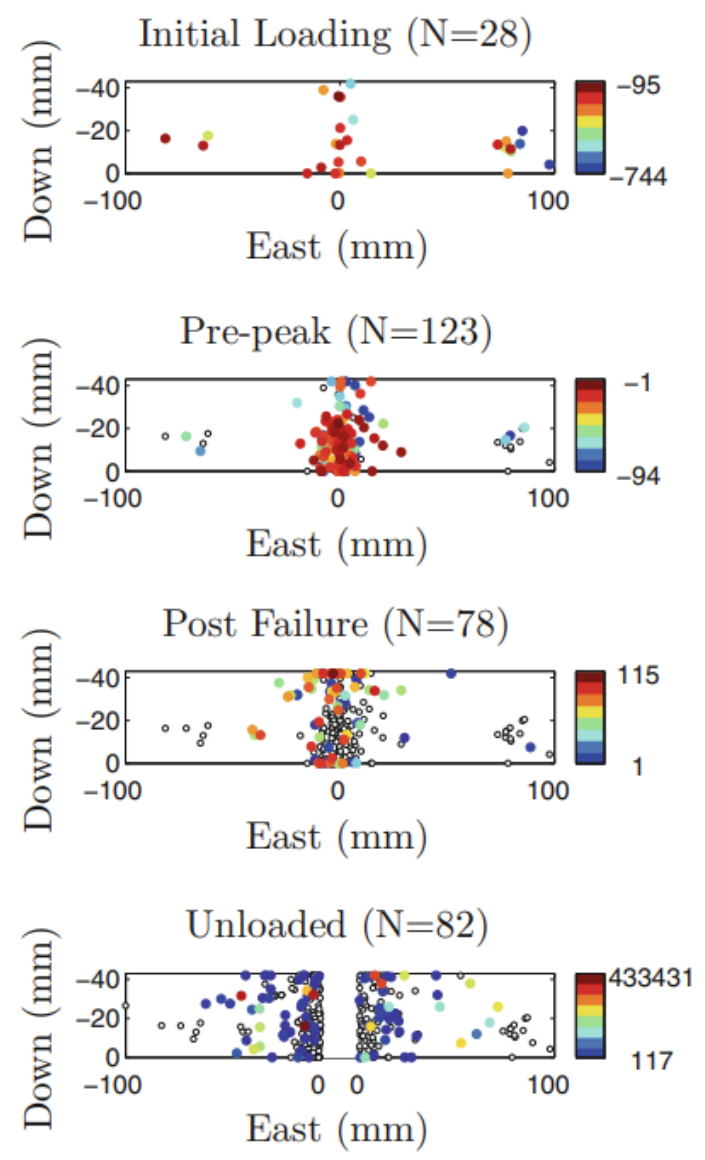


- Range of lifetimes: $O(10^1)$ - $O(10^5)$ seconds
- All cases, transition from declining to increasing event rate occurs just before halfway to breakage

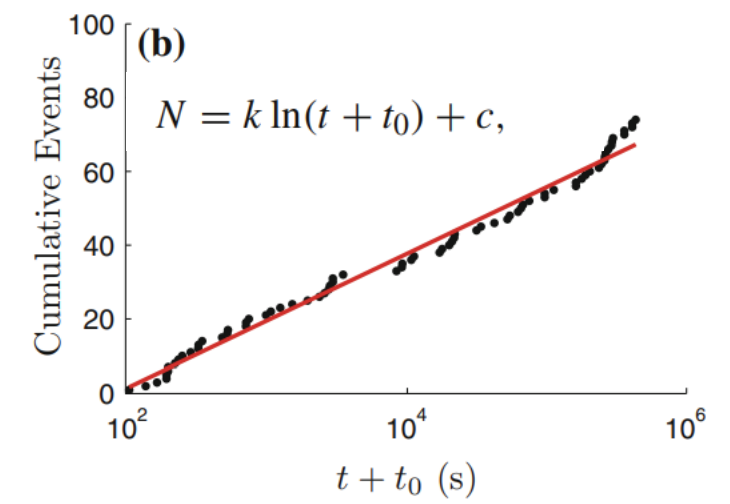
Acoustic Emission Aftershocks



- Increase load to failure
- Separate halves, isolate on rubber mats
- Observe events continuing for hours to days after breakage



Magnitudes follow Gutenberg-Richter law

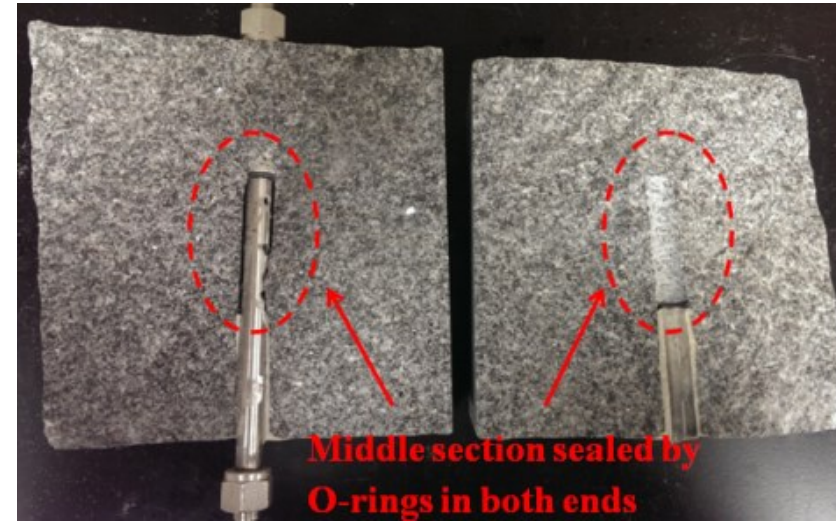


Frequency follows Omori-Utsu law

Laboratory scale

...to...

Field scale



Subsurface Pumped Energy Storage

- Create a storage lens by pumping viscous fluid into the subsurface
- Energy storage: Inflate lens by pumping water during high power production times (i.e. daytime)
- Energy production: Produce water from lens during low production times to drive a turbine (i.e. evening/night)
- Pressure and flowing rate in proportion to the difference between rock density and fluid density



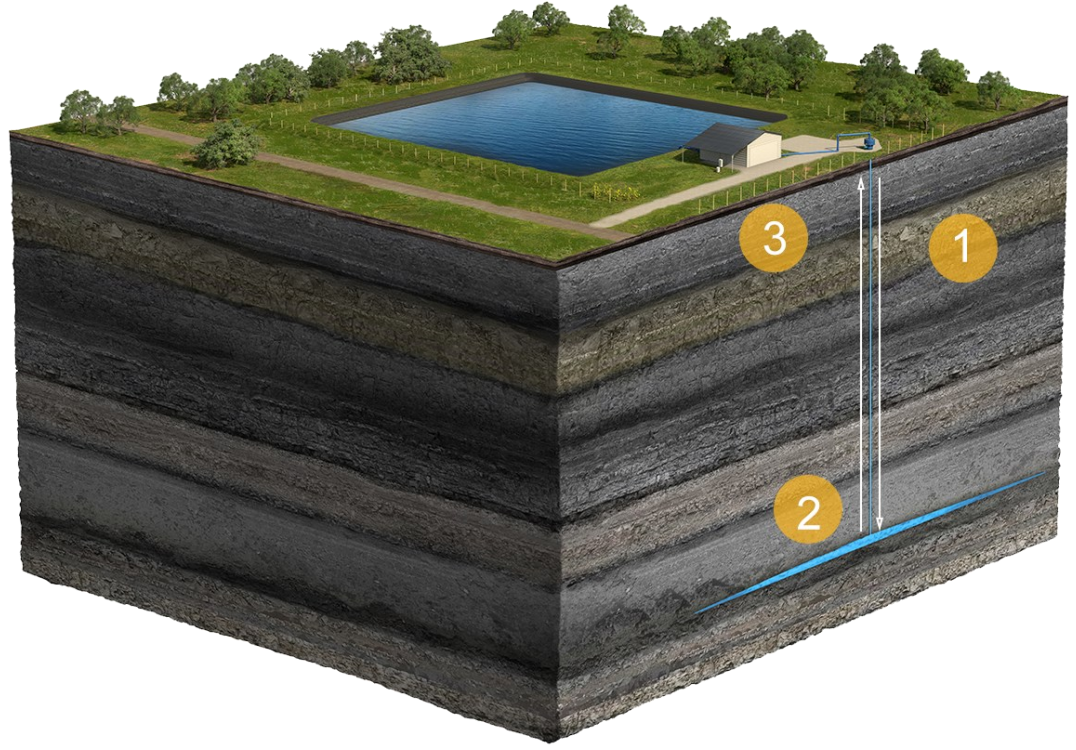
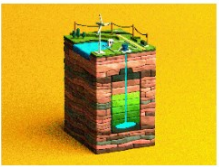
Reporting

ANNALS OF INNOVATION

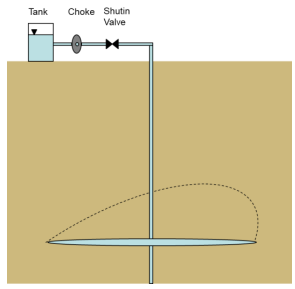
THE RENEWABLE-ENERGY REVOLUTION WILL NEED RENEWABLE STORAGE

Can gravity, pressure, and other elemental forces save us from becoming a battery-powered civilization?

By Matthew Hutson



LOBSTER Governing Equations



Similar to classical hydraulic fracture model

- Elasticity, solved by Displacement Discontinuity Method (elastic half space, circular, horizontal, planar crack) with addition of **bridge stress**
- Continuity Equation **including matrix and pressure dependent fissure leakoff** (local fluid mass balance, solved by finite difference method)
- Fluid flux equation valid for all Reynold's numbers
- Tip Boundary Conditions

$$p_{f,i} - \sigma_o = -\frac{E'}{R} \sum_{j=1}^m A_{ij} w_j - s_i(w_i)$$

$$\frac{\partial w}{\partial t} + \frac{1}{r} \frac{\partial}{\partial r} (rq) + 2v_{\ell,mf} = 0, \quad q = \langle v \rangle w$$

$$q = -\frac{w^3}{12\mu f_D(Re, v)} \frac{\partial p_f}{\partial r}$$

$$w(R, t) = 0, \quad q(R, t) = 0$$

Unique to pumped subsurface storage

- Mixed inlet boundary condition (choke coupling wellhead pressure to flowing rate, can obtain this via classical energy equation from Fluid Mechanics)

Also can switch inlet condition to injection to simulate reinflation of existing lens

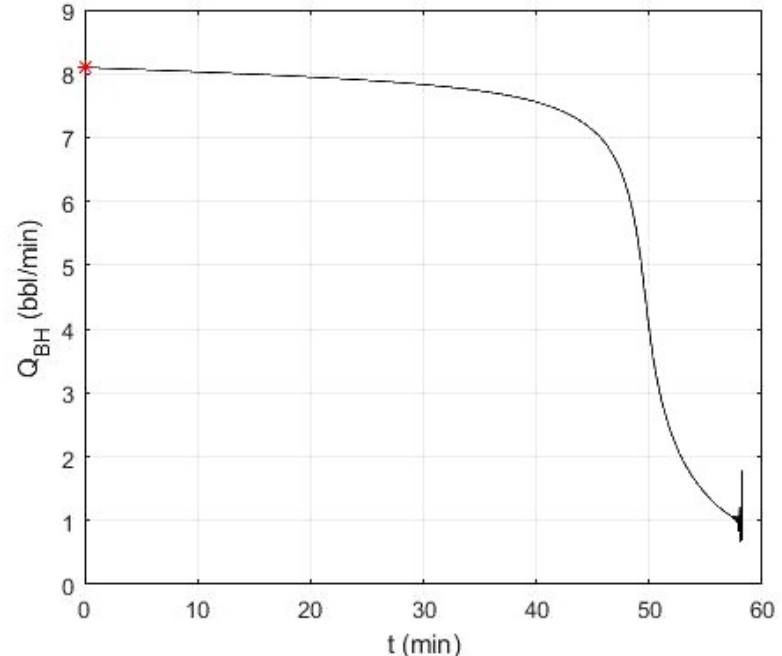
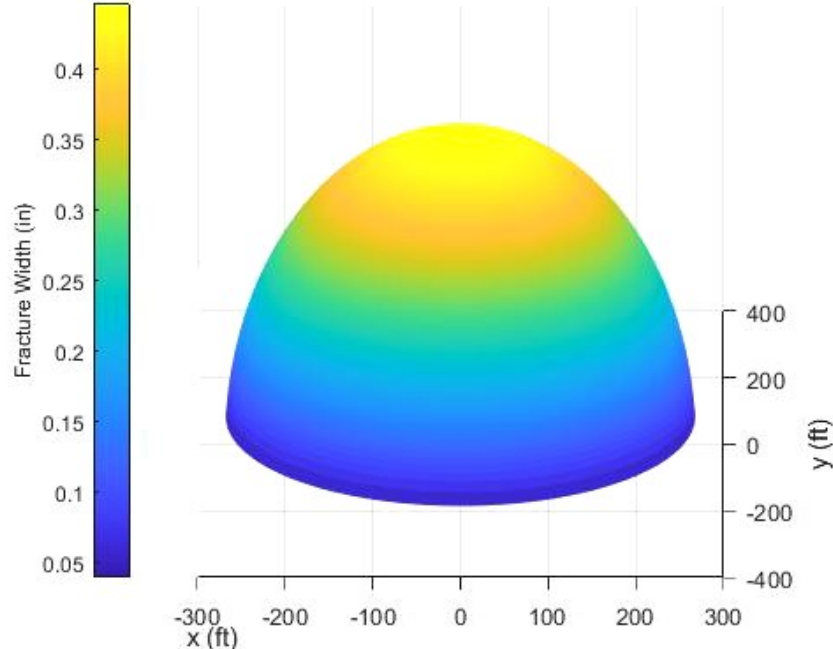
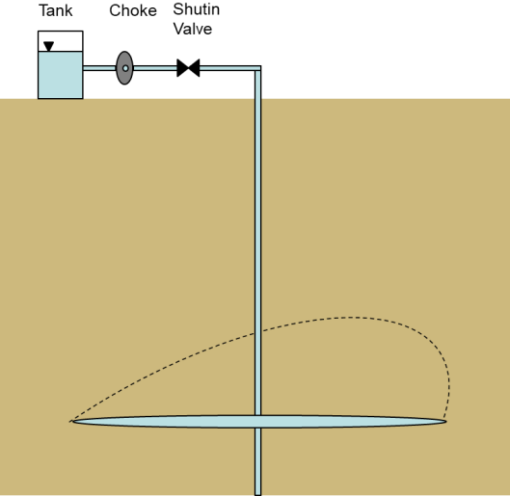
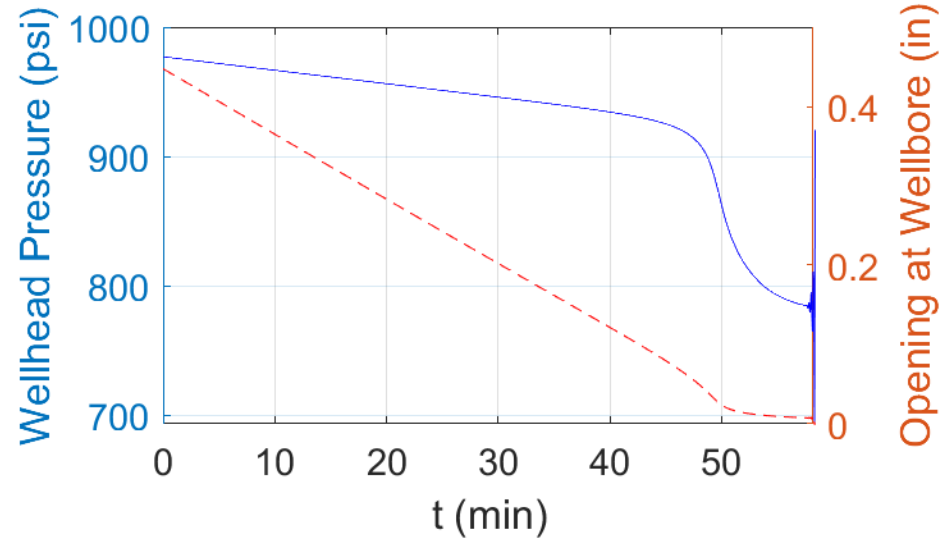
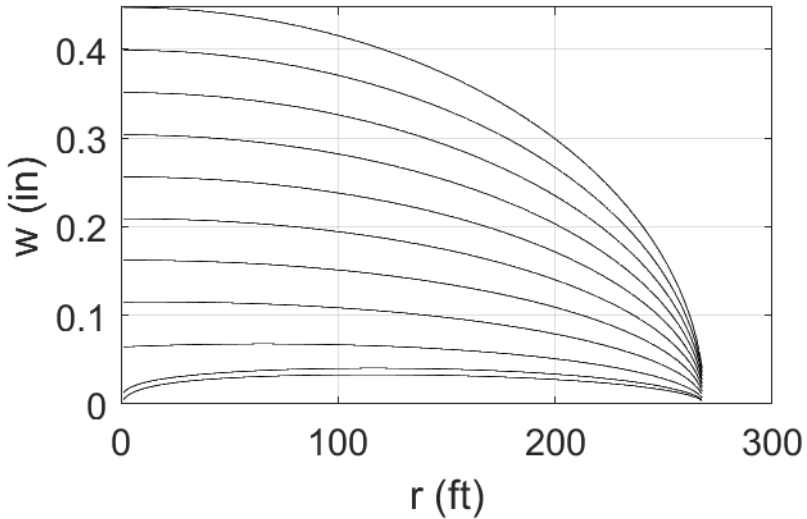
$$Q_{BH} = F(d_o, f_g, \rho_f, H, h_s, p_{net})$$

