AI-based identification of coupling regions for local and non-local one-dimensional coupling approaches

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Motivation

- Peridynamic simulations are computational expensive
- Applying local boundary conditions in non-local models is not trivial

Solution:

- Coupling of non-local and local models
  - Apply boundary conditions in the local region
  - Apply peridynamics in the **region** where we have crack and fractures

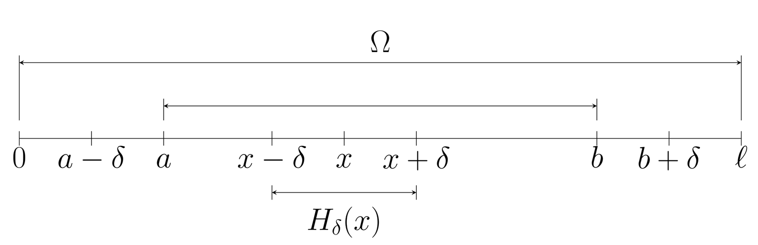
Can we use machine learning to identify the peridynamic region?

Model Problem

Classical linear elasticity model in 1D (with cross-sectional area $A = 1$):

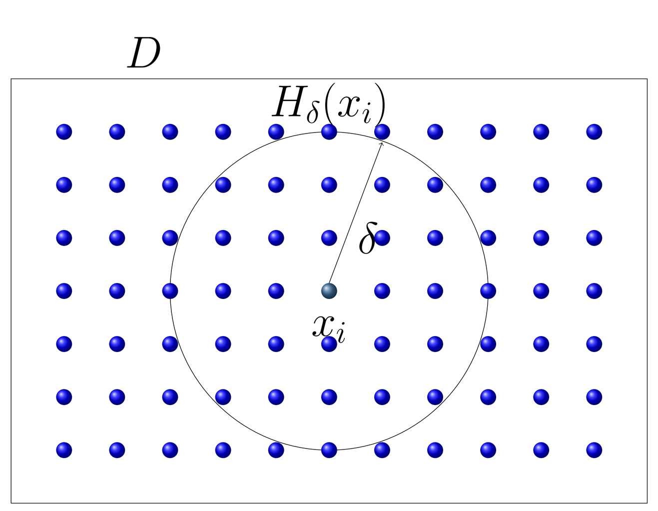
$$\begin{aligned} -E''(x) &= f_b(x), \quad \forall x \in \Omega = (0, \ell), \\ (x) &= 0, \quad \text{at } x = 0 \\ E'(x) &= g, \quad \text{at } x = \ell \end{aligned}$$

Coupling with peridynamic model:
Nonlocal model in $= (a, b) \subset \Omega$ where $\delta =$ horizon.



Background

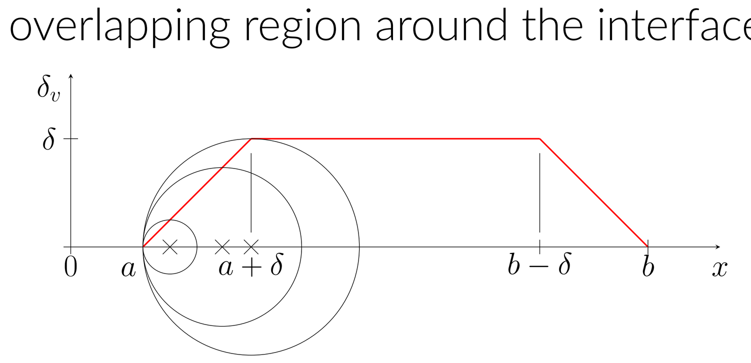
Peridynamics: (NLM)



- A non-local alternative formulation of classical continuum mechanics
- No differentials of displacement fields are used, which makes it an attractive framework for modeling and simulating fracture mechanics applications [1].

Variable Horizon Coupling Method (VHCM):

The main idea of the method is to make the horizon δ decrease to zero as one approaches the interfaces $x = a$ and $x = b$ so as to avoid the need to introduce an overlapping region around the interfaces [2].



Variable horizon function:

$$\delta_v(x) = \begin{cases} x - a, & a < x \le a + \delta \\ \delta, & a + \delta < x \le b - \delta \\ b - x, & b - \delta < x < b \end{cases}$$

