Simulations of the effect of neutral dynamics in magnetic nozzle expansions

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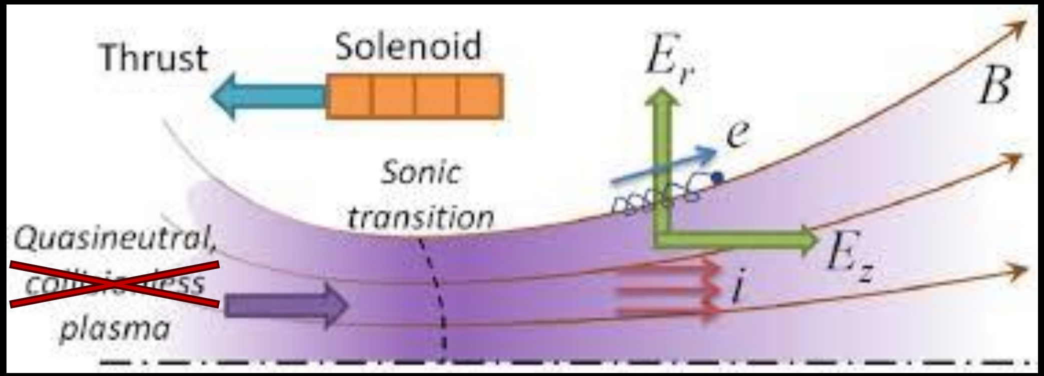
Motivation

- Background pressure is known to play a role on magnetic nozzle performance.
  - Collisions, ionization
- Previous MN studies were collisionless and ignored ionization, charge exchange, and electron collisions
  [Ahedo, Merino, Phys Plasmas 17, 073501 2010 and follow up articles]
- New DGFEM code to overcome previous limitations and explore other plasma expansion configurations (e.g. magnetic arch)
- Preliminary study to account for collisions and ionization in the nozzle



Model equations

- Three-Fluid Model:
  - Quasineutral plasma
  - $\beta = 0$, negligible self induced magnetic field.
  - Massless, polytropic ($p_e \propto n_e^{\gamma}$), fully magnetized electrons (when collisionless).
  - Cold, singly-charged ions.
  - Euler equations for neutrals
  - Ionization and Charge-Exchange Collisions, electron collisions.

$$\frac{\partial n}{\partial t} + \nabla \cdot (n\boldsymbol{u_e}) = S_i$$

$$0 = -\nabla n T_e + en\,\nabla\phi - en\boldsymbol{u_e} \times \boldsymbol{B} - \boldsymbol{R_e}$$

$$\frac{\partial n}{\partial t} + \nabla \cdot (n\boldsymbol{u_i}) = S_i$$

$$m\frac{\partial n\boldsymbol{u_i}}{\partial t} + m\nabla \cdot (n\boldsymbol{u_i} \otimes \boldsymbol{u_i}) = -en\,\nabla\phi + en\,\boldsymbol{u_i} \times \boldsymbol{B} + S_i\,\boldsymbol{u_n} + S_{cex}\,(\boldsymbol{u_n} - \boldsymbol{u_i})$$

$$\frac{\partial n_n}{\partial t} + \nabla \cdot (n_n\boldsymbol{u_n}) = -S_i$$

$$m\frac{\partial n_n\boldsymbol{u_n}}{\partial t} + m\nabla \cdot (n_n\boldsymbol{u_n} \otimes \boldsymbol{u_n}) = -\nabla p_n T_n - S_i\,\boldsymbol{u_n} + S_{cex}\,(\boldsymbol{u_i} - \boldsymbol{u_n})$$

$$\frac{\partial E_n}{\partial t} + \nabla \cdot [(\boldsymbol{E_n} + \boldsymbol{p_n}) \cdot \boldsymbol{u_n}] = 0$$