$$\lim_{r \rightarrow R_w} 2\pi r q = -Q_{BH}$$

- w =width
- p_f =fluid pressure
- q =fluid flux
- Q_{BH} =volumetric outflow rate
- f_g =stress gradient
- H =depth
- R =radius
- μ =fluid viscosity
- E' =plane strain modulus
- ρ_f =fluid density
- d_o =choke aperture
- C_d =choke shape factor
- $p_{net} = p_f f_g H$
- $p' = f_g - \rho_f g$

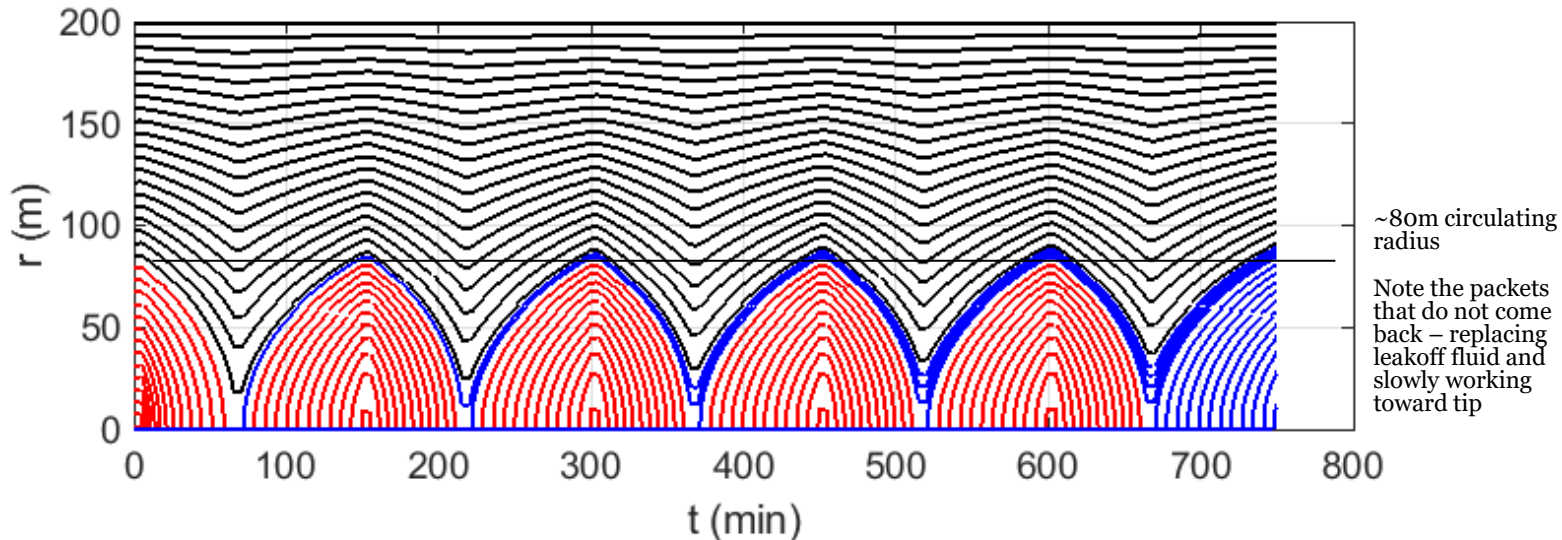
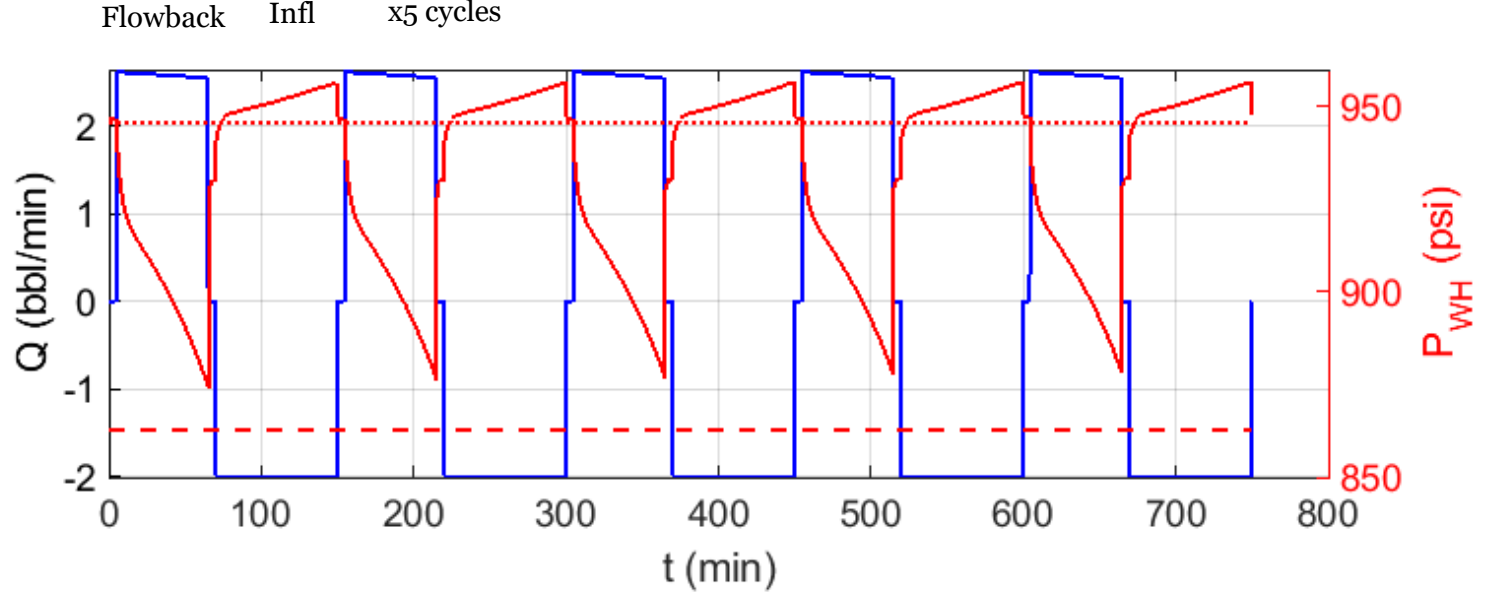
Behavior of a Lens when Emptied to the Point of Pinching

- Initial quasi-steady flow rate and WH pressure (hence also power)
- Gives way at large time to rapid decline – “Pinching”
- Avoid by keeping sufficient surplus volume and designing flowback rate to be small enough



Example: 92% Efficiency, 40 kWh Design

- 40 kW, 60 minute flowback, 1378 ft depth with $fg=1.07\text{psi/ft}$ and 0.4" choke
- **$R=200\text{m}$, $V_i=800\text{bbl}$, $Q_{inj}=2\text{bbl/min}$**
- Pressure dependent leakoff with critical pressure at 1.13 psi/ft
- Cycles 160bbl, 5bbl loss per cycle (**97% fluid efficiency**)
- Returns 42.5 kWh energy from 46 kWh input (**92% RTE**)



Packets starting in and staying in lens
Packets injected to and staying in lens
Packets produced out of the lens

Normalization of Compliance by (Full-Space) Elastic Solution

Volume and width from LEFM:

$$V_{lens} = \frac{16p_{net}R^3}{3E'}, \quad w_o = \frac{8p_{net}R}{\pi E'}$$

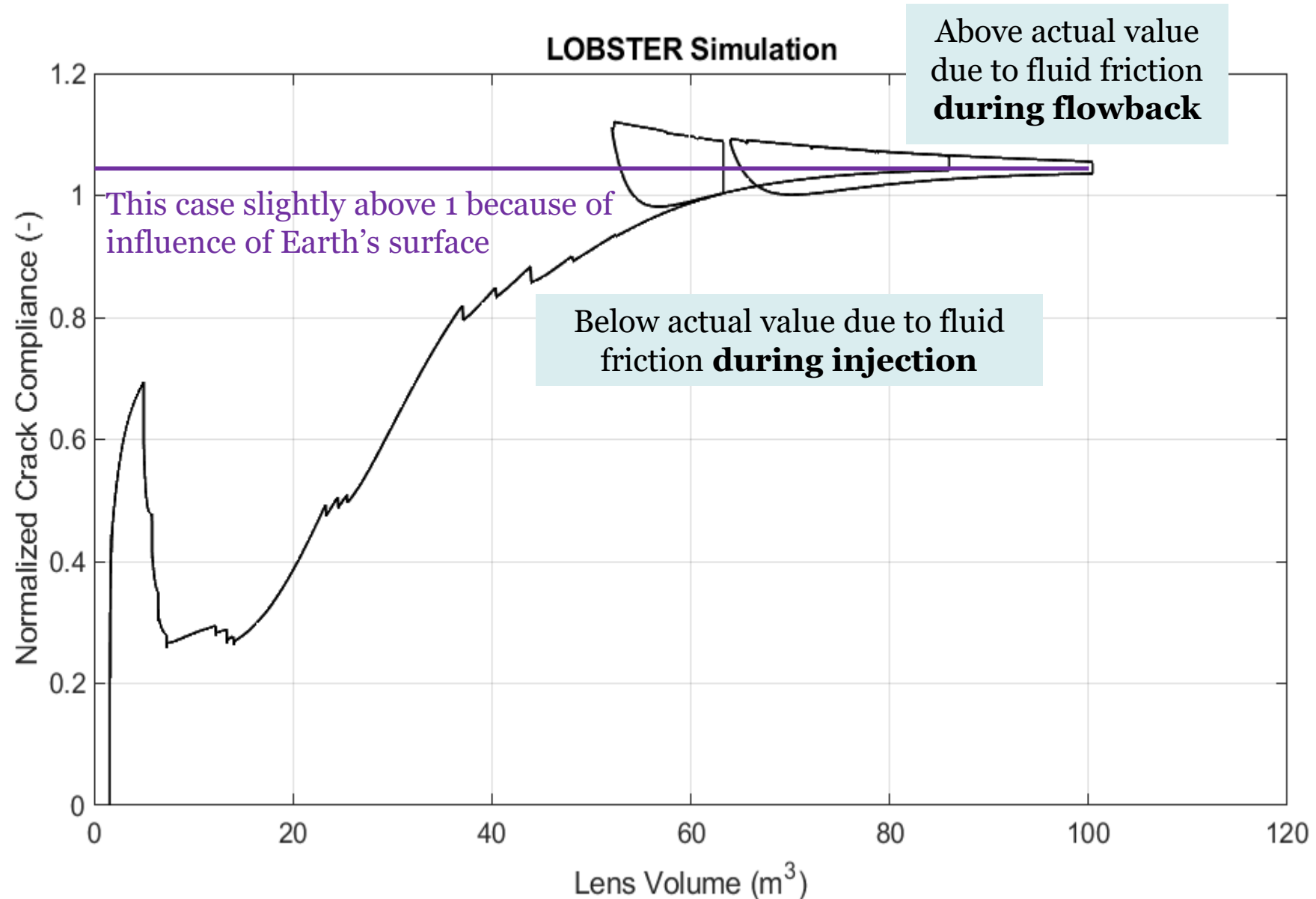
Elastic Compliance:

$$\Rightarrow \frac{w_o}{p_{net}} = \frac{2^{5/3}3^{1/3}}{\pi} \frac{V_{lens}^{1/3}}{E'^{2/3} p_{net}^{1/3}}$$

