Macroscopic response of a Hall thruster discharge from an axial-radial PIC model

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Contents

Introduction

- HET is a mature technology but physics are still not fully understood (instabilities, wall interaction, non-Maxwellian VDF,...)
- Why kinetic (PIC)?
  - Fluid formulations are not fully consistent and can miss important physics.
- Why 2D PIC axial-radial?
  - 1D PIC models
    - Affordable computational cost
    - Strong assumptions due to missing directions
  - 3D PIC models
    - Potentially captures all physics of the discharge
    - Too computationally expensive
  - 2D PIC models
    - Good compromise
    - Axial: main direction of the discharge
    - Radial: wall effects

“Macroscopic plasma analysis from a 1D-radial kinetic results of a Hall thruster discharge”. PSST 30.11 (2021):115011.

“Kinetic plasma dynamics in a radial model of a Hall thruster with a curved magnetic field”. PSST 31.11 (2022):115003.

Objectives:

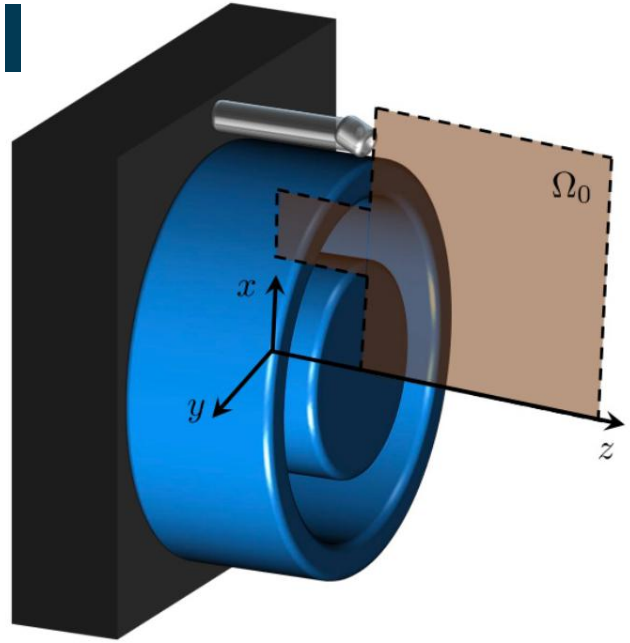
1. Address the validity of some of the assumptions taken by 1D PIC models.

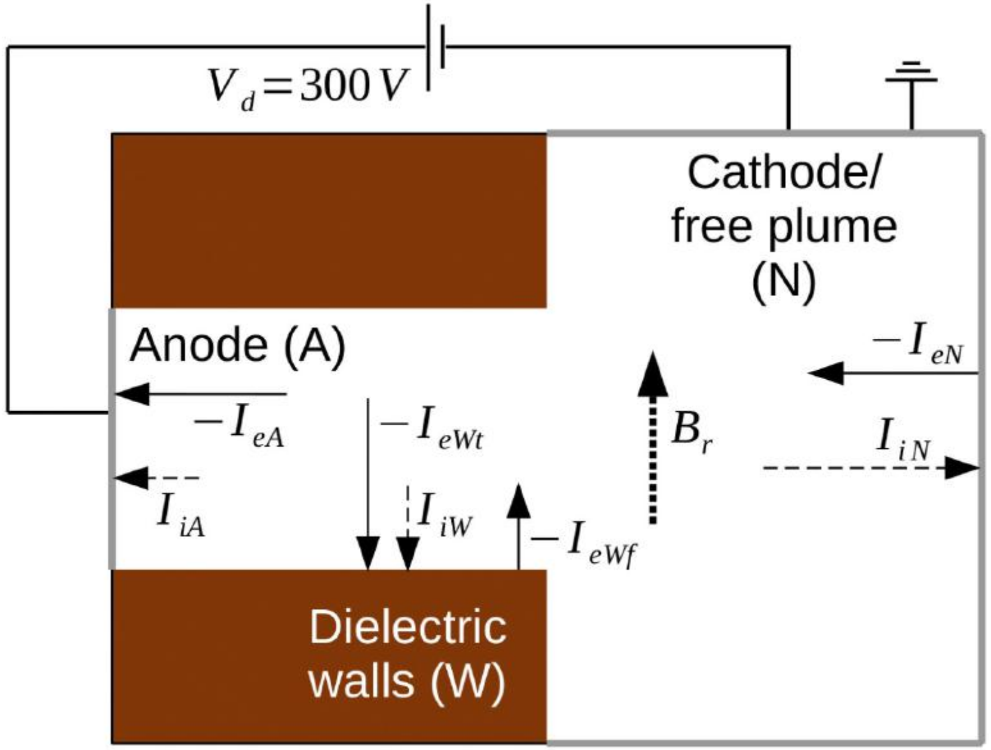
2. Compare the kinetic results with standard fluid formulation highlighting the main differences.

The kinetic model

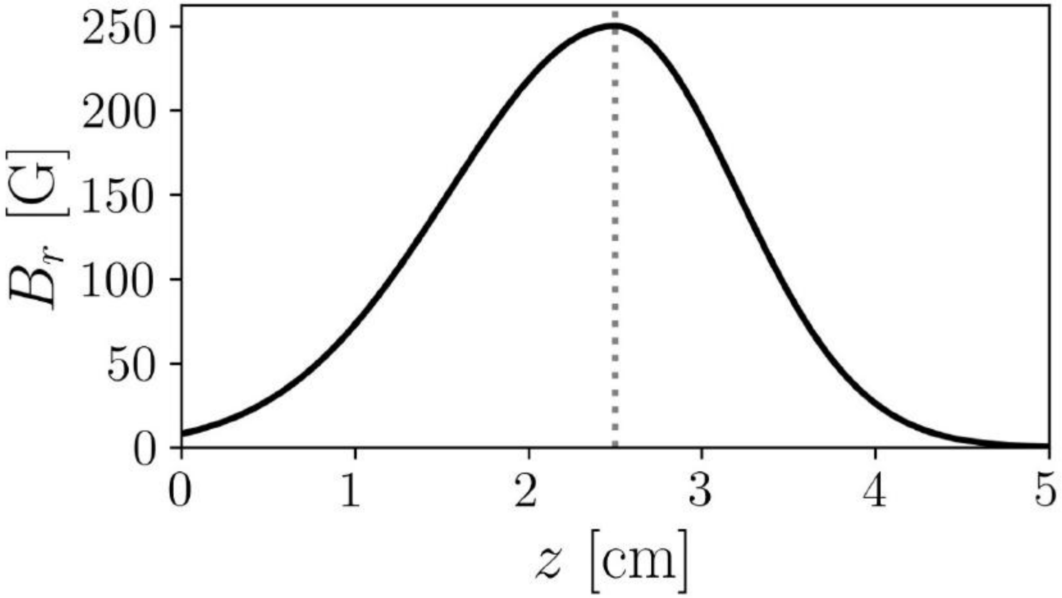
- Electric and magnetic fields
  - Electrostatic assumption.
  - Externally imposed magnetic field, ***B***.
  - Self-consistent solution of the electric field, ***E***.