Collision Models

- Ionization:

$$S_i = n\,\nu_i = n\,n_n c_e \sigma_i$$

$$\sigma_i = \sigma_{i0}\left(1 + \frac{T_e E_i}{(T_e + E_i)^2}\right) e^{-E_i/T_e}$$

$$\sigma_{i0} = 5 \times 10^{-20}\ m^{-2} \quad E_i = 12.1\ e\,V$$

- Charge Exchange:

$$S_{cex} = n\,\nu_{cex} = n\,n_n c_{in} \sigma_{cex}$$

$$\sigma_{cex} = \sigma_{cex\,0}\left(1 - 0.2\ \log\left(\frac{c_{in}}{1\ km/s}\right)\right)$$

$$\sigma_{cex\,0} = 81 \times 10^{-20}\ m^{-2}$$
$$c_{in} = |\boldsymbol{u}_i - \boldsymbol{u}_n|$$

- Electrons (Only considered a posteriori):

$$\nu_e = \nu_{ei} + \nu_{en}$$

$$\nu_{en} = n_n c_e \sigma_{en}$$
$$\sigma_{en} = 27 \times 10^{-20}\ m^2$$

$$\nu_{ei} = n\,R_{ei}$$
$$\frac{R_{ei}}{10^{-12}\ m^3 s^{-1}} = 2.9\left(\frac{1eV}{T_e}\right)^{3/2} \log\Lambda$$
$$\log\Lambda \sim 9 + \frac{1}{2}\log\left[\left(\frac{10^{18} m^{-3}}{n_e}\right)\frac{T_e}{1eV}\right]$$

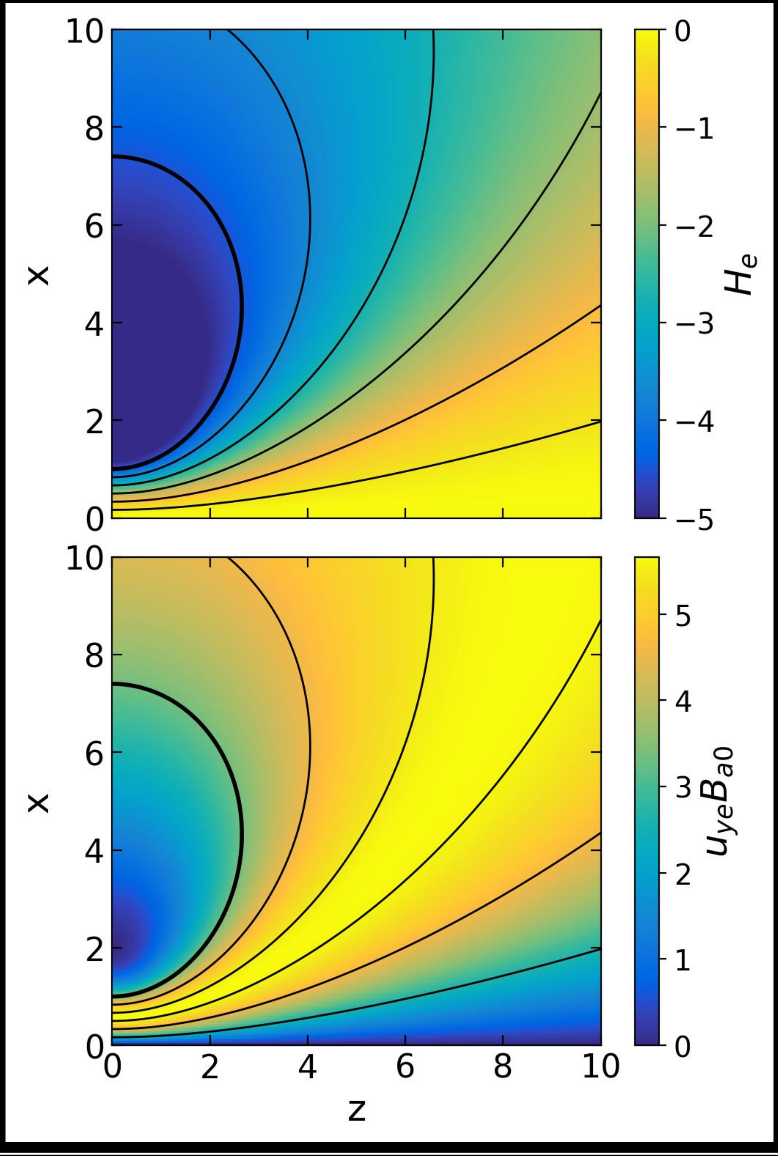
[Enrique Bello-Benítez and Eduardo Ahedo 2021 *Plasma Sources Sci. Technol.* **30** 035003]

