Oscillations and instabilities in a propulsive magnetic nozzle

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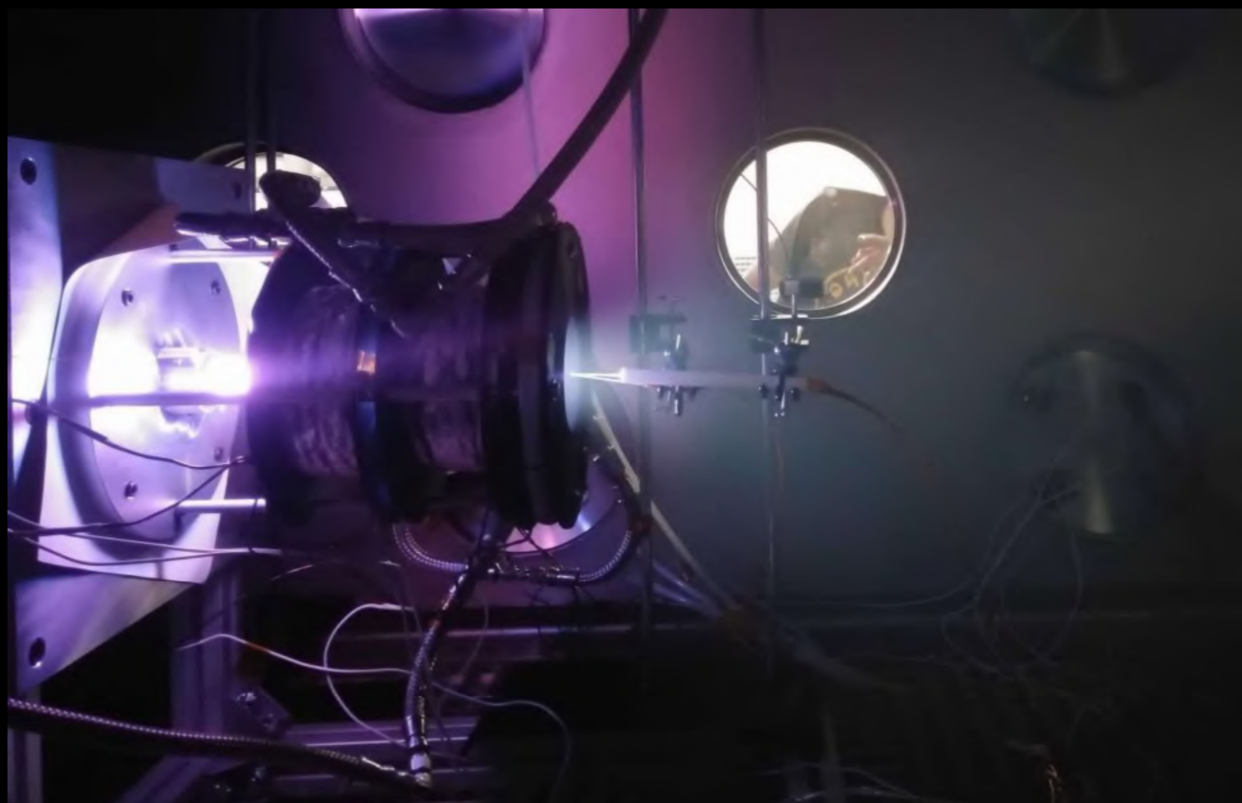
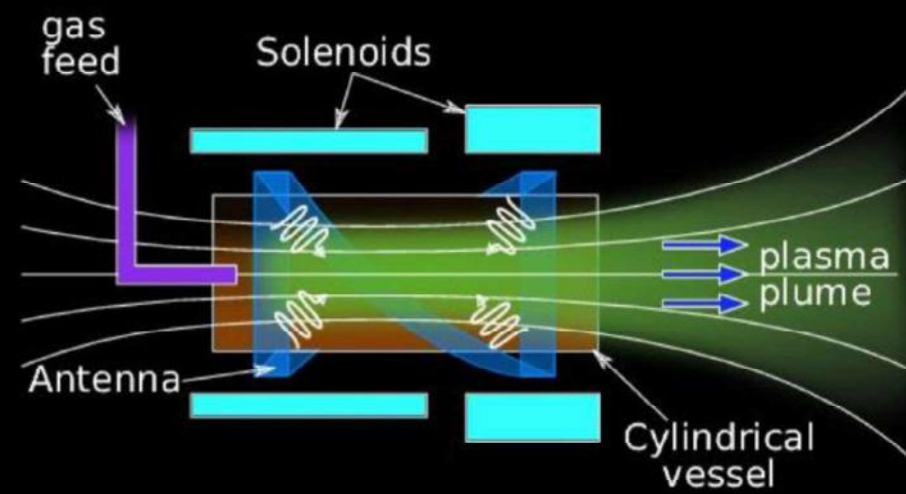
Azimuthal oscillations in a magnetic nozzle

- Helicon plasma thruster consists of
  - Cylindrical dielectric ionization chamber
  - EM inductor (“antenna”)
  - Converging-diverging applied B field → Magnetic nozzle