$$\varepsilon \nabla^2 \phi = -\rho_{el} \qquad \boldsymbol{E} = -\nabla \phi$$



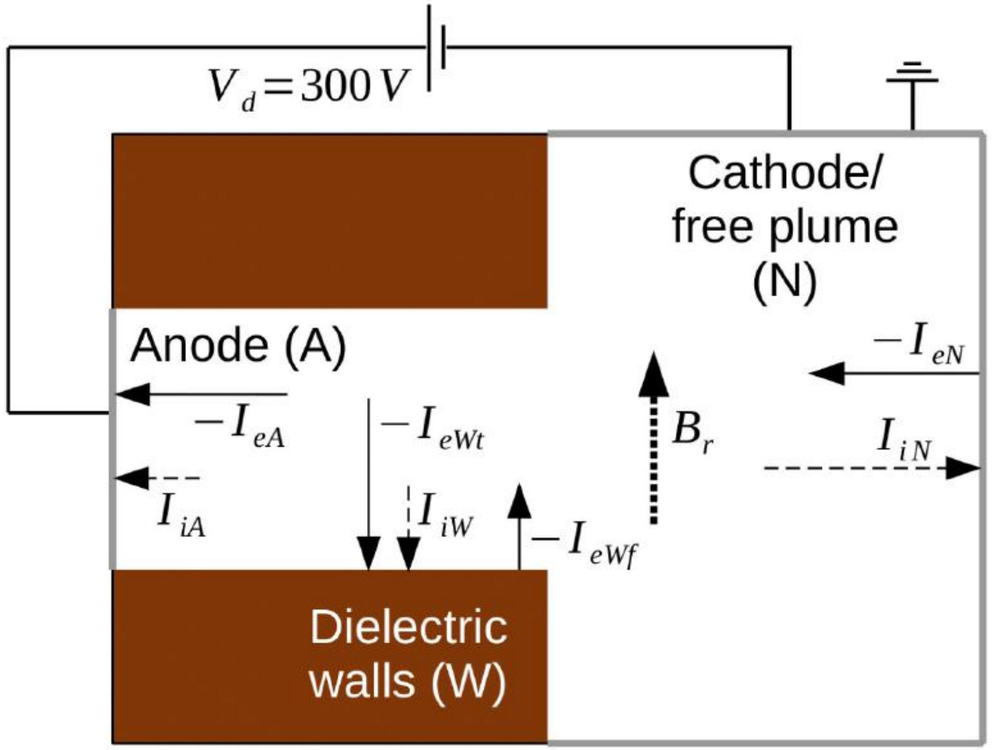


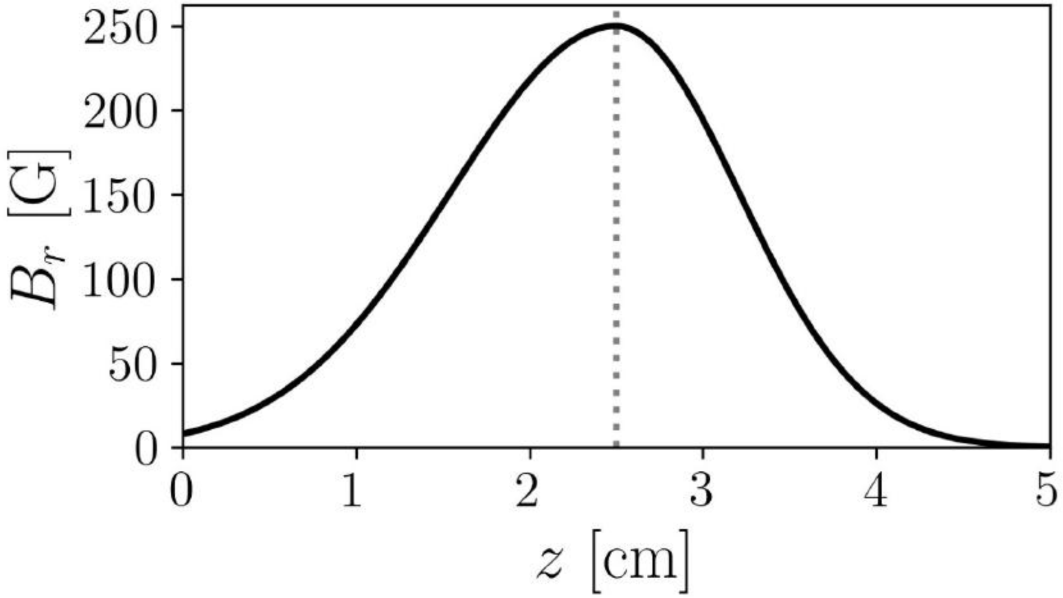
- Particle electrons and singly charged xenon ions
- Fluid neutral background
  - $n_n(z)$ follows a simple depletion law, computed using the steady-state mass continuity equation.
  - u_{zn}=500 m/s and T_n=0.
- Electrically connected anode and cathode
  - I_d continuity is satisfied at any instant
- SEE from lateral dielectric walls