Model equations

- R_e is a resistive term which we will take into account a posteriori.
- In the $R_e = 0$ limit, electron equations are found to be algebraic:
    - Electron momentum equation yields the thermalized potential (H_e) and azimuthal velocity (u_{ye})

$$H_e = \frac{\gamma T_{e0}}{\gamma - 1}\left[\left(\frac{n}{n_0}\right)^{\gamma-1} - 1\right] - e\phi$$

$$u_{ye}(\psi_B) = \frac{-1}{eB}\frac{\partial H_e}{\partial \mathbf{1}_\perp}$$



Electron Equations

- A posteriori estimation of the importance of electron collisions.
  - Resistive force in electron momentum equations leads to:

$$0 = -\boldsymbol{\nabla} H_e - e u_{e\theta} B \, \mathbf{1}_\perp + e u_{e\perp} B \, \mathbf{1}_\theta - \boldsymbol{R}_e$$

$$\boldsymbol{R}_e = m_e \nu_e \boldsymbol{u}_e = -eB\chi^{-1}\boldsymbol{u}_e$$

χ → Hall parameter

  - Projected onto magnetic field frame:

$$\frac{\partial H_e}{\partial 1_\parallel} = -eB\chi^{-1}u_{e\parallel}$$

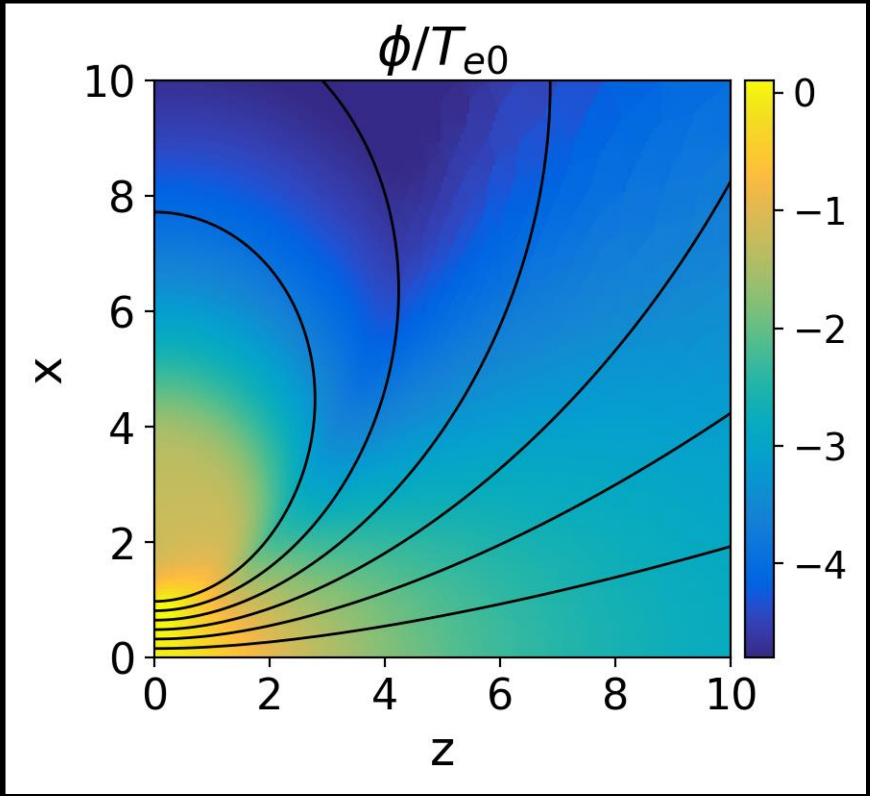
$$u_{e\perp} = \chi^{-1}u_{e\theta}$$

$$\Delta u_{e\theta} = u_{e\theta} - u_{e\theta}\Big|_{\nu_e=0} = -\chi^{-2}u_{e\theta}$$