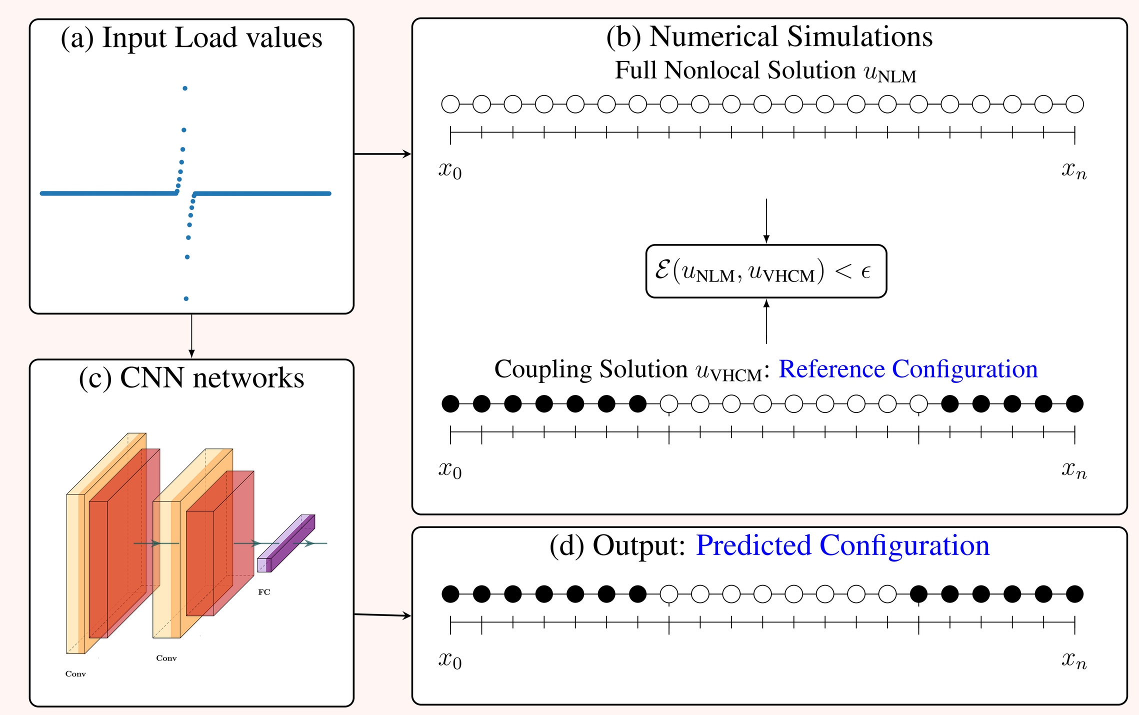
$$\bar{\kappa}(x)\delta_v^2(x) = \kappa\delta^2, \quad \forall x \in$$

References

[1] Silling, Stewart A. "Reformulation of elasticity theory for discontinuities and long-range forces." Journal of the Mechanics and Physics of Solids 48.1 (2000): 175-209.
[2] P. Diehl and S. Prudhomme. "Coupling approaches for classical linear elasticity and bond-based peridynamic models." Journal of Peridynamics and Nonlocal Modeling, 4(3):336–366, 2022.

Structure of Investigation

The input data consists of the second derivative of the load $f_b(x)$. The output of the process is a label associated with each node. A coupling configuration is added to the training set when it induces a coupled solution whose error, with respect to the fully nonlocal solution, is below a given tolerance. CNN architecture is used as deep neural network model.



Loads

The load functions employed in this study include load functions inducing discontinuous solutions with a finite jump at the discontinuity point. The investigation was extended to include the family of loads characterized by solutions of polynomial expressions of degree 3 and lower; which induce full local behavior.

