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Improving Penetration Forecasts Using Social Interactions Data

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We propose an approach for using individual-level data on social interactions (e.g., number of recommendations received by consumers, number of recommendations given by adopters, number of social ties) to improve the aggregate penetration forecasts made by extant diffusion models. We capture social interactions through an individual-level hazard rate in such a way that the resulting aggregate penetration process is available in closed form and nests extant diffusion models. The parameters of the model may be estimated by combining early aggregate penetration data with social interactions data collected from a sample of consumers in as few as one time period. We illustrate our approach by applying it to the mixed influence model (Bass model) and the more recent asymmetric influence model. A field study conducted in collaboration with a consumer packaged goods company and a marketing research company confirms that incorporating social interactions data using the proposed approach has the potential to result in improved aggregate penetration forecasts in managerially relevant settings.

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1. Introduction

Consider a manager interested in forecasting the aggregate penetration of a new product based on data available early after its launch (our discussions with managers confirmed the managerial importance of such forecasts; see §4). Given the robust finding that social interactions influence adoption (see, among others, Chevalier and Mayzlin 2006; Du and Kamakura 2011; Godes and Mayzlin 2004, 2009; Iyengar et al. 2011; Nam et al. 2010; Trusov et al. 2009), it would seem legitimate for this manager to posit that social interactions data have the potential to help better forecast the penetration of this new product. Moreover, it would be easy for this manager to collect individual-level social interactions data, using traditional surveys or other tracking tools developed more recently. This leads to the following question, which guides the present paper: How may individual-level social interactions data be incorporated into aggregate penetration forecasts?[1]

One first possible source of answers to this question lies in traditional aggregate diffusion models such as the Bass model (Bass 1969), also referred to as the mixed influence model (MIM; Mahajan and Peterson 1985), and its many extensions. However, it is not obvious a priori how individual-level social interactions data may be incorporated into the calibration of extant aggregate diffusion models. Consider, for example, the hazard rate of the MIM, $h(t) = p + qF(t)$, where p and q are the coefficients of external influence and the coefficient of internal influence, respectively, and $F(t)$ is the cumulative penetration at time t (proportion of ultimate adopters who have already adopted). Suppose, for example, that data were available on the number of social ties of a group of consumers, the number of recommendations received by these consumers, as well as which of these consumers have adopted the innovation and how many recommendations these adopters gave in turn to other consumers. A likelihood function for these data may not be derived readily from the MIM. This is because the MIM, like most extant aggregate diffusion models, does not capture the impact of individual recommendations on adoption.

A second potential source of answers lies within structural models of diffusion that are based on utility maximization. This approach to modeling diffusion has become increasingly popular in the marketing

[1] In this paper we focus on modeling trial as opposed to repeat sales or total sales, and on producing postlaunch as opposed to prelaunch diffusion forecasts. We leave extensions to repeat sales and to prelaunch forecasts to future research.

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literature. It is particularly well suited for situations in which consumers anticipate changes in price and quality levels (e.g., Dubé et al. 2011, Gordon 2009, Nair 2007, Song and Chintagunta 2003) and for markets with indirect network effects (Dubé et al. 2010, Shriver 2015). However, to the best of knowledge, this line of research has not yet provided systematic ways to incorporate social interactions data into diffusion forecasts. Indeed, modeling social interactions data within a utility-maximization framework is very challenging, since little is known on what motivates consumers to recommend products to other consumers.

As a third possible approach, one may also consider aggregating the individual-level social interactions data and using them to enrich a vector autoregression (VAR) model (Trusov et al. 2009). However, such an approach would require longitudinal data on penetration and social interactions over a fairly large number of time periods. Collecting such data is challenging in situations in which social interactions may happen both online and offline, i.e., social interactions may not be tracked automatically using online tools.

A fourth potential approach relies on agent-based models (ABMs) of diffusion (e.g., Garber et al. 2004). In particular, to uncover the structure of the underlying social network based on aggregate penetration data, Dover et al. (2012) developed an approach for calibrating ABMs. However, although this approach allows incorporating individual-level *sociometric* data (i.e., data on social connections), it is not designed to incorporate *social interactions* data (i.e., interactions that take place over these social connections). Nevertheless, we will test this approach empirically in §4.

A fifth potential approach is that of van der Lans et al. (2010), who use a branching Markov process to model and predict the spread of viral email campaigns. However, such campaigns involve a specific type of social interactions that follow a different set of processes from the ones typically assumed in extant diffusion models. For example, social interactions in the van der Lans et al. (2010) model take the form of emails inviting other consumers to the campaign. As a result, adoption at the individual level is only a function of *whether* a social interaction took place (i.e., an invitation was received), but not how many. A branching process then becomes the appropriate modeling framework (i.e., each participant with an unopened invitation email may or may not participate, and then invite a certain number of new consumers to the campaign), resulting in an aggregate diffusion process different from that of extant diffusion models. This modeling framework is less appropriate in domains in which each social interaction may have an impact on adoption (e.g., adoption is a function of the number of recommendations received). Moreover, the van der Lans et al. (2010) calibration procedure is optimized for viral email campaigns in which firms have extensive, longitudinal, individual-level data on adoption (time-stamped decisions to participate in the campaign) and social interactions (time-stamped invitation emails sent through the firm's referral system with known senders and recipients).

A sixth potential source of answers is the approach of Dellarocas et al. (2007). These authors develop a modified version of the MIM tailored to the entertainment industry. They estimate the parameters of this extended model for a set of movies and link the diffusion parameters to a set of covariates that describe each movie, including measures related to online word of mouth. This link between diffusion parameters and movie covariates enables them to produce diffusion forecasts for any new movie characterized by a set of covariates. However, this approach relies on analogies between innovations and therefore requires access to a fairly large data set of related past innovations, including longitudinal penetration data and social interaction data for each innovation. In cases where such data are not available, this approach is limited.

Finally, a potential source of answers come from a set of diffusion models that have captured social interactions by specifying an individual-level hazard rate (see, e.g., Du and Kamakura 2011, Iyengar et al. 2011, Nam et al. 2010, Van den Bulte and Lilien 2001). However, such individual-level models are typically not well suited to *forecast* future aggregate penetration, because this would require individual-level data (e.g., sociometric data, geographic data) on all potential adopters in the market (more details are provided in §2 below).

Table 1 summarizes these extant approaches for including social interactions data into penetration forecasts. (We review in §2 other research that has linked social interactions to diffusion but that did not focus on forecasting.) Despite the size and diversity of the diffusion literature, to the best of our knowledge, no practical method has been proposed and tested to leverage individual-level social interactions data to improve early penetration forecasts when the following conditions are met: (i) no data on past related innovations are available, (ii) social interactions data are only available from a sample of consumers, (iii) social interactions data are available for as few as a single time period. These conditions are likely to be met when social interactions may take place both online and offline, making it more challenging to track social interactions in the entire target market over extended periods of time. (According to the Keller Fay Group, 90% of conversations about brands happen offline; see Keller and Fay 2012.) In particular, many tracking tools available today for capturing both online and offline social interactions

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Table 1 Extant Approaches for Including Social Interactions Data Into Penetration *Forecasts*

Approach	Usable with no data from past related innovations	Usable with social interactions data from a sample of consumers	Usable with social interactions data from a single period
VAR	Yes	Yes	No
ABM	Yes	Yes	Yes
Van der Lans et al. (2010)	Yes	Yes	No
Delarocas et al. (2007)	No	Yes	Yes
Extant individual-level hazard rate models	Yes	No	Yes
Proposed approach	Yes	Yes	Yes

Notes. An example of a paper using a VAR approach includes Trusov et al. (2009). The ABM may be calibrated using the approach proposed by Dover et al. (2012). Although this approach allows for incorporating individual-level *sociometric* data (i.e., data on social connections), it is not designed to incorporate *social interactions* data (i.e., interactions that take place over these social connections). Examples of extant individual-level hazard rate models include Du and Kamakura (2011), Iyengar et al. (2011), Nam et al. (2010), and Van den Bulte and Lilien (2001).

rely on surveys (e.g., http://www.kellerfay.com, http://www.chatthreads.com, http://www.bzzagent.com, http://www.shespeaks.com, http://www.vocalpoint.com).

The present paper attempts to close that gap in the literature. Our approach is also to capture social interactions through an individual-level hazard rate. However, we do so in a particular way such that (i) closed-form expressions for the resulting aggregate penetration process are available, and (ii) this aggregate penetration process nests extant diffusion models. The first characteristic enables estimating the parameters of the model by combining early aggregate penetration data with social interactions data coming from a sample of consumers. We are then able forecast future penetration based on these parameters. Our approach specifies a conditional individual-level hazard rate, models the process that generates the variable on which the hazard rate is conditioned, and integrates over the distribution of this variable and over heterogeneity in the population to derive aggregate penetration. Although our paper may not be the first to follow these general steps, to the best of our knowledge, it is the first to apply them to the integration of social interactions data into the calibration of extant diffusion models.

We illustrate our approach by applying it to the discrete-time versions of the MIM (Bass model) and the more recent asymmetric influence model (AIM; Van den Bulte and Joshi 2007). We then conduct a field study in collaboration with a consumer packaged goods (CPG) company and a marketing research company. We find that incorporating individual-level social interactions data using the proposed approach results in improved aggregate penetration forecasts.

2. Related Work

Most aggregate diffusion models used in marketing may be traced back to the Bass model (Bass 1969), also referred to as the MIM (Mahajan and Peterson 1985), and its antecedents (e.g., Mansfield 1961). This model has been extended, for example, to account for heterogeneity across potential adopters and asymmetric influence between different segments of potential adopters (Lehmann and Esteban-Bravo 2006, Muller and Yogev 2006, Van den Bulte and Joshi 2007). However, at early stages of the diffusion process, extant aggregate diffusion models are not very useful to forecast future penetration based on aggregate penetration data only (see, e.g., Hauser et al. 2006, Mahajan et al. 1990, Van den Bulte and Lilien 1997). A common solution to this problem is to complement aggregate penetration data with additional penetration data, coming, for example, from a sample of consumers (Schmittlein and Mahajan 1982, Sinha and Chandrashekaran 1992) or from past analogous innovations (Bass et al. 2001, Hahn et al. 1994, Lenk and Rao 1990, Roberts et al. 2005, Sood et al. 2009, Sultan et al. 1990, Talukdar et al. 2002, Trusov et al. 2013). The framework proposed in this paper is not incompatible with the use of such auxiliary data: it allows using individual-level data on social interactions, *in addition* to any other source of data. More generally, it is essential to note that our approach does not consist in developing a new model that is meant to replace other models, but rather in augmenting the estimation of extant diffusion models with individual-level data on social interactions.