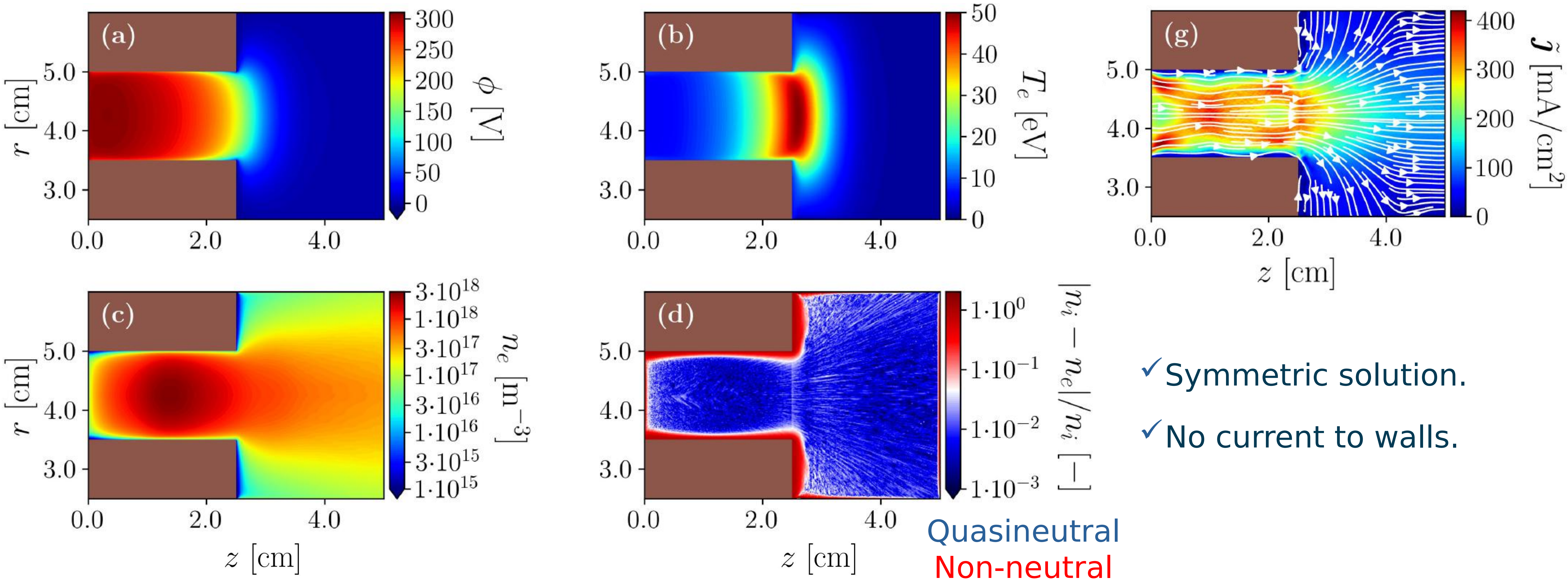
The kinetic model

- **Collisions**
  - Ionization collisions resolved self-consistently.
  - Anomalous electron diffusion (isotropic).
- **Simplified configuration**
  - Radial magnetic field. $\boldsymbol{B}=B_r\boldsymbol{1}_r$
  - Only *true* secondary electrons. $\delta_s=E/E_c$
  - No cylindrical effects (symmetry).
- **Acceleration techniques**
  - Steady-state neutrals → No breathing mode O(1 ms).
  - Vacuum permittivity, $\varepsilon=\varepsilon_0 f_D^2$ → larger Δt, Δx.
- **Numerical parameters (f_D=5)**
  - $\Delta z=\Delta r=0.1$ mm → $N_z=500$, $N_r=350$.
  - $\Delta t=5$ ps, $t_{sim}=50$ μs → $N_t=10\,000\,000$.
  - $n_{e0}=n_{i0}=4\cdot10^{17}$ m^{-3} (20 particles per cell).





Steady-state macroscopic response



✓Symmetric solution.

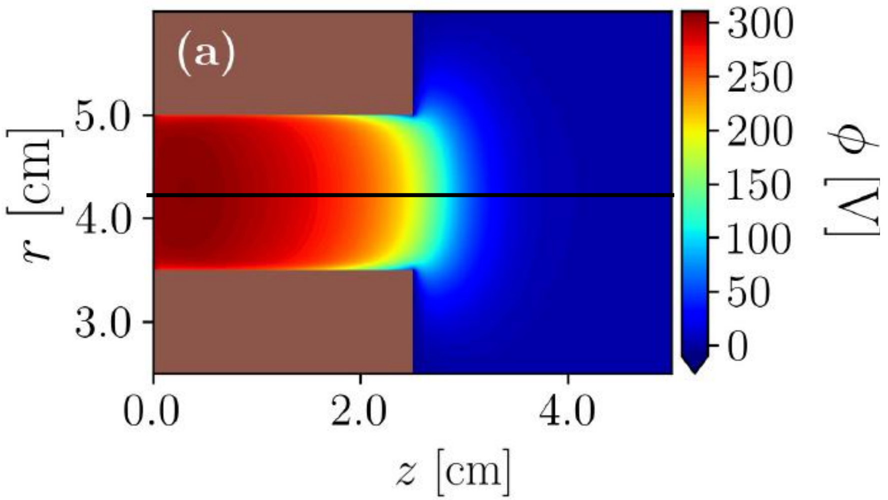
✓No current to walls.

Quasineutral

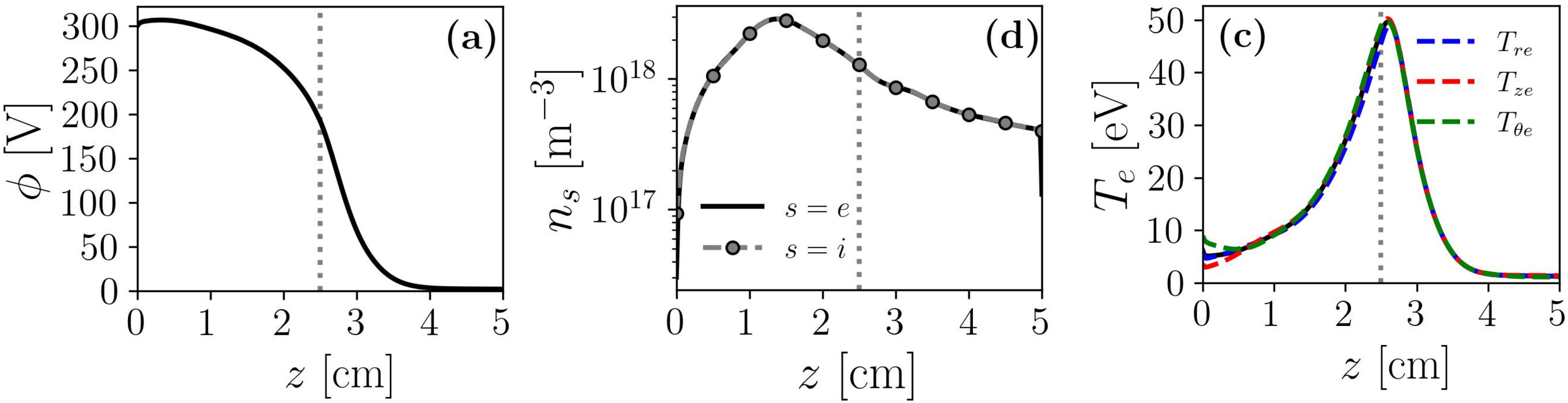
Non-neutral

➢ Debye sheaths at the walls → Thin non-neutral regions with strong gradients of ϕ and n_e

