

SIMULATIONS OF THE EFFECT OF NEUTRAL DYNAMICS IN MAGNETIC NOZZLE EXPANSIONS

Diego García Lahuerta, Mario Merino, Eduardo Ahedo

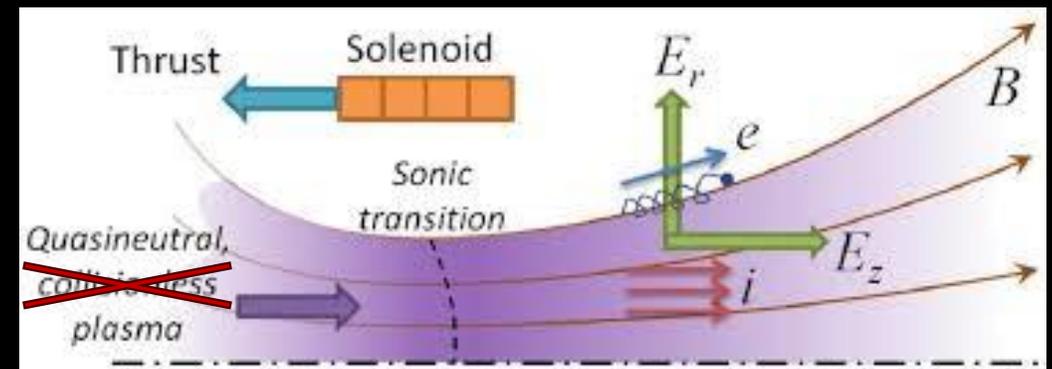
*Equipo de Propulsión Espacial y Plasmas (EP2),
Universidad Carlos III de Madrid, Leganés, Spain*

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MOTIVATION

- Background pressure is known to play a role on magnetic nozzle performance.
 - Collisions, ionization
- Previous MN studies were collisionless and ignored ionization, charge exchange, and electron collisions [Ahedo, Merino, Phys Plasmas 17, 073501 2010 and follow up articles]
- New DGFEM code to overcome previous limitations and explore other plasma expansion configurations (e.g. magnetic arch)
- Preliminary study to account for collisions and ionization in the nozzle



MODEL EQUATIONS

- Three-Fluid Model:
 - Quasineutral plasma
 - $\beta = 0$, negligible self induced magnetic field.
 - Massless, polytropic ($p_e \propto n_e^\gamma$), fully magnetized electrons (when collisionless).
 - Cold, singly-charged ions.
 - Euler equations for neutrals
 - Ionization and Charge-Exchange Collisions, electron collisions.

$$\frac{\partial n}{\partial t} + \nabla \cdot (n\mathbf{u}_e) = S_i$$

$$0 = -\nabla n T_e + en \nabla \phi - en\mathbf{u}_e \times \mathbf{B} - \mathbf{R}_e$$

$$\frac{\partial n}{\partial t} + \nabla \cdot (n\mathbf{u}_i) = S_i$$

$$m \frac{\partial n\mathbf{u}_i}{\partial t} + m \nabla \cdot (n\mathbf{u}_i \otimes \mathbf{u}_i) = -en \nabla \phi + en \mathbf{u}_i \times \mathbf{B} + S_i \mathbf{u}_n + S_{cex} (\mathbf{u}_n - \mathbf{u}_i)$$

$$\frac{\partial n_n}{\partial t} + \nabla \cdot (n_n \mathbf{u}_n) = -S_i$$

$$m \frac{\partial n_n \mathbf{u}_n}{\partial t} + m \nabla \cdot (n_n \mathbf{u}_n \otimes \mathbf{u}_n) = -\nabla p_n T_n - S_i \mathbf{u}_n + S_{cex} (\mathbf{u}_i - \mathbf{u}_n)$$

$$\frac{\partial E_n}{\partial t} + \nabla \cdot [(E_n + p_n) \cdot \mathbf{u}_n] = 0$$