Past research using social interactions data in a diffusion framework has focused primarily on analyzing and quantifying the impact of social interactions on sales and diffusion, as opposed to using social interactions to improve *forecasts* (e.g., Chevalier and Mayzlin 2006; Duan et al. 2008; East et al. 2006; Godes and Mayzlin 2004, 2009; Liu 2006; Trusov et al. 2009). Consequently, the models used in these papers do not always produce out-of-sample forecasts, which is the intent of the present paper. Previous attempts to model social interactions in a way that may produce such forecasts include Van den Bulte and Lilien (2001), who model the diffusion of the drug Tetracycline across a community of 121 physicians by capturing the structure of the physicians' social network and modeling the effect on physician i of the adoption of another physician j to

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which i is connected. Related papers include Iyengar et al. (2011), Nair et al. (2010), and Strang (1991). However, generating out-of-sample aggregate penetration forecasts using this type of approach requires mapping the complete social network of the potential market. Therefore, it is applicable only to networks or groups that are smaller than those typically encountered in contexts such as consumer products. Du and Kamakura (2011), Manchanda et al. (2008), and Nam et al. (2010) model the influence of a potential adopter's nearest neighbors (based on geographical distance) on adoption. Although this approach does not require any sociometric data, using it to generate out-of-sample aggregate penetration forecasts requires knowing the location of all potential adopters. Our approach is consistent with many of these papers in that it relies on the specification of an individual-level hazard rate. However, one key difference is that we are able to provide closed-form expressions for the aggregate diffusion process implied by this individual-level hazard rate. This enables calibrating the model using a combination of aggregate penetration data and individual-level social interactions data collected from a *sample* of consumers, and then producing out-of-sample aggregate penetration forecasts based on the model.

3. Incorporating Social Interactions Data Into Extant Diffusion Models

Our approach is to extend existing aggregate diffusion models in a way that explicitly captures the generation of social interactions and their impact on adoption at the individual level. We focus on extending models that have been studied and validated by many researchers over a long period of time, rather than attempt to develop new, fundamentally different diffusion models. In this paper we chose to illustrate our approach on the best-known aggregate diffusion model, the MIM (Bass 1969), and one of its more recent extensions, the AIM (Van den Bulte and Joshi 2007). Therefore, we make specific assumptions that allow nesting these models while deviating as little as possible from them. In Appendix A we show how these assumptions may be relaxed to generate a broader range of diffusion models.

For simplicity, in the remainder of this paper we focus on *recommendations* between consumers as the primary source of social interactions. We define a recommendation as an event in which a consumer who has adopted the innovation recommends it to another consumer. (That other consumer may or may not have adopted already. If that other consumer has already adopted, then the recommendation will have no effect on adoption.) In our field study, we measured recommendations by asking consumers to keep track, during one week, of the number of people from whom they received recommendations/to whom they gave recommendations. Our approach may be applied to other forms of social interactions as well, such as observing other consumers using the innovation, etc. When appropriate, we note how such other forms of social interactions may be captured by modifying the definition of the parameters of the models.

For ease of exposition we start with the extension of the MIM (Bass 1969). We next turn to the more recent AIM (Van den Bulte and Joshi 2007) and provide an extension. We summarize the parameters of the extended models in Table 2.

3.1. Extending the Discretized Mixed Influence Model (Bass Model)

To incorporate individual-level social interactions data into aggregate penetration forecasts, we model adoption conditional on the number of recommendations received as well as the generation of recommendations, both at the individual level. We do so in such a way that the resulting aggregate diffusion process is given in closed form, and that the discretized MIM is nested. We first describe the specification of the probability of adoption conditional on the number of recommendations received, and of the generation of recommendations. Next, we provide closed-form expressions for the resulting aggregate diffusion process and show formally that the discretized MIM is nested within the extended model.

Table 2 **List of Variables**

Name of variable in extended MIM	Similar variables in extended AIM	Definition
p	p_1, p_2	Captures the effect of external forces on adoption
q	q_1, q_2	Probability of adopting based on one recommendation
g_{it}	$g_{it}^{1\to1}, g_{it}^{1\to2}, g_{it}^{2\to2}$	Number of recommendations given by consumer i in period t
r_{it}	$r_{it}^{1\to1}, r_{it}^{1\to2}, r_{it}^{2\to2}$	Number of recommendations received by consumer i in period t
$ties_i$	$ties_i^{1\to2}, ties_i^{1\to1}, ties_i^{2\to2}$	Number of social ties of consumer i
a	$a^{1\to1}, a^{1\to2}, a^{2\to2}$	Probability that an adopter recommends the innovation to each of his or her social ties in each period following adoption
f_t	f_t^1, f_t^2	Marginal aggregate penetration in period t
F_t	F_t^1, F_t^2	Cumulative aggregate penetration by the end of period t

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3.1.1. Adoption Conditional on Number of Recommendations Received. We index consumers by i. Let the parameter t index (discrete) time periods. Let r_{it} be the number of recommendations received by consumer i in period t. We are interested in specifying the probability that consumer i adopts in period t given that he or she has not adopted yet, as a function of the number of recommendations r_{it}. Each recommendation has some probability of leading to adoption, and the consumers may also adopt based on other, "external" factors. We denote as q the probability that a potential adopter would adopt based on one recommendation, in the absence of external effects. We denote as p the probability that a potential adopter would adopt based on external effects, in the absence of recommendations. The following discrete-time conditional hazard rate $h(r_{it})$ follows directly from these assumptions. This hazard rate is comparable to the hazard rates assumed by agent-based models (Garber et al. 2004, Goldenberg et al. 2002):

$$h(r_{it}) = 1 - (1-p)(1-q)^{r_{it}}. \tag{1}$$

This conditional hazard rate is equal to one minus the probability of "resisting" the innovation, which is equal to the probability of resisting the external forces and resisting the influence of r_{it} recommendations. The parameters p and q capture similar forces as the parameters of the MIM, with p capturing external effects and q capturing internal effects. Note that the hazard rate in Equation (1) does not assume that it only takes one recommendation for adoption to take place. Instead, each recommendation has a probability q of triggering adoption.

3.1.2. Generation of Recommendations. We now specify the generation of recommendations given by consumer i in period t, g_{it}, in a way that allows nesting the MIM. Consider consumer i, who has adopted the innovation on or before period $t-1$. We denote the number of social ties this individual has in the social network that is relevant to the diffusion of the innovation under study, as $ties_i$.[2] This quantity may be measured, for example, using sociometric surveys (see, e.g., Coleman et al. 1966, Iyengar et al. 2011, Nair et al. 2010). Future research may explore measuring this quantity based on alternative sources of data. We denote as a the probability that a consumer recommends the innovation to each of his or her ties in each period conditional on having adopted the innovation. In each period, the consumer either recommends or does not recommend the innovation to each of his or her ties. These assumptions lead to g_{it} following a binomial distribution, where the number of draws is the number of ties, $ties_i$, and the success probability is the probability that an adopter would recommend the product to each of his or her ties in each period, a. Formally,

$$g_{it} \sim \text{Bin}(ties_i, a)$$
$$\Rightarrow \quad P(g_{it} \mid ties_i) = \binom{ties_i}{g_{it}} a^{g_{it}} (1-a)^{ties_i - g_{it}}, \tag{2}$$

where $P(g_{it} \mid ties_i)$ is the probability mass function of the variable g_{it} conditional on the number of social ties, $ties_i$.

Note that other forms of social interactions, different from recommendations, may be captured as well by modifying the definition of the parameter a. For example, if social influence works through potential adopters observing other consumers using the innovation, the parameter a may be defined as the probability that an adopter will be using the innovation while interacting with each of his or her ties.

We also specify the number of recommendations *received* by a potential adopter i in period t, r_{it}. The above assumptions imply that this variable follows a binomial distribution. The number of draws equals the number of social ties of consumer i, and the success probability equals the probability that each of these ties would recommend the product to i in period t. This latter probability is expressed as the probability that a given tie would recommend the product to consumer i conditional on the tie having adopted (captured by the parameter a introduced above), multiplied by the probability that the tie has adopted on or before period $t-1$, captured by the cumulative penetration in period $t-1$, denoted by F_{t-1}. This cumulative penetration equals the probability that a randomly selected consumer in the potential market has adopted the innovation by period $t-1$. Formally,

$$r_{it} \sim \text{Bin}(ties_i, aF_{t-1})$$
$$\Rightarrow \quad P(r_{it} \mid ties_i) = \binom{ties_i}{r_{it}} (aF_{t-1})^{r_{it}} (1-aF_{t-1})^{ties_i - r_{it}}, \tag{3}$$

where $P(r_{it} \mid ties_i)$ is the probability mass function of the variable r_{it} conditional on the number of social ties $ties_i$. Note that Equations (2) and (3) do not assume that all adopters will recommend the product, but rather that each adopter has some probability of recommending the product to each of his or her ties in each period.[3]

[2] We only consider social ties between consumers in the potential market. See, for example, Trusov et al. (2010) for another paper in which a relevant social network (in their case, a network of influence) is defined based on a subset of the ties that exist in a more general social network (in their case, a friendship network).