Divide compliance by elastic solution:

$$\text{let } V_{lens} = \eta V_{inj}, \quad E'_{app} = BE'_{meas}$$

$$\chi = \frac{w_o}{\eta^{1/3} V_{inj}^{1/3}} \frac{E'^{2/3}}{p_{net}^{2/3}} \frac{\pi}{2^{5/3} 3^{1/3}} = \frac{1}{B^{2/3}}$$

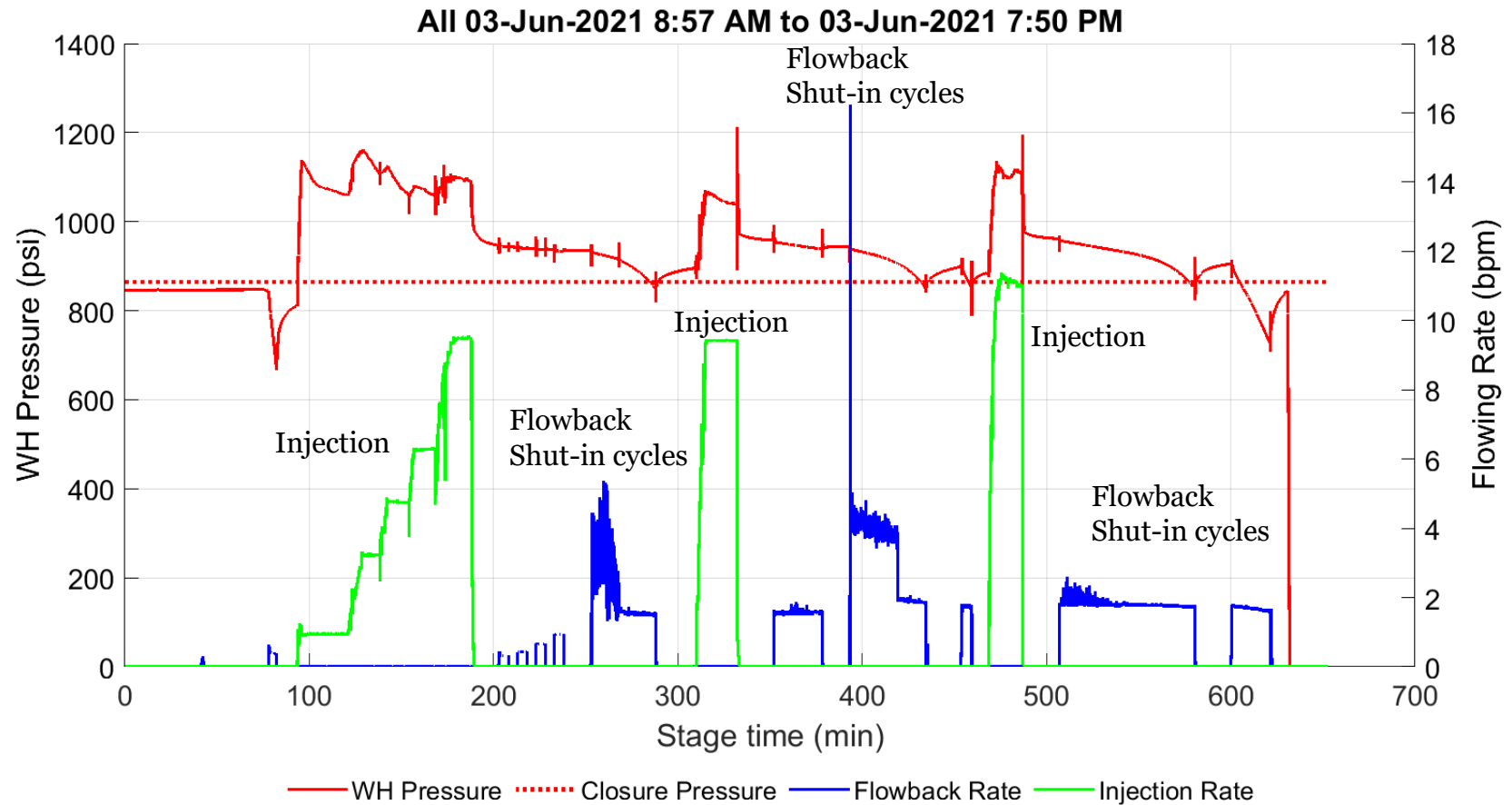


Case Study: Western Canadian Sedimentary Basin

- Create lens
 - Day 1: 1900 bbl (300 m³) initial injection
 - Day 2: additional 400 bbl (65 m³) injection
- Two months later, multiple reinflation and flowback cycles



Injection and Flowback Test Cycles



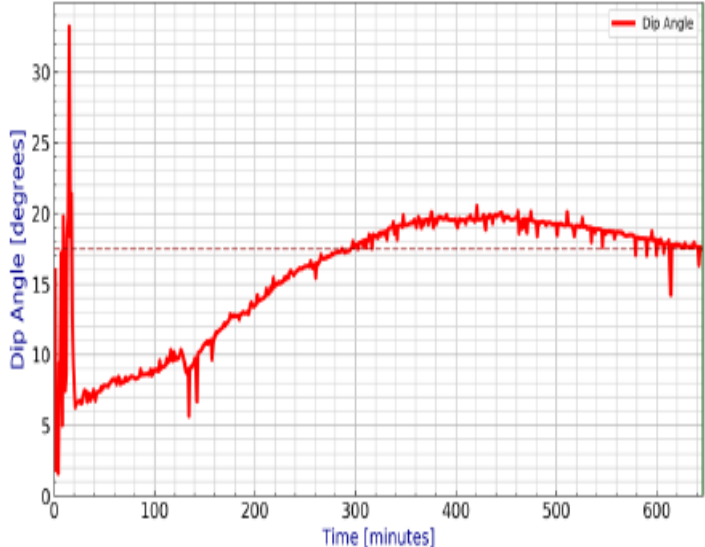
Summary:

- Flow #1
 - Drawdown to 4600 kPa (670 psi)
- Inflate #1
 - 64.2 m3, up to 1.5 m3/min
- Multichoke Test (MCT) #1
 - MCT 1a: 08/64" 5 mins, no pinch
 - MCT 1b: 10/64" 5 mins, no pinch
 - MCT 1c: 12/64" 5 mins, no pinch
 - MCT 1d: 16/64" 5 mins, no pinch
- Flow to pinch (FTP 1)
 - FTP 1a 18/64" 15 mins, no pinch, gasified
 - FTP 1b 20/64" 20 mins, imminent pinch
- Inflate #2
 - 1.5 m3/min for 30 m3
- FTP 2
 - FTP 2a: 20/64" 24 mins, no pinch
 - FTP 2b: 22/64" 40 mins, imminent pinch
 - FTP 2c: 22/64" 5 mins, imminent pinch
- Inflate #3
 - 2.0 m3/min for 30 m3
- FTP 3
 - FTP 3a: 22/64" 77 mins, imminent pinch
 - FTP 3b 22/64" 20 mins, imminent pinch

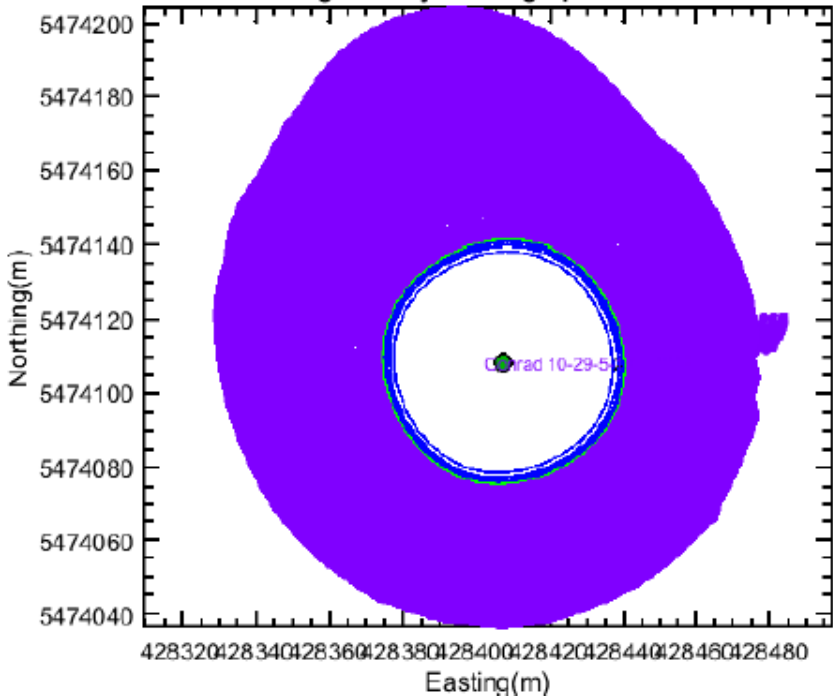
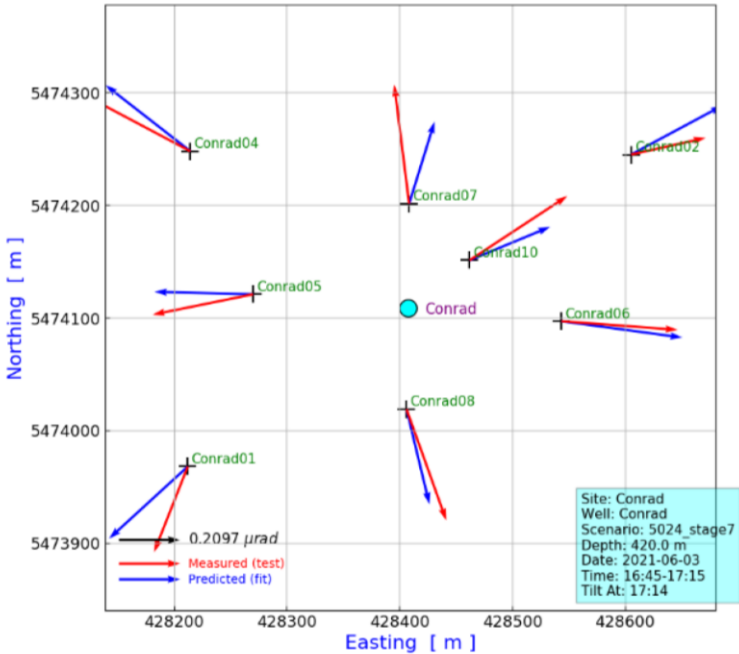
- Closure stress shown here for 1.07 psi/ft
- Flowback 1a, 2a, 2b, and 3a impacted by N₂ gas

Orientation and Shape from Tiltmeter Interpretation

- Tilt data indicative of horizontal (~10 deg dip) lens
- Extended Kalman Filter interpretation based on coupled planar hydraulic fracture model shows nearly circular shape