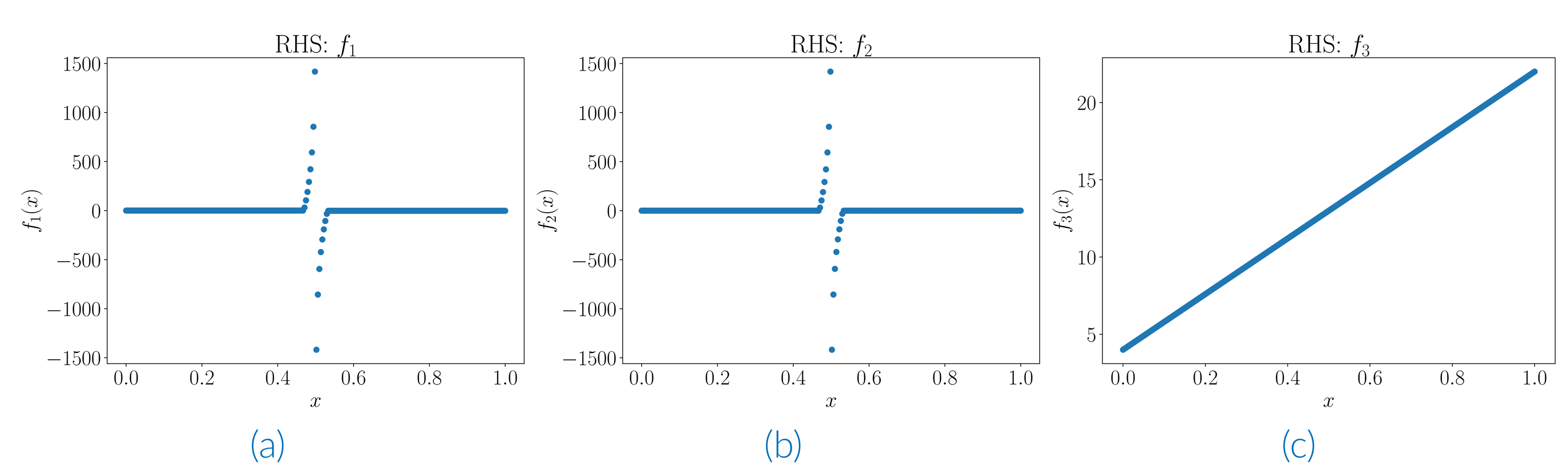


Figure 1. RHS functions $f_i(x), i = 1..3$.

Results

Case 1: Full Input Vector

- **Input**: Full Load Vector
- **Output**: Label (NLM or LM) for each discrete node

	Train	Test	Validation	Total
Case 1	2313	463	308	3084

Prediction Evaluation: Estimate nonlocal region borders (a, b) from predicted output -> Solution estimation based on full nonlocal and coupling algorithms -> Error calculation between Full nonlocal and Coupling solution

Results (Cont.)

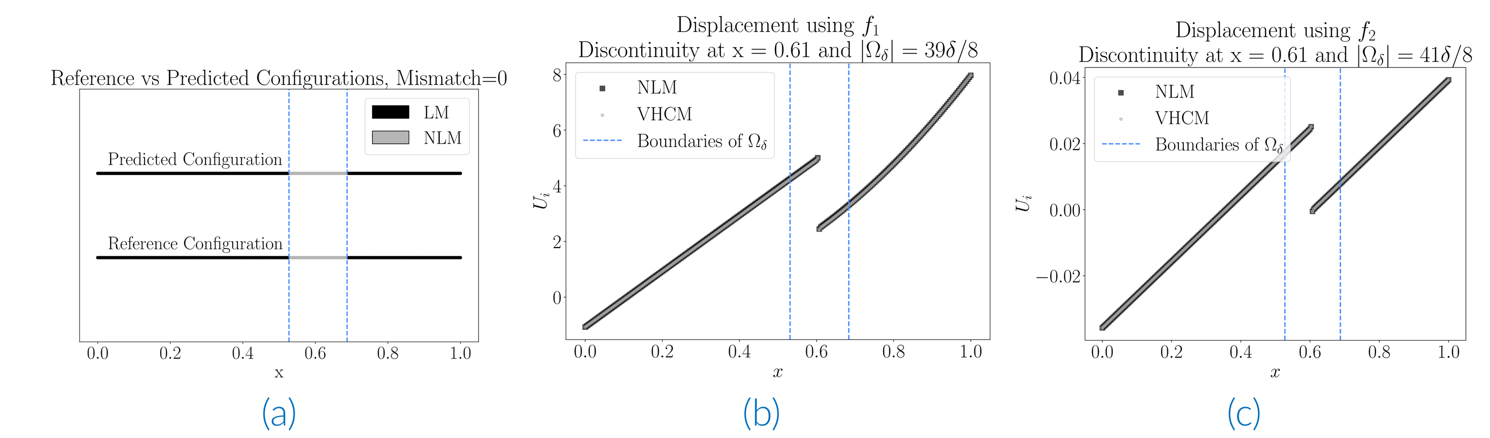


Figure 2. Error estimation after prediction (a) Comparison between the reference and predicted configurations of local and nonlocal regions. (b) displacement fields u_{NLM} (■) and u_{VHCM} (•) using load f_1. (c) displacement fields u_{NLM} and u_{VHCM} using load f_2. **Error estimation:** $\mathcal{E}_{f_1}(u_{NLM}, u_{VHCM}) = 4.036 \times 10^{-4}$. and $\mathcal{E}_{f_2}(u_{NLM}, u_{VHCM}) = 4.035 \times 10^{-4}$.