[3] We note that our specification distinguishes between ties and recommendations. Ties reflect the social network of consumers and describe relatively stable dyadic relationships. Following past research (e.g., Iyengar et al. 2010, Nair et al. 2010, Van den Bulte and Lilien 2001), we assume that $ties_i$ is constant for each consumer

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3.1.3. Aggregate Diffusion Process. The parameters of the individual-level hazard rate specified above may be calibrated with individual-level data only. However, we are able to provide closed-form expressions for the aggregate diffusion process implied by this individual-level hazard rate; i.e., we show how the individual-level processes captured in Equations (1)–(3) may be aggregated to obtain closed-form expressions of the aggregate diffusion process. This closed-form integration enables calibrating the model using a combination of individual-level social interactions data and aggregate penetration data, and then producing out-of-sample aggregate penetration forecasts based on the model. We drop the subscript i when integrating over the distribution of consumers in the population. Let $P(ties)$ denote the probability mass function (i.e., distribution across consumers) of the number of social ties. Let f_t^{ties} and F_t^{ties} be, respectively, the marginal and cumulative aggregate penetration in period t among consumers with *ties*, and let $f_t = \sum_{ties} f_t^{ties} P(ties)$ and $F_t = \sum_{ties} F_t^{ties} P(ties)$ be the marginal and cumulative penetration in the potential market. The marginal penetration f_t^{ties} is equal to the proportion of nonadopters among consumers with *ties* before period t, $1 - F_{t-1}^{ties}$, multiplied by the expected value of the hazard rate in period t among these consumers, where the expected value is taken over r_t, the number of recommendations received during period t. We have the following:

$$\begin{aligned} f_t^{ties} &= (1 - F_{t-1}^{ties}) E_{r_t}[h(r_t) \mid ties] \\ &= (1 - F_{t-1}^{ties}) \sum_{r_t=0}^{ties} h(r_t) P(r_t \mid ties). \end{aligned} \tag{4}$$

Given a number of social ties, *ties*, the number of recommendations received, r_t, may vary between 0 and *ties*, which explains the summation from 0 to *ties* in the above equation. The hazard rate corresponding to each possible value of r_t, given by Equation (1), is weighted by the probability of that value of r_t occurring, given by Equation (3).

This equation provides a closed-form expression for the marginal penetration in period t given the cumulative penetration in the previous period. Marginal penetration in any period unconditional on past penetration is obtained recursively, *without* using any simulation or numerical approximation.

3.1.4. Relation to Mixed Influence Model. Finally, we show how the discretized MIM may be obtained as a special case, in which the number of social ties is assumed to be homogeneously equal to 1. Under the assumption that $ties = 1$ for all consumers, the number of recommendations received by a potential adopter in period t, r_t, is 1 with probability aF_{t-1} and 0 with probability $(1 - aF_{t-1})$. The expected value of the hazard rate over r_t becomes equal to the hazard rate of the discretized MIM, with $p^{\text{MIM}} \equiv p$ and $q^{\text{MIM}} \equiv q(1-p)a$:[4]

$$\begin{aligned} &E_{r_t}[h(r_t) \mid ties = 1] \\ &\quad = pP(r_t = 0 \mid ties = 1) \\ &\qquad + (1 - (1-p)(1-q))P(r_t = 1 \mid ties = 1) \\ &\quad = p(1 - aF_{t-1}) + (p + q(1-p))aF_{t-1} \\ &\quad = p + q(1-p)aF_{t-1}. \end{aligned} \tag{5}$$

Note that this special case is presented here only to establish that the model described in Equations (1)–(4), which we will refer to as the extended MIM, nests the original (discretized) MIM. We will not set the parameter *ties* to 1 in our field application.

We show in Online Appendix D (in the electronic companion, available at http://papers.ssrn.com/sol3/papers.cfm?abstract_id=2380725) that when *ties* follows any general distribution, the *first-order linear approximation* in q of the expected value of the hazard rate of the extended MIM among consumers with *ties* is equal to the hazard rate of the discretized MIM, with $p^{\text{Bass}} \equiv p$ and $q_{ties}^{\text{Bass}} \equiv q(1-p) \cdot ties \cdot a$.[5]

We note that these nesting results are a direct consequence of modeling the hazard rate conditional on the number of social interactions, and that they would typically not hold if the hazard rate were conditioned on other factors (e.g., income, number of advertising exposures). Nesting follows from the fact that the process generating the number of recommendations (Equation (3)) is a function of the cumulative penetration F_{t-1}. If the variable r_{it} in Equation (1) were not the number of social interactions and were not a function of cumulative penetration, the expected hazard rate in Equation (5) would typically not be a function of cumulative penetration and would not nest the MIM.

We also note that our nesting result does not imply that the MIM necessarily assumes that the number of social ties is assumed to be homogeneously equal to 1. Indeed, while this assumption is sufficient to reconstruct the Bass model under the approach proposed

in the relevant time frame. Recommendations, on the other hand, describe events that occur between consumers linked in that social network. Social ties and recommendations are different constructs that may both be measured. Capturing them separately allows for incorporating data on both of these constructs into penetration forecasts.

[4] Although the hazard rate in the continuous-time MIM is a function of F_t, F_{t-1} is used in the discrete-time version.

[5] This suggests an alternative extension of the MIM where the individual-level hazard rate would be $h(ties) = p + q(1-p) \cdot ties \cdot a \cdot F_{t-1}$. This model could be calibrated using a combination of aggregate penetration data, individual-level adoption data, and sociometric data (the parameter *ties*). However, this model would not accommodate data on the number of recommendations received or given. We have tested this model on our field study data, and we found that it did not perform better than the traditional MIM.

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here, it is not necessary. Other sets of assumptions have been shown to give rise to the MIM as well (see, e.g., Goldenberg et al. 2009). Our research also shows that the MIM may be viewed as the first-order linear approximation of a diffusion model in which social ties may follow any distribution.

3.1.5. Identification. We now discuss identification issues; i.e., we discuss conditions under which the parameters of the model are uniquely identified. As mentioned above, the number of social ties of a set of consumers, $\{ties_i\}$, may be measured directly, for example, using sociometric surveys. The parameters p, q, and a are identified when the following additional individual-level data are available from a sample of consumers for at least one time period: adoption status (i.e., whether each consumer has adopted the innovation) at the beginning and end of the period, the number of recommendations received during the period by each consumer who had not adopted yet at the beginning of that period, and the number of recommendations given during the period by each consumer who had already adopted at the beginning of that period. The number of recommendations given (conditional on adoption) does not depend on p or q, which allows identifying the probability of recommending the innovation, parameter a, from p and q.[6] Similarly, adoption during the period conditional on the number of recommendations received does not depend on a, which allows identifying p and q from a. The parameters p and q are identified from each other because the number of recommendations received influences the hazard rate only through q, and not p. We verify identification using simulations, reported in Online Appendix F (in the electronic companion). In particular, we simulate data with the same structure as the data in our field study, using 16 different sets of values of the parameters. We estimate the model using the same procedure as in our field study and show that the true parameter values are well recovered.

3.1.6. Relaxing Some of the Assumptions. Finally, we highlight a set of assumptions made only for ease of exposition and to nest extant models in a parsimonious fashion. A list of these assumptions is provided in Table 3. These assumptions may be relaxed. In Appendix A we introduce a more general diffusion model (i) in which all the assumptions listed in Table 3 are relaxed, (ii) that accepts the models presented in this paper as special cases, and (iii) for which closed-form expressions of the aggregate diffusion process are still available.

First, the special cases considered in this paper assume heterogeneity in the parameter *ties*, but homogeneity in p and q within each segment (the extended AIM model presented next assumes the existence of multiple segments). The general model in Appendix A assumes instead that these parameters are distributed across consumers according to any joint discrete probability distribution (allowing, e.g., a positive correlation between p and *ties*). Second, the special cases considered in this paper assume that the probability that an adopter will recommend the product to each of his or her ties is constant over time. The general model in Appendix A captures nonuniform influence (Easingwood et al. 1983) by making the parameter a a function of the period at which the adoption occurred and of the current period. Letting the parameter a be a function of the current period also allows capturing the impact of time-varying covariates (e.g., marketing mix variables) on the generation of recommendations. The model may be extended further to model the impact of marketing mix variables on other parameters (Bass et al. 1994, Horsky and Simon 1983, Kalish and Sen 1986, Robinson and Lakhani 1975). Third, the conditional hazard rate in Equation (1) assumes that the number of recommendations relevant to the adoption decision of consumer i in period t is the number of recommendations received in the same period by this consumer, r_{it}. The general model in Appendix A assumes instead that adoption in period t is influenced by any linear combination of the number of recommendations received by i in each period 1 to t (e.g., number of recommendations received in period $t-1$, cumulative number of recommendations received, higher weight on more recent recommendations, etc.). Fourth, whereas social ties are assumed to be symmetric (A connects to B implies that B connects to A) in the special cases considered in this paper, the general model in Appendix A allows for asymmetries in social ties. Fifth, whereas recommendations are assumed to always have a positive impact on adoption in this paper, the general model in Appendix A allows for the existence of both positive and negative recommendations (Mahajan et al. 1984).

3.2. Extending the Discretized Asymmetric Influence Model

The MIM (Bass 1969) is probably the best-known aggregate diffusion model in marketing, and it has been used in a large number of applications. Since it was introduced, many theoretical developments have been published. One of the latest models proposed in the literature is the AIM of Van den Bulte and Joshi (2007). This model assumes the existence of two segments with asymmetric influence on one another (see also

[6] The number of recommendations received provides additional data that allow identifying the parameter a. In our field study, we found that similar estimates of a were obtained with or without including the number of recommendations received into the likelihood function. The analysis reported in §4 includes these data. Future research may explore additional sources of data that would allow estimating a, including surveys.

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Table 3 Relaxable Assumptions

Assumption	Implication for extended MIM	Possible relaxation
Homogeneous diffusion parameters within a segment	$\{p, q\}$ are homogeneous	Parameters follow any discrete probability distribution across consumers
Uniform influence over time: the probability that an adopter recommends the innovation is not a function of when adoption took place or of the current time period	a constant over time	Recommendation probability depends on the period at which the adoption occurred and on the current period
Only recommendations from period t influence adoption at t	Hazard rate is a function of r_{it}	Recommendations from periods 1 to t may have an impact on adoption at time t; recommendations are weighted based on the number of periods since they occurred
Social ties are symmetric	The number of recommendations received and the number of recommendations given are (different) functions of the same parameter $ties_i$	Social ties are asymmetric
All recommendations are positive	The hazard rate is monotonically increasing in the number of recommendations	Recommendations may be positive or negative

Notes. The above assumptions were made to nest the discretized versions of the MIM and AIM in a parsimonious fashion. However, they are not necessary to derive closed-form expressions of the aggregate diffusion process. Appendix A presents a more general model that relaxes all these assumptions.