Steady-state macroscopic response



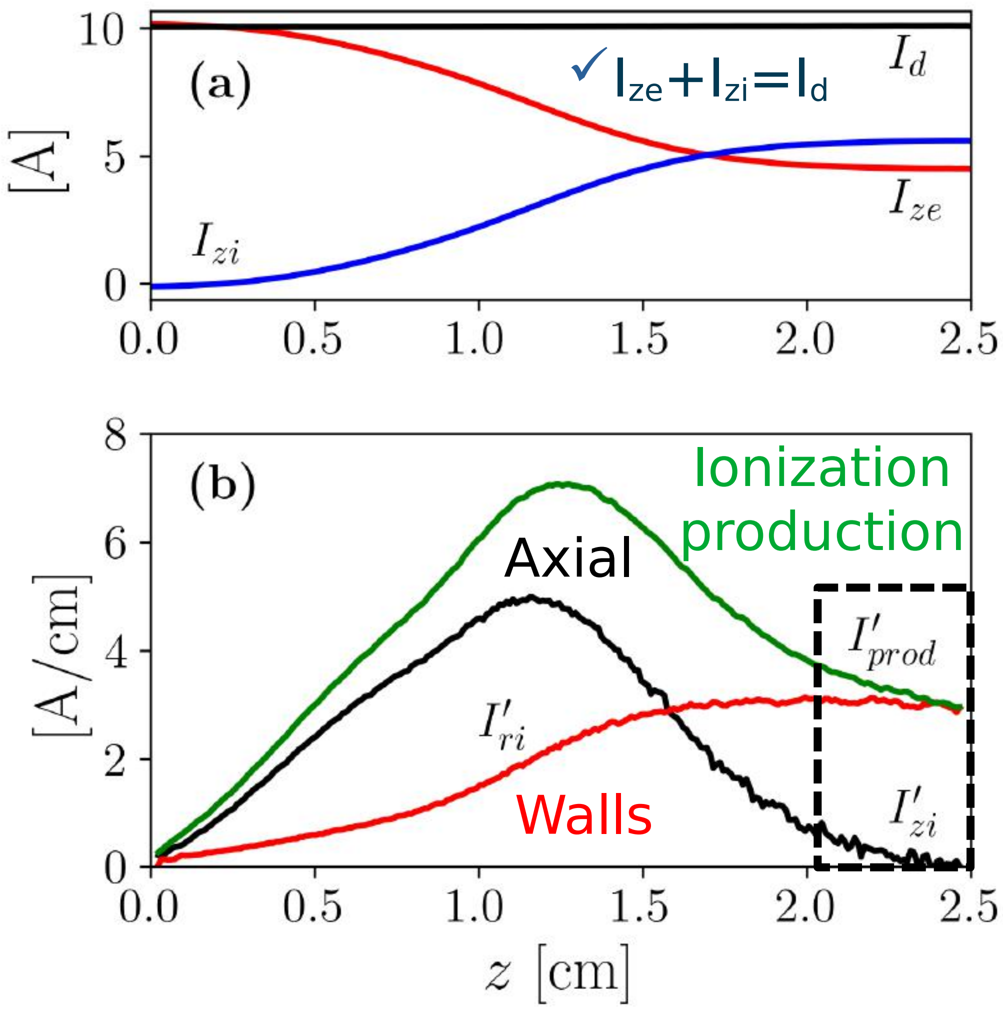
- Axial profiles of main macroscopic variables along mid-radius M.
- Dotted lines mark the thruster exit location (z=2.5 cm)
- Same trends than those obtained by 1Dz fluid and kinetic models.



1Dr code validation

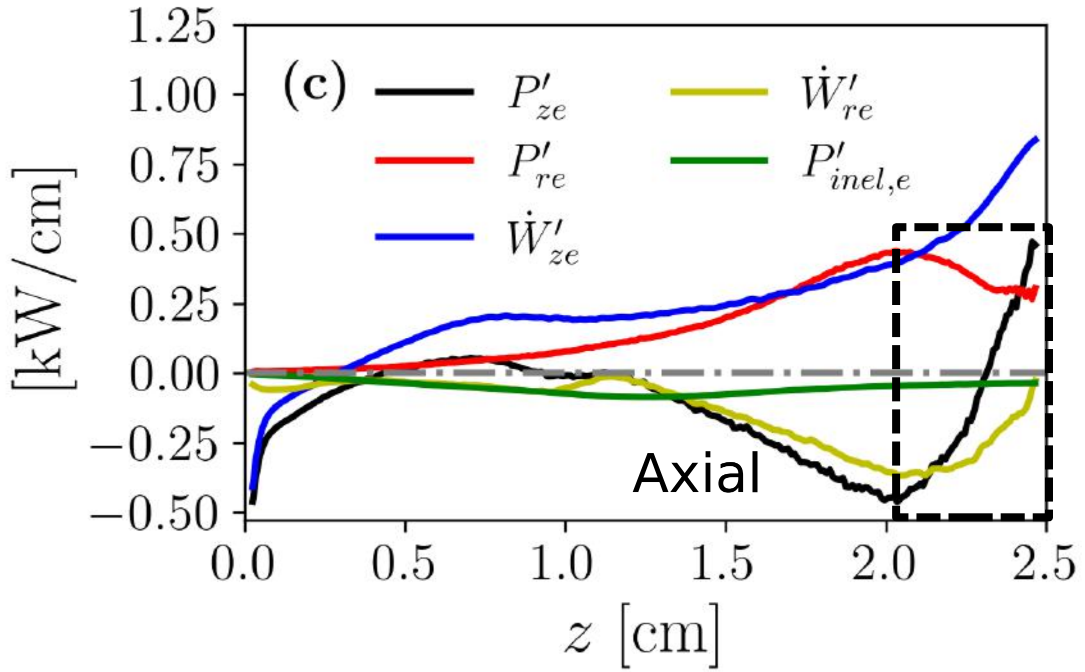
- Current and continuity balances

$$\frac{dI_{zi}}{dz}=\frac{dI_{prod}}{dz}-\frac{dI_{ri}}{dz}$$



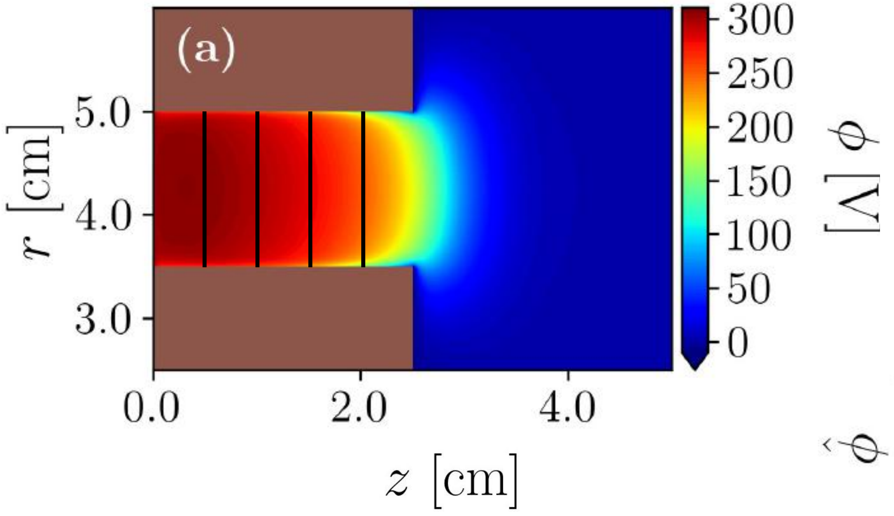
- Electron energy balance

$$\frac{dP_{ze}}{dz}=\frac{d\dot{W}_{ze}}{dz}+\frac{dP_{inel,e}}{dz}-\frac{d}{dz}\left(P_{re}-\dot{W}_{re}\right)$$