Case 2: Window Based Case

- **Input**: Window load
- **Output**: Label (NLM or LM) for each discrete node

	Train	Test	Validation	Total
Case 2	109885	22959	15312	148156

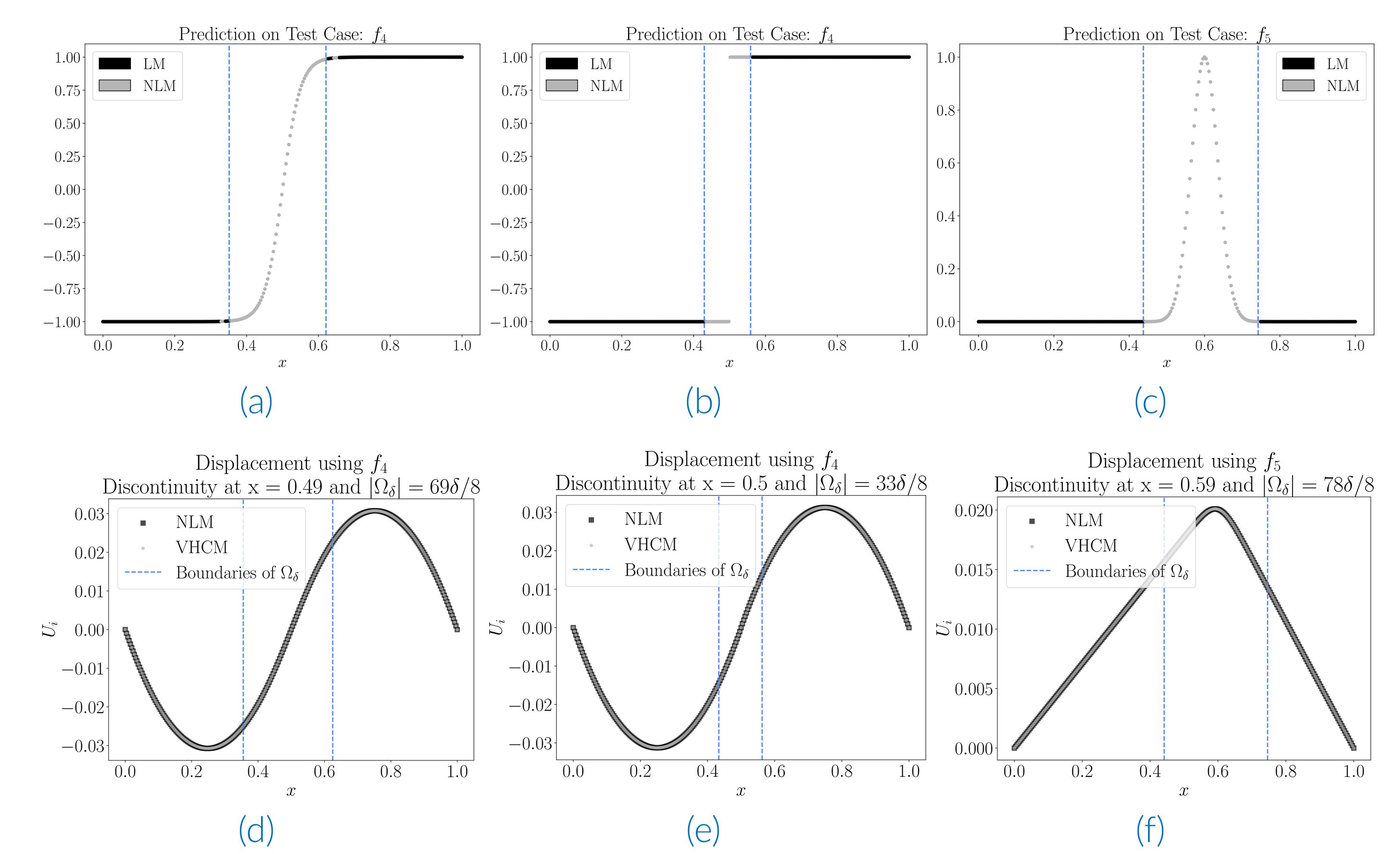


Figure 3. Nonlocal region detection and error estimation after prediction for general test cases. **(Top)** Predicted configuration of local and nonlocal regions. **(Bottom)** Displacement fields u_{NLM} and u_{VHCM}. **Error Estimation: (a-d)** $f_4 = tanh((x - 0.5)/t), t = 0.05$. The error $\mathcal{E}(u_{NLM}, u_{VHCM})$ is 1.73×10^{-4}. **(b-e)** $f_4 = tanh((x - 0.5)/t), t = 0.0005$. The error $\mathcal{E}(u_{NLM}, u_{VHCM})$ is 1.75×10^{-4}. **(c-f)** $f_5 = e^{-(20x-10-c)^2}, c = 2$. The error $\mathcal{E}(u_{NLM}, u_{VHCM})$ is 9.93×10^{-5}.

Conclusion And Outlook

Conclusion: This study shows the proof of concept for the AI-based identification of overlapping regions for the coupling of local and nonlocal models. Our model demonstrated robust performance, as reflected by high evaluation metrics, with an accuracy of 0.98 and an F1-score of 0.97.

Outlook: The approach will be extended for higher dimensional geometries, while incorporating damage via bond breaking in two dimensions facilitates a more realistic simulation, enhancing the model's fidelity and applicability.

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