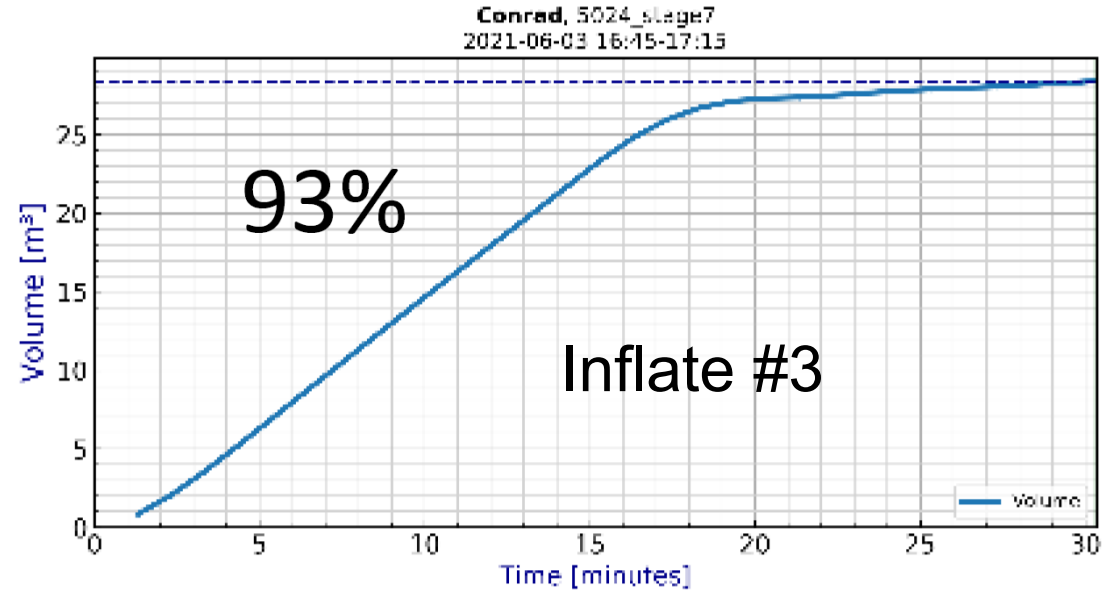
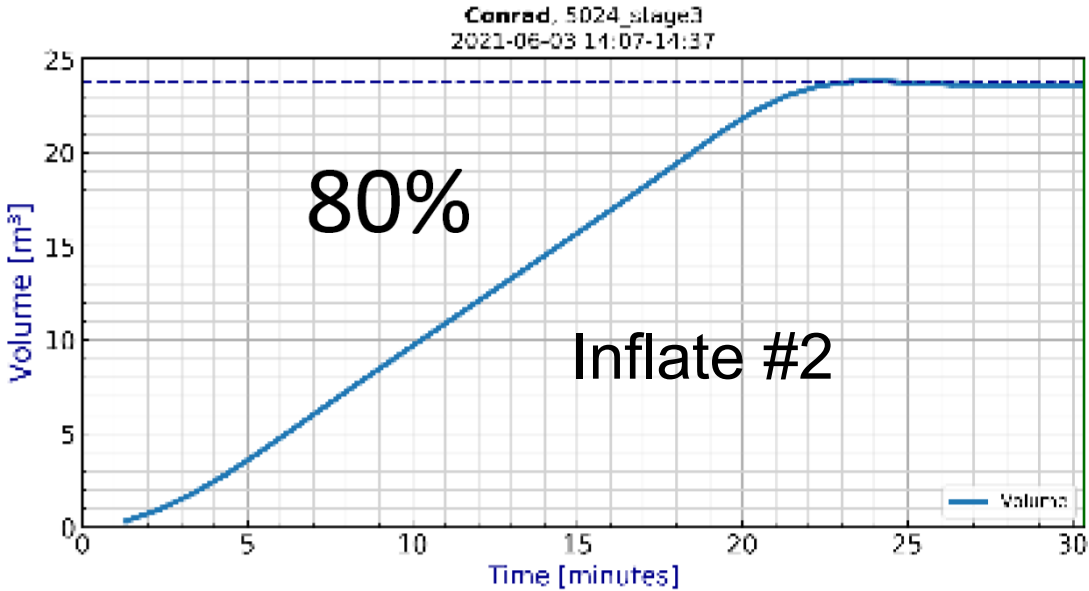
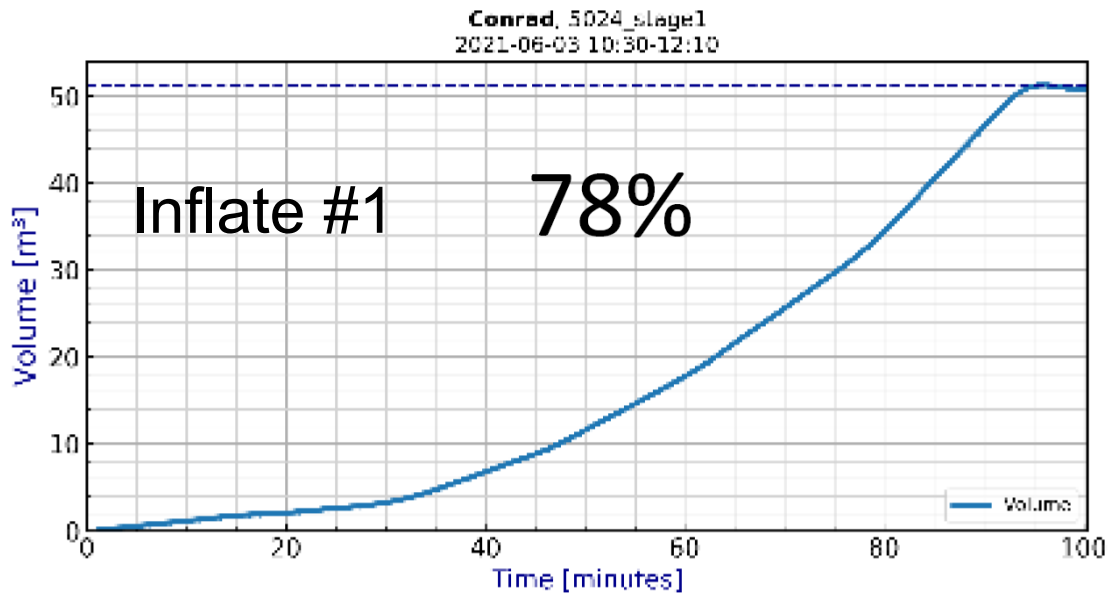


Inflate #3
30 m3

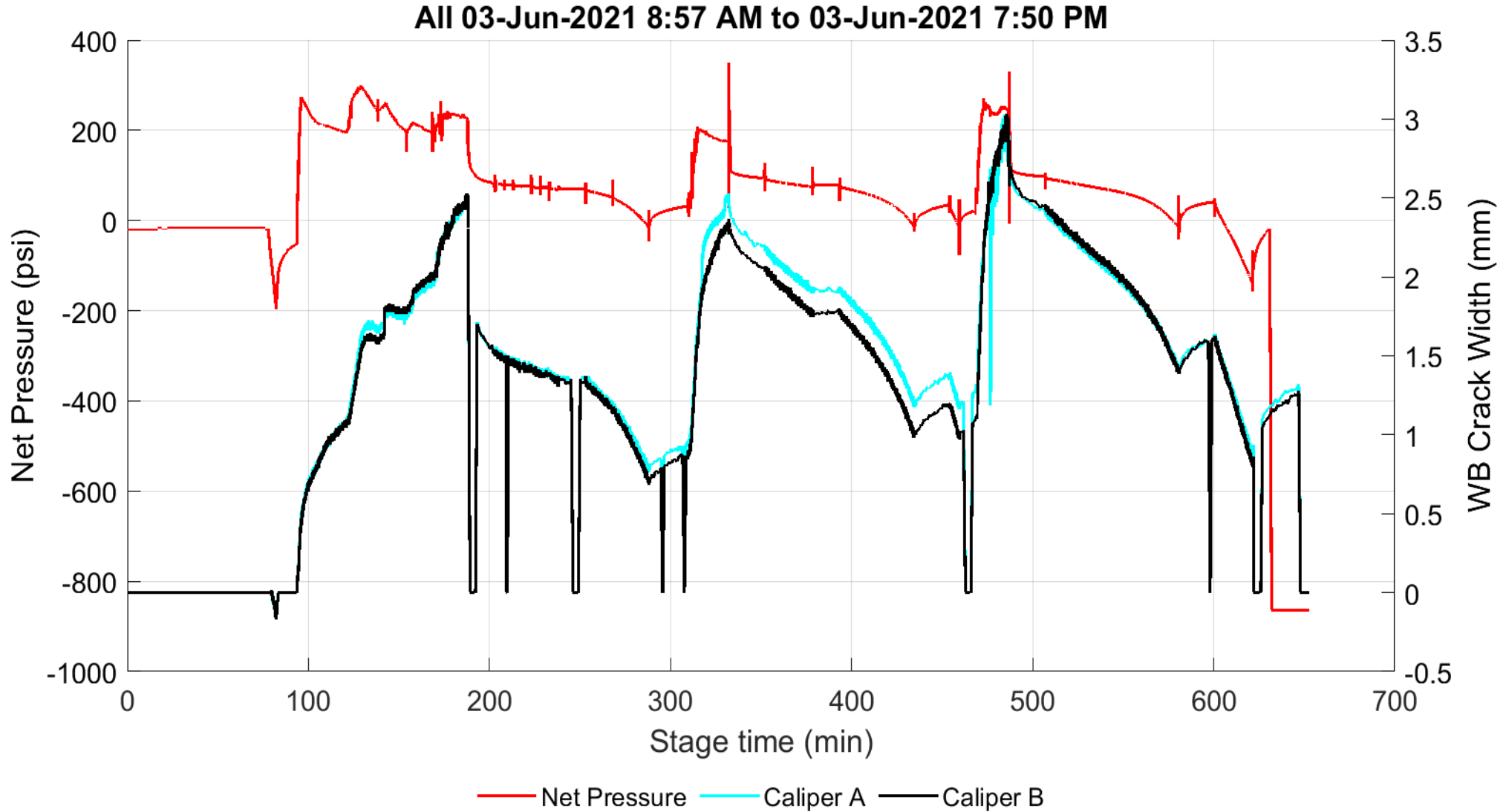


Tiltmeter Inferred Volumes

- Fluid leakoff 22% of total for first cycles, improves to 7% of total for later cycles
- Fluid efficiency improves from 78% to 93% with successive injections

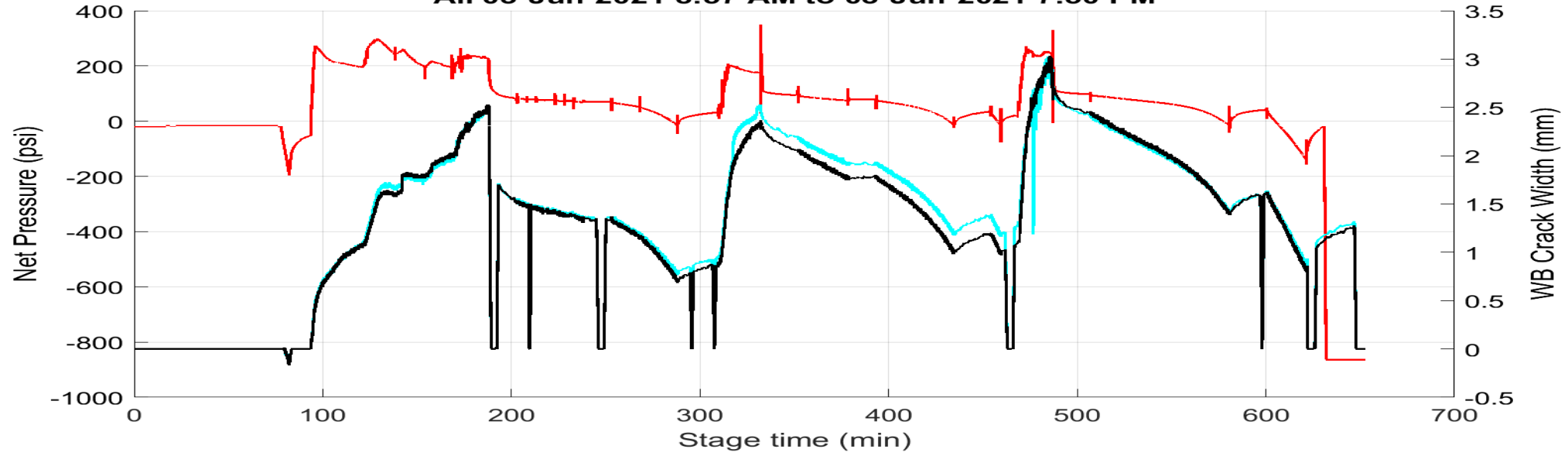


Caliper Lens Width Measurement at Wellbore



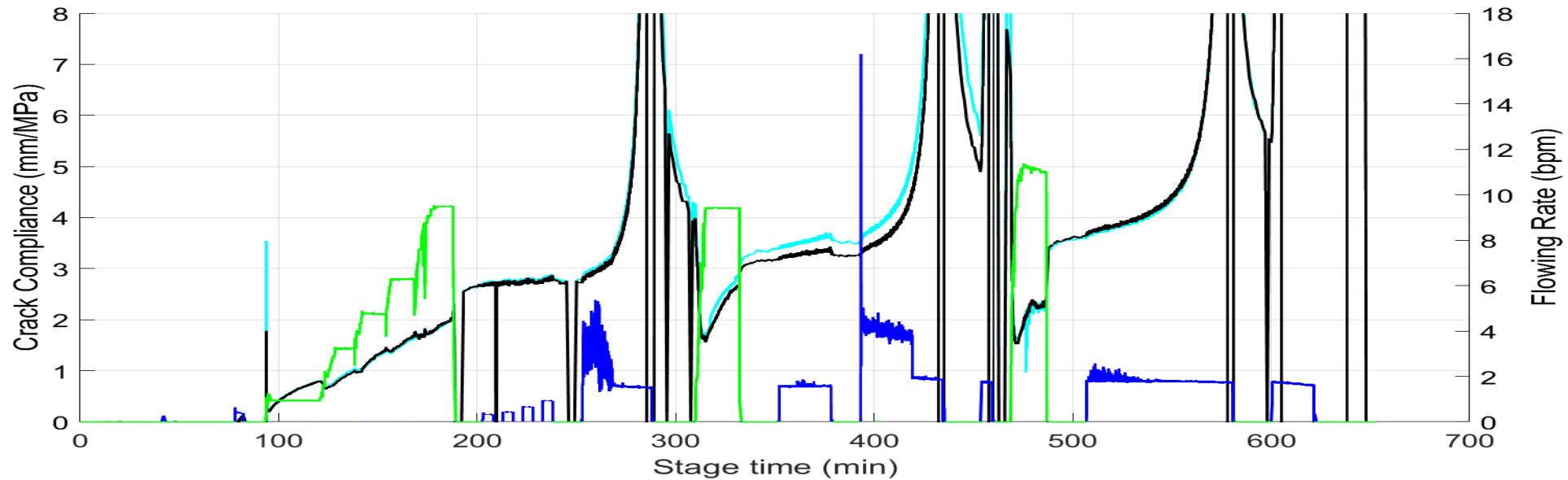
Field Measurement of Crack Compliance

All 03-Jun-2021 8:57 AM to 03-Jun-2021 7:50 PM



— Net Pressure — Caliper A — Caliper B

$$C = \frac{w}{P_{net}}$$



— Compliance (Caliper A) — Compliance (Caliper B) — Flowback Rate — Injection Rate