- 1D radial model assumptions
  - No axial fluxes $dI_{ze}/dz=dP_{ze}/dz=0$

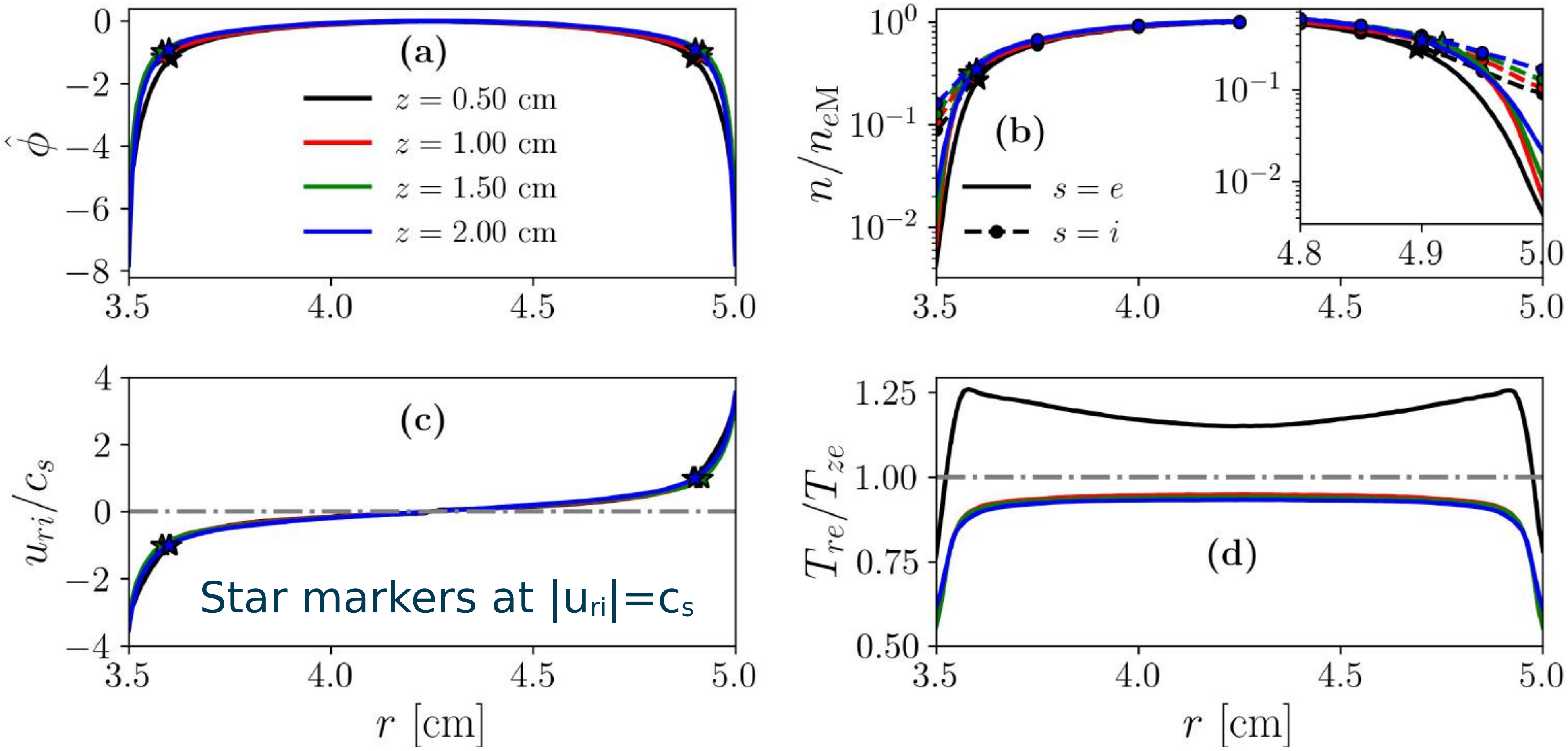
1Dr code validation



- Subscript M refers to the channel mid-radius
- Ion sonic point approximately matches the transition to the sheath

$$\hat{\phi} = e(\phi - \phi_M)/T_e$$

- Sheath potential drop is related to the local electron temperature
- Electron temperature anisotropy
  - $T_{re} > T_{ze}$ near the anode
  - $T_{re} < T_{ze}$ elsewhere