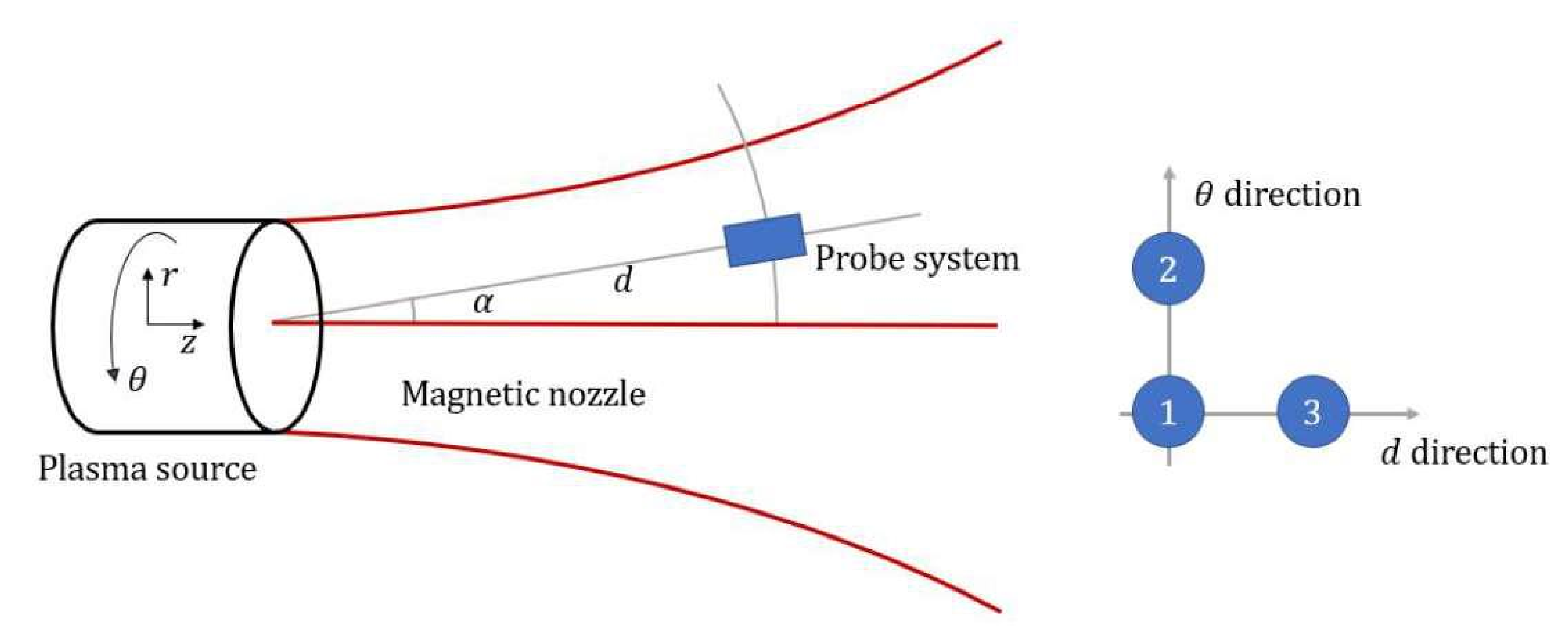


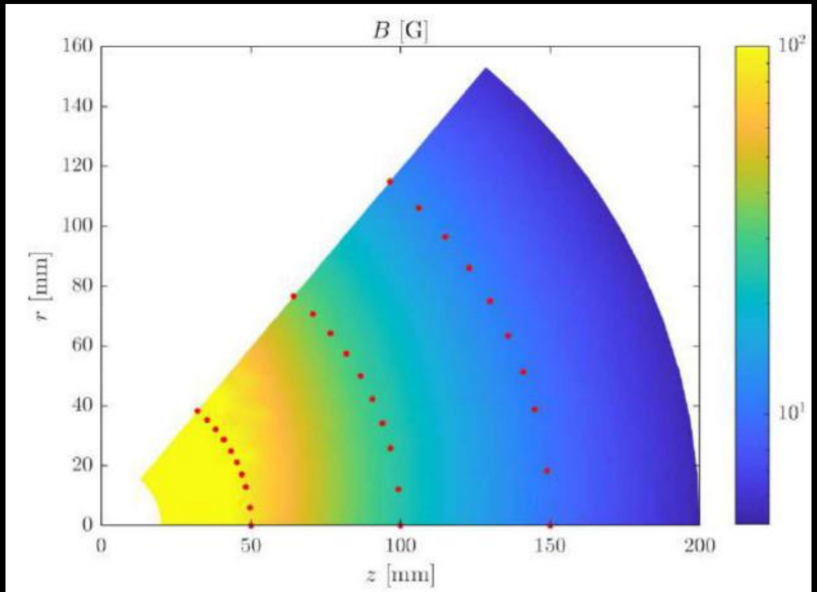


- Losses to lateral walls are large and suggest anomalous transport
- Gradient-driven drift instabilities are a candidate mechanism for that enhanced transport
- Previous works identify oscillations in the magnetic nozzle plasma plume

Experimental setup

- HPT prototype running at 5 sccm Xe, 450 W EM power
- Max. B field (magnetic throat): 750 G
- Vacuum chamber: 1.5 m diameter, 3.5 m length, 10^{-5} mbar during operation
- 3 floating, cylindrical tungsten probe tips, 1 cm apart (along θ and d directions)
- Probe system displaced with radial-polar arm





Cross correlation analysis

- Averaging over 30 realizations to estimate the cross correlation spectrum between each 2 probes :

$$C_{12}(\omega) = X_1(\omega) X_2^*(\omega)$$

- Mean and deviation of log power $c = \log C$:

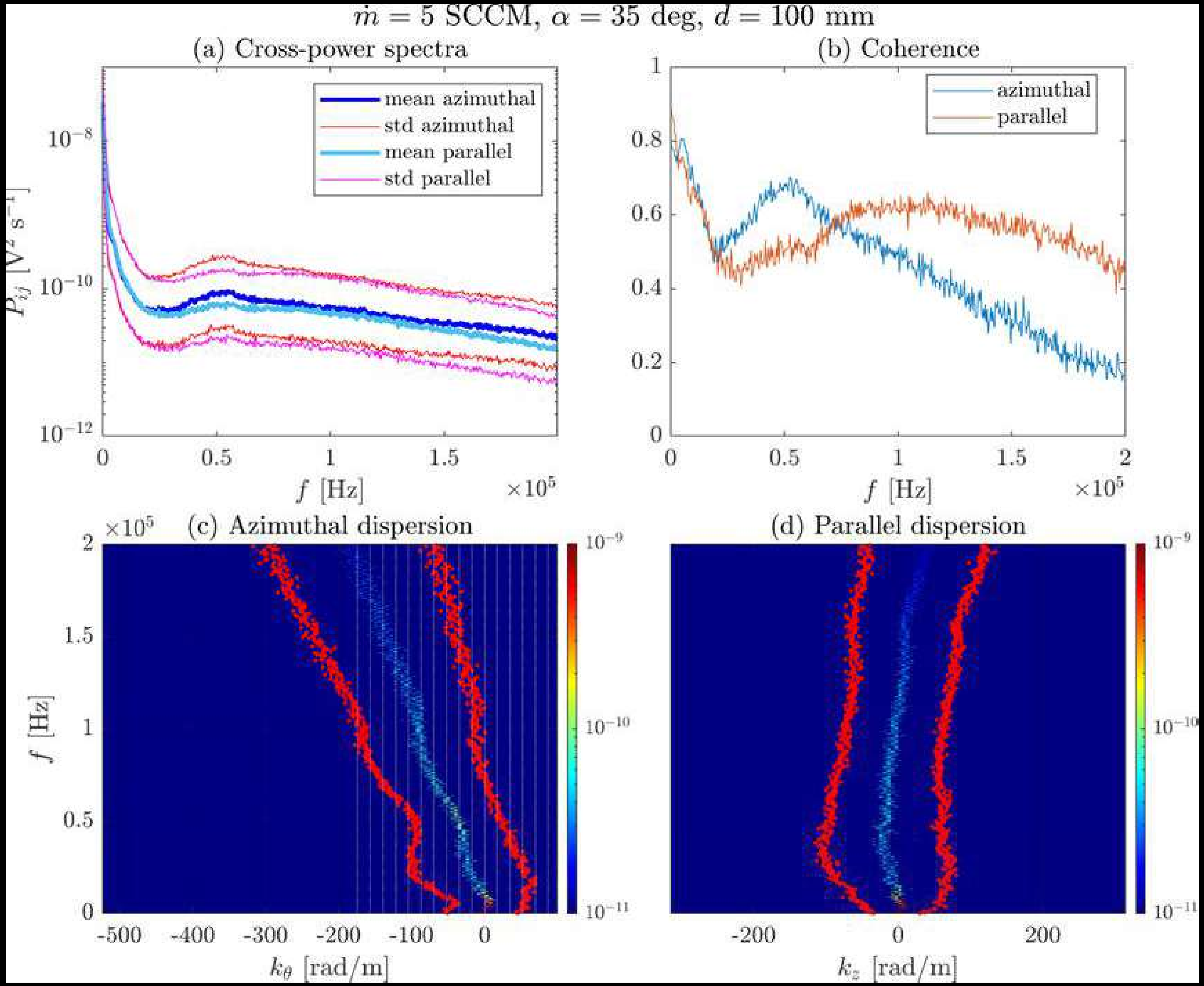
$$\bar{\mu}_c = \frac{1}{n}\sum_k^n c_k \qquad \bar{\sigma}_c = \sqrt{\frac{1}{n-1}\sum_k^n (c_k - \bar{\mu}_c)^2}$$

- Mean and deviation phase difference (circular statistics):

$$\tilde{z} = \frac{1}{n}\sum_k^n \exp(i\phi_k) \qquad \tilde{\mu}_\phi = \arg \tilde{z} \qquad d_1 = \frac{1}{n}\sum_k^n |\phi_k - \mu_{\tilde{\phi}}|$$

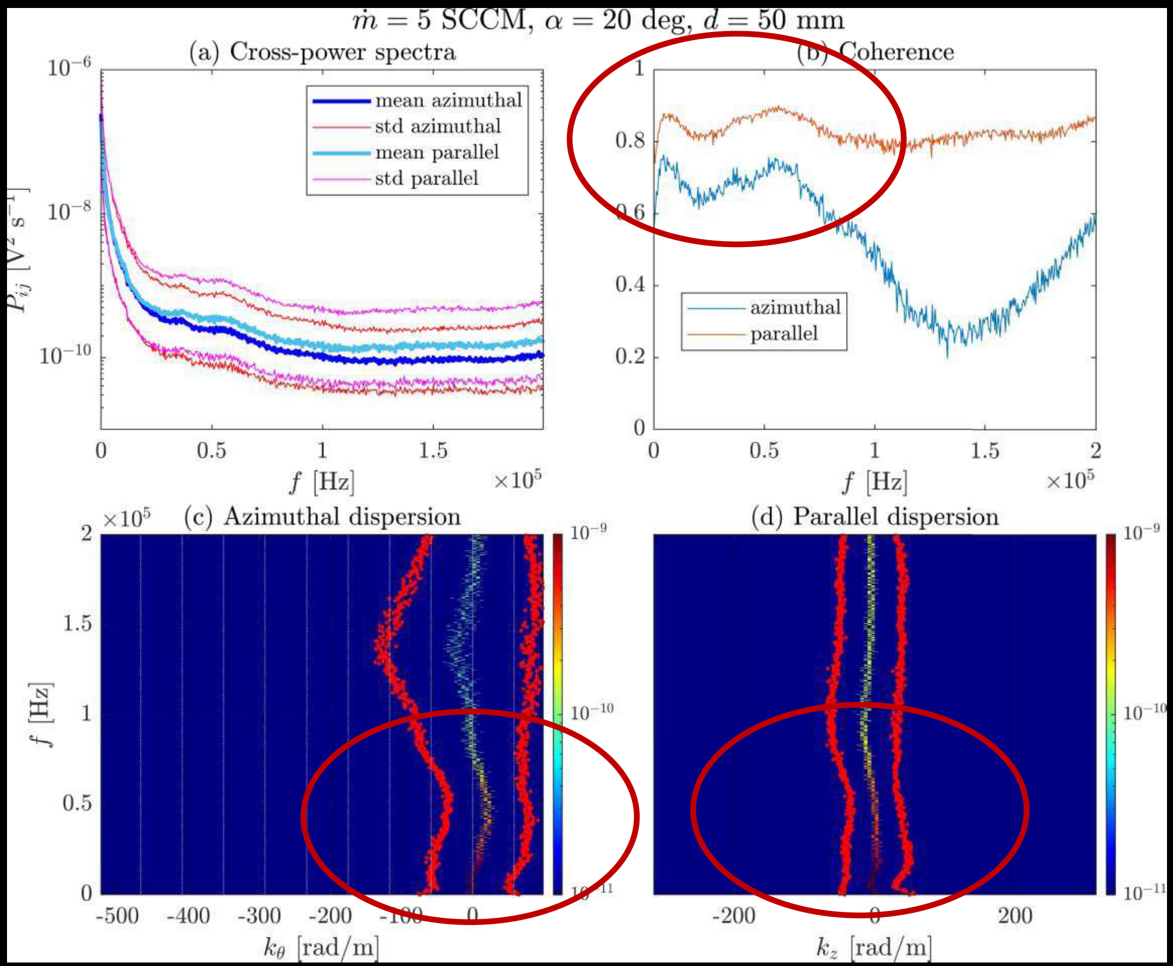
- Coherence (normalized cross correlation spectrum):