Field Compliance Compared to Elastic Compliance

Elastic Compliance:

$$V_{lens} = \frac{16p_{net}R^3}{3E'}, \quad w_o = \frac{8p_{net}R}{\pi E'}$$

$$E' = \frac{E}{1-\nu^2}$$

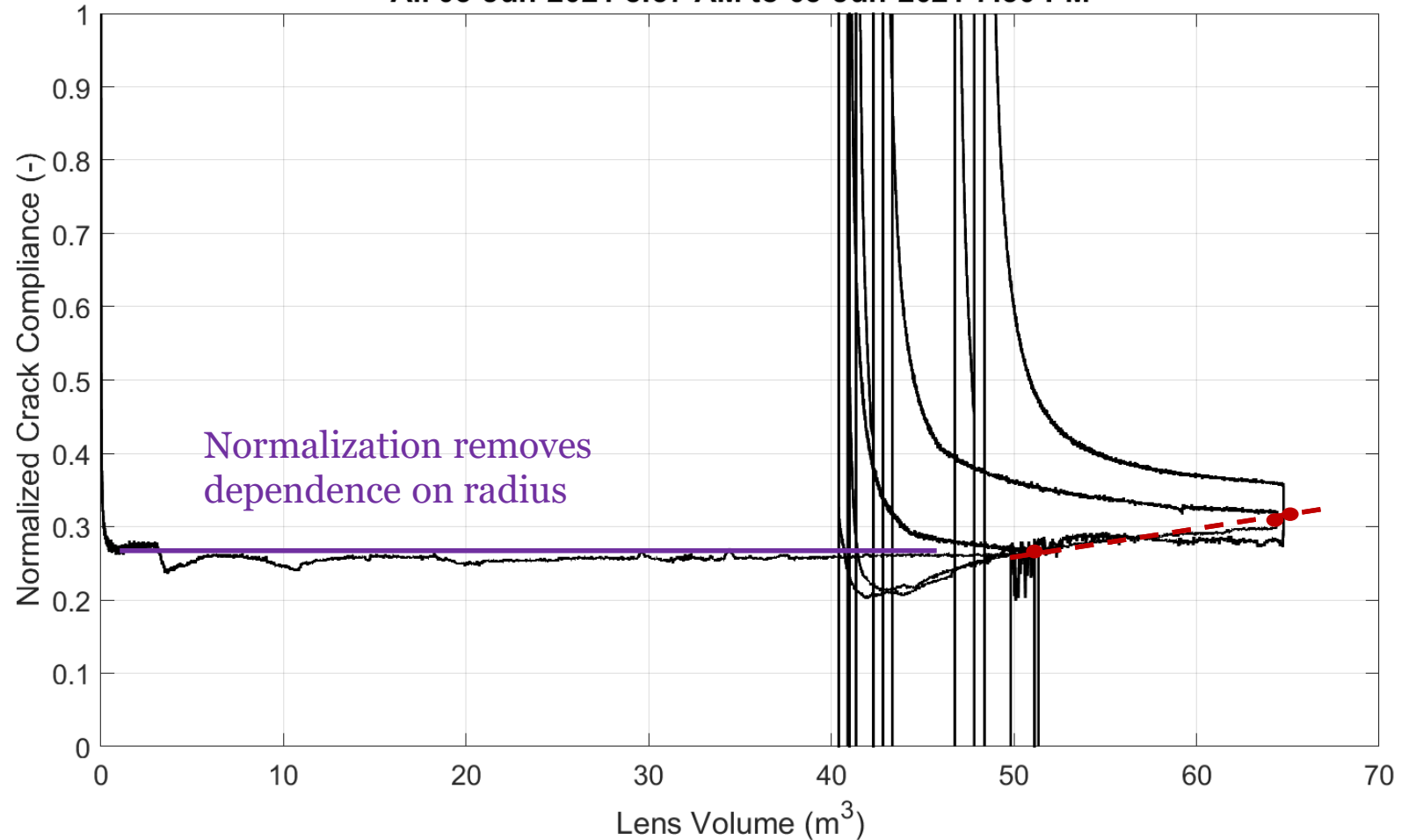
$$\Rightarrow \frac{w_o}{p_{net}} = \frac{2^{5/3}3^{1/3}}{\pi} \frac{V_{lens}^{1/3}}{E'^{2/3} p_{net}^{1/3}}$$

$$\text{let : } V_{lens} = \eta V_{inj}, \quad \boxed{E'_{app} = BE'_{meas}}$$

$$\chi = \frac{w_o}{\eta^{1/3} V_{inj}^{1/3}} \frac{E'^{2/3}}{p_{net}^{2/3}} \frac{\pi}{2^{5/3} 3^{1/3}} = \boxed{\frac{1}{B^{2/3}}}$$

Apparent elastic stiffness is around 8x higher than sonic log value of 15 GPa

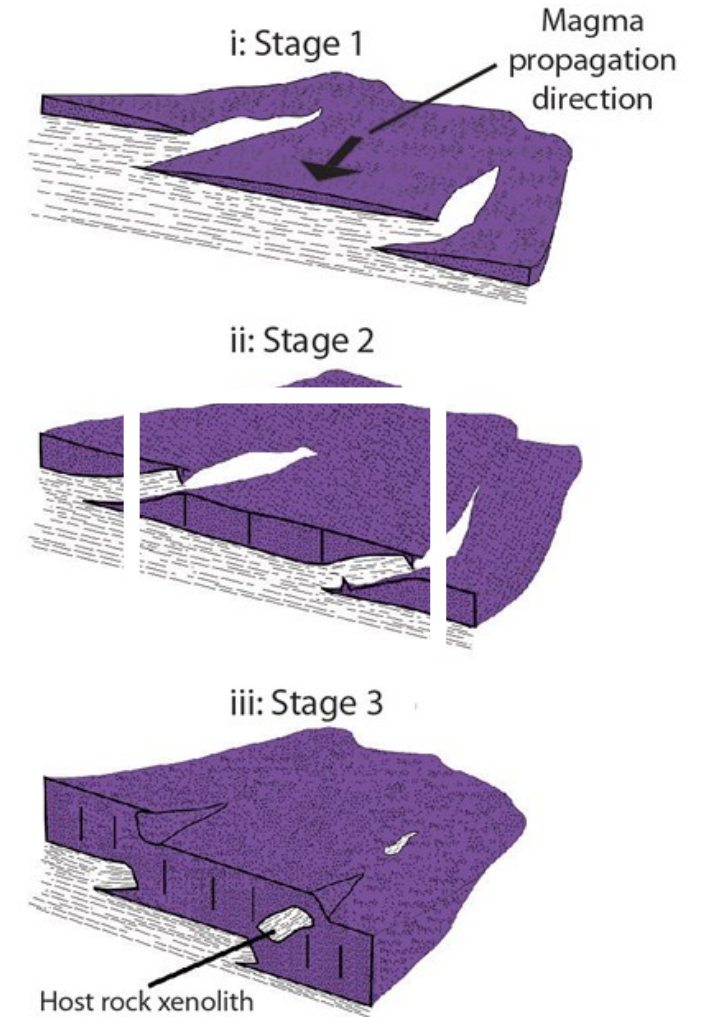
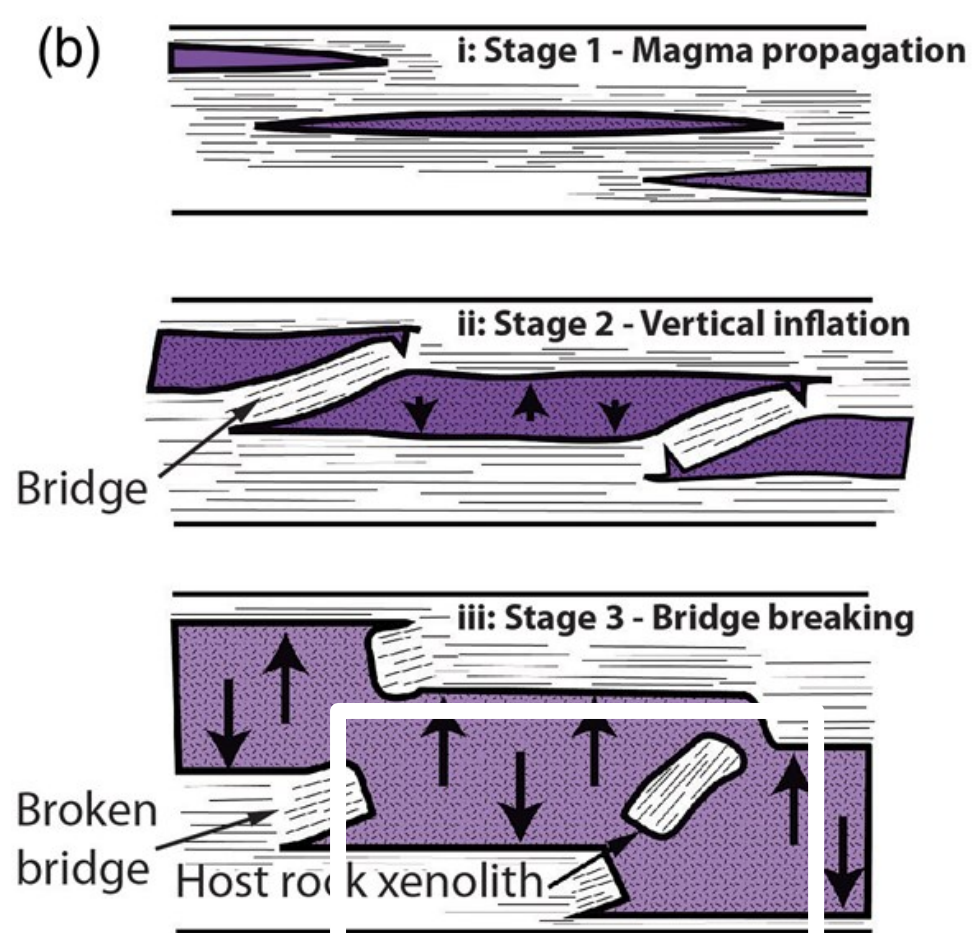
All 03-Jun-2021 8:57 AM to 03-Jun-2021 7:50 PM



Upward trend of compliance with multiple cycles – in LOBSTER this is captured by bridge breaking

Bridge Connections – Geological Evidence and Model

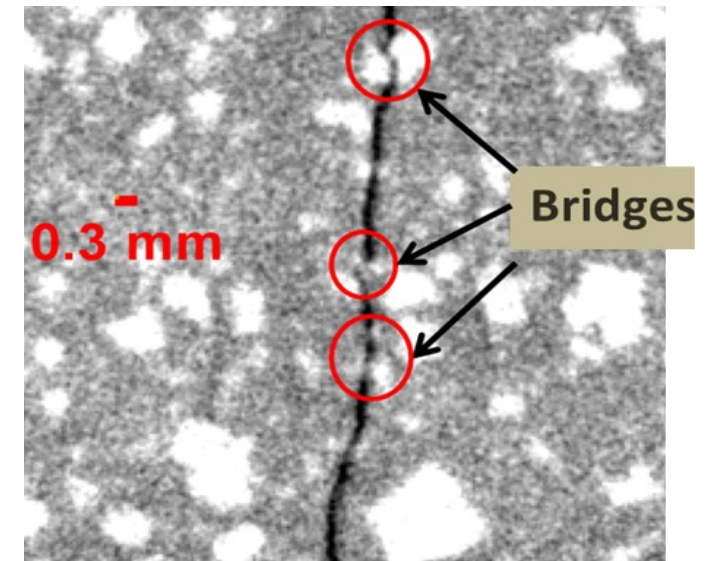
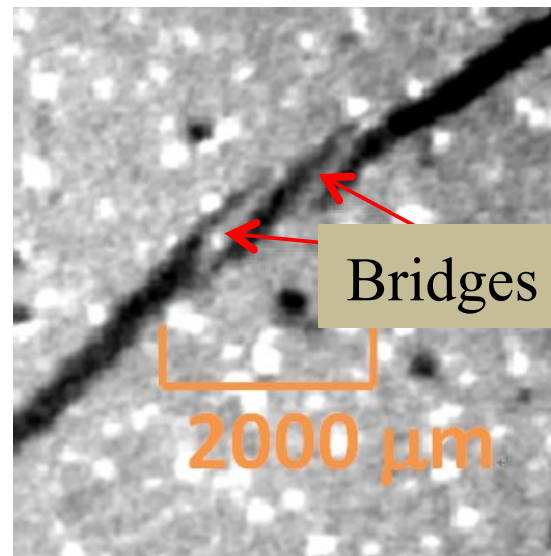
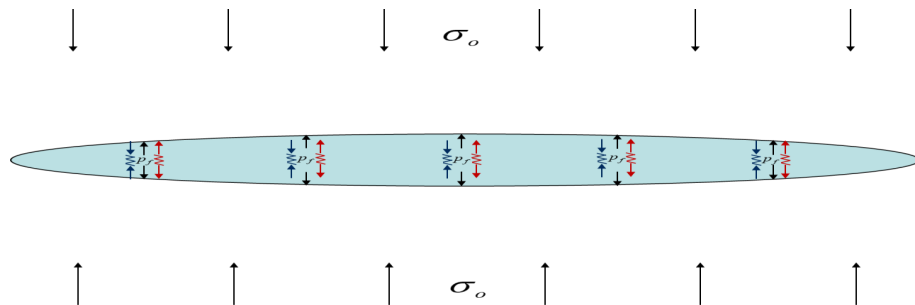
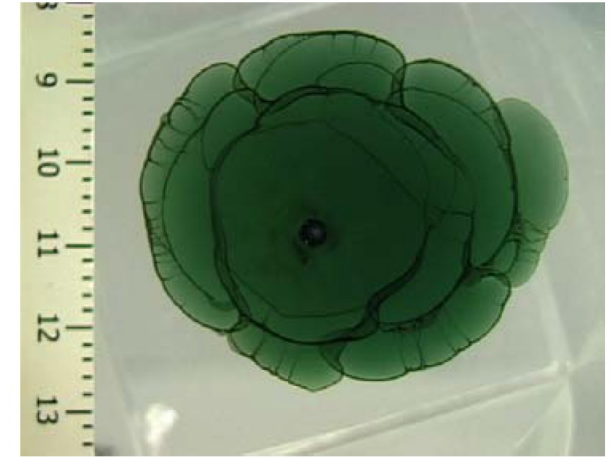
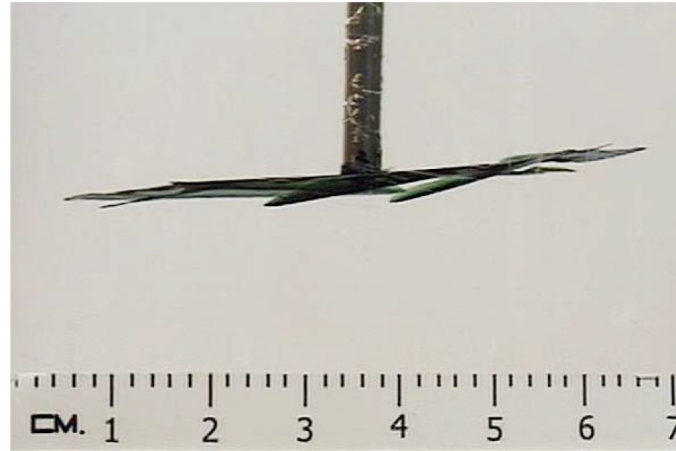
- Segmentation with subsequent overlap leads to bridges
- Continued inflation can break bridges



Bridge Connections – Evidence at Lab Scale

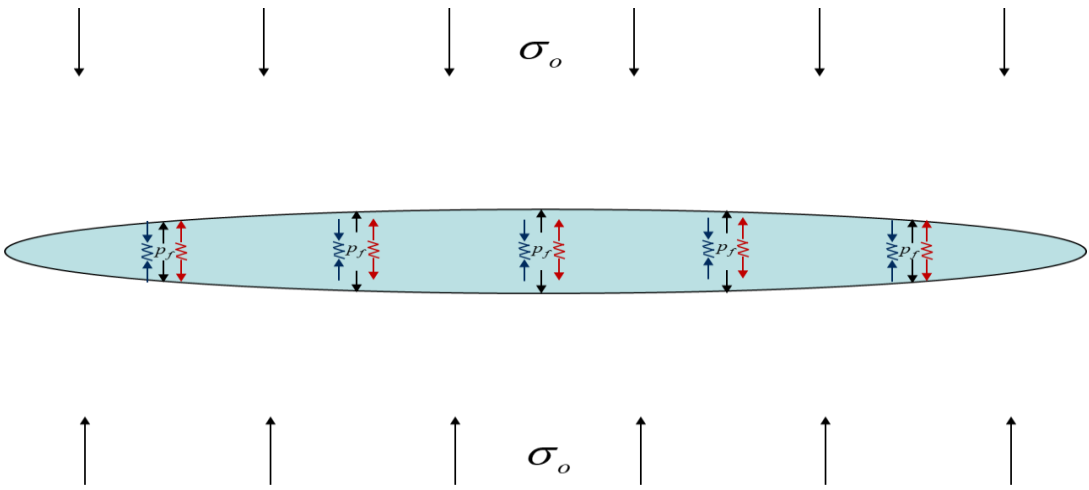
- Crack front bifurcation during growth leads to segmentation and hence intact rock bridges cutting through the lens
- Could act like tension springs that stiffen the lens, making it less compliant

Ruiting Wu, PhD Thesis, 2006 (Georgia Tech).