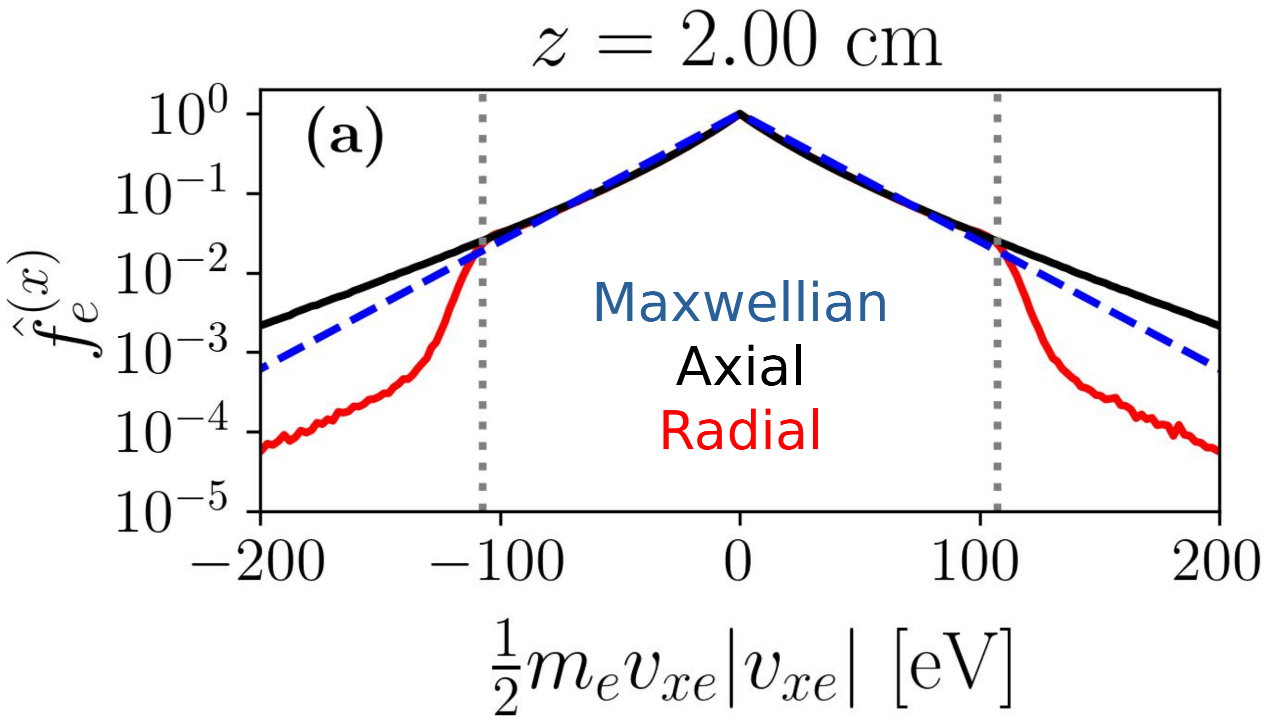


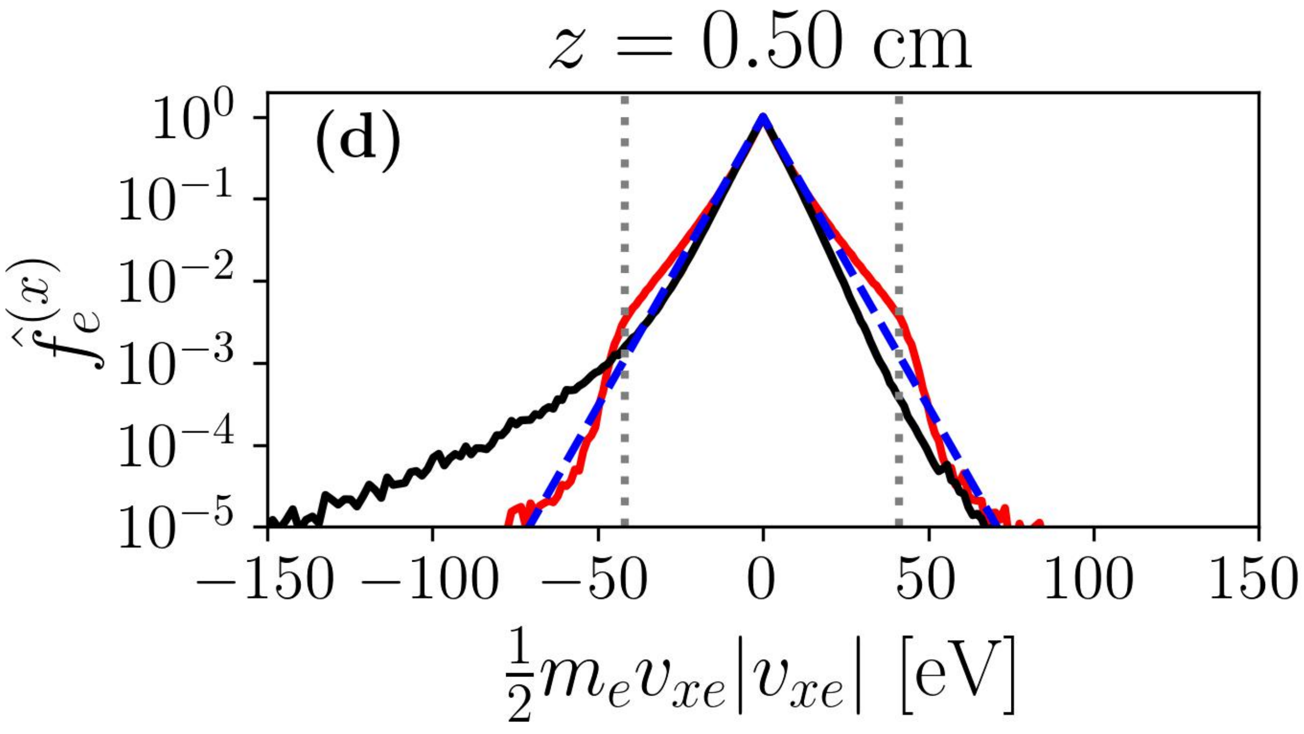
The electron VDF at M

- Near exit region
  - Axial VDF is near-Maxwellian.
  - Radial VDF is depleted beyond the energy required for wall collection.

Wall collection energy ≡$e(\phi_M-\phi_W)$

- Near anode region
  - Axial VDF is asymmetric due to electrons collected at the anode.
  - Radial VDF is still depleted beyond the energy required for wall collection.

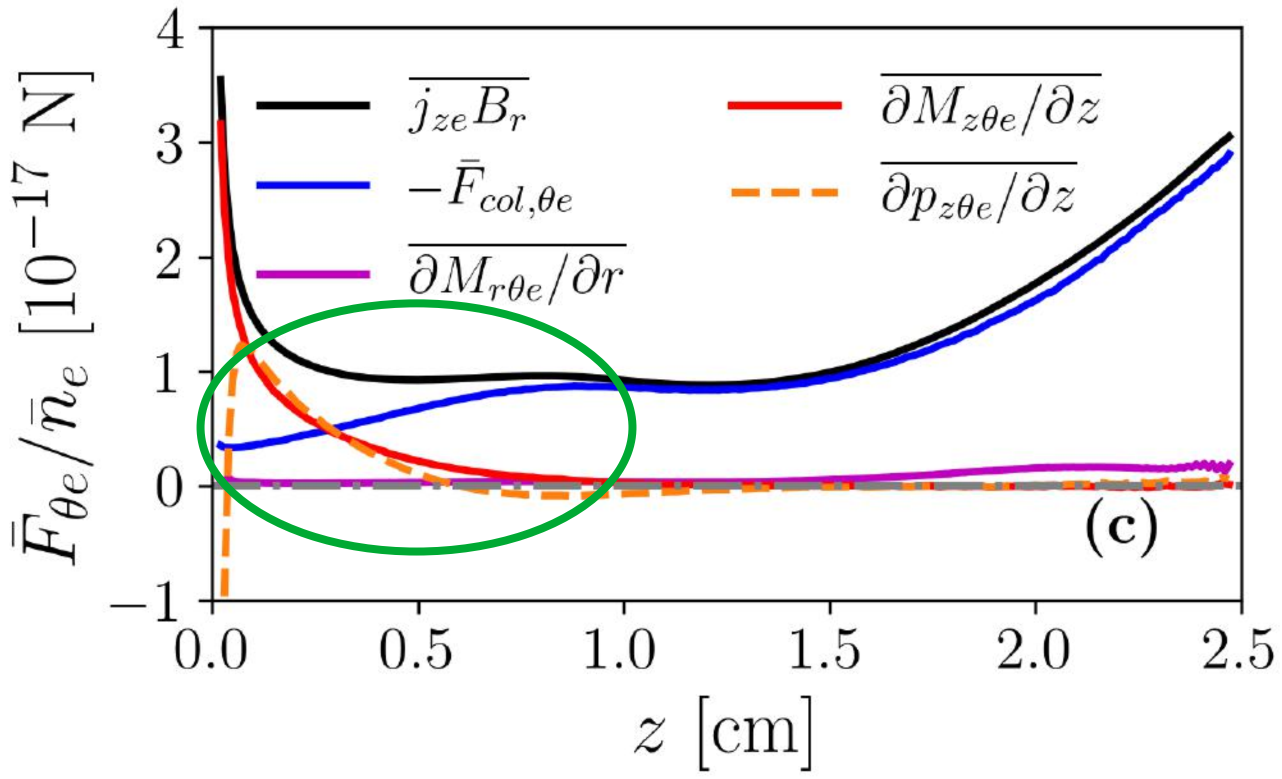




Non-Maxwellian electron VDF features have a strong effect on wall interaction parameters

