OXBRIDGE APPLICATIONS

MAT
MINI-PAPER

MOCK EXAMINATION 1

Oxford Aptitude Test for Mathematics

Before your test – a few pointers on timing…

<u>A guide to the MAT</u>
The Oxford Mathematics Aptitude Test is 2½ hours long and split into 2 sections, a multiple choice section consisting of 10 questions worth 4 marks each (40 total) and 4 longer questions requiring working worth 15 marks each (60 total) for the full test to be marked out of 100.

<u>Multiple Choice</u>
It is advised to spend no longer than 1 hour on the multiple choice section (aim to spend around 5 minutes on each question) and leave the majority of time for the longer and more detailed 2nd section. The answers will need to be filled into the table attached and (although it is not discouraged to express your working) marks are awarded solely for correct answers.
Because of this, it is advised to work through the multiple choice section quickly and not dwell on questions that take too long! In the final 10 minutes these can be revisited with a fresh mind and an answer can often be found. It is important not to leave any of these questions in the table blank as there are no lost marks for wrong answers. If stuck for time, go with the best guess!

<u>The Longer Questions</u>
These questions should take the bulk of your time (aim to spend around 20 minutes on each question) in the exam since this is where marks are harder to pick up. Each applicant has to answer 4 questions but which 4 are designated by your choice of course.
The questions are usually a good mix of algebra, calculus, geometry and some more abstract mathematics.

Time Allowed: usually 2 ½ hours.

For candidates applying for Mathematics, Mathematics & Statistics, Computer Science, Mathematics & Computer Science, or Mathematics & Philosophy.

Instructions to Candidates
The first question on the test is multiple-choice and contains 10 parts each worth 4 marks. Marks are given solely for the correct answers, though candidates are encouraged to show any working in the space provided. Questions 2-7 are longer questions, each worth 15 marks, and candidates will need to show their working. Candidates will answer only 4 of these questions, depending upon which subjects they are taking.

- No calculators, formula sheets or dictionaries are permitted during the test.
- Only answers written in the booklet will be marked. There are spare blank pages at the end of the test paper, but working on these pages will not be marked.
- Further credit cannot be gained by attempting questions other than those appropriate to the degree applied for.

Please turn over.

Section 1: multiple choice

For what range of values of k does the line y=k intersect the graph $y=x^3+4x^2-3x+1$ exactly 3 times?

a) k<13/27 **b) 13/27<k<19** **c) -3<k<1/3** **d) 19<k**

Section 2: longer questions

Let q,r be real coefficients.

i) Show that $x^3-3xqr+q^3+r^3$ can be written as $P(x)(x+q+r)$ where $P(x)$ is a quadratic polynomial.

ii) Show that $2P(x)$ can be expressed as a sum of 3 perfect squares.

A perfect square is an expression of the form $(x-s)^2$

It is given that the equations ax^2+bx+c and bx^2+cx+a have common root k where a,b and c are real and non-zero and $ac\neq b^2$.

iii) Show that $a^2-bc=k(b^2-ac)$ and determine a similar expression for k^2.

iv) Hence show that $(a^2-bc)^2=(b^2-ac)(c^2-ab)$ and therefore $a^3-3abc+b^3+c^3=0$

v) Deduce that either $k=1$ or ax^2+bx+c and bx^2+cx+a are identical.

Turn over for the solutions...

Section 1: multiple choice

Solution:
The line $y=k$ is a horizontal line, intersecting the y axis at k. From our knowledge of cubic polynomials, we know that $y\to-\infty$ as $x\to-\infty$ and $y\to\infty$ as $x\to\infty$. We can therefore eliminate a) and d) since they can't intersect more than once for very large or very small values of k.
Next, calculate the stationary points of the cubic by solving $3x^2+8x-3=0$ to obtain $x=⅓$and $x=-3$. These are the x values at each of the stationary points, however we are interested in the range of y values! They can be calculated simply from the original cubic to find that $y=13/27$ at $x=⅓$and $y=19$ at $x=-3$.
Hence the solution is b)
Remember that since these are shorter questions and no working is required, a process of elimination method can sometimes be very helpful and save time!

Section 2: longer question

i) It is easiest to factorise this expression by inspection,

so let $x^3-3xqr+q^3+r^3=(dx^2+ex+f)(x+q+r)$ and calculate the respective values of d,e and f. Immediately we can see by expanding the right hand expression that d=1 and that $f(q+r)= q^3+r^3$, hence $f=q^2+r^2-qr$. From examining the coefficient of x on both sides, we also see that $f+e(q+r)=3pq$ and hence (since we know f) that $e=-(q+r)$.
Therefore we have that $x^3-3xqr+q^3+r^3=(x^2-qx-rx+q^2+r^2-qr)(x+q+r)$.

ii) From before, $2P(x)=2(x^2-qx-rx+q^2+r^2-qr)$

$=2x^2+2q^2+2r^2-2qx-2rx-2qr=(x-q)^2+(x-r)^2+(q-r)^2$

iii) Since k is a root of both equations, we have $ak^2+bk+c=bk^2+ck+a=0$.

Multiplying one side by b and the other side by a and then subtracting one from the other gives us that $abk^2+ack+a^2-(abk^2+b^2k+bc)=a^2-bc+k(ac-b^2)=0$.
Hence $a^2-bc=k(b^2-ac)$.
Similar manipulation to eliminate k gives us that $(c^2-ab)=k^2(b^2-ac)$.

iv) From before we have $a^2-bc=k(b^2-ac)$ and $(c^2-ab)=k^2(b^2-ac)$. Hence, squaring the first expression, equating the two and simplifying, we get the required expression,

$(a^2-bc)^2=(b^2-ac)(c^2-ab)$.
By expanding both sides we get $a^4-2a^2bc+b^2c^2=b^2c^2-ac^3-ab^3+a^2bc$ which can further be simplified by cancelling and dividing by a to the expression $a^3-3abc+b^3+c^3=0$ as required.

v) Using the factorisation in part i), we have that $(a^2+b^2+c^2-ab-ac-bc)(a+b+c)=0$.

Therefore either $(a+b+c)=0$ and hence $k=1$ or $(a^2+b^2+c^2-ab-ac-bc)$
$=(a-b)^2+(a-c)^2+(b-c)^2=0$ and hence $a=b=c$ (since $y^2 \geq 0$ for all y)