Arbitrary order virtual element methods for high-order phase-field modeling of dynamic fracture

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Motivation

Materials failure in the form of fracture under extreme loading conditions happens in many engineering applications such as hydraulic fracturing, vehicle and space shuttle crash accidents, and more. These applications motivate us to develop accurate modeling and efficient prediction of fracture propagation in brittle and ductile materials under dynamic loading.
Moreover, it is often necessary to use small triangular elements to resolve intricate features of the geometry. As a result, this may require small time steps given the CFL condition for explicit time stepping. Polygonal elements can alleviate such difficulties because they often prevent the need to refine spatial discretization even in the presence of complicated geometrical features.

Governing equations

The governing equations of the high-order phase-field fracture model:

$$\rho\ddot{\boldsymbol{u}} - \nabla\cdot\boldsymbol{\sigma}(\boldsymbol{u},d) - \mathbf{f} = \mathbf{0}, \quad \text{on } \Omega\times(0,T],$$
$$\alpha_2\Delta^2 d - \alpha_1\Delta d + \alpha_0 d + g'(d)\mathcal{H}_t = 0, \quad \text{on } \Omega\times(0,T],$$

where $\alpha_2 = G_c\ell_0{}^3/2, \alpha_1 = G_c\ell_0$, and $\alpha_0 = G_c/(2\ell_0)$.

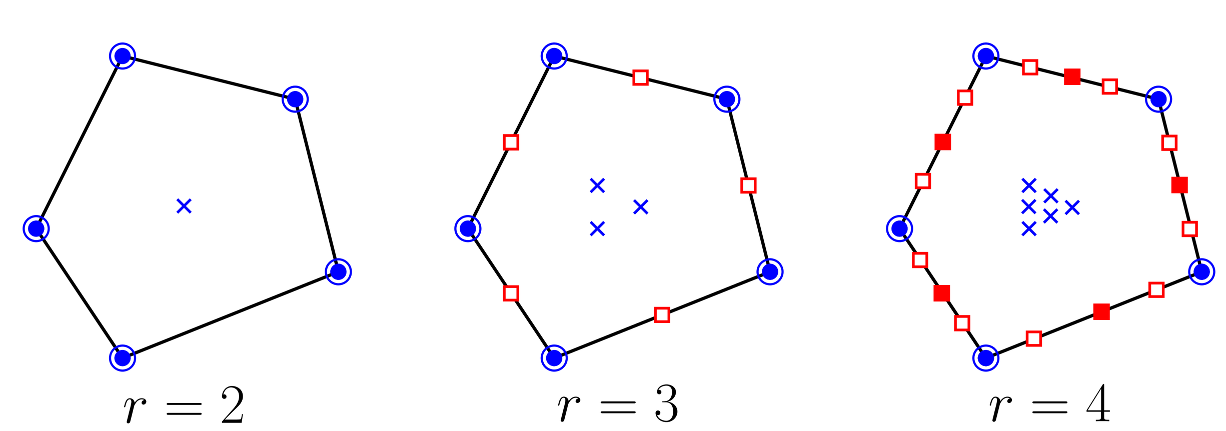
H^2-conforming VEM for phase-field

[D1]: for $r \geq 2$, $d_h(\mathbf{x}_{\mathrm{v}})$, $\partial_x d_h(\mathbf{x}_{\mathrm{v}})$, $\partial_y d_h(\mathbf{x}_{\mathrm{v}})$ $\quad \forall \mathrm{v}$ of $\partial\mathrm{E}$;

[D2]: for $r \geq 4$, $\frac{1}{|\mathrm{e}|}\int_{\mathrm{e}} q d_h \,\mathrm{d}S \quad \forall q \in P_{r-4}(\mathrm{e})$, and $\forall \mathrm{e} \in \mathcal{E}_{\mathrm{E}}$;

[D3]: for $r \geq 3$, $\int_{\mathrm{e}} q\partial_n d_h \,\mathrm{d}S \quad \forall q \in P_{r-3}(\mathrm{e})$, and $\forall \mathrm{e} \in \mathcal{E}_{\mathrm{E}}$;

[D4]: for $r \geq 2$, $\frac{1}{|\mathrm{E}|}\int_{\mathrm{E}} d_h d \,\mathrm{d}V \quad \forall q \in P_{r-2}(\mathrm{E})$.