Lehmann and Esteban-Bravo 2006, Muller and Yogev 2006). These two segments, labeled as "innovators" and "imitators," are such that innovators are only influenced by other innovators, whereas imitators are influenced both by innovators and by imitators.

We show how our approach also allows extending the AIM. The same assumptions made in the extended MIM and listed in Table 3 are made in the extended AIM for ease of exposition and to nest extant models in the most parsimonious fashion. These assumptions may still be relaxed, and the general model presented in Appendix A accepts the extended AIM as a special case. Following Van den Bulte and Joshi (2007), we refer to the innovators segment as segment 1 and to the imitators segment as segment 2. Like in the original model, we assume that the proportion of innovators in the potential market is given by θ. Details are provided in Online Appendix C (in the electronic companion). In particular, we show that the discretized AIM is a special case of the extended AIM under a specific set of assumptions on the distribution of social ties. Moreover, we show that the discretized AIM is a first-order linear approximation of the extended AIM when social ties follow any discrete distribution.

4. Field Study

In the previous section we developed a fairly general approach for nesting extant diffusion models within an individual-level hazard rate model that captures explicitly the generation of social interactions as well as their impact on adoption, and for which the resulting aggregate diffusion process is available in closed form. We have illustrated this approach by applying it to the MIM and the asymmetric influence model. We now describe a field study that demonstrates how individual-level social interactions data may be combined with aggregate penetration data to calibrate these extended models in practice. Our objective is not to reach substantive insights on a particular market or innovation, but rather to provide a proof of concept of our approach, and assess whether it has the potential to improve aggregate penetration forecasts in a managerially relevant way.

Because we did not propose a new model that is meant to replace other models, but rather an approach for augmenting the estimation of extant diffusion models, our main focus is on comparing models calibrated without social interactions to their extended counterparts, not to alternative models. Nonetheless, we made our best efforts to compare our approach to all relevant benchmarks. As mentioned earlier, our approach enables researchers to leverage social interactions data even when the following conditions are met: (i) no data on past related innovations are available, (ii) social interactions data are only available from a sample of consumers, (iii) social interactions data are available for as few as a single time period. These conditions are satisfied in our field study. Therefore, the set of possible benchmarks against which to compare our approach is limited. However, we are able to calibrate an agent-based model using the approach developed by Dover et al. (2012). This approach relies only on sociometric data to complement the aggregate penetration data.

4.1. Setup and Data

The applicability and implementation of our approach was refined based on two initial field studies (details available from the authors). Our main study was conducted between 2011 and 2012 in collaboration with a major U.S.-based CPG manufacturer and a marketing research company that specializes in buzz marketing and social interactions. The manufacturer was interested in the penetration of a new cooking product. This product offered a significantly new benefit and represented an

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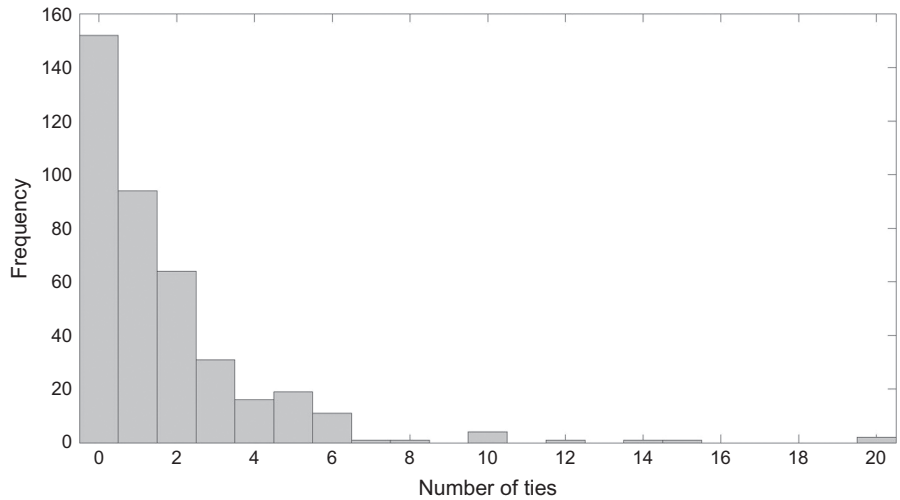
Figure 1 **Distribution Across Consumers of the Number of Ties**



innovation in that category. For confidentiality reasons, we will refer to this new product as PROD.[7]

Our social interactions data came from a tracking study administered through a proprietary platform developed by ChatThreads, a marketing research company. The respondents were 398 consumers from PROD's target market. These consumers were not given any free sample of the product and were not instructed or incentivized to recommend the product to anyone. Each consumer was asked to keep track for one week of all the recommendations received/given for the new product using their mobile phone and in a post-tracking survey. Consumers were recruited in a random order between the 8th week and the 17th week after the launch of PROD (i.e., each consumer was tracked for exactly one week during that window). We index respondents by i, define one time period as one week, and denote by t_i the tracking period for respondent i ($t_i \in \{8, 9, \ldots, 17\}$). Out of all respondents, 146 had tried PROD before their tracking period and 252 had not. We label the first group as "initial triers" and the second as "initial nontriers." Each initial nontrier reported the number of people from whom he or she received a recommendation for PROD during period t_i, which we denote as r_{it_i}, and whether he or she tried (i.e., purchased at least once) PROD during t_i, which we capture using a binary variable y_{it_i} equal to 1 if respondent i purchased PROD at least once during t_i. Each initial trier reported the number of people to whom he or she recommended PROD during period t_i, which we denote as g_{it_i}. Finally, we measured $ties_i$ by asking each respondent to indicate the number of people in his or her social network who would be interested in PROD. In the remainder of this paper we refer to $\{r_{it_i}, ties_i\}$ for initial nontriers and $\{g_{it_i}, ties_i\}$ for initial triers as the individual-level social interactions data, and to $\{y_{it_i}\}$ for initial nontriers as the individual-level adoption data.

In addition to these data, we received aggregate penetration data for PROD from an independent professional market research company, for four-week periods ending at $t = 4, 8, \ldots, 48$. Aggregate penetration is measured as the proportion of households in the market who purchased PROD for the first time in each period. The company was interested in predicting the penetration of PROD during the rest of its first year (12 four-week periods) based on the data available around the time of the tracking study. Further discussion with the brand manager in charge of PROD confirmed that penetration after one year is a key managerial metric for CPG companies, and that managers are typically interested in predicting this quantity a few months after the launch of the product. We use the first six aggregate penetration data points ($t = 4, 8, 12, 16, 20, 24$) for calibration and the remaining six for validation. We later check the robustness of the results when using instead the first four, five, seven, and eight data points for calibration and the rest for validation.

4.2. Descriptive Statistics

The median value of $ties_i$ across respondents was 1, and the average was 1.699. Figure 1 plots the distribution of $ties_i$ across respondents. The distribution has a long tail, although most consumers report having two or fewer people in their social network that would be interested in PROD.

We first focus on recommendations received and their impact on behavior. The average value of r_{it_i} among

[7] There is evidence that social interactions play a significant role in the penetration of CPG products, making this setting a reasonable one for testing the approach developed in this paper. In their classic study, Katz and Lazarsfeld (1955) found that approximately one-third of brand switching for household goods involves personal influences, and Du and Kamakura (2010) found empirical evidence for social contagion across a wide range of CPG categories.

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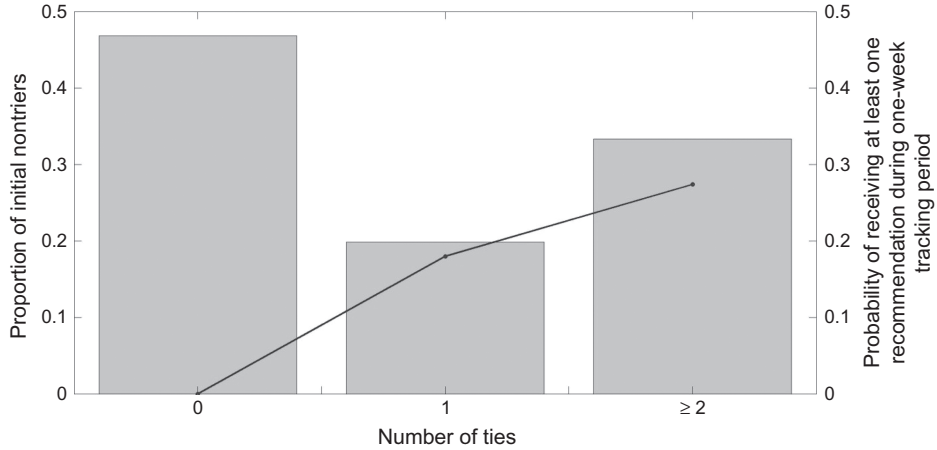
Figure 2 **Number of Recommendations Received vs. Number of Ties for Initial Nontriers**

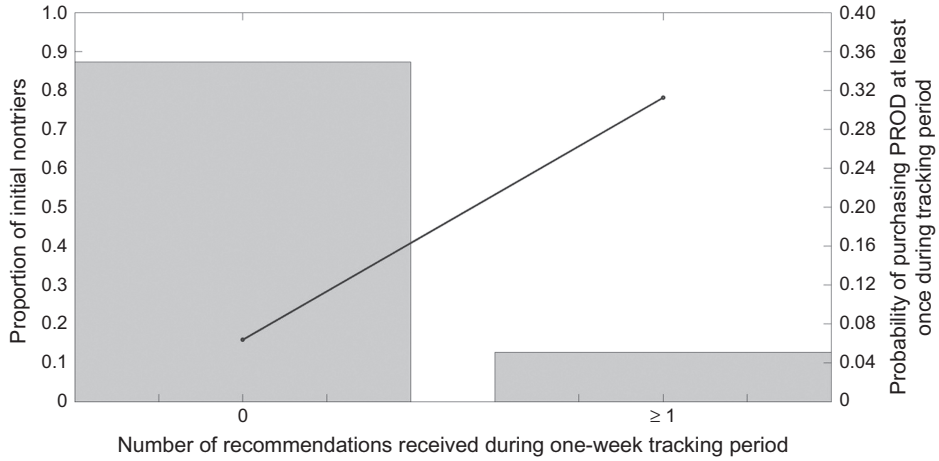


Notes. The bar chart plots the distribution of the number of ties. The line plots the probability of receiving at least one recommendation during the one-week tracking period.

initial nontriers was 0.163, with 87.30% receiving no recommendation during their one-week tracking period, 9.92% receiving one, 2.38% receiving two, and 0.40% receiving four. Figure 2 plots the proportion of initial nontriers who received at least one recommendation, as a function of their number of ties. Consistent with Equation (3), we see that consumers with more ties are more likely to receive recommendations. The rank correlation between $ties_i$ and r_{it_i} among initial nontriers was significantly positive ($\rho = 0.349$, $p < 0.01$). Figure 3 shows the impact of receiving recommendations on behavior. Consistent with Equation (1), we see that initial nontriers who received at least one recommendation during the one-week tracking period had a higher probability of purchasing PROD at least once during that period. The average value of y_{it_i} among initial nontriers was 0.0952. The average value of y_{it_i} among initial nontriers for whom $r_{it_i} = 0$ was 0.0636, and the average value of y_{it_i} among initial nontriers for whom $r_{it_i} > 0$ was significantly higher at 0.3125 ($z = 4.48$, $p < 0.01$).