COLLISION MODELS

- Ionization:

$$S_i = n v_i = n n_n c_e \sigma_i$$

$$\sigma_i = \sigma_{i0} \left(1 + \frac{T_e E_i}{(T_e + E_i)^2} \right) e^{-E_i/T_e}$$

$$\sigma_{i0} = 5 \times 10^{-20} \text{ m}^{-2} \quad E_i = 12.1 \text{ eV}$$

- Charge Exchange:

$$S_{cex} = n v_{cex} = n n_n c_{in} \sigma_{cex}$$

$$\sigma_{cex} = \sigma_{cex0} \left(1 - 0.2 \log \left(\frac{c_{in}}{1 \text{ km/s}} \right) \right)$$

$$\sigma_{cex0} = 81 \times 10^{-20} \text{ m}^{-2}$$

$$c_{in} = |\mathbf{u}_i - \mathbf{u}_n|$$

- Electrons (Only considered a posteriori):

$$v_e = v_{ei} + v_{en}$$

$$v_{en} = n_n c_e \sigma_{en}$$

$$\sigma_{en} = 27 \times 10^{-20} \text{ m}^2$$

$$v_{ei} = n R_{ei}$$

$$\frac{R_{ei}}{10^{-12} \text{ m}^3 \text{ s}^{-1}} = 2.9 \left(\frac{1 \text{ eV}}{T_e} \right)^{3/2} \log \Lambda$$

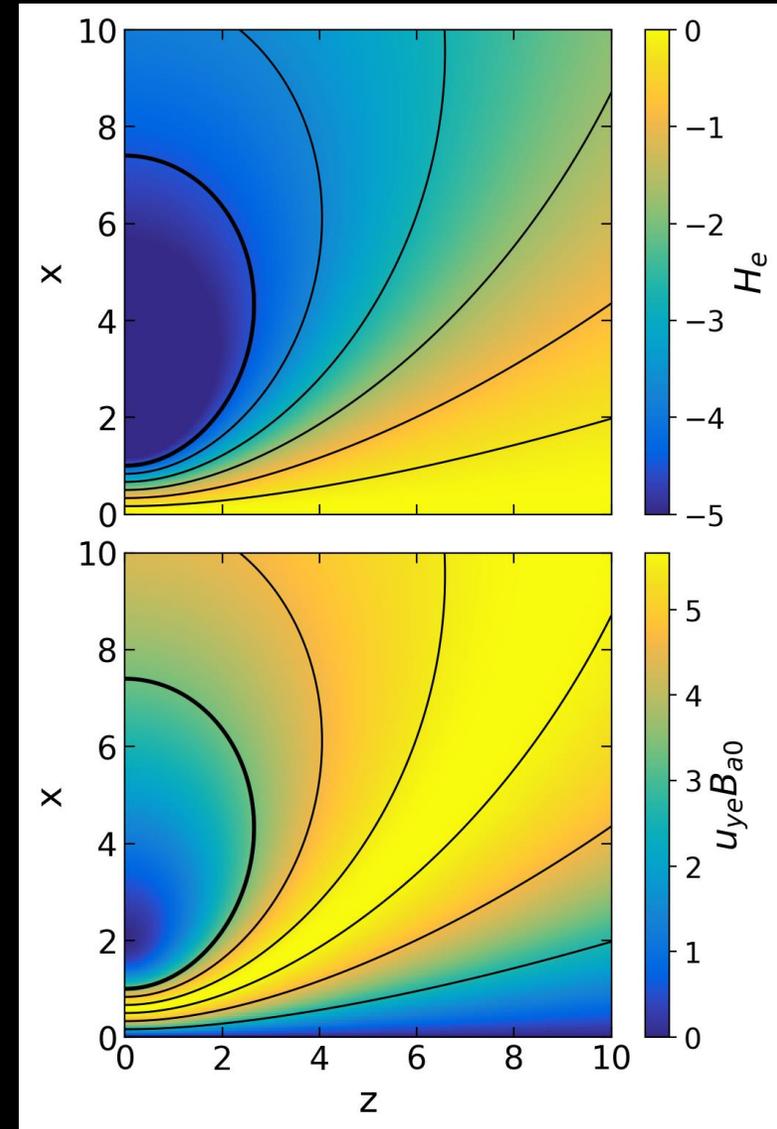
$$\log \Lambda \sim 9 + \frac{1}{2} \log \left[\left(\frac{10^{18} \text{ m}^{-3}}{n_e} \right) \frac{T_e}{1 \text{ eV}} \right]$$