The degrees-of-freedom of the scalar conforming virtual element spaces, ($r = 2, 3, 4$) on a pentagon, which approximates the scalar field solving the high-order phase-field equation. The solid blue circles and empty blue circles are **[D1]**, solid red squares are **[D2]**, empty red squares are **[D3]**, and blue crosses are **[D4]**.

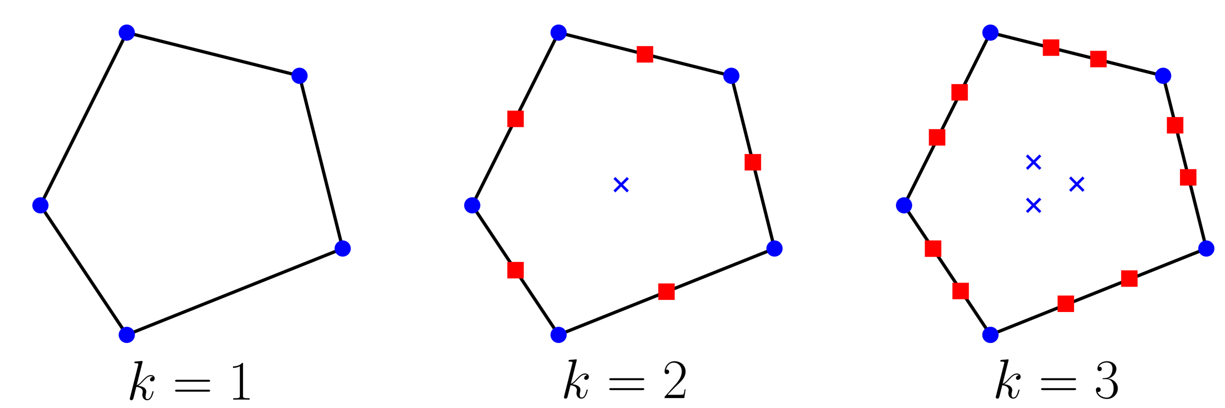
H^1-conforming VEM for displacement

[U1]: the values of u_h^i at the vertices of E;
[U2]: the edge polynomial moments of u_h^i of order up to $k-2$ on each one-dimensional edge $e \in \mathcal{E}_{\mathrm{E}}$:

$$\frac{1}{|\mathrm{e}|}\int_{\mathrm{e}} u_h^i\, m \,\mathrm{d}S, \quad \forall m \in M_{k-2}(\mathrm{e}), \forall \mathrm{e} \in \mathcal{E}_{\mathrm{E}};$$

[U3]: the element polynomial moments of u_h^i of order up to $k-2$ on E:

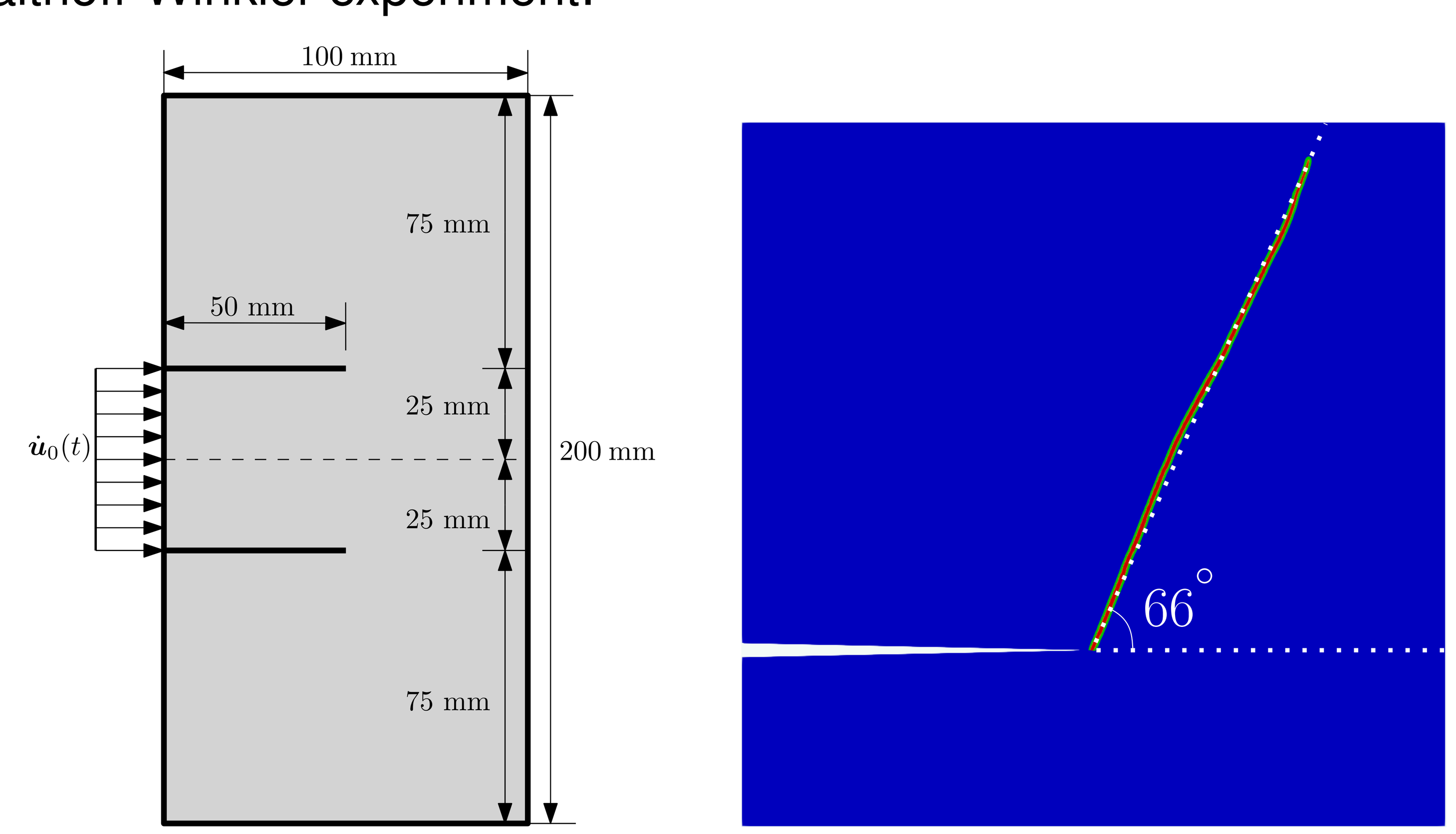
$$\frac{1}{|\mathrm{E}|}\int_{\mathrm{E}} u_h^i\, m \,\mathrm{d}V, \quad \forall m \in M_{k-2}(\mathrm{E}).$$



The degrees-of-freedom of the scalar conforming virtual element spaces, ($k = 1, 2, 3$) on a pentagonal cell, which approximates the components of the vector-valued displacement field $\boldsymbol{u}$, solving the momentum balance equation. Blue circles, red squares, and blue crosses are **[U1]**, **[U2]**, and **[U3]**, respectively.