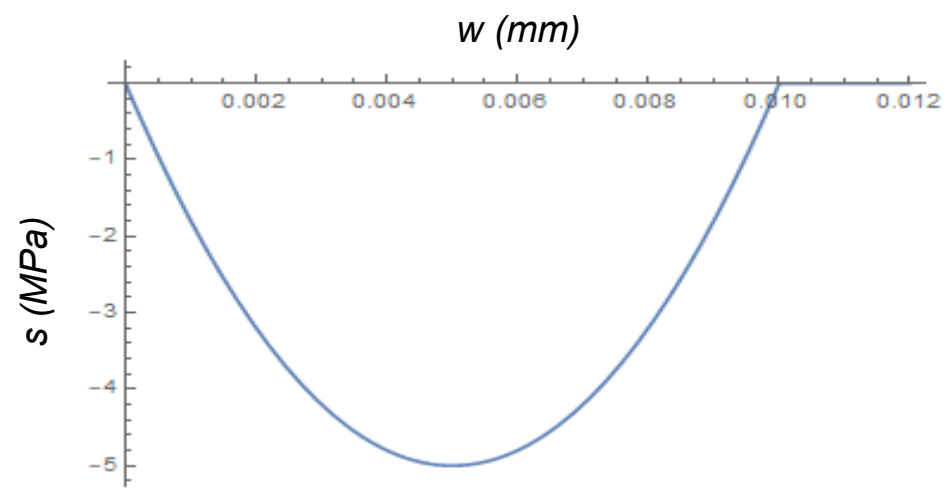
Dyskin, A.V., E. Pasternak, J. He, M. Lebedev & B. Gurevich, 2016. The role of bridge cracks in hydraulic fracturing. In: Proc. 10th Int. Conf. on Structural Integrity and Failure (SIF2016), Kotousov A, Ma J (eds), Adelaide, Australia, 2016, Paper #6.
He, J., E. Pasternak, A.V. Dyskin, M. Lebedev & B. Gurevich, 2017b. The constricted effect of bridges in hydraulic fracturing. ACAM 9.

Bridge Condition



$$p_{f,i} - \sigma_o = -\frac{E'}{R} \sum_{j=1}^m A_{ij} w_j - s_i(w_i)$$

$$s(w) = E \left[-\eta_T \left| \frac{w_T - w_{\max}}{w_T} \right|^{\alpha_T} \frac{w}{w_T} \hat{H}(w_T - w_{\max}) \right]$$

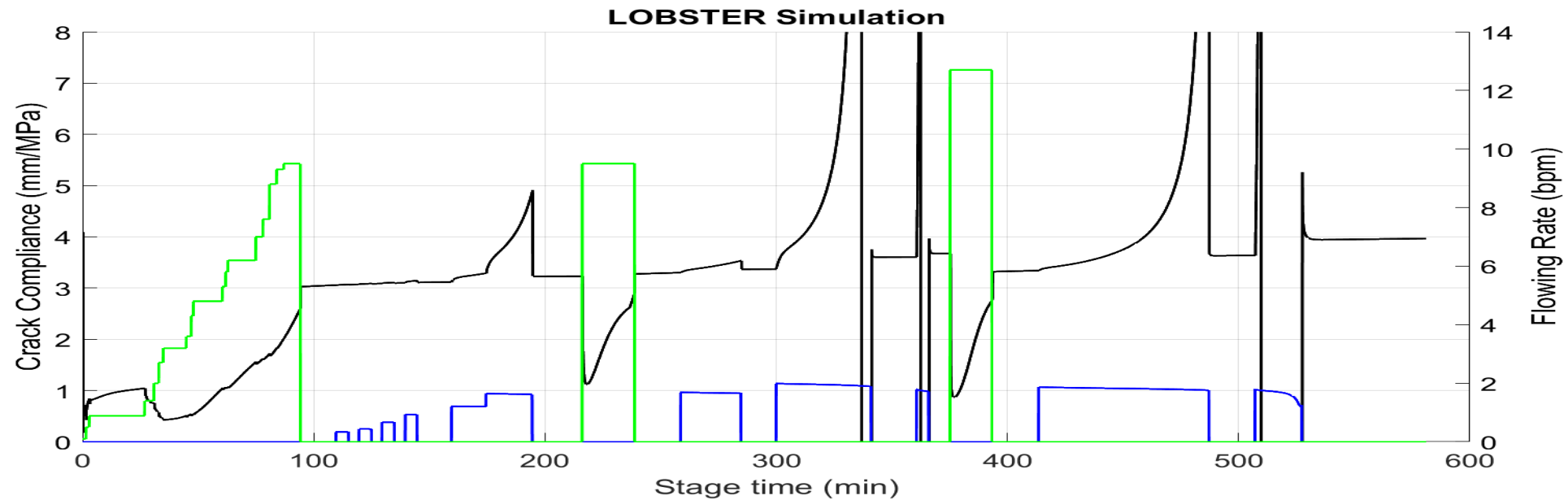
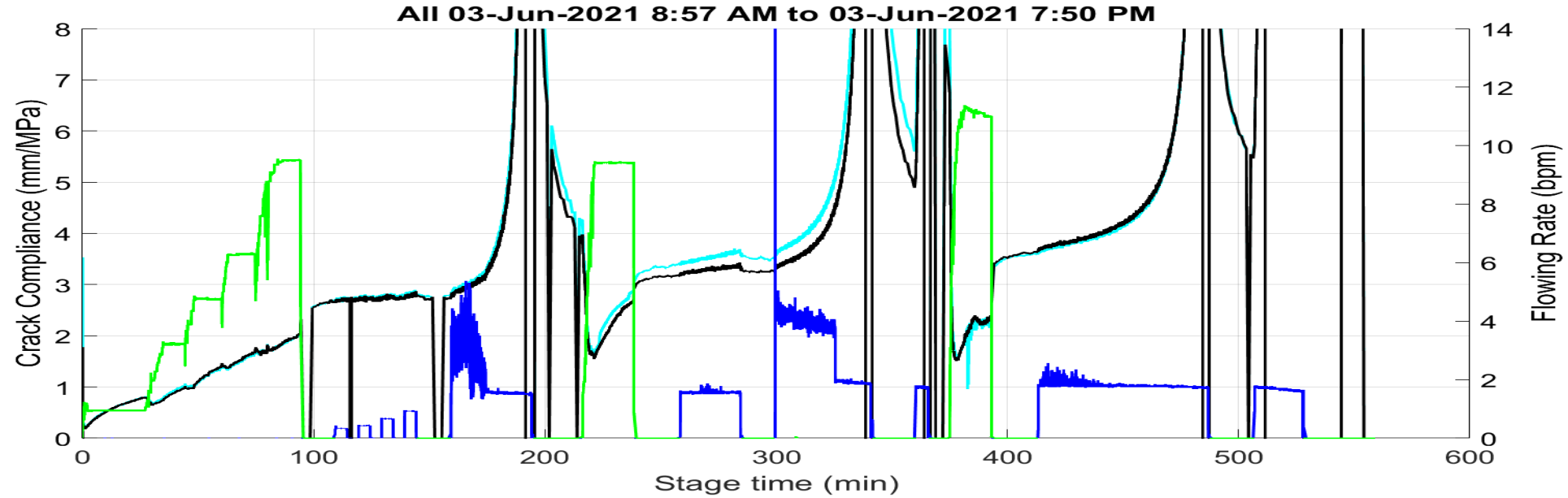


Bridges:

- Linear elastic stretching of bridges
- Non-linear decrease of bridge area as bridges permanently break
- History dependence – solution “remembers” largest width ever attained at each location
- Result: Increasing tension as width increases, and then decreasing tension as bridges break

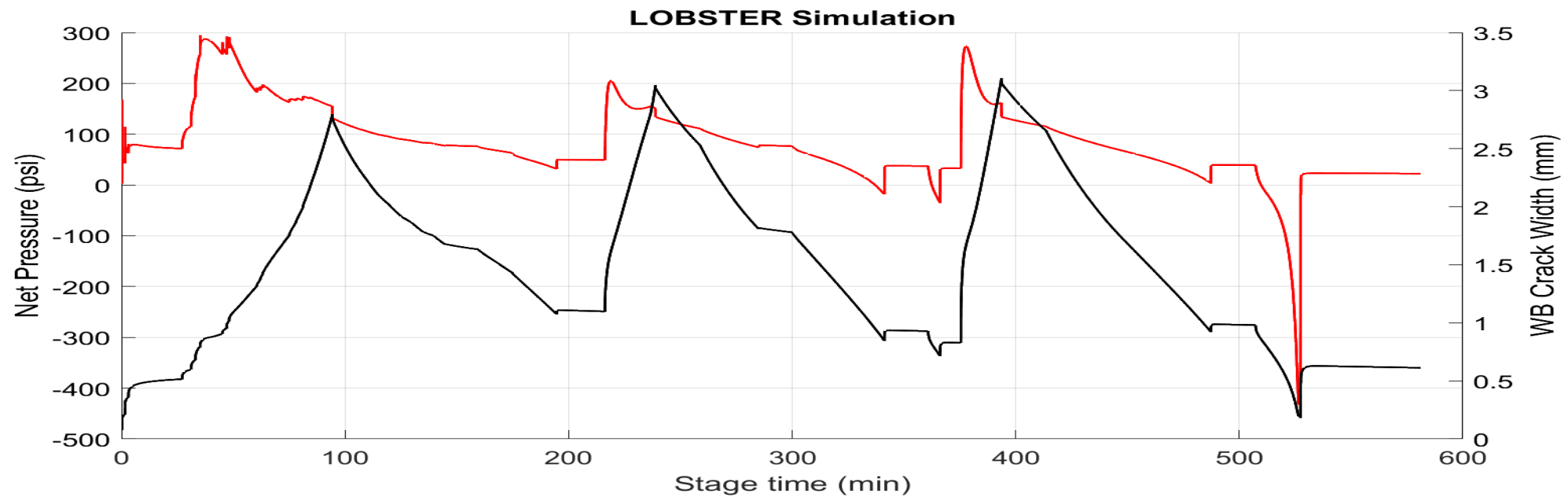
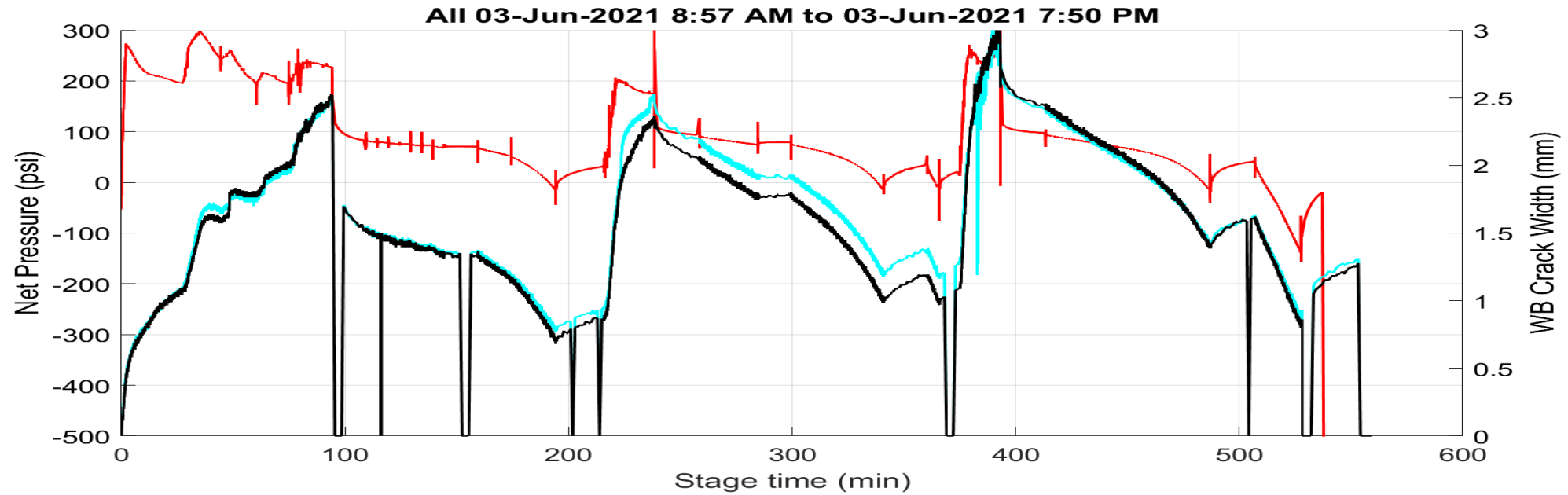
Bunger, A. P., Lau, H., Wright, S., & Schmidt, H. (2023). Mechanical model for geomechanical pumped storage in horizontal fluid-filled lenses. *International Journal for Numerical and Analytical Methods in Geomechanics*, 47(8), 1349-1372.