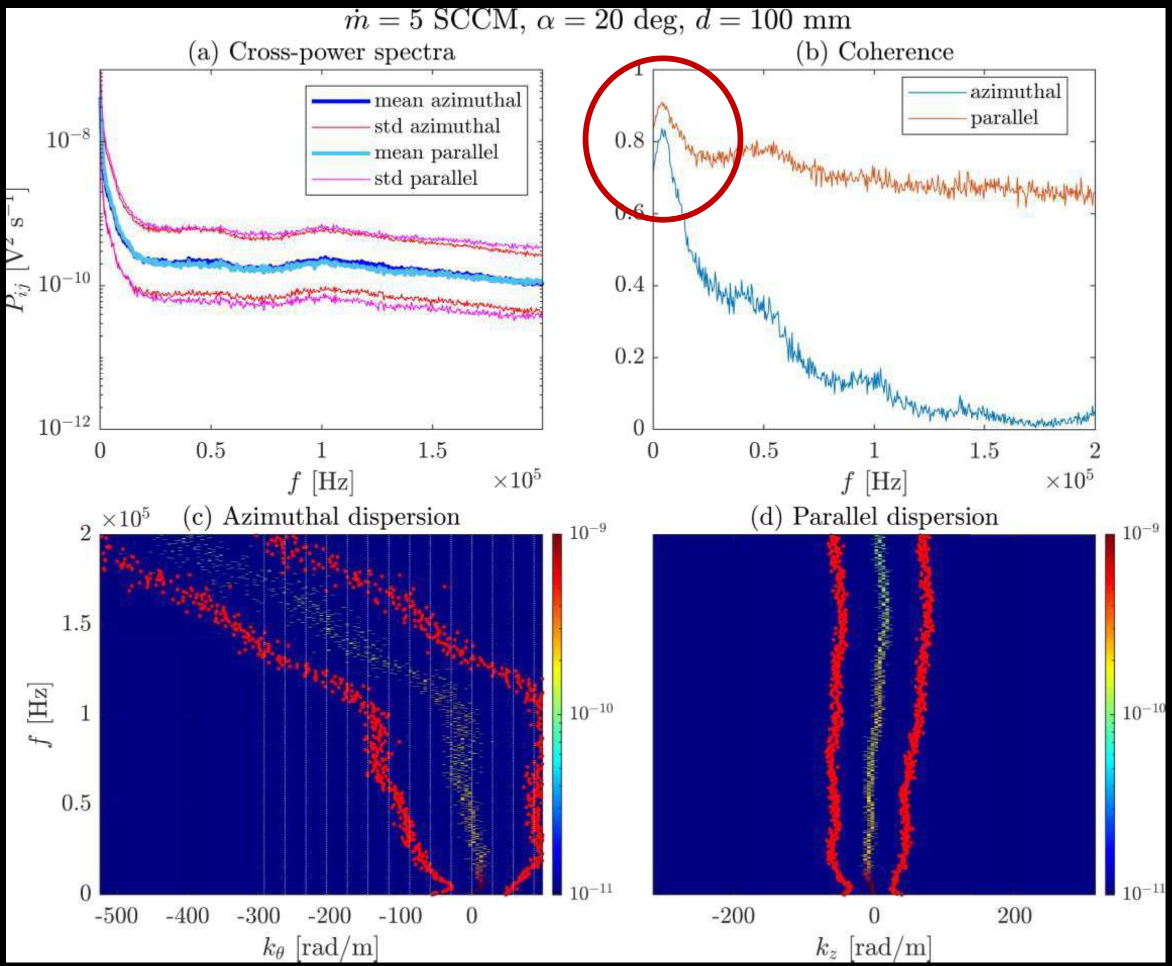
$$\hat{C}_{12}(\omega) = \frac{|C_{12}(\omega)|}{\sqrt{|X_1(\omega)|^2 |X_2(\omega)|^2}}$$



Cross correlation analysis

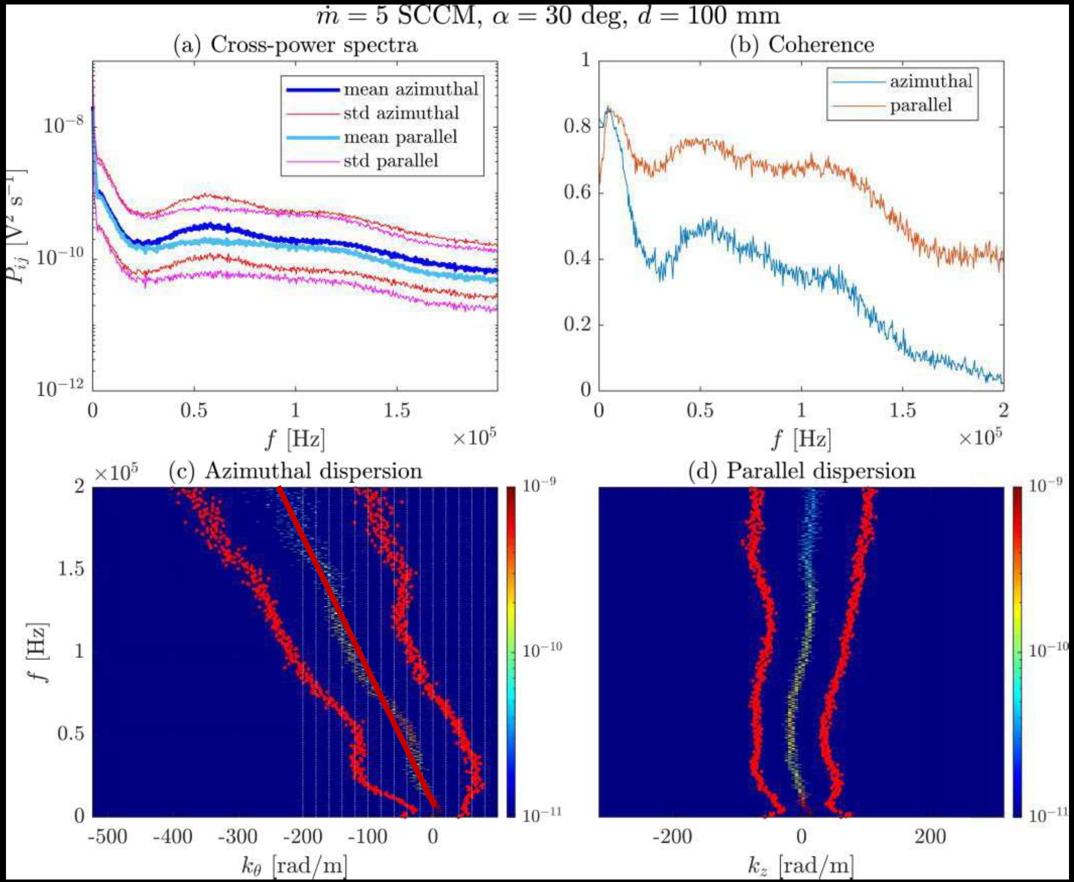
- At low α angles ($\alpha = 20$ deg), good coherence up to 100 kHz at 50 mm; not so good at 100 mm
- Dispersion relation in that range has $m = 0 \div 1$, $k_\parallel \simeq 0$, corresponding to essentially global oscillations