Dynamic crack propagation

Kalthoff-Winkler experiment:



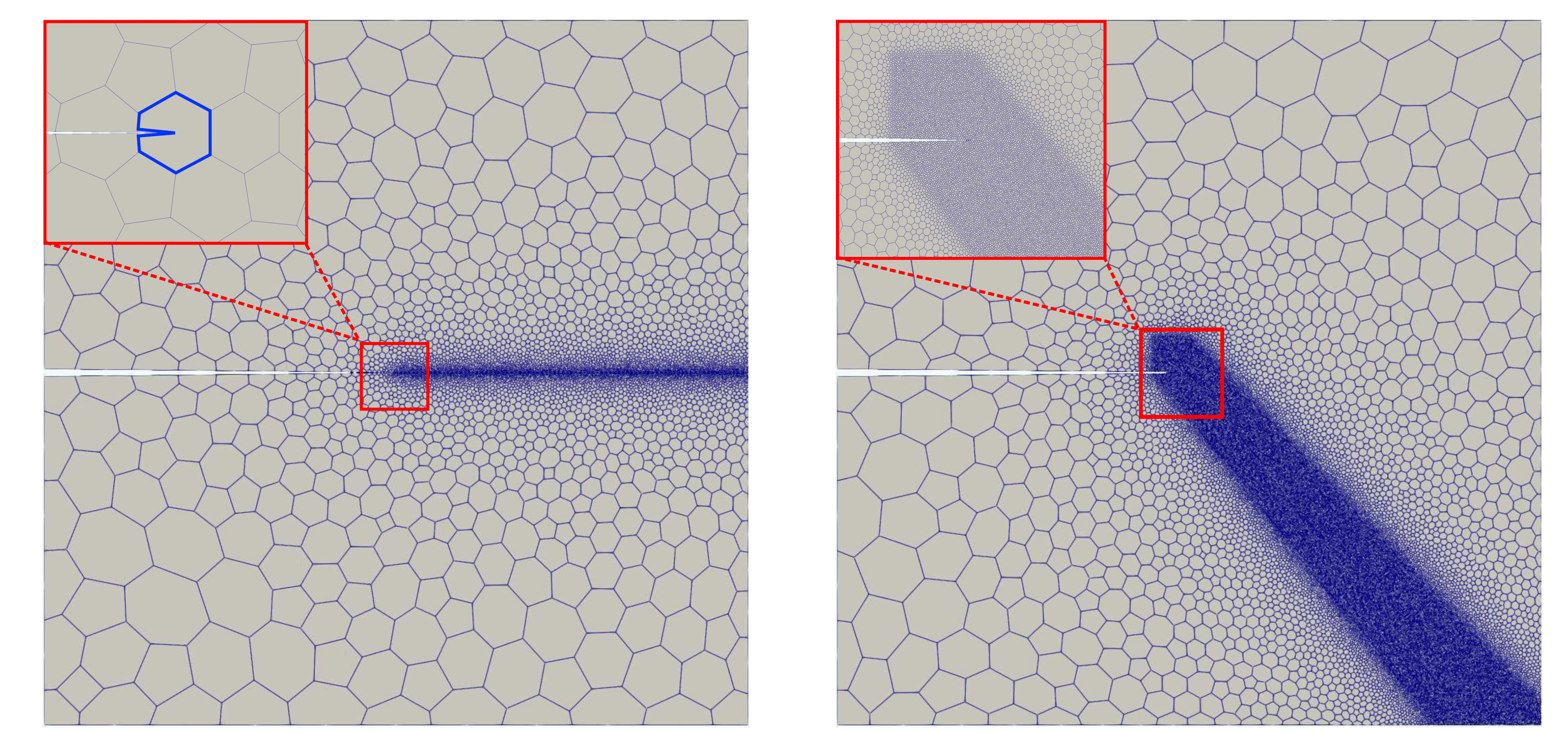
Geometry and boundary conditions

Crack pattern

The angle between the crack propagation direction and the initial notch plane is 66° which agrees with experimental results.

