OXBRIDGE APPLICATIONS

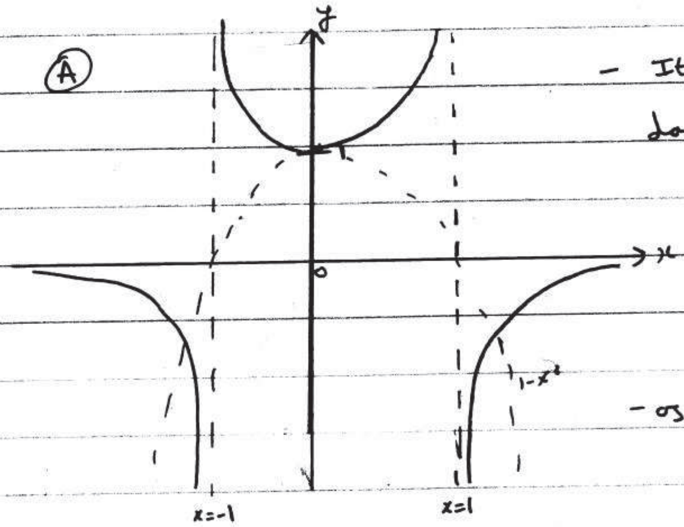
PAT - ANSWERS

MINI-PAPER

1 ~~sketch~~ $\frac{x^2+2}{1-x^2}$:

First, we can simplify the fraction: $\frac{x^2+2}{1-x^2} = \frac{x^2-1+3}{1-x^2} = \frac{3}{1-x^2} - 1$.

now we plot $\frac{1}{1-x^2}$, then scale it by 3 and shift it down by 1.

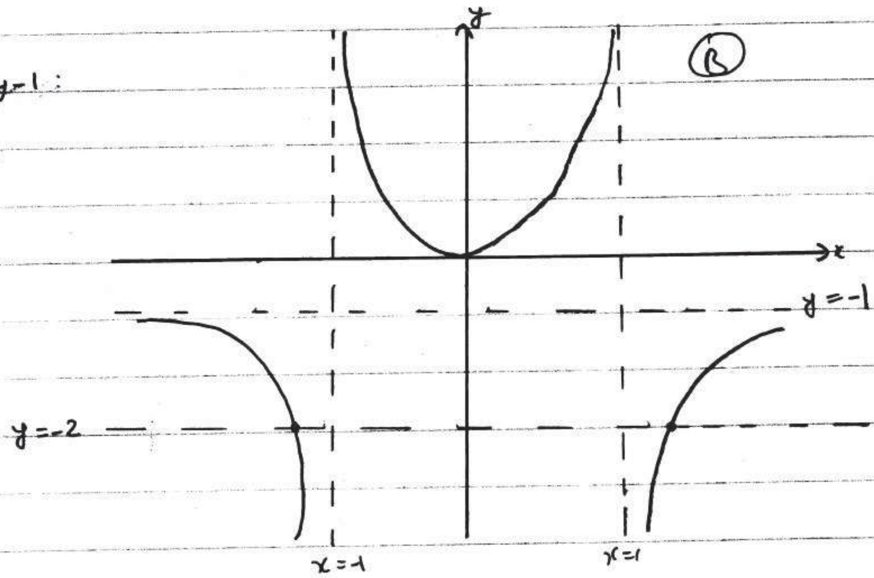


- It is helpful to first plot $y=1-x^2$, as shown dashed, then consider $\frac{1}{1-x^2}$.

- at $x = \pm 1$, $1-x^2=0$, so $\frac{1}{1-x^2}$ diverges.

- as $x \to \pm\infty$, $\frac{1}{1-x^2} \to 0$, as shown.

now, scaling it by $\times 3$ and shift by -1:



Hence or otherwise find the values of x for which $\frac{x^2+2}{1-x^2} > -2$:

As shown on (B), we draw in the line $y=-2$, and see there are two, disjointed, regions of x where this is true. Finding the intersection:

$$\frac{x^2+2}{1-x^2} = -2$$

$$\therefore \quad x^2+2 = -2 + 2x^2$$

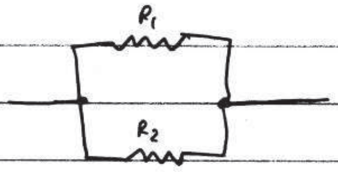
$$\therefore \quad 4 = x^2$$

$$\therefore \quad x = \pm 2$$

so for $x > 2$, $x < -2$, it is true.

<u>But</u> it is also true for $-1 < x < 1$ - this is easy to miss if you don't draw a sketch first!

ie: $x < -2, \; -1 < x < 1, \; x > 2$ is the answer.

2 

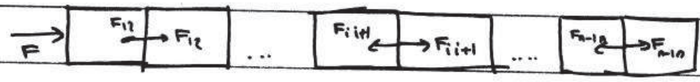
$$\frac{1}{R_{Tot}} = \frac{1}{R_1} + \frac{1}{R_2}$$

ie $\frac{1}{R_{tot}} > \frac{1}{R_1}$ $\therefore$ $R_{tot} < R_1$ (since $\frac{1}{R_1} + \frac{1}{R_2} > \frac{1}{R_1}$)

& $\frac{1}{R_{tot}} > \frac{1}{R_2}$, $\therefore$ $R_{tot} < R_2$. (since $\frac{1}{R_1} + \frac{1}{R_2} > \frac{1}{R_2}$)

So $R_{tot} < R_1, R_2$, the answer is C.

3



(a) i The row of blocks accelerate at $a = \frac{F}{nM}$. since every block has this acceleration, the net force on each block, using $F=ma$, is $\frac{F}{n}$.

ii taking the first block & resolving forces, $F - F_{12} = \frac{F}{n}$

$$\therefore F_{12} = F\left(1-\frac{1}{n}\right).$$

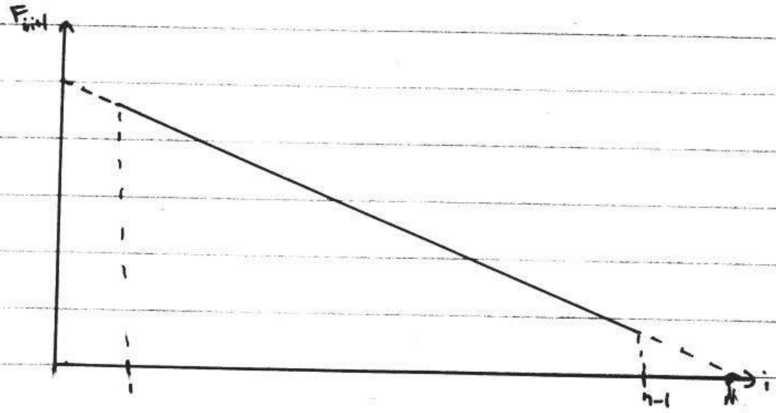
taking the second block: $F_{12} - F_{23} = \frac{F}{n}$

Substituting in F_{12}: $\therefore$ ~~$F_{23} - F(1-\frac{1}{n}) = \frac{F}{n}$~~ $F\left(1-\frac{1}{n}\right) - F_{23} = \frac{F}{n}$

$$\therefore F_{23} = F - \frac{2F}{n} = F\left(1-\frac{2}{n}\right)$$

and so on, $F_{34} = F\left(1-\frac{3}{n}\right)$, etc, i.e $F_{i,i+1} = F\left(1-\frac{i}{n}\right)$

(b)



ie the contact forces decrease the further down the row we get

3 (c) It begins to resemble a rod, the many discrete blocks approach a continuum. our result tells us the internal stresses decrease down the rod as we push it.