[Enrique Bello-Benítez and Eduardo Ahedo 2021 *Plasma Sources Sci. Technol.* **30** 035003]

MODEL EQUATIONS

- R_e is a resistive term which we will take into account a posteriori.
- In the $R_e = 0$ limit, electron equations are found to be algebraic:
 - Electron momentum equation yields the thermalized potential (H_e) and azimuthal velocity (u_{ye})

$$H_e = \frac{\gamma T_{e0}}{\gamma - 1} \left[\left(\frac{n}{n_0} \right)^{\gamma-1} - 1 \right] - e\phi$$
$$u_{ye}(\psi_B) = \frac{-1}{eB} \frac{\partial H_e}{\partial \mathbf{1}_\perp}$$



ELECTRON EQUATIONS

- A posteriori estimation of the importance of electron collisions.
 - Resistive force in electron momentum equations leads to:

$$0 = -\nabla H_e - eu_{e\theta}B \mathbf{1}_\perp + eu_{e\perp}B \mathbf{1}_\theta - \mathbf{R}_e$$

$$\mathbf{R}_e = m_e \nu_e \mathbf{u}_e = -eB\chi^{-1}\mathbf{u}_e$$

$\chi \rightarrow$ Hall parameter

- Projected onto magnetic field frame:

$$\frac{\partial H_e}{\partial \mathbf{1}_\parallel} = -eB\chi^{-1}u_{e\parallel}$$

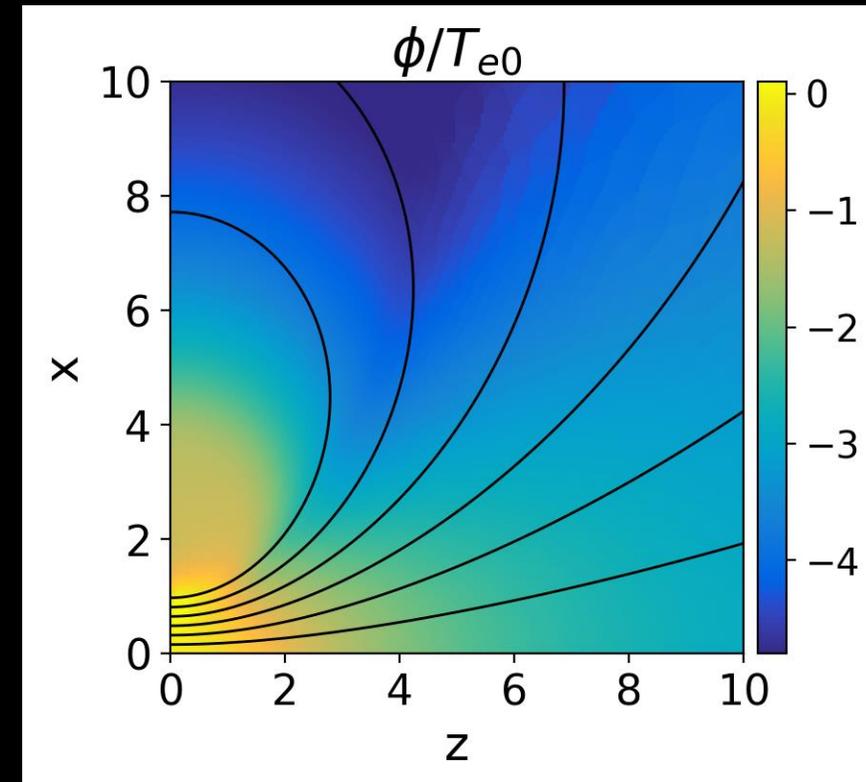
$$u_{e\perp} = \chi^{-1}u_{e\theta}$$

$$\Delta u_{e\theta} = u_{e\theta} - u_{e\theta} \Big|_{\nu_e=0} = -\chi^{-2}u_{e\theta}$$