Field Data Indication of Bridges: Conrad LC5 Example



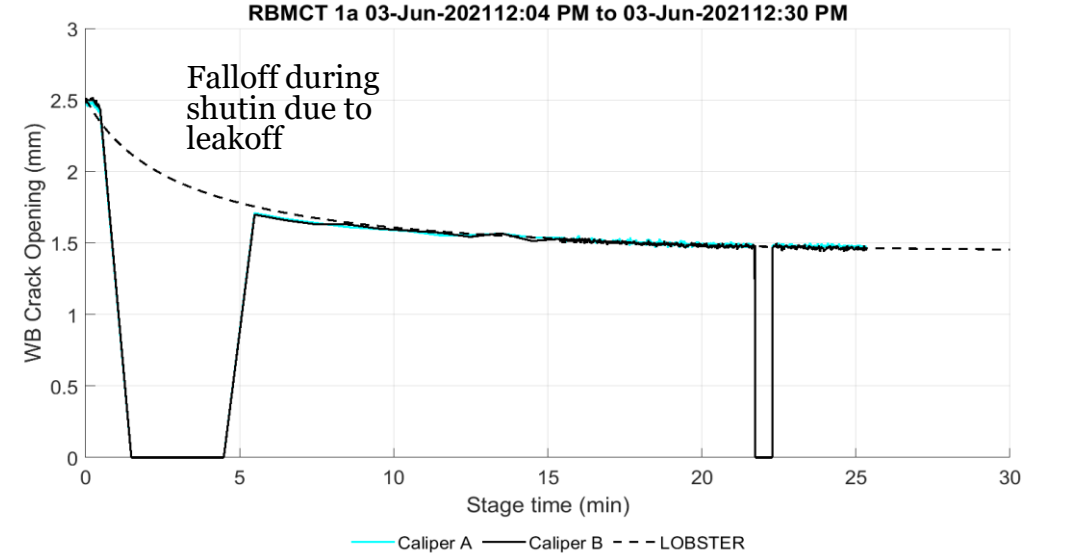
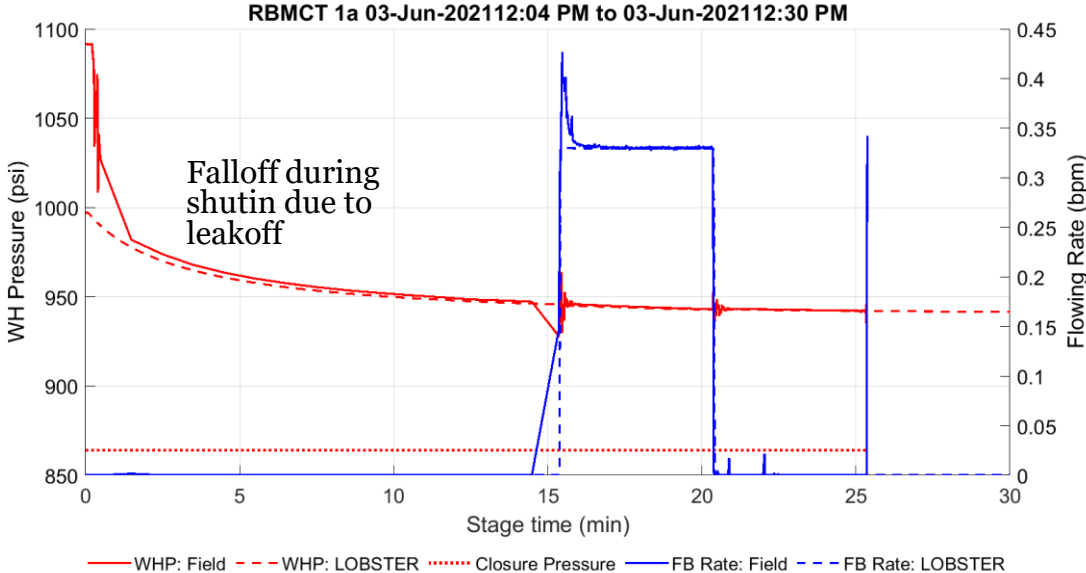
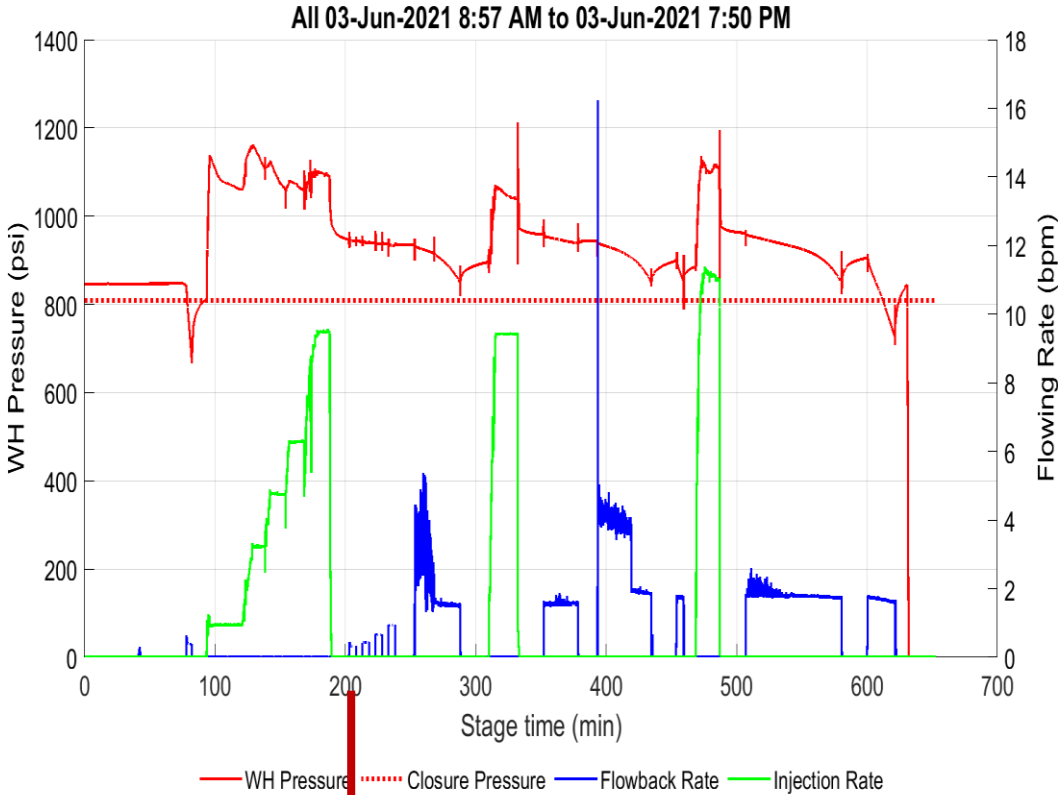
— Compliance (Caliper A) — Compliance (Caliper B) — Flowback Rate — Injection Rate

Field Data Indication of Bridges: Conrad LC5 Example

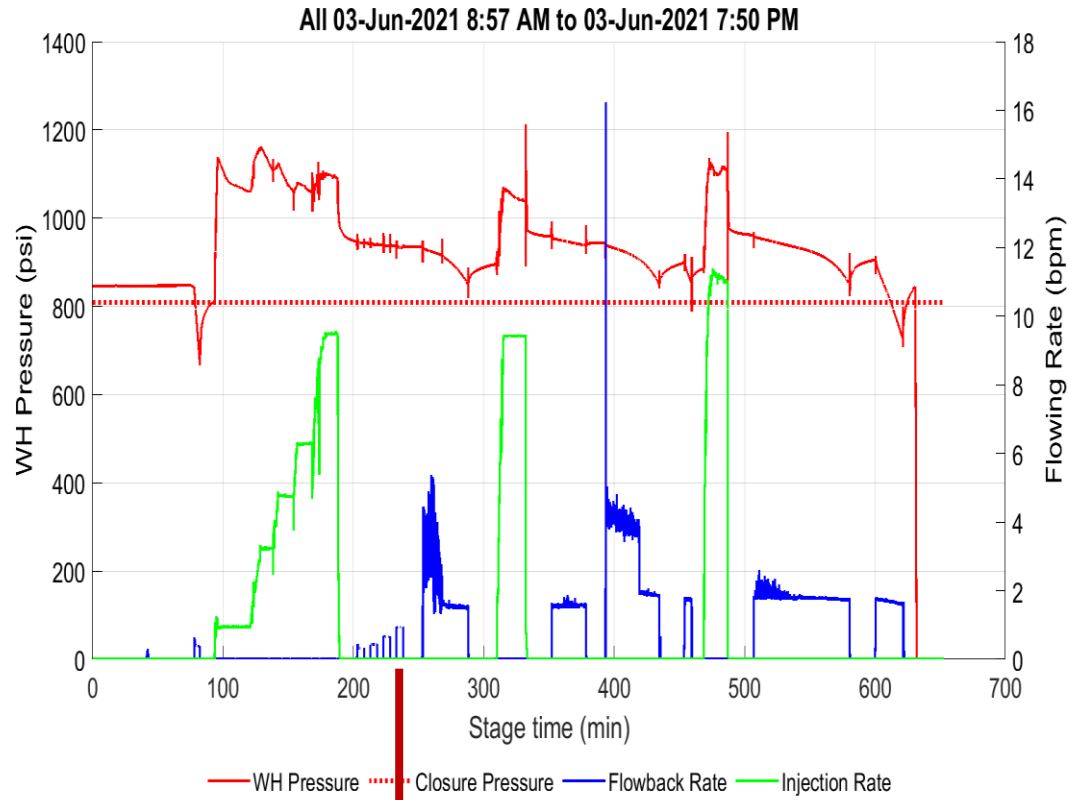


— Net Pressure — Caliper A — Caliper B

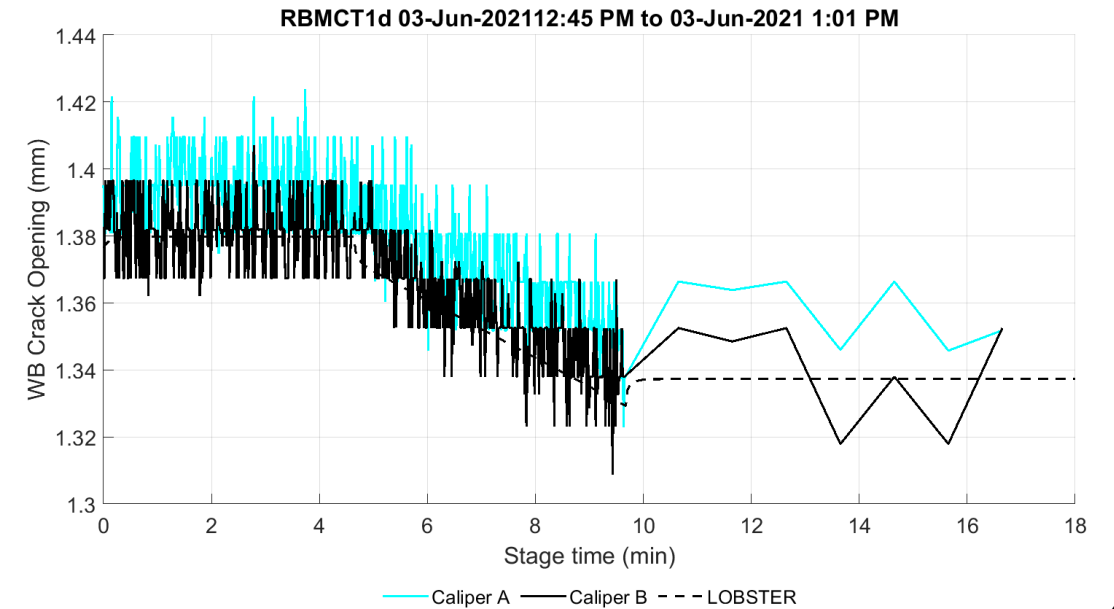
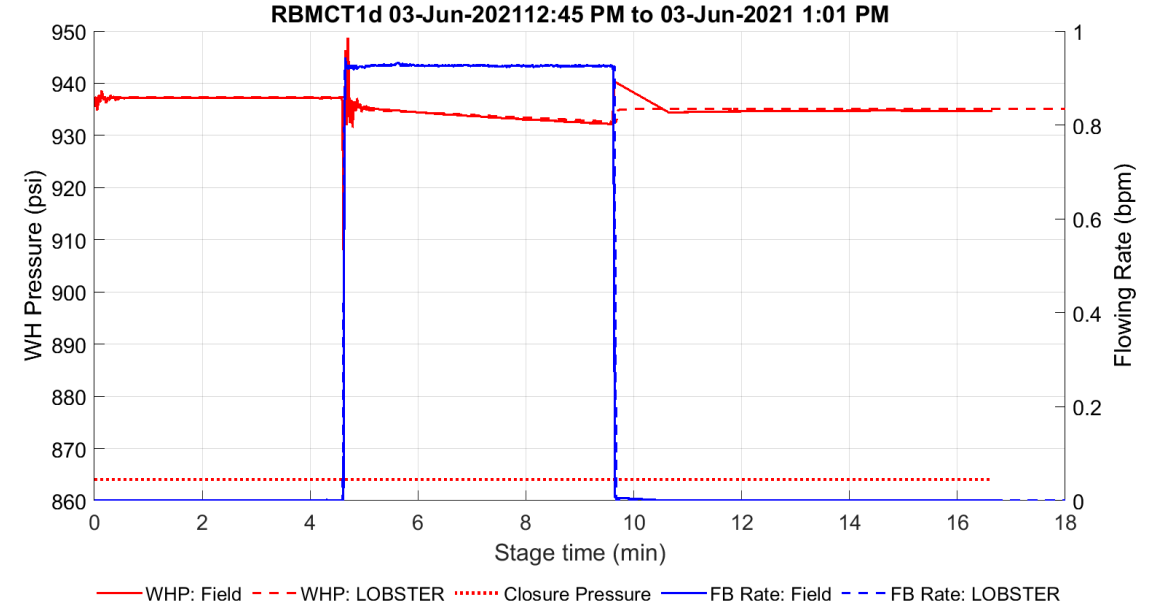
Fit between Model and Data: MCT1a