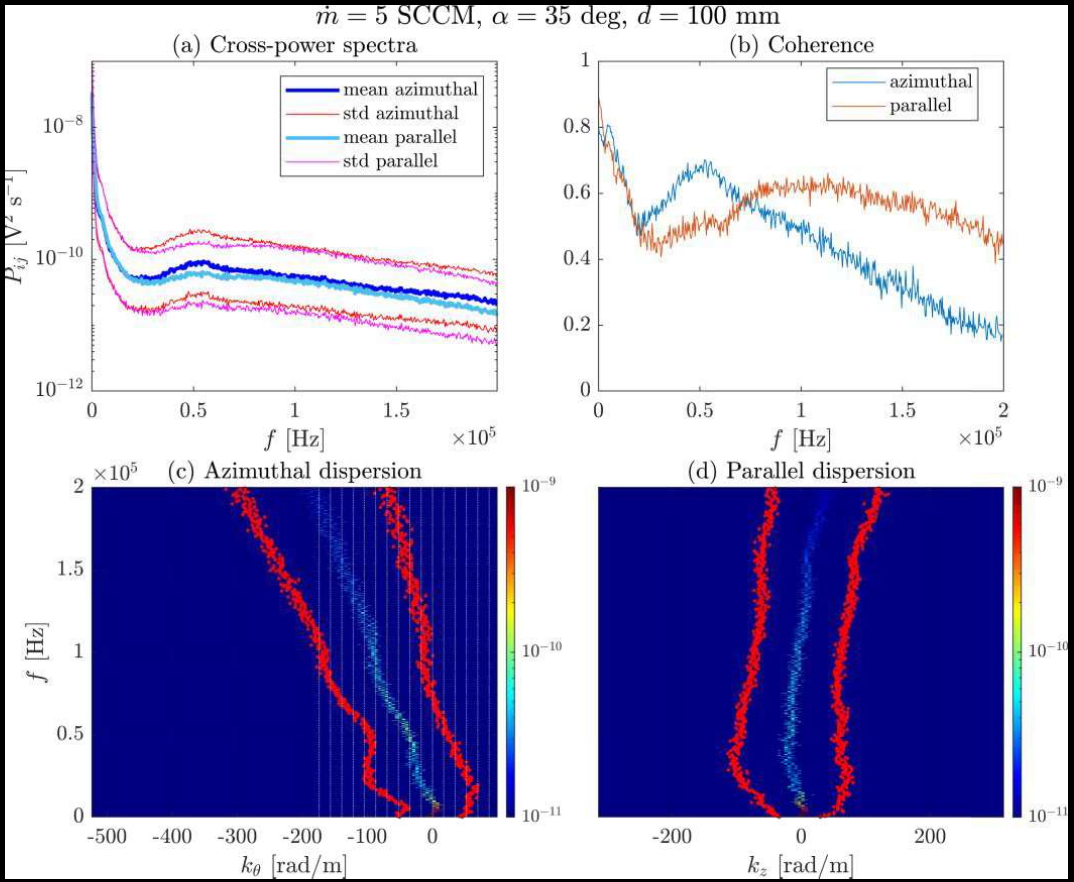




Cross correlation analysis

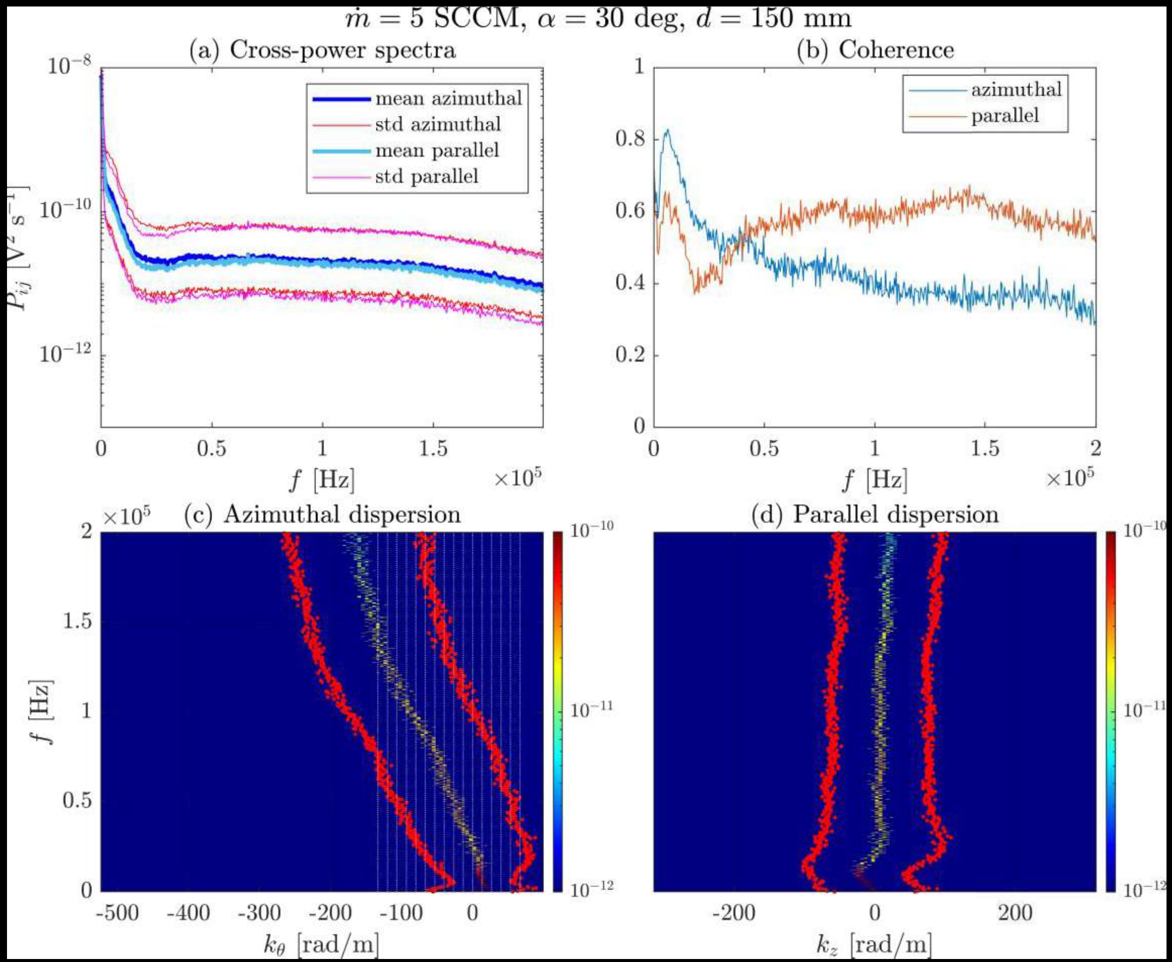
- At larger angles ($\alpha = 30 \div 35$) and $d = 100 \div 150$ mm, a dominantly-azimuthal mode is found at $f < 100$ kHz with
  - $m = 1 \div 4$, $k_{\parallel} < 10$ rad/m
  - Peak in CSD power at ~60 kHz; harmonic at ~120 kHz