- This equations act as leading order corrections to electron velocity. One can observe:
  - Perpendicular corrections are $\mathcal{O}(\chi^{-1})$ while azimuthal correction is $\mathcal{O}(\chi^{-2})$
  - Parallel velocity results from solving continuity equation and is affected by $u_{e\perp}$ and S_i

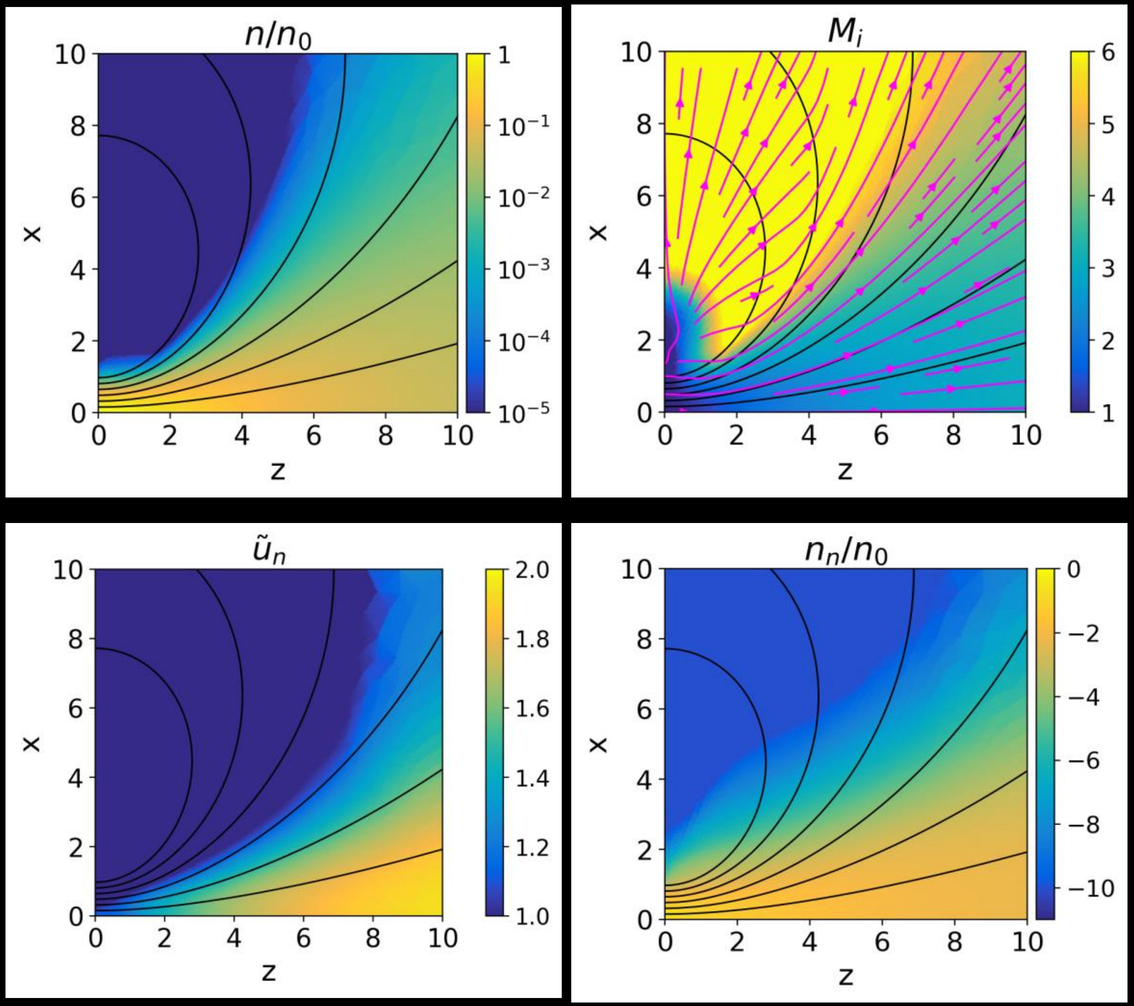
Simulation Setup

- Numerical setup:
  - Discontinuous Galerkin discretization (FEniCS)
    - Order 1 elements in this work.
    - Runge Kutta time-stepping (and final solve for steady state)
  - Unstructured mesh (Gmsh):
    - Cell diameter such that the nozzle throat is resolved in 40 cells.
- Physical parameters:
  - Sonic axial velocity
  - Gaussian density profile:
    - $\mathrm{n}(0) = \mathrm{n}_0$ and $\mathrm{n}(\mathrm{R}_0) = 10^{-3} n_0$ for ions.
    - $\mathrm{n_n}(0) = \alpha \mathrm{n}_0$ and $\mathrm{n_n}(\mathrm{R}_0) = \alpha 10^{-3} n_0$ for neutrals.
    - α calculated to match ionization percentage in the source.
      - $\eta_u \in [1, 0.95, 0.5]$ (utilization at the source)
  - $T_{e0} = 10$ eV (fixed for all cases)
  - Coil radius: $R_L = 2R_0$
  - $\gamma_e = 1.2$