- These equations act as leading order corrections to electron velocity. One can observe:
 - Perpendicular corrections are $\mathcal{O}(\chi^{-1})$ while azimuthal correction is $\mathcal{O}(\chi^{-2})$
 - Parallel velocity results from solving continuity equation and is affected by $u_{e\perp}$ and S_i

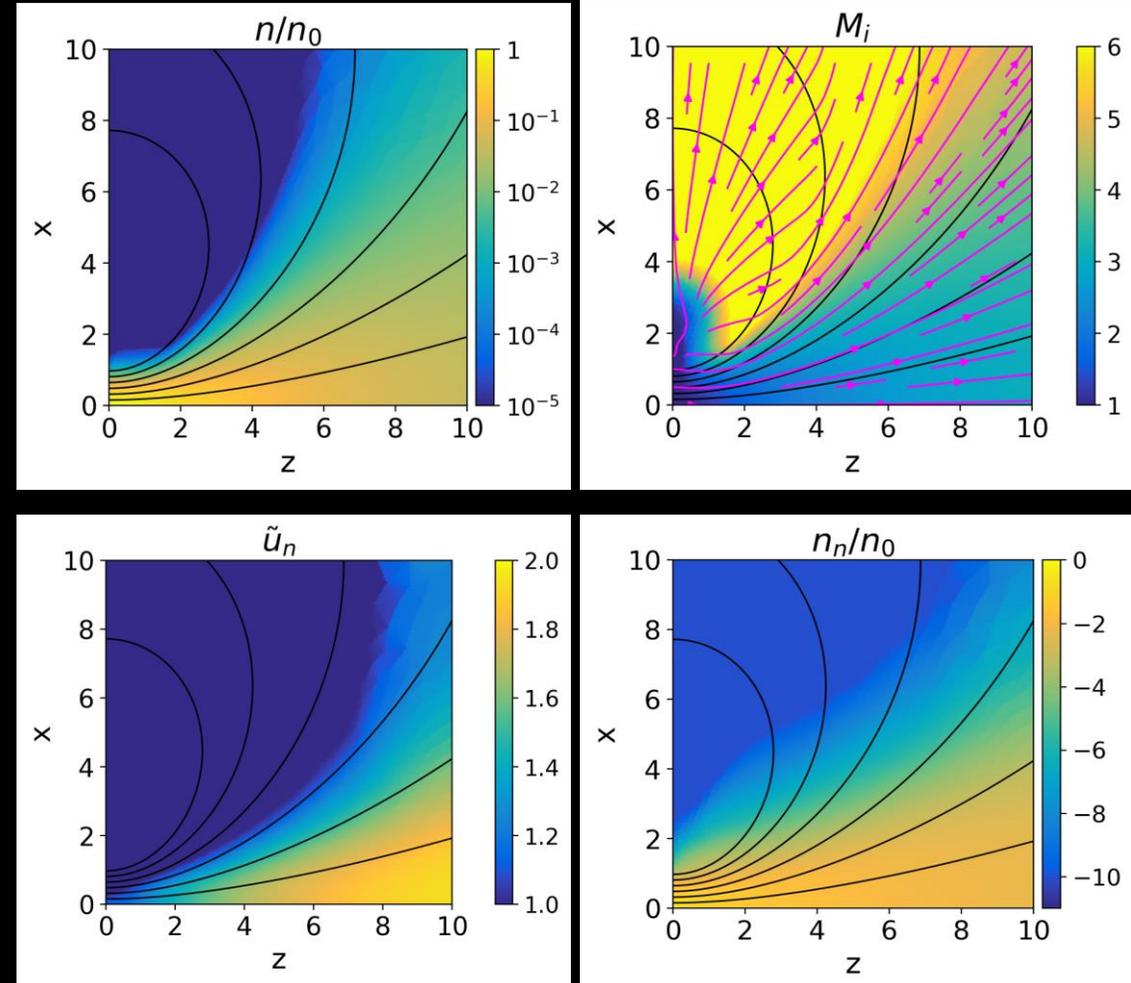
SIMULATION SETUP

- Numerical setup:
 - Discontinuous Galerkin discretization (FEniCS)
 - Order 1 elements in this work.
 - Runge Kutta time-stepping (and final solve for steady state)
 - Unstructured mesh (Gmsh):