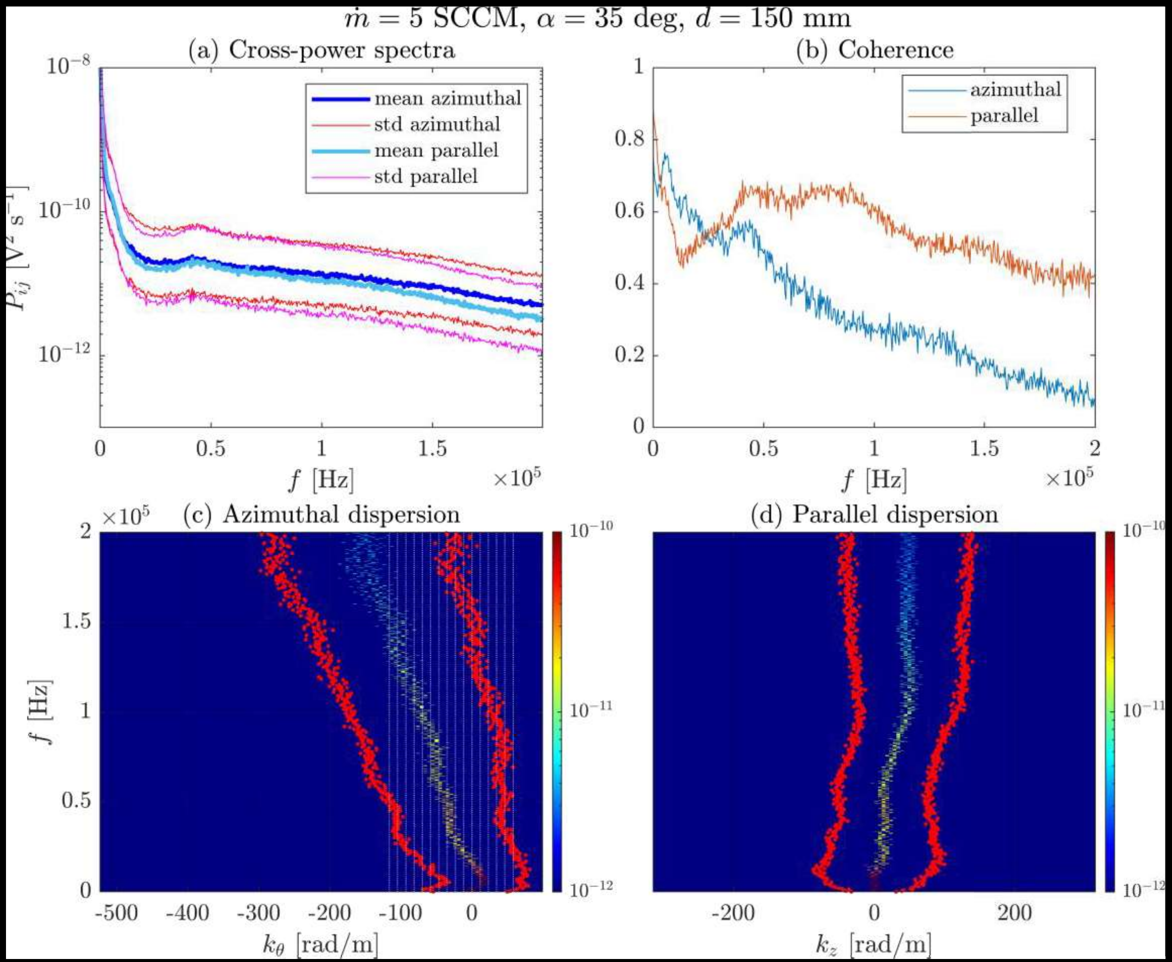




Cross correlation analysis

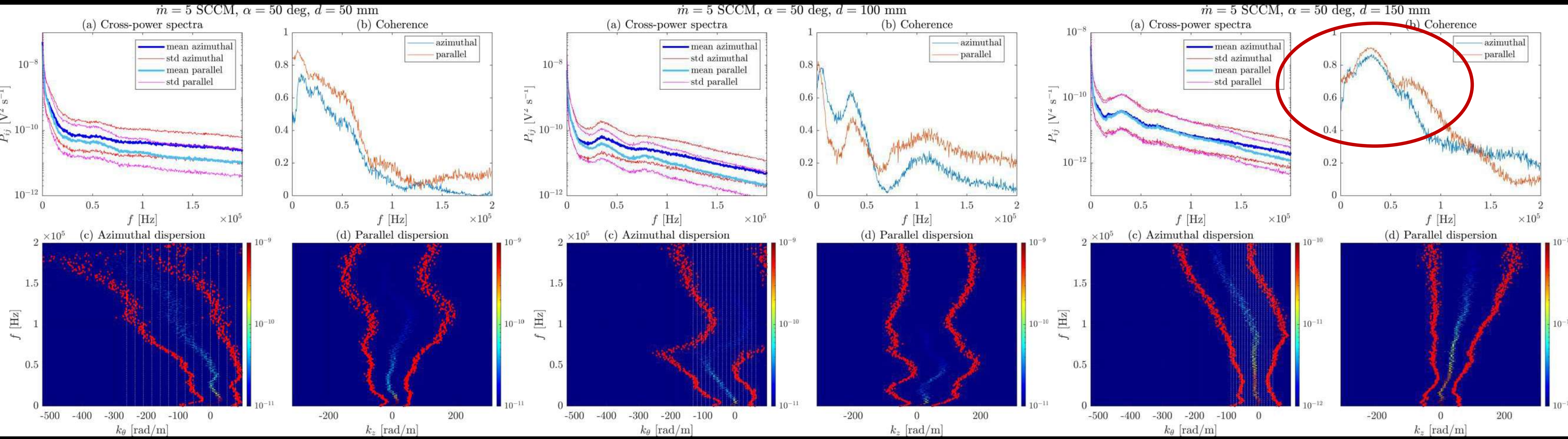
Similar features but flatter spectrum found at $d = 150$ mm





Cross correlation analysis

- Behavior at large α is more complex.
  - Good coherence at low frequencies far downstream
  - $m = 1 \div 2$, $k_{\parallel}$ has a dispersion relation

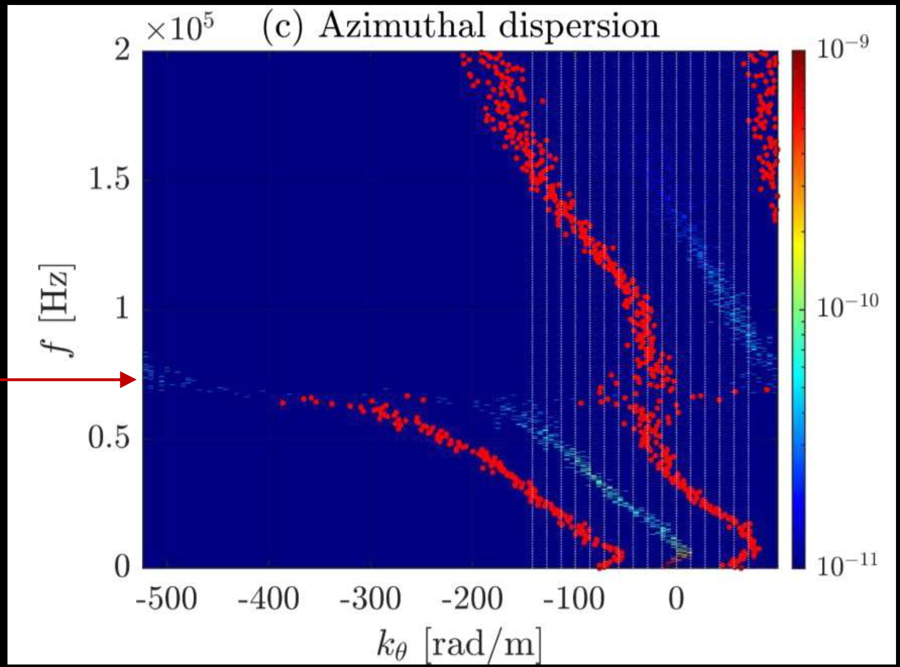


Summary of findings - and issues

- 10 kHz Oscillations found everywhere, with $m \simeq 0$ and $k_{\parallel} \simeq 0$
- 40-60 kHz Oscillations at intermediate α angles downstream, with $m < 4$ and $k_{\parallel}$ likely nonzero
- 40-60 kHz Oscillations at large $\alpha \simeq 50$ deg (plume periphery), with $m < 2$ and $k_{\parallel} \neq 0$

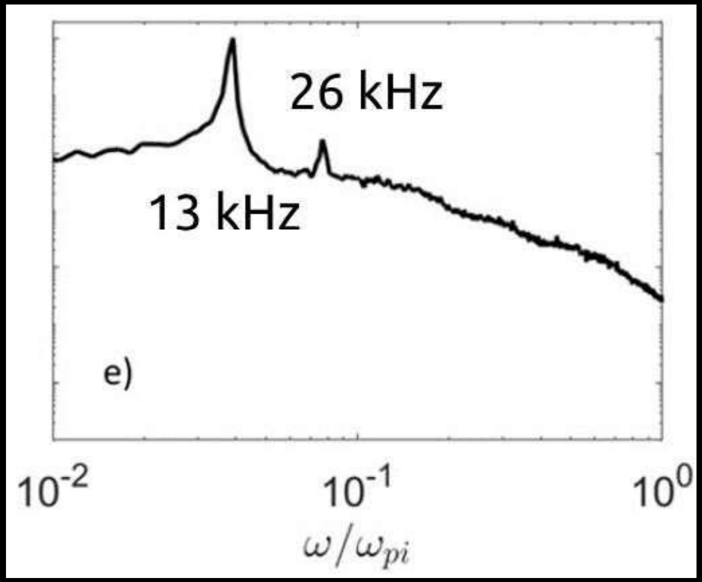
- Phase velocities ($< 10^5$ m/s) comparable to estimated density-gradient and ExB drift velocities