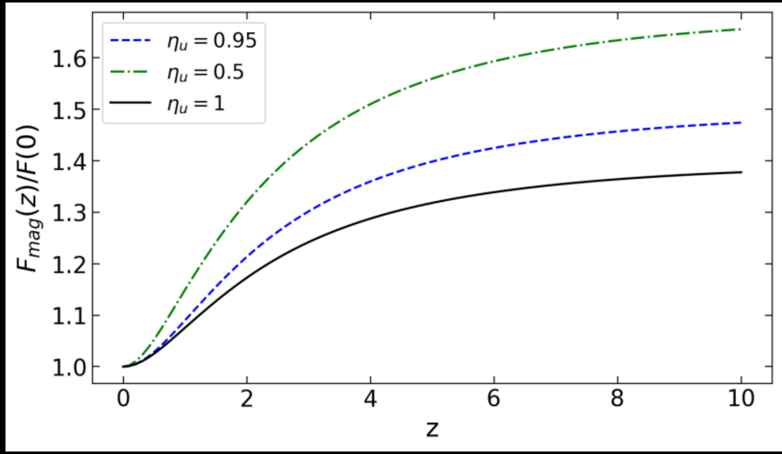
Results - $\eta_u = 0.95$

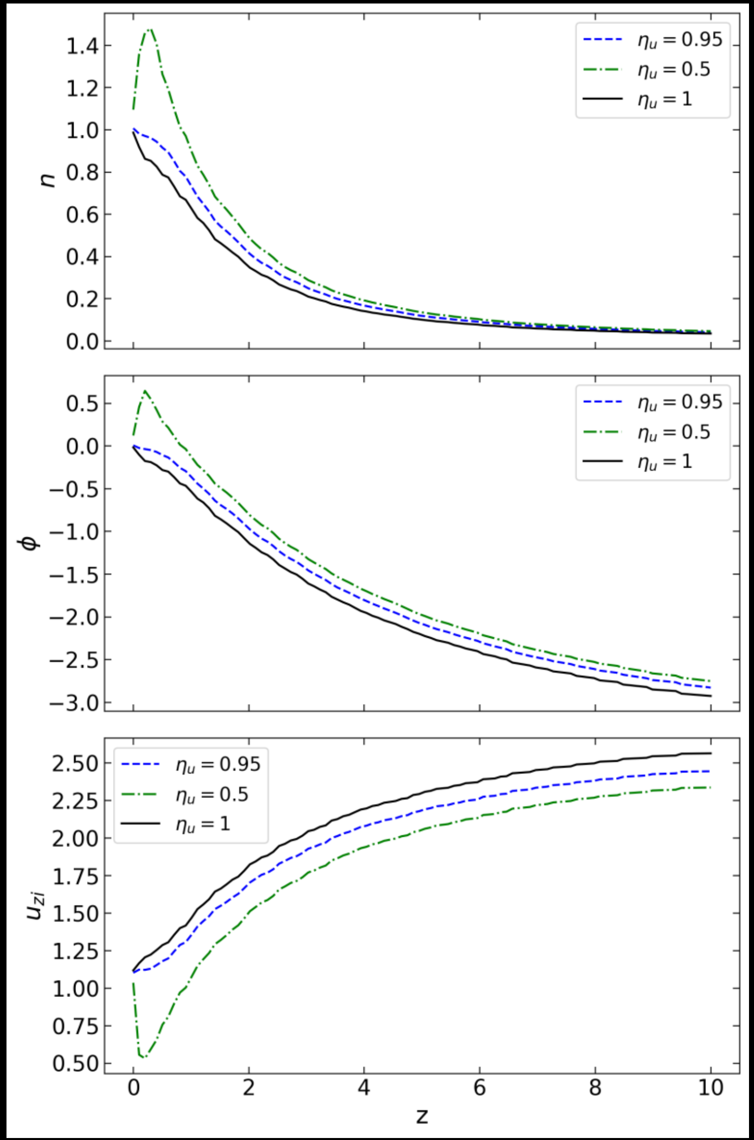
- Overall characteristics of the discharge not much affected by collisions and ionization at $\eta_u = 0.95$
- Ions:
  - Expansion follows the magnetic field lines initially then separates inwards due to increasing ion Mach number.
  - Consistent with previous results [Merino, Ahedo, PSST 23 (2014) 032001]
- Neutrals:
  - Charge exchange collisions accelerate neutrals downstream adding to their thermal expansion.



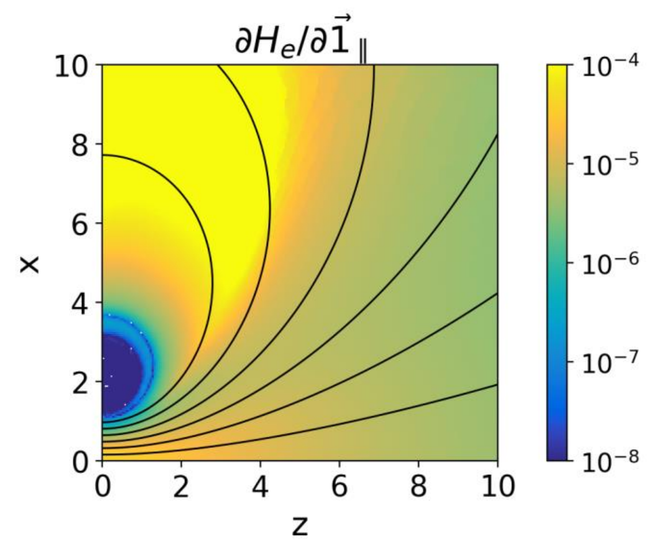
Results - Parametric Study

- Decreasing η_u → more neutrals near the exit:
  - External ionization increases plasma density and decreases ion velocity locally
- Total potential fall essentially unchanged (we are holding T_e constant).
- Beam current and density increases downstream,
- Magnetic thrust scales as $enu_{\theta e}B$, and since $u_{\theta e}$ is not affected, magnetic F/F_0 increases (again, for T_e fixed)
  - But for fixed power and total mass flow rate, F_0 depends on η_u and T_e. These essential trends are not studied here!

