OXBRIDGE
APPLICATIONS

ANSWER LIKE A PRO!
MATHS AND HARD SCIENCES

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MATHS AND HARD SCIENCES: QUESTIONS AND ANSWERS

In this resource, we have chosen eight past-interview questions, and asked a Physics graduate, Mathangi, to explain how she would go about answering each of these tricky subject-specific questions.

Eight truly tricky questions… how would YOU answer?

1. If I am in a room with 5 people and guess all their birthdays, what is the probability of getting (only) one correct?
2. Prove m3-m is always divisible by 6.
3. Right now, as you sit and talk to me, explain how we know a centripetal force exists?
4. I have a three litre and a five litre bottle. How can I get four litres of liquid?
5. How high can I go up a mountain having only eaten a Mars bar?
6. How would you explain what 'momentum' means, to a non-physicist?
7. Explain the principle of how the global positioning system (GPS) operates.
8. How much water has to go through a hydroelectric power station in order for me to make a cup of coffee?

We asked Mathangi, our graduate Physics expert and part-time blogger, how she would approach these questions…

1. If I am in a room with 5 people and guess all their birthdays, what is the probability of getting (only) one correct?

A birthday is only one date in a year. For ease of calculation, I'll first take a year to mean 365 days. The probability of guessing a birthday right is (1/365). Therefore the probably of not guessing a birthday correctly, i.e. guessing wrong, is 1-(1/365) which is (364/365).

Guessing the 5 birthdays are independent events, so I can multiply the probabilities together.

Finally, there are 5 different ways that I can get one birthday right, so I'd have five repetitions of (1/365)*(364/365))*(364/365) *(364/365) *(364/365)

So, the full answer: 5 (1/365) $(364/365)^4$

Other methods include the binomial theorem where you'd take just the term for when n=5 and r=1, and probability being (1/365).

2. Prove m3-m is always divisible by 6.

Using the method of 'Proof by induction', I'll first start off with the basic case, and test the expression for when m=1. 13-1 = 0 satisfies the expression. Then, rearrange:
m3-m = m(m2-1) = m(m+1)(m-1), i.e. (m-1) m (m+1)

Next, I want to test what happens when I try the next number up from m, i.e. (m+1) in the expression. This gives me m (m+1) (m+2).

Now I want to see what the 'jump' is in the successive terms of the series that this expression gives, i.e. the difference between the two consecutive terms, as calculated for m and (m+1):

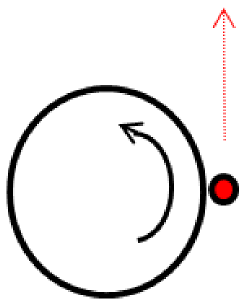
m (m+1) (m+2) – (m-1) m (m+1)
m(m+1) {m+2 – (m-1)} = m(m+1) {3}

There is clearly a factor of 3. But also, m and (m+1) are consecutive numbers, so one of these two numbers will be even, so we can pull out a factor of 2 as well. Hence, the difference has a factor of 3*2 = 6.

And the expression works for the case m=1. So starting there, each successive term has a factor of 6. Therefore, the given expression is always divisible by 6.

3. Right now, as you sit and talk to me, explain how we know a centripetal force exists?

There is force, in addition to Gravity, that keeps us on the Earth as the Earth spins on its axis. Gravity is simply the attractive force between two bodies with mass: We are pulled towards the centre of the earth because of its significant mass. But this has nothing to do with the Earth spinning on its axis. The effect of this additional rotation on us is to make us want to fly off the surface of the earth, on a tangent.



The fact that we don't do this, and are kept pulled into the Earth when we really feel like flying out of it is due to the Centripetal force, also towards the centre of the Earth, that arises due to its rotation.

Extra: Why is it not strictly true to say that one planet orbits another?

Two masses at a distance from each other would both exert a gravitational force on the other, pulling it closer. The extent and size of each force depends on the relative masses of the bodies, but the result of both bodies attracting each other leads to both of them orbiting around their common centre of mass, which is a point in between. In the case of two equal masses, the centre of mass will be exactly halfway. If they were different masses, the centre of orbit would like closer to the heavier mass, but strictly speaking, not at the exact same location as the heavier mass as that would mean that the other body has no mass!