- Coherence is not large (~ 1) in some of these
- Dispersion relation plots with bends could suggest multiple waves coexist
  - If two or more oscillations coexist at same ω, 2-probe method is unable to resolve them
- Jumps exist at some, but not all locations, around ω_{lh}

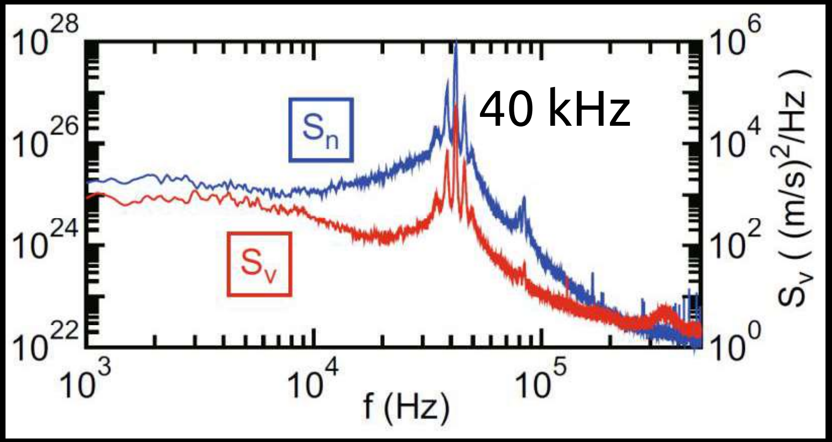


Dispersion relation discussion

- Other works (below) also found azimuthal oscillations in similar setups
  - Hepner et al: suggest ECDI
  - Takahashi: suggests oscillations are a magnetosonic wave



Hepner et al ECRT
Appl. Phys. Lett. 116, 263502 (2020);
Proposes outward electron transport



Takahashi's HPT
Scientific Reports (2022) 12:20137;
Proposes inward electron transport

Linear Local Stability Analysis

- 3D wave dispersion relation for inhomogeneous plasma
  - Obtained from fluid approach
    - Locally Cartesian set of coordinates $(r, \theta, z) \rightarrow (x, y, z)$

$$-i\omega_s n_s + u_{sx}\frac{\partial n_0}{\partial x} + u_{sx0}\frac{\partial n_s}{\partial x} + n_0 \nabla \cdot \boldsymbol{u}_s + n_s \frac{\partial u_{sx0}}{\partial x} = \nu_p n_s$$

$$-i\omega_s \boldsymbol{u}_s + u_{sx}\frac{\partial \boldsymbol{u}_{s0}}{\partial x} + u_{sx0}\frac{\partial \boldsymbol{u}_s}{\partial x} = -\left(\frac{\nabla \cdot \Pi}{nm}\right)^{(1)}_s - \frac{q}{m}\nabla\phi_1 + \frac{q}{|q|}\omega_{cs}\boldsymbol{u}_s \times \boldsymbol{b} - \mathrm{N}_s \boldsymbol{u}_s$$

    - $\boldsymbol{B} \parallel \hat{z}$

  - Inertial, collisional and FLR effects considered for the electron flow
    - Stress tensor (Π) comprised of both gyrotropic and gyroviscous parts

$$\Pi_e = p_\perp \boldsymbol{I} + \left(p_\parallel - p_\perp\right)\boldsymbol{bb} + \hat{\Pi}_e$$

Linear Local Stability Analysis

- 3D wave dispersion relation for inhomogeneous plasma
  - Obtained from fluid approach

$$\frac{k^2 c_s^2}{\omega_{Pi}^2}\left(\frac{\omega_{Pi}^2}{\omega_i^2}-1\right)=\frac{\omega_D+\omega_B+D_\perp+D_\parallel}{\omega_e+\omega_D+\omega_B+D_\perp+D_\parallel}$$

$$\omega_e \equiv \omega+\omega_E+\omega_D$$

  - Drift frequencies:

$$\omega_D \equiv -k_\perp \frac{c_e^2}{\omega_{ce}}\frac{\partial \ln n_0}{\partial x}$$

$$\omega_E \equiv -k_\perp \frac{E_{x0}}{B_0}$$

$$\omega_B \equiv -k_\perp \frac{c_e^2}{\omega_{ce}}\frac{\partial \ln B_0}{\partial x}$$

- Inertial, collisional and FLR effects are accounted for
  - For perpendicular propagation:

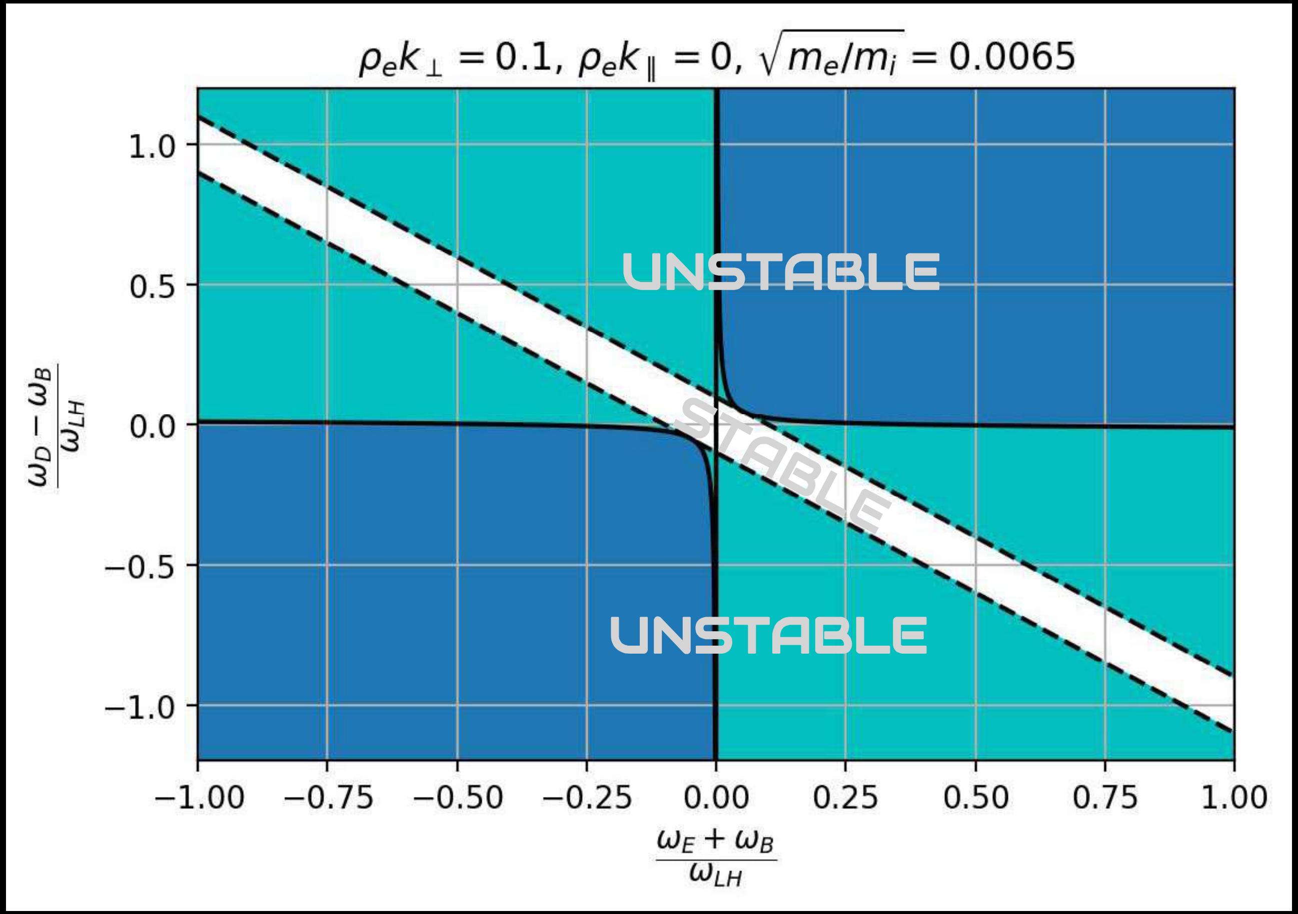
$$D_\perp = D_\perp\left(\rho_e k_\perp, \nu_\perp, \omega, \omega_E, \omega_D, \omega_B\right)$$

  - For parallel propagation:

$$D_\parallel = D_\parallel\left(\rho_e^2 k_\parallel^2, \rho_e^2 k_\perp^2, \nu_\parallel, \omega, \omega_E, \omega_D, \omega_B\right)$$

Linear Local Stability Analysis

- Delimited parametric regions for instabilities to occur
- For perpendicular propagation:
  - Blue region for collisionless instability
  - Dashed lines delimit region for collisional instability