We now turn to recommendations given. The average value of g_{it_i} among initial triers was 1.110, with 50.68% giving no recommendation during their one-week tracking period, and {23.29%, 16.44%, 3.42%, 2.05%, 2.05%, 0.68%, 0.68%, 0.68%} giving, respectively, $\{1, 2, 3, 4, 5, 6, 12, 20\}$ recommendations. Figure 4 plots the average number of recommendations given by initial triers as a function of their number of ties. Consistent with Equation (2), we see that initial triers with more ties gave on average more recommendations during the one-week tracking period. The rank correlation between $ties_i$ and g_{it_i} among initial triers was significantly positive ($\rho = 0.653$, $p < 0.01$). Finally, Figure 5 plots the marginal aggregate penetration from $t = 4$ to $t = 48$.

Figure 3 **Trial vs. Recommendations Received for Initial Nontriers**



Notes. The bar chart plots the proportion of initial nontriers who received no recommendation and at least one recommendation during the one-week tracking period. The line plots the corresponding probabilities of purchasing PROD at least once during the one-week tracking period.

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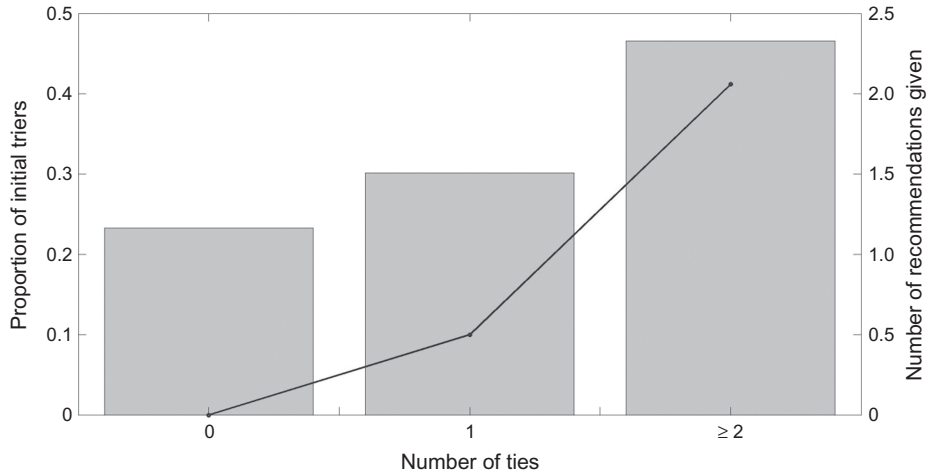
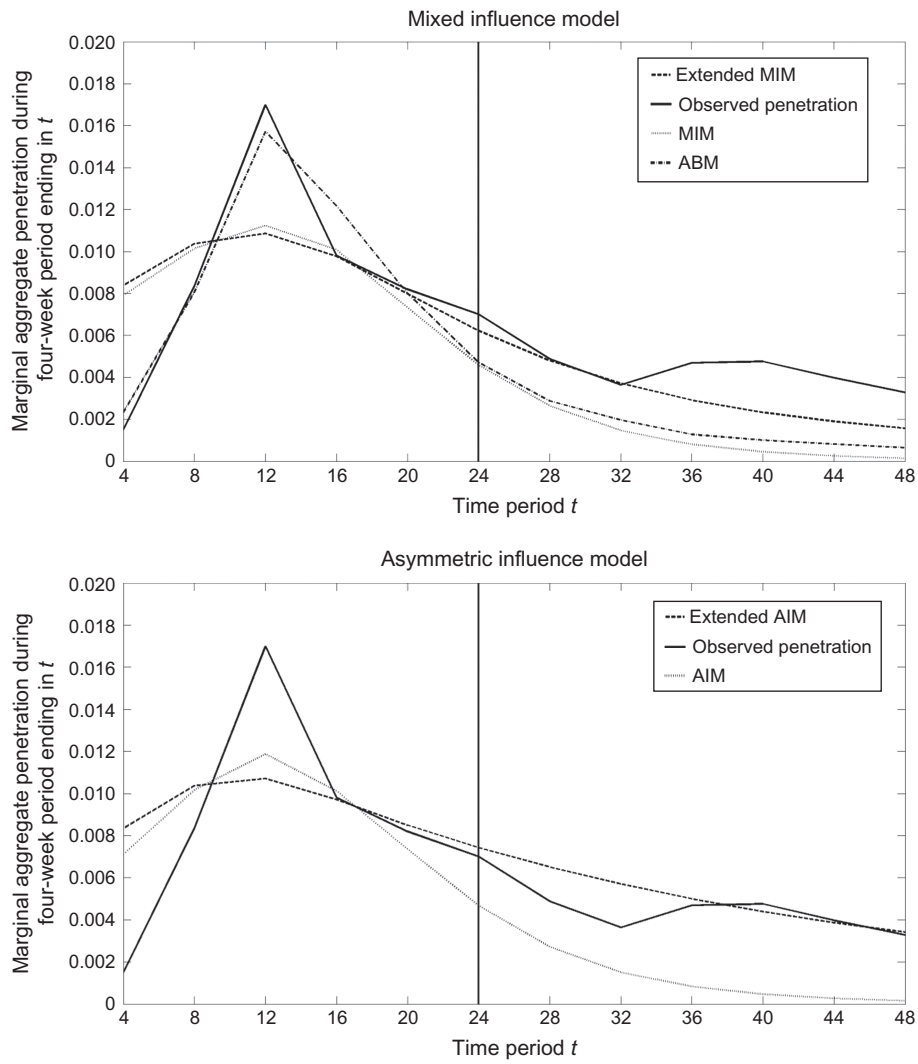
Figure 4 **Recommendations Given vs. Number of Ties for Initial Triers**



Notes. The bar chart plots the distribution of the number of ties among initial triers. The line plots the average number of recommendations given by initial triers as a function of the number of ties.

Figure 5 **Actual Marginal Penetration Curve vs. Fitted Marginal Penetration Curves**



Note. The vertical line separates calibration from validation periods.

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Overall, these descriptive statistics are consistent with previous literature that found that social interactions have an impact on purchasing behavior, and suggest that social interactions data may be linked to sociometric data. This suggests that the type of social interactions data considered throughout this paper and measured in our field study indeed have the potential to improve penetration forecasts.

4.3. Calibration

We calibrate the extended models (extended MIM, extended AIM) using all the data described above: the individual-level social interactions data (number of recommendations received/given, number of ties), the individual-level adoption data, and the calibration aggregate penetration data. We calibrate the original models (MIM, AIM) based on the individual-level adoption data and the aggregate penetration data. We use a similar procedure (Bayesian Markov chain Monte Carlo (MCMC) with noninformative priors) for all these models (for other uses of Bayesian MCMC in diffusion research, see, e.g., Dellarocas et al. 2007, Lenk and Rao 1990). We used 300,000 MCMC iterations, using the first 200,000 as burn-in and saving 1 in every 10 draws. Convergence was assessed through time-series plots of the parameters. Similar priors and numbers of draws were used for all models. Our likelihood function follows directly from the equations provided in the previous section. Details are provided in Appendix B and Online Appendix E (in the electronic companion).

In addition, we calibrate an ABM (e.g., Garber et al. 2004) using the Dover et al. (2012) approach.[8] This benchmark is based on the following hazard rate: $h(n_{it}) = 1-(1-p)(1-q)^{n_{it}}$, where n_{it} is the number of consumers connected to i who have adopted before period t. The data used to calibrate this model are the aggregate penetration data and the distribution of the number of ties. We generate five random networks, each with a potential market in which the number of agents is equal to 100 times the number of consumers in our sample, and with a distribution of the parameter *ties* that matches exactly the distribution in our data. Then we perform a grid search over p, q, m to fit the aggregate penetration data. For each candidate value of p, q, m and each of the five networks, we simulate diffusion based on p and q and multiply by m to estimate aggregate penetration in the overall market. We select p, q, and m to minimize the mean squared error (MSE) between the observed aggregate penetration data and the estimates obtained by averaging over the five simulated networks.[9] We note that this approach is not likelihood based, and confidence intervals are not available.

4.4. Results

We compute the log marginal density of the data under each likelihood-based model (Rossi and Allenby 2003). We compute the MSE and the mean absolute percentage error (MAPE) between the true marginal aggregate penetration in each four-week period and the point estimates provided by each model. We compute the MSE and MAPE for both the calibration and the holdout aggregate penetration data.