4. I have a three litre and a five litre bottle. How can I get four litres of liquid?

Method 1:

- First fill the 3l bottle and pour it into the empty 5l bottle.
- Fill the 3l bottle again, and pour enough to fill 5l bottle. This leaves exactly 1 litre in the 3l bottle.
- Empty the 5l bottle. Pour the remaining 1 litre from the 3l bottle into the 5l bottle.
- Fill the 3l bottle and pour it into the 5l bottle. The 5l bottle now has exactly 4 litres.

Method 2:

- Fill the 5l bottle and pour it into the 3l bottle until it is full. This leaves 2 litres in the 5l bottle.
- Empty the 3l bottle and pour the 2 litres from the 5l bottle into the 3l bottle.
- Fill the 5l bottle again and pour it into the 3l bottle. Since the 3l bottle had 2 litres of water, only one lire is transferred leaving exactly 4 litres of water in the 5l bottle.

5. How high can I go up a mountain having only eaten a Mars bar?

To go up any height requires energy, which will have to come from the Mars bar in this case and not your energy reserves. Typical calorie figures come in hundreds or thousands. [I don't know where to go from here, so I will make a decision and then check if that was acceptable]. I will go for 200 kilocalories. Again, I don't know what the conversion is from calories to joules from the top of my head. But that is a technical detail which I can ask. 1 calorie is 4.2kJ. So, the energy form the Mars bar is around 840×10^6 from my calculations. I will need to convert this to potential energy (mgh), using my mass for m, g = 10 ms^{-2} and find out what h will be, using mgh = 840×10^6 J

6. How would you explain what 'momentum' means, to a non-physicist?

Momentum can be thought of as the resistance that a body has to the force you put on it. Or: the momentum of a body can tell you how difficult it can be to get the body to change what it is doing - the inertia or resistance to change. So, if you think pushing an object, the heavier the object is, the more difficult it is to get it to shift; so the higher the mass, the greater the momentum. Now if you image an object travelling towards you and you want to halt its motion. The more massive the body, the tougher it is to get it to stop. But equally, the faster the body travels, the harder it is to get it to change its course of motion. Think of a car and a tennis ball as an example. The greater the mass, the greater the speed, the greater the momentum and the harder it is to get a body to change its course of motion.

7. Explain the principle of how the global positioning system (GPS) operates.

GPS satellites transmit a signal unique to each, and repeat these signals at regular time intervals that are synchronised across all satellites and receivers, that then detects these signals to calculate its position on Earth.

The receiver can locate those satellites that are in the local sky and in its line of sight. It needs to locate atleast 4 such satellites so that three can then be used to calculate the position of the receiver in 3-D. The satellite signals contain information about both the time of the message sent and the location of the satellite at time of message transmission. The receiver compares the times of these signals with its own internal time to calculate the time lag, and therefore the transmit time of each message. Using this time, and the speed of light, the receiver can calculate the distance to each of the four satellites. If we consider three of these satellites, the receiver sits on the surface of a sphere from the position of each of the satellites, where the radius is the distance calculated previously. The intersection of these spheres gives the 3-D co-ordinate of the receiver. Ideally the four satellites need to be as spread out as possible - at wide angles from each other - to improve accuracy, instead of clustered around the same line of sight from the receiver.

8. How much water has to go through a hydroelectric power station in order for me to make a cup of coffee?

Ignoring assumptions for the moment, I will take a basic route through the question. Later, I can discuss how the situation could be made more realistic (e.g. by taking into account errors, energy losses, etc.)
A hydroelectric power station utilizes the gravitational force of falling water (the dam attached to the power station facilitates this) and converts this input energy into output electrical energy.
So at the heart of it, this is an energy transfer. The gravitational energy from the falling water is converted to electrical energy which is used to heat up the water needed for the cup of coffee.
The gravitational energy is mgh, where m is the mass of water you are looking to find. Take g=10ms-2; for h, the height of the dam, I can compare it to buildings and imagine a dam to be about 100m. The water in one cup of coffee is about 250ml, and therefore has a mass of 250mg. This will be heated by a kettle (typical power is around 1kW), and that's the connection to the electrical energy output from the power station. Typically the kettle will boil for about a minute (so t=60 seconds). Or you can work out a value using $Q=mc\Delta t$ for the cup of coffee.
So I can approximate the answer, using mgh=Pt or Q.