GEOMETRIC MECHANICS FORMULATIONS AND STRUCTURE-PRESERVING DISCRETIZATIONS FOR CONTINUUM MECHANICS AND KINETIC MODELS

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MINISYMPOSIUM

Both continuum mechanics and kinetic (particle) models have an elegant description in terms of geometric mechanics formulations, such as variational, Hamiltonian, metriplectic, GENERIC and port-Lagrangian/Hamiltonian. Based on understanding a system's configuration space and its corresponding symmetries, these geometric descriptions enable accurate representation of both reversible (thermodynamic energy-conserving) and irreversible (thermodynamic entropy-generating) dynamics, along with the correct interconnection/coupling between systems. This can be done in both Lagrangian and Eulerian coordinates, with corresponding conversions between the two representations. Additional insight is gained by working in terms of exterior calculus (differential forms) rather than the usual vector or tensor calculus. Specifically, exterior calculus leads to a clear mathematical representation of the physical field quantities and a rich insight into the properties of their associated functional spaces; along with a clean separation between topological and metric parts of the equations. This is of fundamental importance when constructing numerical discretization models.

Building on this, it is possible to develop numerical models that emulate the fundamental features of these geometric mechanics formulations, leading to many desirable properties. Such methods are known as structure-preserving or mimetic, and are based on discrete versions of exterior calculus that enable the construction of a discrete version of a geometric mechanics formulation. These structure-preserving discretizations display remarkable benefits over more naïve discretizations, including freedom from spurious/unphysical numerical modes, consistent energetics, controlled dissipation of enstrophy or thermodynamic entropy, and stable coupling between subsystems. By constructing discrete methods which mimic the structure of their continuous counterparts, we obtain numerical schemes which are more stable, more interpretable, and more respectful of known dynamical invariants, leading to improved accuracy and physical realism in resulting simulations.

This minisymposium brings together researchers studying and implementing these ideas at both the continuous and discrete levels across a wide range of continuum mechanical and kinetic models, including geophysical fluid dynamics, plasmas, compressible flow, and solid mechanics. We welcome contributions related to all areas of geometric mechanics and structure-preserving discretizations including (but not limited to) novel geometric mechanics formulations for both continuum mechanical and kinetic models; and structure-preserving spatial, temporal and spatiotemporal discretizations such as variational integrators, compatible Galerkin methods (ex. finite element exterior calculus, mimetic Galerkin differences, compatible isogeometric methods), discrete exterior calculus, symplectic/Poisson/metriplectic time integrators, energy-conserving time integrators, and structure-preserving reduced order models.