Results are reported in Table 4, and Figure 5 compares the actual marginal penetration curve with the marginal penetration curves predicted by the various models. We also report point estimates of the parameters in Tables 5 and 6. As seen from Table 4 and Figure 5, neither the original nor the extended models fit the *calibration* aggregate penetration data as well as the ABM. Moreover, the extended models do not necessarily fit the *calibration* aggregate penetration better compared to the original models. This is expected, based on the differences in the data used to calibrate the various models. The parameters of the ABM are estimated to maximally fit the aggregate penetration data, and indeed this model achieves the best in-sample fit on the calibration aggregate penetration data. The original models are calibrated based on the calibration aggregate penetration data and the individual-level adoption data,[10] and the extended models are calibrated based on these data as well as the individual-level social interactions data. Therefore, less emphasis is put in the extended models on fitting calibration aggregate penetration data, possibly resulting in worse fit of these data. Note that because the extended and original models are calibrated based on different sets of data, the standard results on nesting do not apply here.

More importantly, the results suggest that the additional data used in the calibration of the extended models provide additional information that allows improving *out-of-sample* aggregate penetration forecasts. Indeed, Table 4 and Figure 5 show that the extended models fit the *holdout* aggregate penetration

[8] We are indebted to Yaniv Dover for his guidance in implementing this approach, and for confirming that it matches Dover et al. (2012).

[9] We first perform a grid search with a resolution of 10^{-2} with $\{p, q, m\}$ in $[0.01; 0.1] \times [0.1; 0.5] \times [0.04; 0.15]$, and then perform another grid search with a resolution of 10^{-3} around the best-fitting combination.

[10] We also estimated the original models based on the calibration aggregate penetration data only. Fit increased on the calibration aggregate penetration data for the MIM, but not the AIM (the model is poorly identified with six parameters and six data points). In both Cases, similar conclusions were reached regarding out-of-sample penetration forecasts. Details are available from the authors.

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Table 4 Holdout Predictive Ability and In-Sample Fit

Model	Log marginal density	MSE on calibration penetration data · 10^4	MAPE on calibration penetration data	MSE on holdout penetration data · 10^4	MAPE on holdout penetration data	Predicted cumulative aggregate penetration at $t = 48$
MIM	−74.567	0.140	88.354	0.112	77.948	0.0571
Extended MIM	−350.184	0.149	88.803	0.027	32.756	0.0709
AIM	−89.449	0.112	78.823	0.110	77.178	0.0573
Extended AIM	−321.867	0.151	87.878	0.012	18.496	0.0840
ABM	N/A	0.022	20.877	0.082	66.343	0.0596

Notes. The extended models predict *out-of-sample* aggregate penetration data more accurately compared to the original models. The observed cumulative penetration at $t = 48$ was 0.0771. The extended models do not necessarily fit *in-sample* aggregate penetration better than the original model or the ABM. The extended and original models are calibrated based on different sets of data. (Less emphasis is put in the extended models on fitting calibration aggregate penetration data.) Therefore, the standard results on nesting do not apply, and the log marginal density may not be compared between original and extended models. The ABM is calibrated based on the aggregate penetration data only, and therefore fits these data better than the original MIM, which is calibrated based on the aggregate penetration data and the individual-level penetration data.

Table 5 Point Estimates of the Parameters for the MIM, Extended MIM, and ABM

Parameter	Original MIM	Extended MIM	ABM
p	0.031 (0.018)	0.023 (0.005)	0.013 (N/A)
q	0.140 (0.053)	0.147 (0.049)	0.484 (N/A)
a	—	0.452 (0.027)	—
m	0.057 (0.014)	0.083 (0.019)	0.094 (N/A)

Note. Posterior standard deviations are reported in parentheses.

Table 6 Point Estimates of the Parameters for the AIM and Extended AIM

Parameter	Original AIM	Extended AIM
p_1	0.021 (0.011)	0.013 (0.005)
q_1	0.179 (0.075)	0.236 (0.169)
p_2	0.230 (0.289)	0.025 (0.009)
q_2	0.402 (0.255)	0.146 (0.063)
Θ	0.833 (0.152)	0.719 (0.085)
$a^{1\to1}$	—	0.144 (0.041)
$a^{2\to2}$	—	0.818 (0.062)
M	0.058 (0.011)	0.122 (0.067)

Note. Posterior standard deviations are reported in parentheses.

data much better than the original models.[11] Similarly, the ABM does not fit the holdout aggregate penetration data as well as the extended models that incorporate individual-level social interactions data.

As mentioned above, discussions with the brand manager in charge of PROD confirmed that penetration after one year is a key managerial metric for CPG companies, and that managers are typically interested in predicting this quantity a few months after the launch of the product. In our case, the product reached a penetration of 7.71% after 12 four-week periods ($t = 48$). The point estimates of this quantity made by the original models after six four-week periods were off by approximately 2% (5.71% forecast for the MIM and 5.73% for the AIM). The estimate from the ABM was off by a similar amount (forecast of 5.96%). The estimates made by the extended models were both within 0.7% of the truth (7.09% for the extended MIM and 8.40% for the extended AIM). Such increase in predictive ability may result in substantively different managerial actions and translate into a substantial increase in profit, which outweighs the costs involved in collecting the additional data.[12]

Finally, we check the robustness of our results by changing the number of aggregate penetration data points used for calibration from six to four, five, seven, and eight (using the remaining observations for validation). The results, reported in Online Appendix G (in the electronic companion), are consistent with those obtained when using six aggregate penetration data points for calibration.

Therefore, our results demonstrate that complementing aggregate penetration data with individual-level social interactions data using the approach proposed in this paper has the potential to improve aggregate penetration forecasts. Although the general usefulness of this approach may not be established with a single field study, the results are encouraging, and this application provides a proof of concept. We hope that future research will provide additional tests of the proposed approach, with the caveat that our data collection effort spanned a year: the penetration data were tracked for a year, and the social interactions data needed to be collected in the first few months after launch, ruling out the possibility of studying past innovations for which these data were not collected at that time.

[11] Tables 5 and 6 suggest that the original models tend to produce lower estimates of m compared to their extended counterparts. Simulations (available from the authors) showed that the original MIM has a tendency to underestimate m when the extended MIM is the true data generation process. Future research may explore the robustness of this finding and its potential link to the lower out-of-sample predictive performance achieved by the original models.

[12] We also computed observed and predicted *discounted* penetration at $t = 48$ assuming monthly discount rates that correspond to yearly discount rates of 5% and 15%. Similar results were obtained. Details are available from the authors.

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5. Conclusions, Limitations, and Directions for Future Research

Academic researchers have found repeatedly that social interactions influence consumers' purchase decisions. Moreover, individual-level social interactions data have become increasingly easy and inexpensive to collect. This suggests an opportunity for managers to leverage social interactions data to better forecast the penetration of new products. However, to the best of our knowledge, the extant literature offers no practical method for doing so (unless data are available from a set of related previously launched innovations, from the entire set of potential adopters, or for extended periods of time).

The present paper attempts to close that gap. We have proposed an approach for using individual-level social interactions data to improve aggregate penetration forecasts. We have shown how social interactions may be captured through an individual-level hazard rate, developed in such a way that (i) closed-form expressions for the resulting aggregate penetration process are available, and (ii) this aggregate penetration process nests extant diffusion models. The first characteristic enables estimating the parameters of the model by combining early aggregate penetration data with social interactions data coming from a sample of consumers. Future penetration may be then forecasted based on these parameters. Our field study suggests that our approach has the potential to improve managers' ability to forecast penetration when only limited aggregate penetration data are available. We expect this approach to be particularly useful with really new products that have no similar historical comparisons and for products that are new to the firm where no similar products have been launched by the company. Any improvement in forecasting can substantially impact launch and production decisions for innovating firms.

Although our focus in this paper was on forecasting, the proposed approach may also provide a deeper understanding of the mechanisms of the diffusion process, in at least two ways. First, it may enable testing several assumptions on the mechanisms of the diffusion process, for example, by comparing various nested special cases of the general model proposed in Appendix A. Second, by decomposing the adoption process between the generation of recommendations and adoption conditional on recommendations, the proposed approach may provide a deeper understanding of barriers to adoption. In particular, it may help identify innovations for which a small number of recommendations are given versus recommendations that have a low probability of leading to adoption.

We recognize there are limitations to our approach, which provide opportunities for future research. First, we caution managers that the proposed approach may be less useful for innovations that have lower word-of-mouth potential, and that it relies on the level of accuracy with which social interactions are recorded. Second, the proposed approach assumes that the marginal effect of each recommendation (captured by the parameter q) is constant within each consumer. Empirical evidence suggests that this may not be the case (e.g., Leskovec et al. 2007). Third, Equation (2) implies that adopters choose the recipients of their recommendations with replacement (i.e., they may make multiple recommendations to the same consumer). Future research may test the validity of this assumption and propose adequate corrections if necessary. Fourth, whereas our field study provided a proof of concept, future research may further test the usefulness of the proposed approach. In particular, future studies may test different ways of collecting social interactions data and estimating the model parameters. Fifth, the framework itself may be extended, for example, to capture specific network structures (Barabási and Albert 1999, Dover et al. 2012, Shaikh et al. 2007, Trusov et al. 2013, Watts and Strogatz 1998), or different types of ties or relationships (Ansari et al. 2011, Iyengar et al. 2011). Sixth, the models specified in this paper use discrete-time intervals, making the parameters a function of the data frequency. Future research may explore continuous-time versions. Finally, the proposed approach may be extended to capture repeat sales, and the estimation approach may be extended to produce prelaunch forecasts. With the growing ease of collecting social interactions data, the opportunities to study how these data may impact managerial decision making are ripe for marketing researchers to capitalize upon.

Acknowledgments

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Appendix A. General Model

We present here a more general model that nests the models included in this paper and that relaxes some of the assumptions made in this paper to nest extant models. The notations are the same as in the paper, unless indicated otherwise. We consider two segments in the population ("innovators" and "imitators").