The electron momentum equation

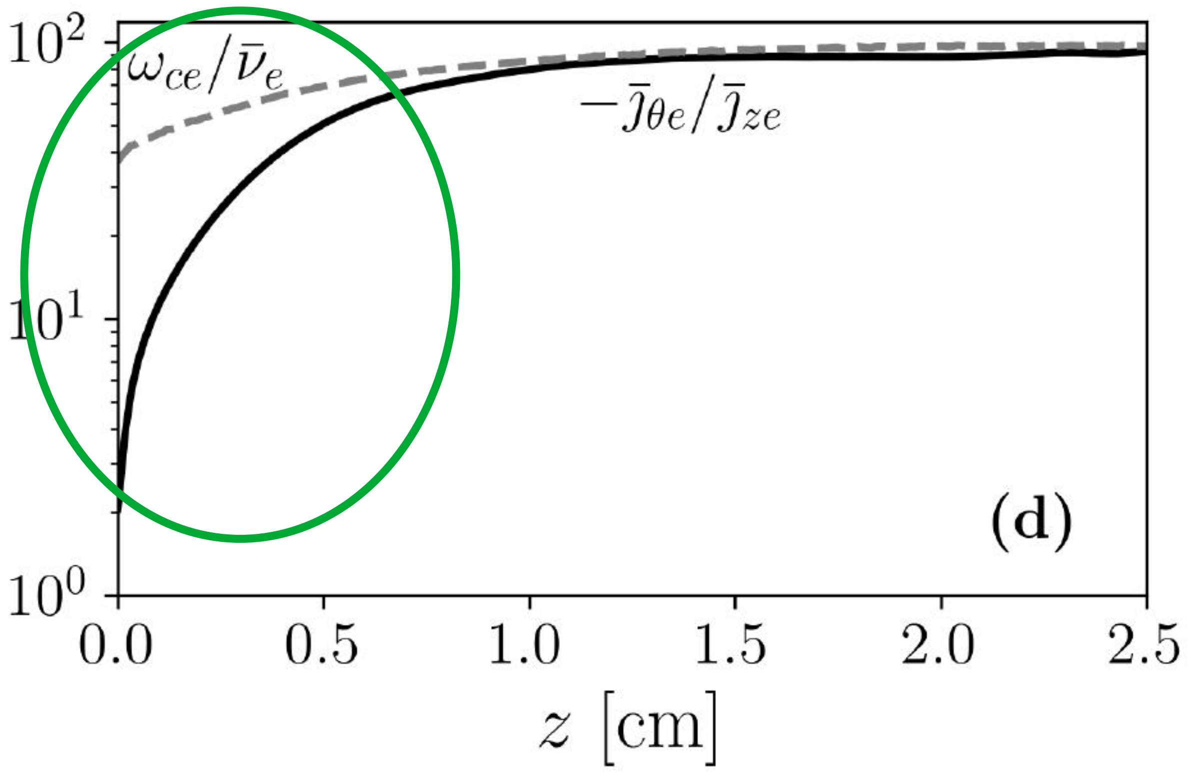
- Standard solutions along z and r directions
  - Radial momentum $e n_e \frac{\partial \phi}{\partial r} \simeq \frac{\partial p_{rre}}{\partial r}$
  - Axial momentum $-j_{\theta e} B_r \simeq \frac{\partial p_{zze}}{\partial z} + e n_e E_z$



- FLR effects along θ direction
  - All the terms in the equation are small.

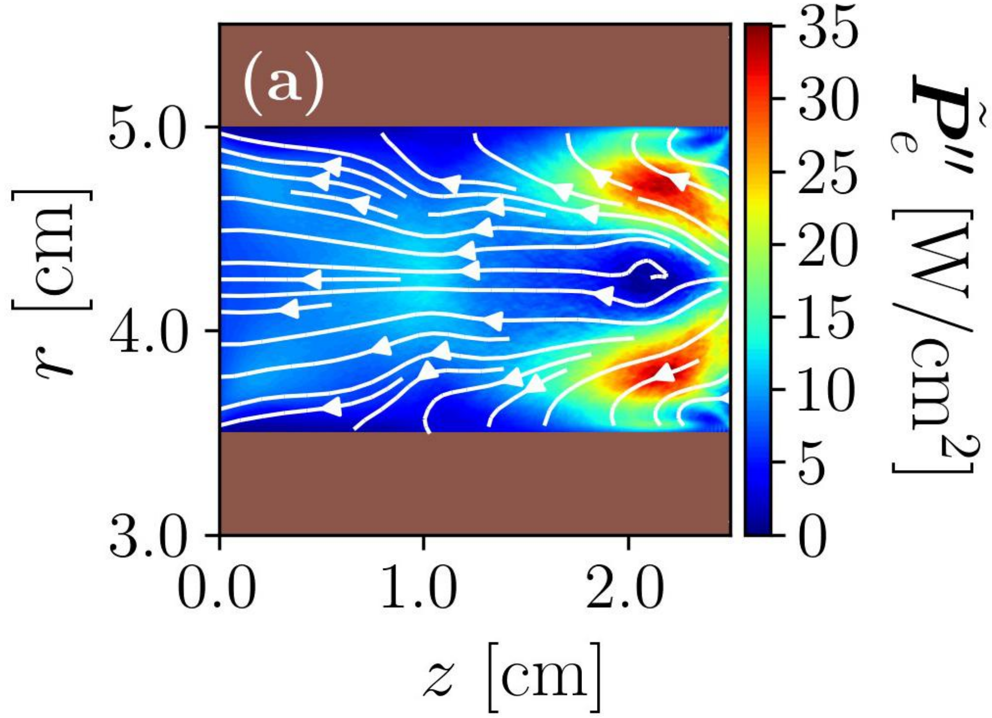
$$j_{ze} B_r = \frac{\partial M_{z\theta e}}{\partial z} + \frac{\partial M_{r\theta e}}{\partial r} + F_{col,\theta e} \qquad \bar{\bar{M}}_e = \bar{\bar{p}}_e + m_e n_e \boldsymbol{u}_e \boldsymbol{u}_e$$