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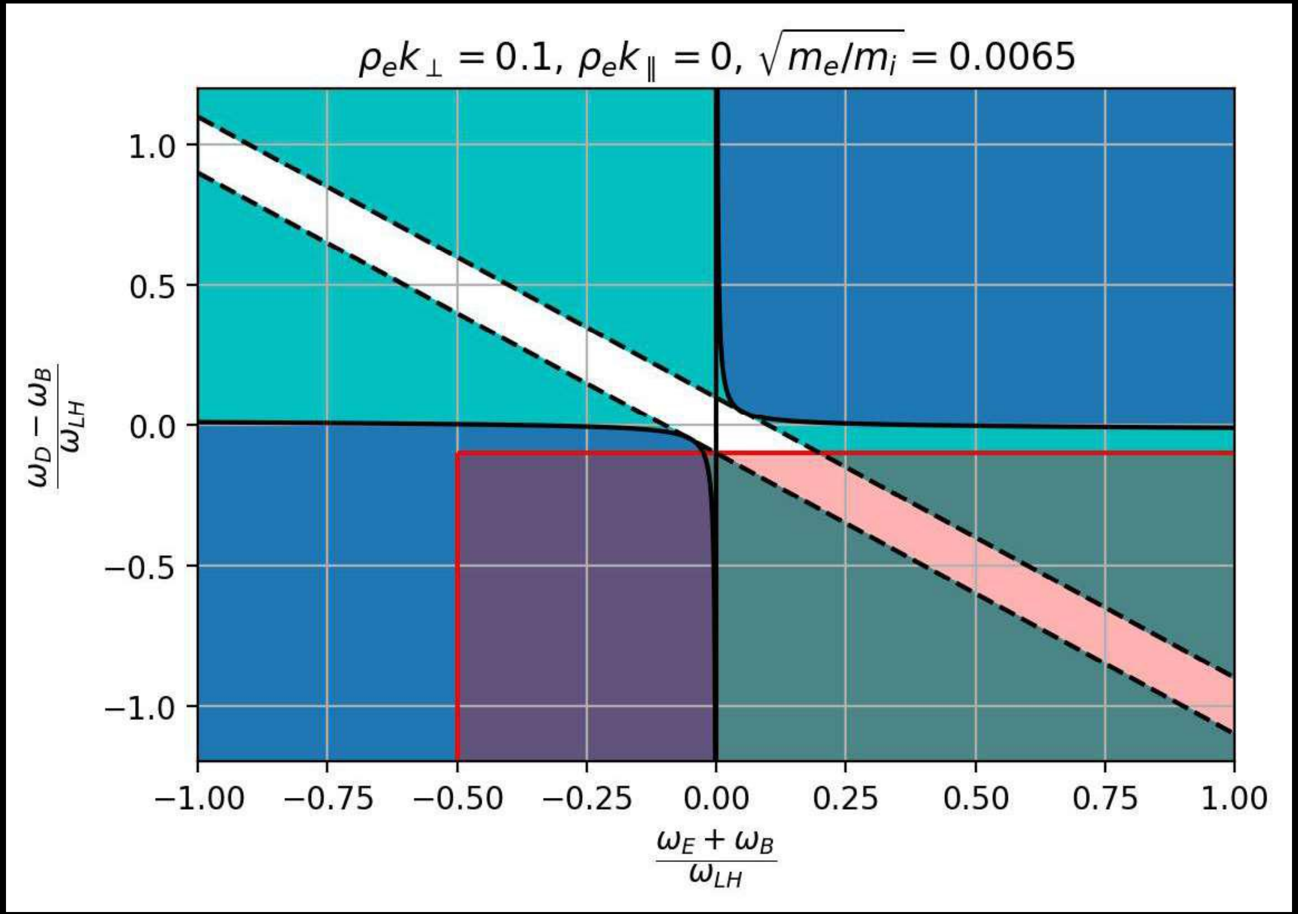

Linear Local Stability Analysis

- Delimited parametric regions for instabilities to occur
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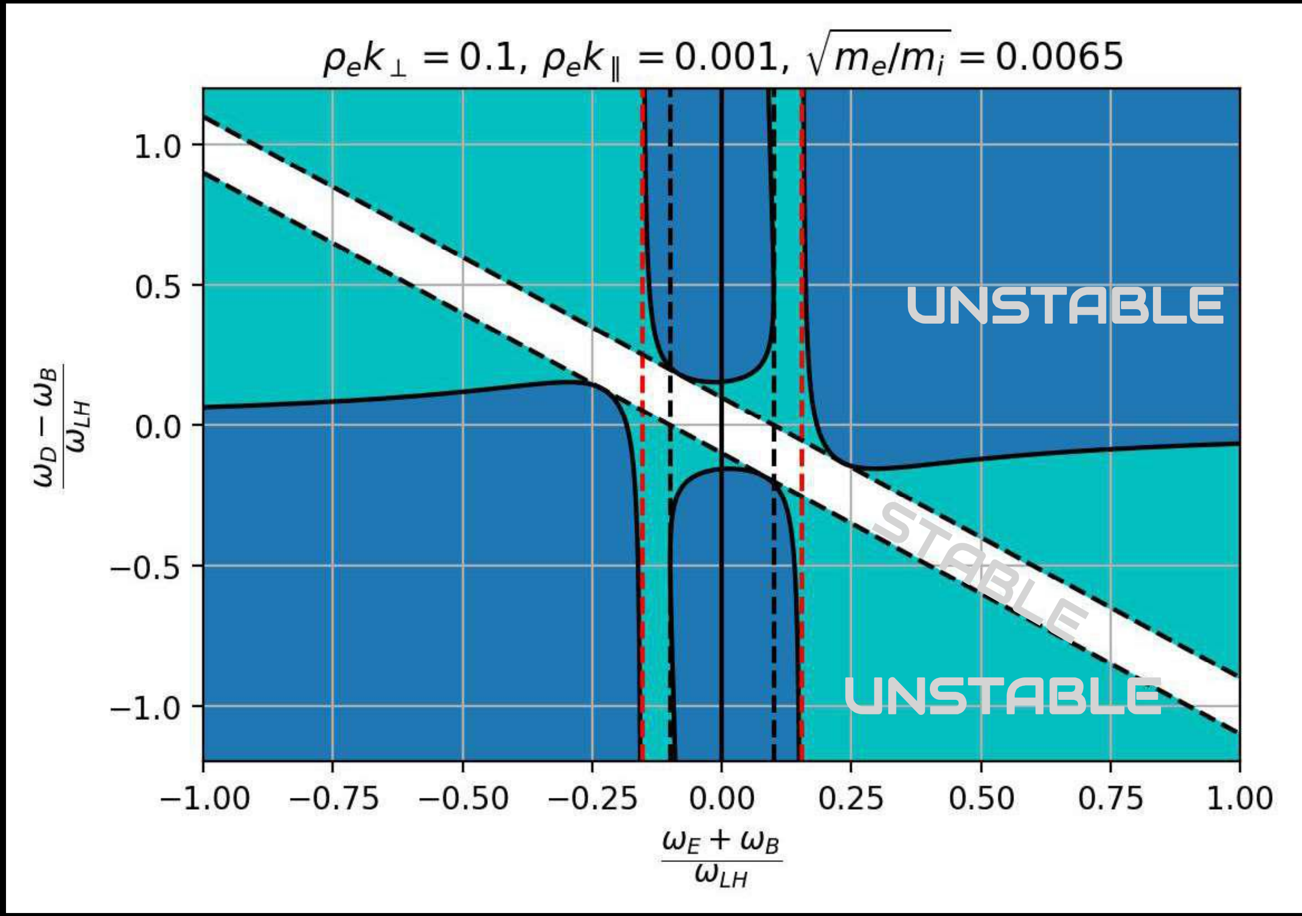
- For both propagation directions:
  - Two different regions for collisonless instability
  - Collisions widen instability regions