The conditional hazard rates in the innovators and imitators segments are written as follows for a consumer indexed by i:

$$h^1(p_i^1, q_i^{+,1}, q_i^{-,1}, \{r_{i,t-\tau}^{+,1\to 1}\}_{\tau=0\ldots t-1}, \{r_{i,t-\tau}^{-,1\to 1}\}_{\tau=0\ldots t-1})$$
$$= [1-(1-p_i^1)(1-q_i^{+,1})^{\sum_{\tau=0}^{t-1}\alpha_\tau^{+,1\to 1} r_{i,t-\tau}^{+,1\to 1}}]$$
$$\cdot (1-q_i^{-,1})^{\sum_{\tau=0}^{t-1}\alpha_\tau^{-,1\to 1} r_{i,t-\tau}^{-,1\to 1}},$$
$$h^2(p_i^2, q_i^{+,2}, q_i^{-,2}, \{r_{i,t-\tau}^{+,1\to 2}\}_{\tau=0\ldots t-1}, \{r_{i,t-\tau}^{+,2\to 2}\}_{\tau=0\ldots t-1},$$
$$\{r_{i,t-\tau}^{-,1\to 2}\}_{\tau=0\ldots t-1}, \{r_{i,t-\tau}^{-,2\to 2}\}_{\tau=0\ldots t-1})$$

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$$= [1-(1-p_i^2)(1-q_i^{+,2})^{\sum_{\tau=0}^{t-1}(\alpha_\tau^{+,1\to 2} r_{i,t-\tau}^{+,1\to 2} + \alpha_\tau^{+,2\to 2} r_{i,t-\tau}^{+,2\to 2})}]$$
$$\cdot (1-q_i^{-,2})^{\sum_{\tau=0}^{t-1}(\alpha_\tau^{-,1\to 2} r_{i,t-\tau}^{-,1\to 2} + \alpha_\tau^{-,2\to 2} r_{i,t-\tau}^{-,2\to 2})}.$$

The superscripts + and − refer to positive and negative recommendations, respectively. The parameter $q_i^{-,k}$ may be interpreted as the probability that a negative recommendation will prevent consumer i in segment k from adopting. The parameter $\alpha_\tau^{+,k\to j}$ (resp., $\alpha_\tau^{-,k\to j}$) captures the effect of a positive (resp., negative) recommendation made τ periods ago by a consumer in segment k to a consumer in segment j. The numbers of recommendations of different types received in period t are given as

$$r_{i,t}^{+,1\to 1} \sim \text{Bin}\left(ties_i^{1\to 1}, \sum_{\tau=1}^{t-1} a_{t,\tau}^{+,1\to 1} f_{t-\tau}^1\right),$$
$$r_{i,t}^{-,1\to 1} \sim \text{Bin}\left(ties_i^{1\to 1}, \sum_{\tau=1}^{t-1} a_{t,\tau}^{-,1\to 1} f_{t-\tau}^1\right)$$

if consumer i is in segment 1 (innovator), and

$$r_{i,t}^{+,1\to 2} \sim \text{Bin}\left(ties_i^{1\to 2}, \sum_{\tau=1}^{t-1} a_{t,\tau}^{+,1\to 2} f_{t-\tau}^1\right),$$
$$r_{i,t}^{+,2\to 2} \sim \text{Bin}\left(ties_i^{2\to 2}, \sum_{\tau=1}^{t-1} a_{t,\tau}^{+,2\to 2} f_{t-\tau}^2\right),$$
$$r_{i,t}^{-,1\to 2} \sim \text{Bin}\left(ties_i^{1\to 2}, \sum_{\tau=1}^{t-1} a_{t,\tau}^{-,1\to 2} f_{t-\tau}^1\right),$$
$$r_{i,t}^{-,2\to 2} \sim \text{Bin}\left(ties_i^{2\to 2}, \sum_{\tau=1}^{t-1} a_{t,\tau}^{-,2\to 2} f_{t-\tau}^2\right)$$

if consumer i is in segment 2 (imitator), where $ties_i^{k\to j}$ refers to the number of "incoming" social ties that consumer i in segment j has with consumers in segment k, i.e., the number of consumers in segment k who may recommend the innovation to consumer i.

The number of positive and negative recommendations given in period t by consumer i who adopted in period $t-\tau$ (where $\tau > 0$) is given by

$$g_{i,(t-\tau)\to t}^{+,1\to 1} \sim \text{Bin}\left(ties_{i,\text{out}}^{1\to 1},\ a_{t,\tau}^{+,1\to 1}\right),$$
$$g_{i,(t-\tau)\to t}^{-,1\to 1} \sim \text{Bin}\left(ties_{i,\text{out}}^{1\to 1},\ a_{t,\tau}^{-,1\to 1}\right),$$
$$g_{i,(t-\tau)\to t}^{+,1\to 2} \sim \text{Bin}\left(ties_{i,\text{out}}^{1\to 2},\ a_{t,\tau}^{+,1\to 2}\right),$$
$$g_{i,(t-\tau)\to t}^{-,1\to 2} \sim \text{Bin}\left(ties_{i,\text{out}}^{1\to 2},\ a_{t,\tau}^{-,1\to 2}\right)$$

if consumer i is in segment 1, and by

$$g_{i,(t-\tau)\to t}^{+,2\to 2} \sim \text{Bin}\left(ties_{i,\text{out}}^{2\to 2},\ a_{t,\tau}^{+,2\to 2}\right),$$
$$g_{i,(t-\tau)\to t}^{-,2\to 2} \sim \text{Bin}\left(ties_{i,\text{out}}^{2\to 2},\ a_{t,\tau}^{-,2\to 2}\right)$$

if consumer i is in segment 2, where $ties_{i,\text{out}}^{k\to j}$ refers to the number of "outgoing" social ties that consumer i in segment k has with consumers in segment j, i.e., the number of consumers in segment j to whom consumer i may recommend the innovation, and $a_{t,\tau}^{+,k\to j}$ and $a_{t,\tau}^{-,k\to j}$ refer to the recommendation probabilities in period t from consumers in segment k who adopted in period $t-\tau$ to consumers in segment j. Note that letting the parameter a be a function of the time period t allows capturing the impact of time-varying marketing mix variables on the generation of recommendations.

Instead of assuming homogeneous parameters in each segment, we let $\{p^1, q^{+,1}, q^{-,1}, p^2, q^{+21}, q^{-2}, ties^{1\to 1}, ties^{1\to 2}, ties^{2\to 2}, ties_{\text{out}}^{1\to 1}, ties_{\text{out}}^{1\to 2}, ties_{\text{out}}^{2\to 2}\}$ be heterogeneous across consumers and distributed according to a discrete distribution with probability mass function g.

Closed-form expressions for the aggregate penetration in the innovator segment among innovators with number of ties $ties^{1\to 1}$ and with p^1, $q^{+,1}$, $q^{-,1}$ are as follows:

$$f_t^{1,ties^{1\to 1},p^1,q^{+,1},q^{-,1}}$$
$$= (1-F_{t-1}^{1,ties^{1\to 1},p^1,q^{+,1},q^{-,1}}) E_{\{r_{t-\tau}^{+,1\to 1}, r_{t-\tau}^{-,1\to 1}\}_{\tau=0\ldots t-1}} [h^1(p^1, q^{+,1},$$
$$q^{-,1}, \{r_{t-\tau}^{+,1\to 1}\}_{\tau=0\ldots t-1}, \{r_{t-\tau}^{-,1\to 1}\}_{\tau=0\ldots t-1}) \,|\, ties^{1\to 1}]$$
$$= (1-F_{t-1}^{1,ties^{1\to 1},p^1,q^{+,1},q^{-,1}}) \cdot \sum_{\{r_{t-\tau}^{+,1\to 1}, r_{t-\tau}^{-,1\to 1}\}_{\tau=0\ldots t-1}} (1-(1-p^1)$$
$$\cdot (1-q^{+,1})^{\sum_{\tau=0}^{t-1}\alpha_\tau^{+,1\to 1} r_{t-\tau}^{+,1\to 1}})(1-q^{-,1})^{\sum_{\tau=0}^{t-1}\alpha_\tau^{-,1\to 1} r_{t-\tau}^{-,1\to 1}}$$
$$\cdot P(\{r_{t-\tau}^{+,1\to 1}\}_{\tau=0\ldots t-1}, \{r_{t-\tau}^{-,1\to 1}\}_{\tau=0\ldots t-1} \,|\, ties^{1\to 1}, f_1^1, \ldots, f_{t-1}^1),$$

where

$$P(\{r_{t-\tau}^{+,1\to 1}, r_{t-\tau}^{-,1\to 1}\}_{\tau=0\ldots t-1} \,|\, ties^{1\to 1}, f_1^1, \ldots, f_{t-1}^1)$$
$$= \prod_{\tau=0}^{t-1} \binom{ties^{1\to 1}}{r_{t-\tau}^{+,1\to 1}} \left(\sum_{\tau'=1}^{t-\tau-1} a_{t,\tau'}^{+,1\to 1} f_{t-\tau-\tau'}^1\right)^{r_{t-\tau}^{+,1\to 1}}$$
$$\cdot \left(1-\sum_{\tau'=1}^{t-\tau-1} a_{t,\tau'}^{+,1\to 1} f_{t-\tau-\tau'}^1\right)^{ties^{1\to 1}-r_{t-\tau}^{+,1\to 1}} \binom{ties^{1\to 1}}{r_{t-\tau}^{-,1\to 1}}$$
$$\cdot \left(\sum_{\tau'=1}^{t-\tau-1} a_{t,\tau'}^{-,1\to 1} f_{t-\tau-\tau'}^1\right)^{r_{t-\tau}^{-,1\to 1}}$$
$$\cdot \left(1-\sum_{\tau'=1}^{t-\tau-1} a_{t,\tau'}^{-,1\to 1} f_{t-\tau-\tau'}^1\right)^{ties^{1\to 1}-r_{t-\tau}^{-,1\to 1}}.$$

Aggregate penetration in the innovator segment is obtained by integrating the above over the distribution of $ties^{1\to 1}$, p^1, $q^{+,1}$, $q^{-,1}$.