 - Cell diameter such that the nozzle throat is resolved in 40 cells.
- Physical parameters:
 - Sonic axial velocity
 - Gaussian density profile:
 - $n(0) = n_0$ and $n(R_0) = 10^{-3}n_0$ for ions.
 - $n_n(0) = \alpha n_0$ and $n_n(R_0) = \alpha 10^{-3}n_0$ for neutrals.
 - α calculated to match ionization percentage in the source.
 - $\eta_u \in [1, 0.95, 0.5]$ (utilization at the source)
 - $T_{e0} = 10$ eV (fixed for all cases)
 - Coil radius: $R_L = 2R_0$
 - $\gamma_e = 1.2$



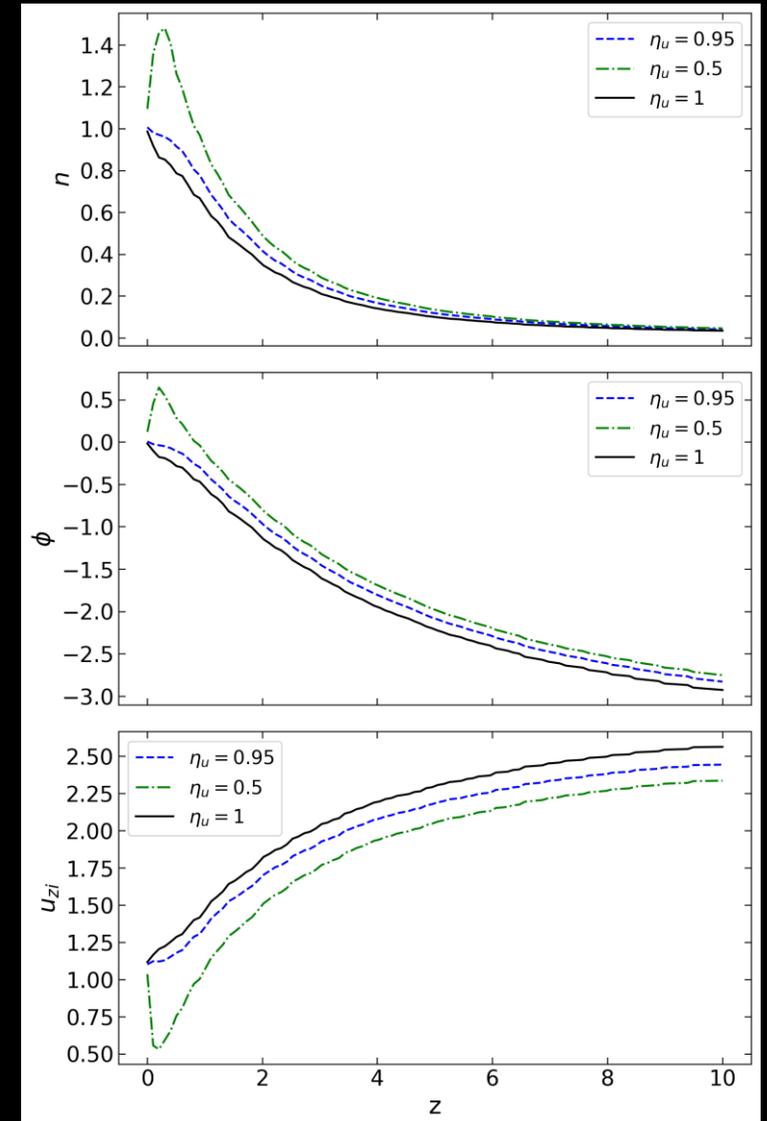
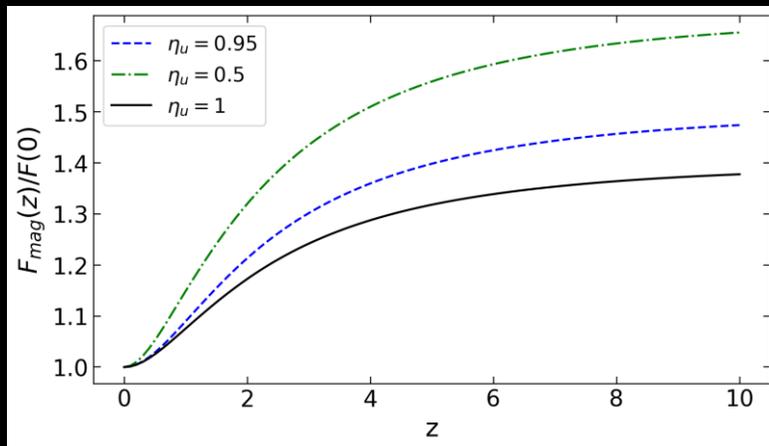
RESULTS - $\eta_u = 0.95$