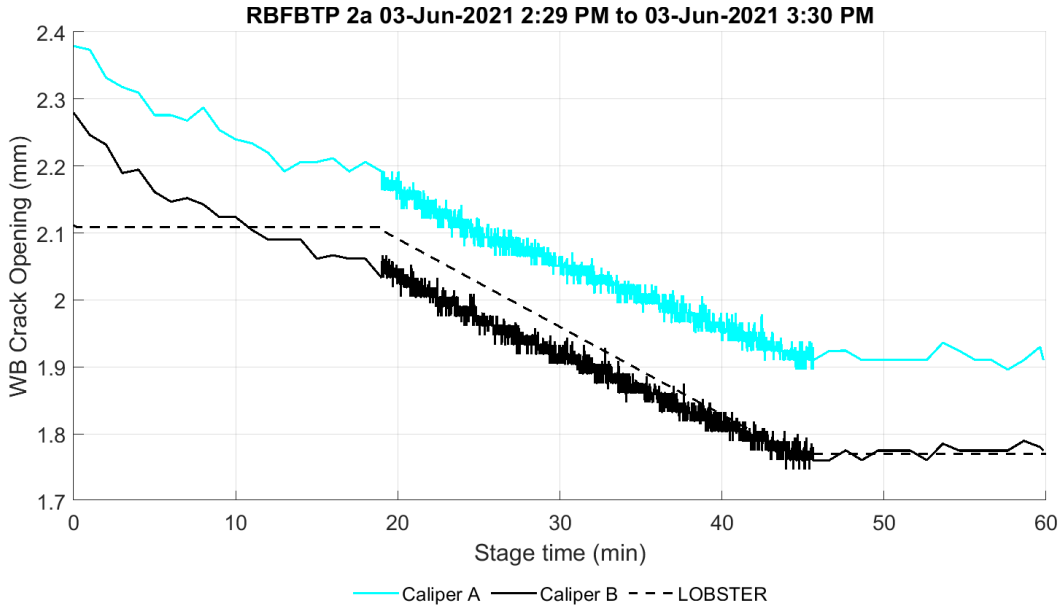
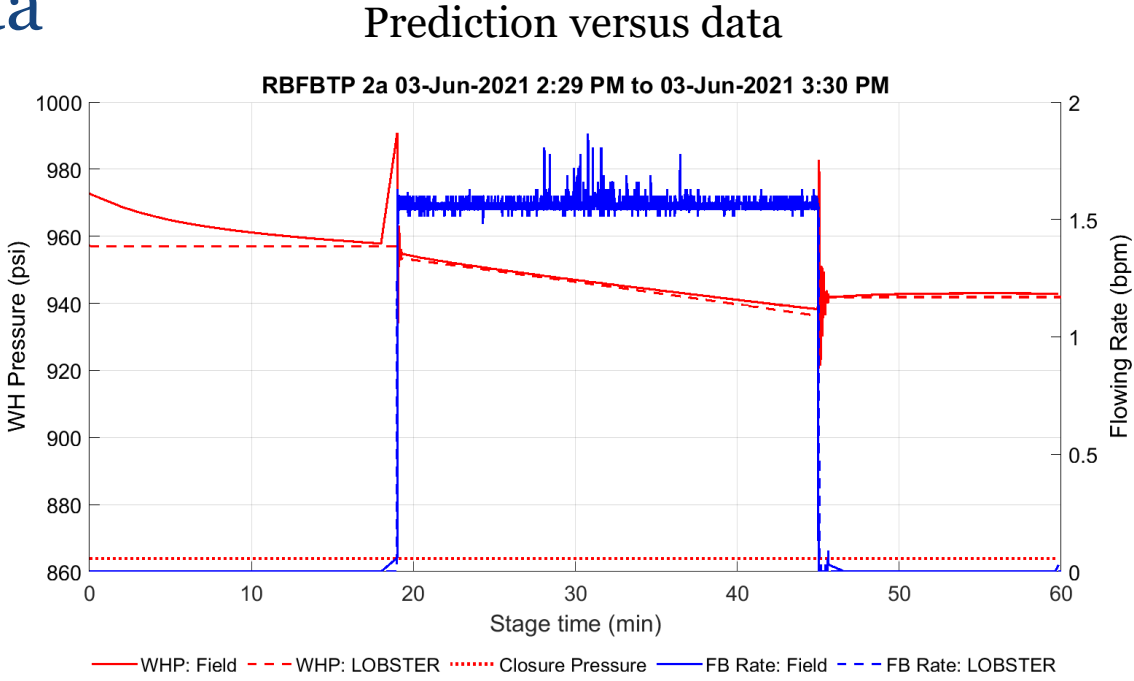
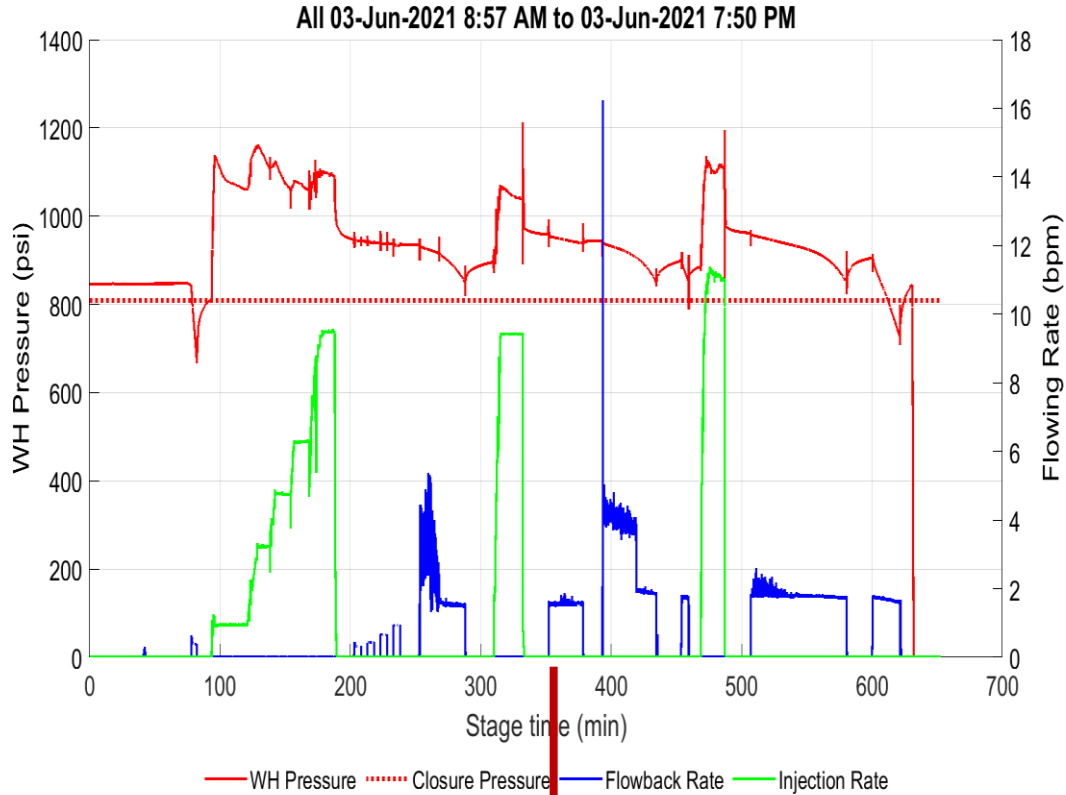
Goodness of Fit between Model and Data



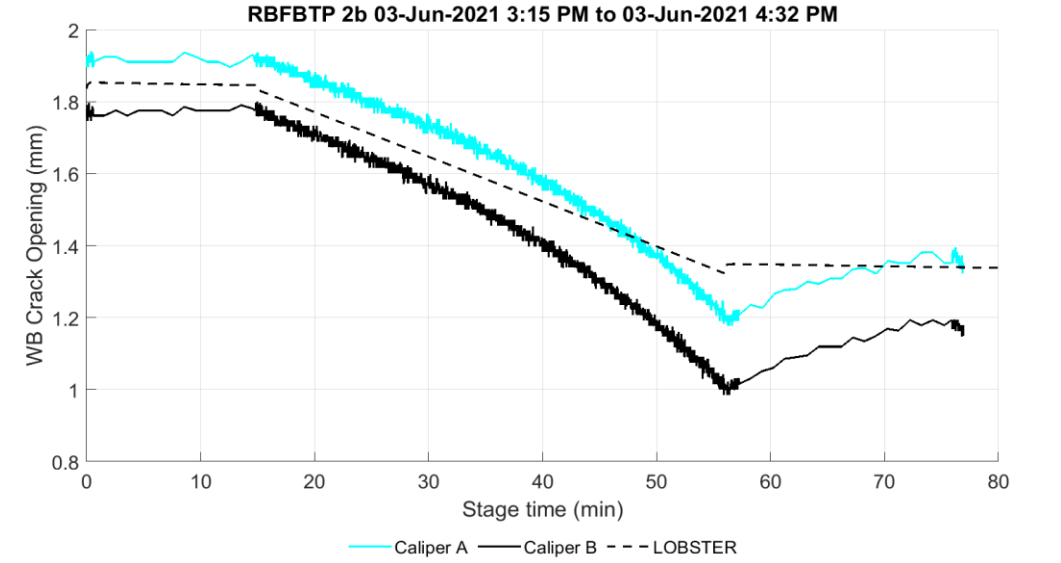
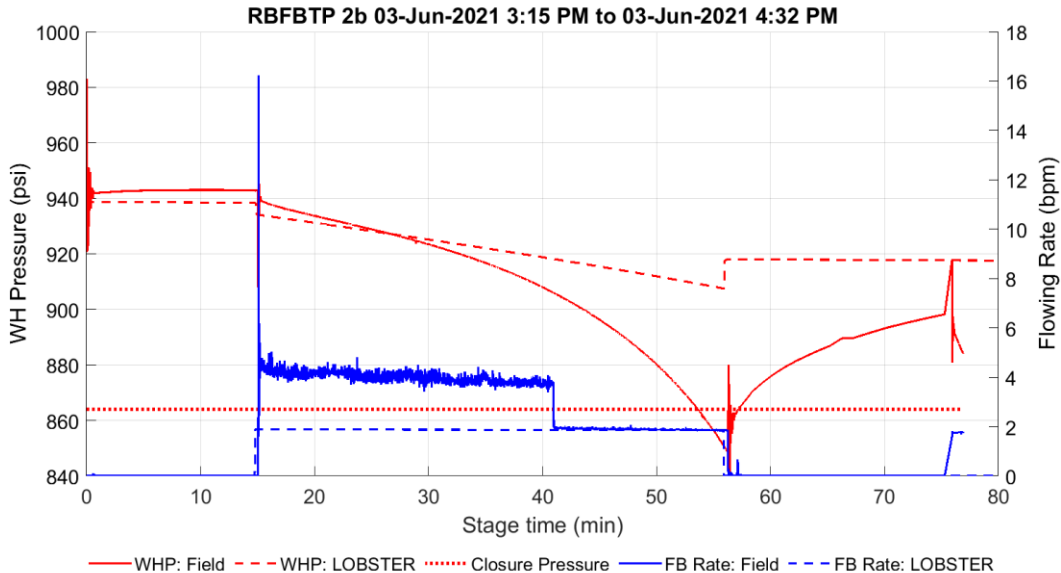
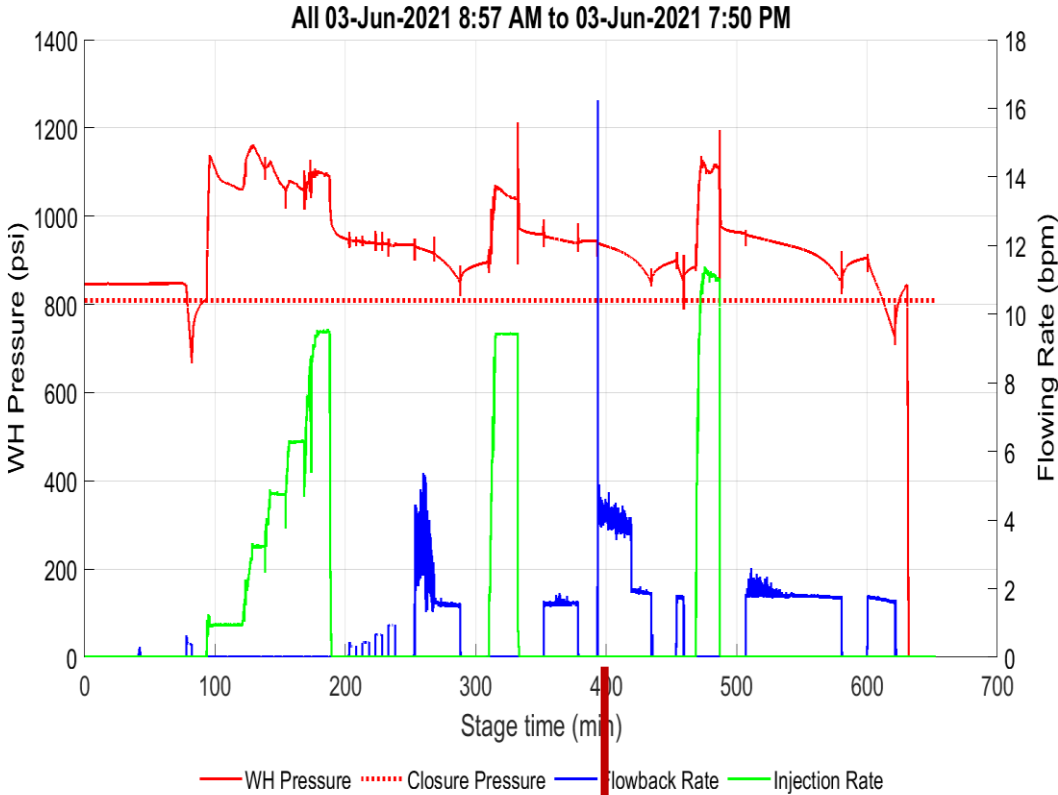
Prediction versus data



Goodness of Fit between Model and Data



Fit between Model and Data: FBTP2b



Conclusions

- Fluid-solid coupling in tip region leads to asymptotic behavior distinct from LEFM
- Rock failure is time-dependent
 - Failure can occur at load levels well below what is required for instantaneous failure
 - Acoustic emission signature shows coalescence of microcracks in leadup to failure
 - Aftershocks are generated from vicinity of failure surface
- Rock fracture involves segmentation resulting in intact bridges
 - Crack compliance is far below prediction from LEFM
 - Will lead to underprediction of pressure and length and overprediction of width when not accounted for in models

Acknowledgments

- Funding and permission to present field test data from Quidnet Energy
- Tiltmeter inversion/interpretation from CSIRO Energy
- Discussion of bridges (prior to this project) with Arcady Dyskin and Elena Pasternak (University of Western Australia), as well as a travel grant for APB from the Australian Research Council (as a part of DP190103260)

