$$\int f(g(x)) \cdot g'(x)\,dx = \int f(u)\cdot du = F(u)+C$$
$$= F(g(x))+C$$

Ex. 1 Integrate

a) $\int 4(3x+1)^{20}\,dx = 4\int (3x+1)^{20}\,dx$

$u = 3x+1$
$du = 3\,dx$
$\frac{1}{3}du = dx$

$= 4\int (u)^{20}\cdot \frac{1}{3}\,du$

$\frac{4}{3}\int u^{20} = \frac{4}{3}\cdot\frac{u^{21}}{21}+C$

$= \frac{4}{63}(3x+1)^{21}+C$

b) $\int \frac{9x}{\sqrt{9+x^2}}\,dx$

$9\int \frac{x}{\sqrt{9+x^2}}\,dx$

$9\int \frac{1}{\sqrt{9+x^2}}\,x\,dx$

$u = 9+x^2$
$du = 2x\cdot dx$
$\frac{1}{2}du = x\,dx$

$9\int \frac{1}{\sqrt{u}}\cdot\frac{1}{2}\,du$

$\frac{9}{2}\int \frac{1}{\sqrt{u}} = \frac{9}{2}\int (u)^{-1/2} = \frac{9}{2}\left(\frac{2}{1}\right)u^{1/2}+C =$

$9(9+x^2)^{1/2}+C$

c) $\int 7xe^{x^2+2}\,du = 7\int e^{x^2+2}\cdot x\,dx$

$u = x^2+2$
$du = 2x\cdot dx$
$\frac{1}{2}du = x\cdot dx$

$7\int e^u\cdot\frac{1}{2}\,du = \frac{7}{2}\int e^u\,du = \frac{7}{2}\cdot e^{x^2+2}+C$

Ex. 1 Integrate

d) $\int \cos x \sin^8 x \, dx$

$u = \sin x$

$du = \cos x \, dx$

$\int u^8 \cdot du = \frac{u^9}{9} + C = \frac{\sin^9 x}{9} + C$

B

e) $\int \frac{6x}{1+x^2}$

$6\int \frac{1}{1+x^2} x \, dx$

$u = x^2 + 1$

$du = 2x \cdot dx$

$\frac{1}{2} du = x \, dx$

$6\int \frac{1}{u} \cdot \frac{1}{2} du$

$\frac{6}{2}\int \frac{1}{u} \, du$

$3\int \ln|u| + C = 3\ln(x^2+1) + C$

f) $\int \frac{6}{1+x^2} dx$

$6\int \arctan$

$6 \arctan + C$

g) $\int \frac{3\ln(9+x)}{9+x} dx$

$$3\int \frac{\ln(9+x)}{9+x}\,dx$$

$$3\int \ln(9+x)\cdot\frac{1}{9+x}\,dx$$

$$u = \ln(9+x)$$

$$du = \frac{1}{9+x}\cdot 1\,dx$$

$$3\int \frac{u^2}{2}\cdot du$$

$$= \frac{3}{2}\left(\ln(9+x)\right)^2 + C$$

b) $$\int \frac{2x^4 - 4x^3 + 5x - 13}{1+x^2}$$

$$\begin{array}{r} 2x^2 - 4x - 2 \\ x^2+0x+1 \overline{)\,2x^4 - 4x^3 + 0x^2 + 5x - 13} \\ \underline{-2x^4 \quad +0x \quad -2x^2} \\ -4x^3 - 2x^2 + 5x - 13 \\ \underline{-4x^3 \quad -0x^2 \quad -4x} \\ -2x^2 + 9x - 13 \\ \underline{+2x^2 \quad -0x \quad +2} \\ 9x - 11 \end{array}$$

$$(t^2+17)^{1/2}\,dx$$

$$u = t^2+17$$

$$du = 2x\,dx$$

$$\frac{1}{2}x\,du = dx$$

$$\frac{2}{3}(u)^{1/2}\cdot\frac{1}{2}x\,du$$

$$\frac{1}{3}(u)^{3/2}$$

$$\frac{1}{3}(t^2+17)^{3/2}x$$

$$\frac{1}{2}+\frac{2}{2} = \frac{3}{2}$$