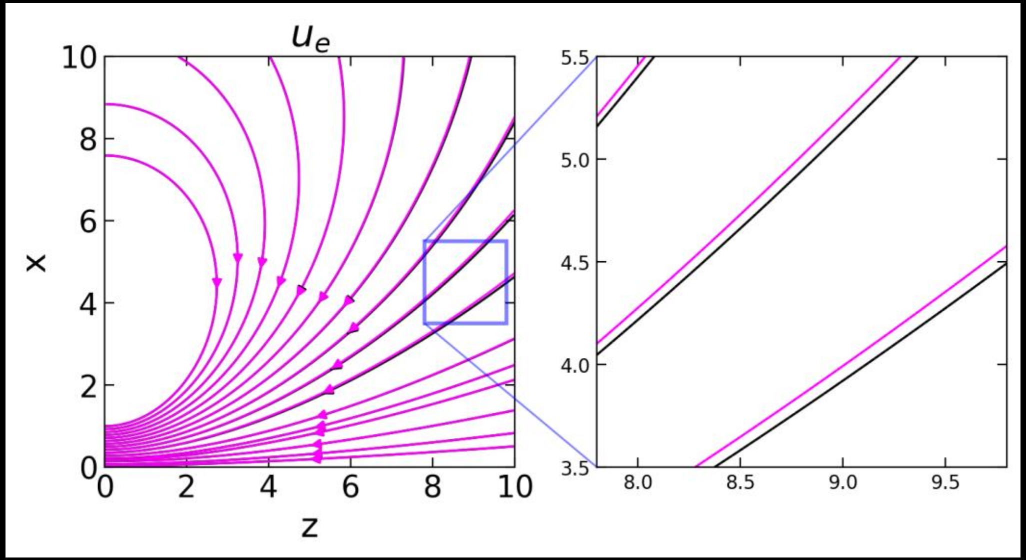




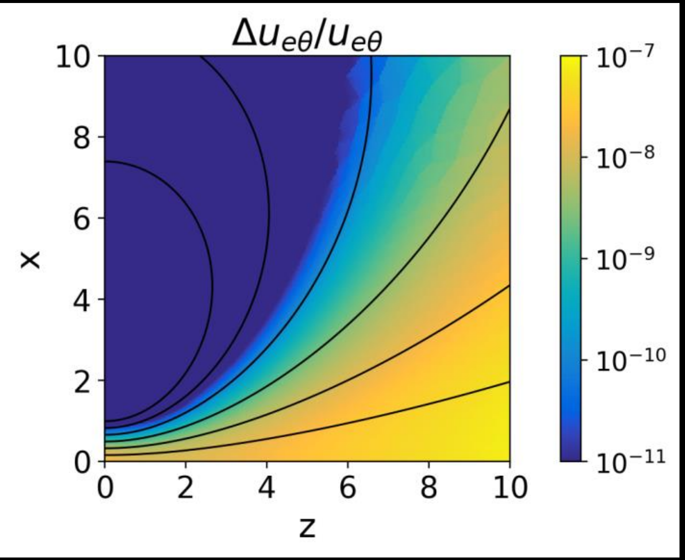
Role of electron Collisions – $\eta_u = 0.95$



- Parallel Component



- Perpendicular Component:
  - In black collisionless electron streamlines.
  - In magenta collisional streamlines.



- Azimuthal Component

Conclusion

- New model in DGFEM allows studying effect of collisions and ionization on MN plasma expansions. Here, we have analyzed the effect of incomplete ionization in the source

- Ion current increases downstream due to late ionization; CEX energizes neutrals; electrons are essentially unaffected in the explored regimes

- Keeping T_e fixed and normalizing with F_0, total thrust increases with presence of neutrals due to mass entrainment
  - However, downstream ionization is a bad thing: those neutrals should have been ionized in the source and undergo acceleration across the full potential fall!
  - Proper power balance taking into account downstream ionization must be carried out to compute T_e consistently

- Background pressure has not been simulated here, but expected to play a similar role: it provides a “free” additional mass flow rate to the thruster by late ionization; will decrease T_e with respect to collisionless case

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Thank you!

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