Quasi-static crack propagation

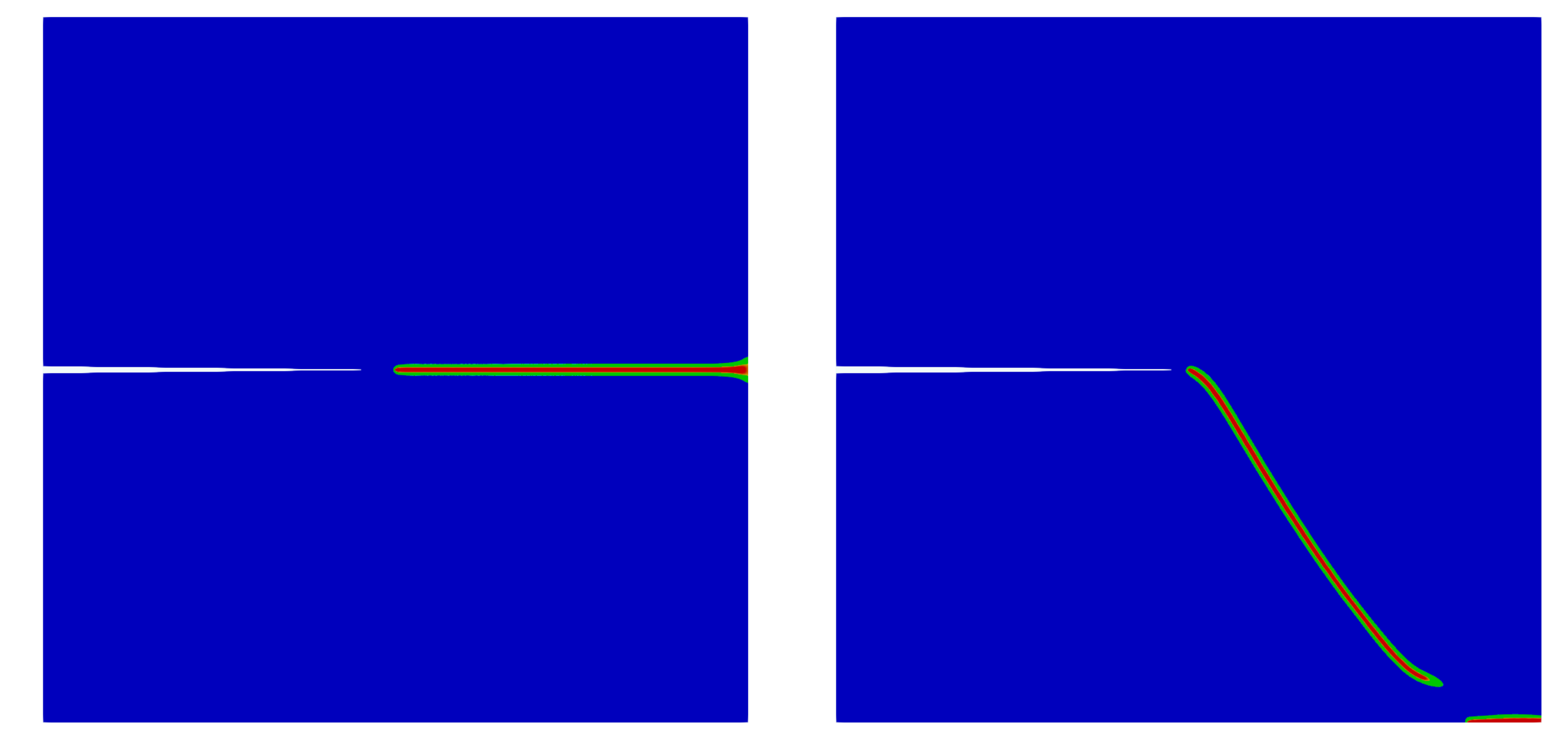
Polygonal mesh:



Tensile

Shear

Crack patterns at the end of the loading:



Tensile

Shear

Conclusions

- We have developed a virtual element framework to solve dynamic fracture problems governed by the high-order phase-field model on polygonal meshes.
- We have verified our numerical framework by simulating benchmark quasi-static tensile and shear tests, and the Kalthoff-Winkler dynamic fracture experiment.

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