Similarly, we have the following in the imitators segment:

$$f_t^{2,ties^{1\to 2},ties^{2\to 2},p^2,q^{+,2},q^{-,2}}$$
$$= (1-F_{t-1}^{2,ties^{1\to 2},ties^{2\to 2},p^2,q^{+,2},q^{-,2}})$$
$$\cdot \sum_{\{r_{t-\tau}^{+,1\to 2}, r_{t-\tau}^{+,2\to 2}, r_{t-\tau}^{-,1\to 2}, r_{t-\tau}^{-,2\to 2}\}_{\tau=0\ldots t-1}} (1-(1-p^2)$$
$$\cdot (1-q^{+,2})^{\sum_{\tau=0}^{t-1}(\alpha_\tau^{+,1\to 2} r_{t-\tau}^{+,1\to 2} + \alpha_\tau^{+,2\to 2} r_{t-\tau}^{+2\to 2})}$$
$$\cdot (1-q^{-,2})^{\sum_{\tau=0}^{t-1}(\alpha_\tau^{-,1\to 2} r_{t-\tau}^{-,1\to 2} + \alpha_\tau^{-,2\to 2} r_{t-\tau}^{-,2\to 2})})$$
$$\cdot P(\{r_{t-\tau}^{+,1\to 2}, r_{t-\tau}^{+,2\to 2}, r_{t-\tau}^{-,1\to 2}, r_{t-\tau}^{-,2\to 2}\}_{\tau=0\ldots t-1}$$
$$|\, ties^{1\to 2}, ties^{2\to 2}, f_1^1, \ldots, f_{t-1}^1, f_1^2, \ldots, f_{t-1}^2),$$

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where

$$
\begin{aligned}
&P(\{r^{+,1\to 2}_{t-\tau}, r^{+,2\to 2}_{t-\tau}, r^{-,1\to 2}_{t-\tau}, r^{-,2\to 2}_{t-\tau}\}_{\tau=0\ldots t-1} \\
&\quad \mid ties^{1\to 2}, ties^{2\to 2}, f^1_1, \ldots, f^1_{t-\tau-1}) \\
&\quad = \prod_{\tau=0}^{t-1} \binom{ties^{1\to 2}}{r^{+,1\to 2}_{t-\tau}} \left(\sum_{\tau'=1}^{t-\tau-1} a^{+,1\to 2}_{t,\tau'} f^1_{t-\tau-\tau'}\right)^{r^{+,1\to 2}_{t-\tau}} \\
&\qquad \cdot \left(1 - \sum_{\tau'=1}^{t-\tau-1} a^{+,1\to 2}_{t,\tau'} f^1_{t-\tau-\tau'}\right)^{ties^{1\to 2} - r^{+,1\to 2}_{t-\tau}} \binom{ties^{2\to 2}}{r^{+,2\to 2}_{t-\tau}} \\
&\qquad \cdot \left(\sum_{\tau'=1}^{t-\tau-1} a^{+,2\to 2}_{t,\tau'} f^2_{t-\tau-\tau'}\right)^{r^{+,2\to 2}_{t-\tau}} \\
&\qquad \cdot \left(1 - \sum_{\tau'=1}^{t-\tau-1} a^{+,2\to 2}_{t,\tau'} f^2_{t-\tau-\tau'}\right)^{ties^{2\to 2} - r^{+,2\to 2}_{t-\tau}} \binom{ties^{1\to 2}}{r^{-,1\to 2}_{t-\tau}} \\
&\qquad \cdot \left(\sum_{\tau'=1}^{t-\tau-1} a^{-,1\to 2}_{t,\tau'} f^1_{t-\tau-\tau'}\right)^{r^{-,1\to 2}_{t-\tau}} \\
&\qquad \cdot \left(1 - \sum_{\tau'=1}^{t-\tau-1} a^{-,1\to 2}_{t,\tau'} f^1_{t-\tau-\tau'}\right)^{ties^{1\to 2} - r^{-,1\to 2}_{t-\tau}} \binom{ties^{2\to 2}}{r^{-,2\to 2}_{t-\tau}} \\
&\qquad \cdot \left(\sum_{\tau'=1}^{t-\tau-1} a^{-,2\to 2}_{t,\tau'} f^2_{t-\tau-\tau'}\right)^{r^{-,2\to 2}_{t-\tau}} \\
&\qquad \cdot \left(1 - \sum_{\tau'=1}^{t-\tau-1} a^{-,2\to 2}_{t,\tau'} f^2_{t-\tau-\tau'}\right)^{ties^{2\to 2} - r^{-,2\to 2}_{t-\tau}}.
\end{aligned}
$$

The above system of equations provides a closed-form expression for the marginal penetration in period t in each segment, given the marginal penetrations in the previous periods. Marginal penetration in any period is obtained recursively, *without* using any simulation or numerical approximation. The extended AIM in this paper is obtained as a special case of this general model in which

- $\alpha^{1\to 1}_\tau = \alpha^{1\to 2}_\tau = \alpha^{2\to 2}_\tau = 0$ for $\tau > 0$ (only recommendations from period t influence adoption at t);
- $a^{+,k\to j}_{t',\tau'} = a^{+,k\to j}_{t,\tau}$ for all $\{t, t', \tau, \tau', k, j\}$ (uniform influence over time);
- the probability mass function g is concentrated at one point (homogeneous parameters in each segment);
- ties are symmetric; and
- all recommendations are positive (i.e., $a^{-,k\to j}_{t,\tau} = 0$ for all $\{t, \tau, k, j\}$).

The extended MIM is obtained as a special case if we assume further that one of the segments is empty (e.g., $\theta = 0$).

Appendix B. Calibration of the Extended MIM and Original MIM

All models except the ABM were estimated using Bayesian MCMC estimation (Rossi and Allenby 2003), with the following uninformative priors: $\sigma^2 \sim IG(r_0/2, s_0/2)$ with $r_0 = 1$, $s_0 = 10^{-10}$, p, q, and a (when applicable) uniform on $[0, 1]$, and m uniform on $[0, 1]$. The Metropolis–Hastings algorithm was used for all the parameters, except for σ, which was drawn directly from its (inverse-gamma distributed) conditional posterior distribution.

We first describe the calibration of the extended MIM (referred to with the superscript *EXT-MIM*). The likelihood for the individual-level adoption data and the social interactions data is as follows for consumer i if he or she is an initial nontrier:

$$
\begin{aligned}
&P^{\text{EXT-MIM}}(y_{it_i}, r_{it_i} \mid ties_i, p, q, a) \\
&\quad = P^{\text{EXT-MIM}}(y_{it_i} \mid r_{it_i}, p, q) P^{\text{EXT-MIM}}(r_{it_i} \mid ties_i, p, q, a) \\
&\quad = (1 - (1-p)(1-q)^{r_{it_i}})^{y_{it_i}} ((1-p)(1-q)^{r_{it_i}})^{1-y_{it_i}} \binom{ties_i}{r_{it_i}} \\
&\qquad \cdot (aF^{\text{EXT-MIM}}_{t_i-1})^{r_{it_i}} (1 - aF^{\text{EXT-MIM}}_{t_i-1})^{ties_i - r_{it_i}},
\end{aligned}
$$

where the conditional hazard rate is taken from Equation (1), the likelihood for the number of recommendations received is taken from Equation (3), and the cumulative penetration $F^{\text{EXT-MIM}}_t$ is based on Equation (4).

Based on Equation (2), the likelihood for the social interactions data is as follows for consumer i if he or she is an initial trier:

$$
P^{\text{EXT-MIM}}(g_{it_i} \mid ties_i, a) = \binom{ties_i}{g_{it_i}} a^{g_{it_i}} (1-a)^{ties_i - g_{it_i}}.
$$

Finally, we specify a likelihood function for the aggregate penetration data. We make the standard assumption (see, e.g., Srinivasan and Mason 1986) that the marginal aggregate penetration in the four weeks ending in period t, S_t, is equal to the penetration predicted by the model, plus a normal i.i.d. noise that captures the effects of sampling errors, excluded variables, and misspecifications of the density function (see Srinivasan and Mason 1986, pp. 170–171):

$$
S_t = m(F^{\text{EXT-MIM}}_t - F^{\text{EXT-MIM}}_{t-4}) + \varepsilon_t, \quad \varepsilon_t \sim N(0, \sigma^2),
$$

where m is the (estimated) market potential, and $F^{\text{EXT-MIM}}_t$ is based on Equation (4).

Therefore, our complete likelihood function is as follows:

$$
\begin{aligned}
&P^{\text{EXT-MIM}}(\{y_{it_i}, r_{it_i}\}_{i \text{ is initial nontrier}}, \{g_{it_i}\}_{i \text{ is initial trier}}, \\
&\qquad \{S_t\}_{t=1,\ldots,6} \mid \{ties_i\}, p, q, a, m, \sigma) \\
&\quad = \prod_{i \text{ is initial nontrier}} P^{\text{EXT-MIM}}(y_{it_i}, r_{it_i} \mid ties_i, p, q, a) \\
&\qquad \cdot \prod_{i \text{ is initial trier}} P^{\text{EXT-MIM}}(g_{it_i} \mid ties_i, a) \\
&\qquad \cdot \prod_{t=1}^{6} P^{\text{EXT-MIM}}(S_t \mid \{ties_i\}, p, q, a, m, \sigma).
\end{aligned}
$$

We next describe the calibration of the original MIM (referred to with the superscript *MIM*). The individual-level likelihood for initial nontrier i becomes

$$
P^{\text{MIM}}(y_{it_i} \mid p, q) = \left(\frac{F^{\text{MIM}}_{t_i} - F^{\text{MIM}}_{t_i-1}}{1 - F^{\text{MIM}}_{t_i-1}}\right)^{y_{it_i}} \left(1 - \frac{F^{\text{MIM}}_{t_i} - F^{\text{MIM}}_{t_i-1}}{1 - F^{\text{MIM}}_{t_i-1}}\right)^{1-y_{it_i}},
$$

where $F^{\text{MIM}}_t = (1 - \exp(-(p+q)t))/(1 + (q/p)\exp(-(p+q)t))$ is the penetration given by the MIM.

The likelihood function for the aggregate penetration data is

$$
S_t = m(F^{\text{MIM}}_t - F^{\text{MIM}}_{t-4}) + \varepsilon_t, \quad \varepsilon_t \sim N(0, \sigma^2).
$$

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Therefore, the complete likelihood function for the original MIM is as follows:

$$P^{\mathrm{MIM}}(\{y_{it_i}\}_{i \text{ is initial nontrier}}, \{S_t\}_{t=1,\ldots,6} \mid p, q, m, \sigma)$$
$$= \prod_{i \text{ is initial nontrier}} P^{\mathrm{MIM}}(y_{it_i} \mid p, q) \prod_{t=1}^{6} P^{\mathrm{MIM}}(S_t \mid p, q, m, \sigma).$$

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