- Overall characteristics of the discharge not much affected by collisions and ionization at $\eta_u = 0.95$
- Ions:
 - Expansion follows the magnetic field lines initially then separates inwards due to increasing ion Mach number.
 - Consistent with previous results [Merino, Ahedo, PSST 23 (2014) 032001]
- Neutrals:
 - Charge exchange collisions accelerate neutrals downstream adding to their thermal expansion.

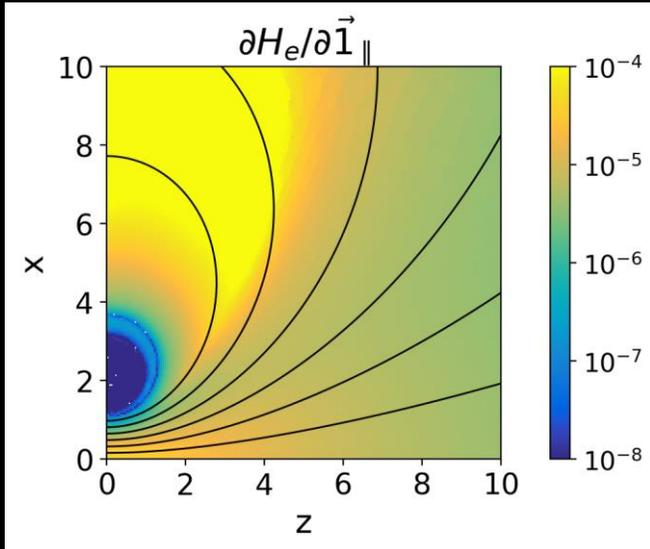


RESULTS - PARAMETRIC STUDY

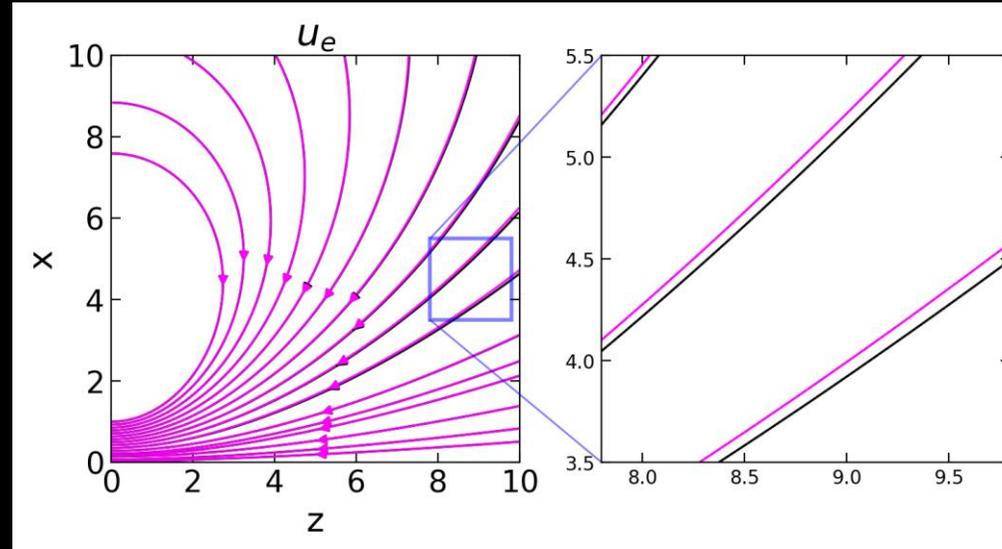
- Decreasing $\eta_u \rightarrow$ more neutrals near the exit:
 - External ionization increases plasma density and decreases ion velocity locally
- Total potential fall essentially unchanged (we are holding T_e constant).
- Beam current and density increases downstream,
- Magnetic thrust scales as $enu_{\theta e}B$, and since $u_{\theta e}$ is not affected, magnetic F/F_0 increases (again, for T_e fixed)
 - But for fixed power and total mass flow rate, F_0 depends on η_u and T_e . These essential trends are not studied here!



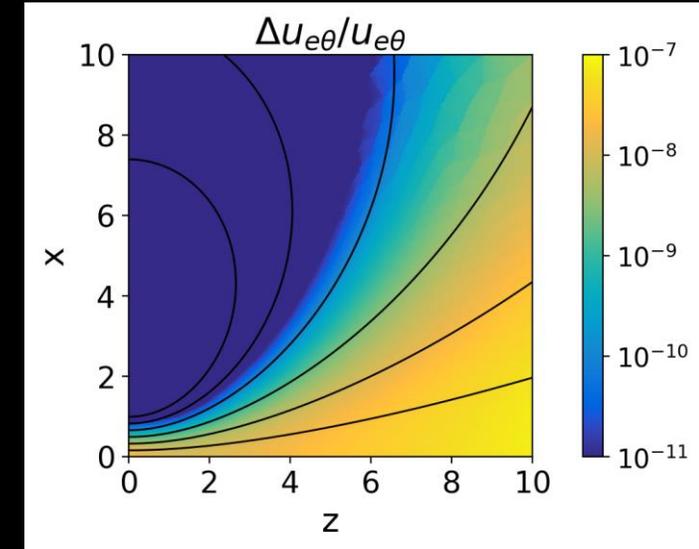
ROLE OF ELECTRON COLLISIONS - $\eta_u = 0.95$



• Parallel Component



• Perpendicular Component:
• In black collisionless electron streamlines.
• In magenta collisional streamlines.



• Azimuthal Component

CONCLUSION

- New model in DGFEM allows studying effect of collisions and ionization on MN plasma expansions. Here, we have analyzed the effect of incomplete ionization in the source
- Ion current increases downstream due to late ionization; CEX energizes neutrals; electrons are essentially unaffected in the explored regimes
- Keeping T_e fixed and normalizing with F_0 , total thrust increases with presence of neutrals due to mass entrainment
 - However, downstream ionization is a bad thing: those neutrals should have been ionized in the source and undergo acceleration across the full potential fall!
 - Proper power balance taking into account downstream ionization must be carried out to compute T_e consistently
- Background pressure has not been simulated here, but expected to play a similar role: it provides a “free” additional mass flow rate to the thruster by late ionization; will decrease T_e with respect to collisionless case

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THANK YOU!

dieggarc@ing.uc3m.es