  - Classical solution neglects $\bar{\bar{M}}_e$



The electron energy flux vector

- Complex 2D behavior
- Dominant contributions
  - Heat flux, $\mathbf{q}$
  - Enthalpy flux, $(5/2)T_e n_e \mathbf{u}_e$



$$\boldsymbol{P}_e'' = \frac{1}{2} m_e u_e^2 n_e \boldsymbol{u}_e + \frac{5}{2} T_e n_e \boldsymbol{u}_e + \bar{\bar{\tau}}_e \cdot \boldsymbol{u}_e + \boldsymbol{q}_e$$

$$\bar{\bar{\tau}}_e = \bar{\bar{p}}_e - p_e \bar{\bar{I}}$$ Gyrostress tensor

- Axial energy flux ($B\perp$)
  - Near the exit: $(5/2)T_e n_e u_{ze} \gg q_{ze}$
  - Near the anode: $(5/2)T_e n_e u_{ze} \sim q_{ze}$
  - A conductive (Fourier) law $q_{ze} \simeq -K_e(dT_e/dz)$, seems appropriate only in the near exit region.
- Radial energy flux ($B\parallel$)
  - Enthalpy flux and heat flux are close in all locations in the channel.
  - Heat flux seems to follow a convective-type law. $q_{re} \propto (5/2)T_e n_e u_{re}$

Conclusion and future work

- 2D simulation of a simplified HET discharge.
- The electron VDF is non-Maxwellian and its features are different in the near-anode and near-exit regions. This has important implications on the electron behavior.
  - Wall interaction magnitudes.
  - Full pressure tensor.
  - Complex heat flux vector.
- 1D model assumptions in the near-exit region.
  - Qualitative matching of plasma profiles.
  - $dI_{ze}/dz=0$ has been proven to be a good approximation in the near exit region.
  - Neglecting axial fluxes in the 1Dr power balance is questionable.
  - Radial effects are difficult to model in 1Dz models.

- Future work
  - Realistic/complex magnetic topology.
  - Include other collisional processes.
  - Realistic near plume.

- Other research
  - Penning discharge benchmark.
  - Turbulent transport studies

“Two-dimensional kinetic simulation of electrostatic instabilities in a Hall plasma”. IEPC-2022-314

Thank you! Questions?

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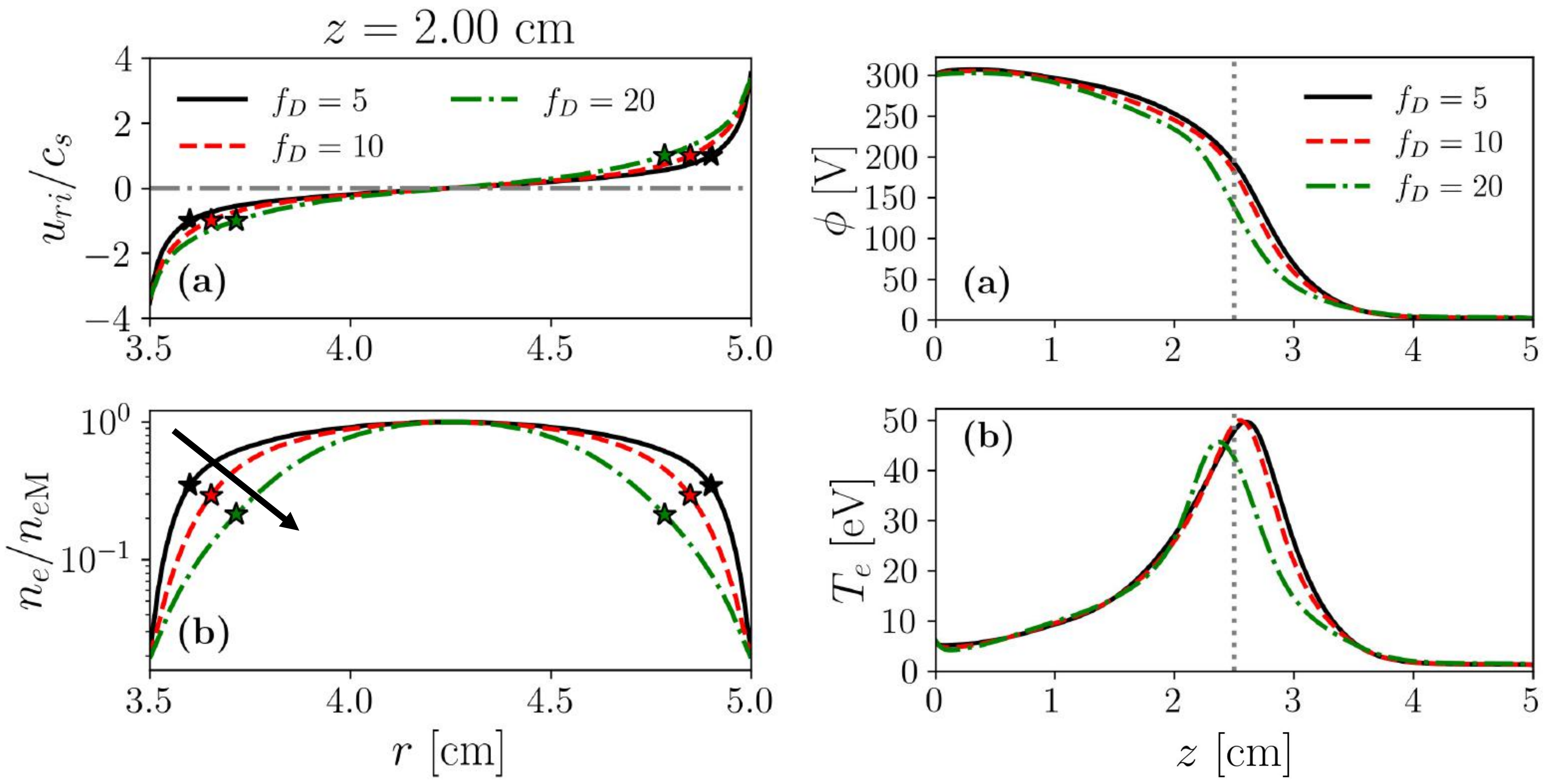
Backup slides

Effects of an augmented vacuum permittivity

Radial profiles

Axial profiles



- Sheath thickness, $\lambda_D \propto f_D$
- Smoother gradients
- Physical ordering
  - Space: $\lambda_D < r_{Le} < L$
  - Time: $1/\omega_{pe} < 1/\omega_{ce}$
- f_D=20: limit case
- f_D=10: good enough
- f_D=5: fine details
- Computational cost scales $\propto f_D^{-3}$