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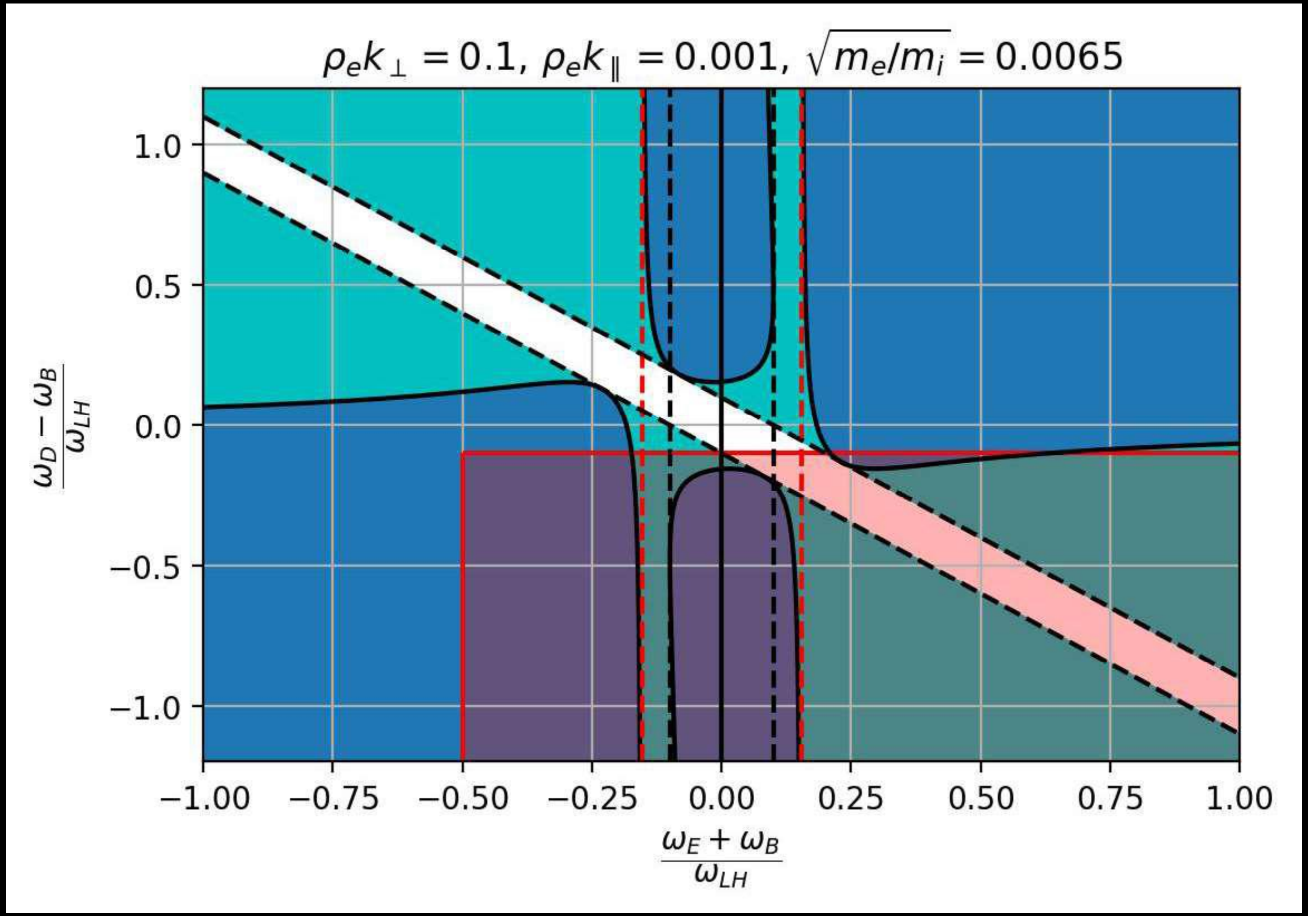

Summary and way forward

- At low frequency ranges (< 100 kHz), floating probe measurements show existence of mainly azimuthal oscillations with low m number (and likely nonzero $k_\parallel$). Oscillations are more apparent downstream, at moderate and high angles α from axis. Likely, various types of oscillations are present at the same time.
- Present measurements are partially inconclusive: low coherence, “dirty” dispersion relation plots
- Adding a 3rd probe could help discriminate the coexistence of multiple waves (which would be affecting current results)
- Phase velocity is compatible with drift velocities
- Local linear dispersion analysis in slab geometry including collisionality, $k_\parallel$, and gyroviscous terms shows:
  - Collisionless regions of instability are affected by $k_\parallel$
  - Collisions add a weak instability almost everywhere in the parametric plane
  - Our operating point corresponds with one such weak instability condition, which could justify experimental observations
- Effect of oscillations/instabilities on $\perp$ transport still unclear

Acknowledgments

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Thank you!

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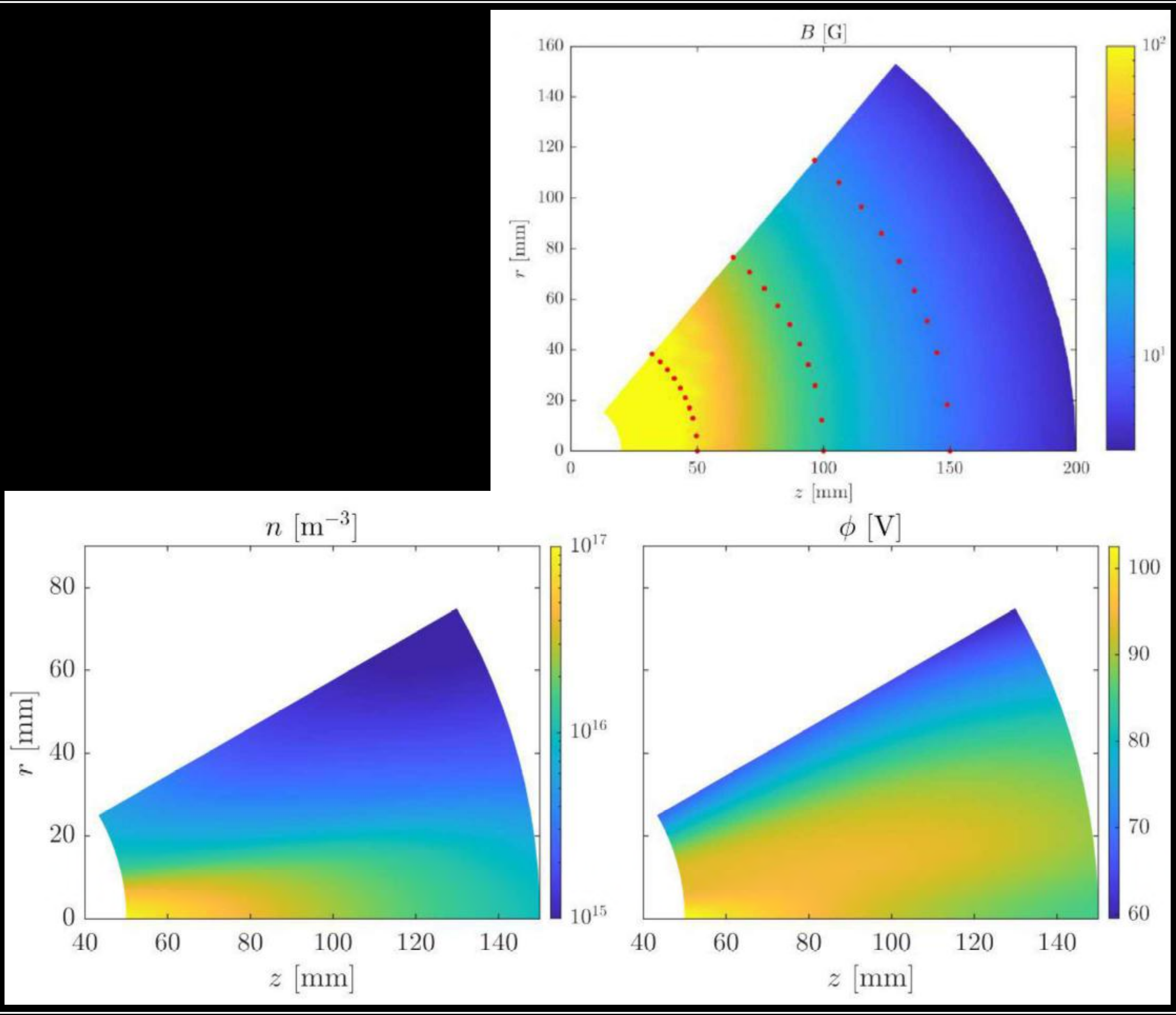
ZARATHUSTRA

Steady-state results

- $B \simeq 10 \div 100$ G
  - $\omega_{ce} \simeq 2.8 \cdot 10^7 \div 2.8 \cdot 10^8$ Hz
  - $\omega_{ci} \simeq 1.2 \cdot 10^2 \div 1.2 \cdot 10^3$ Hz (Xenon)
  - $\omega_{lh} \simeq 5.7 \cdot 10^4 \div 5.7 \cdot 10^5$ Hz
- $n \simeq 10^{15} \div 10^{17}$ m^{-3}
  - $\omega_{pe} = 9.0 \cdot 10^8$ Hz
- $T_e \simeq 12.5$ eV
  - $c_s = 3 \cdot 10^3$m/s

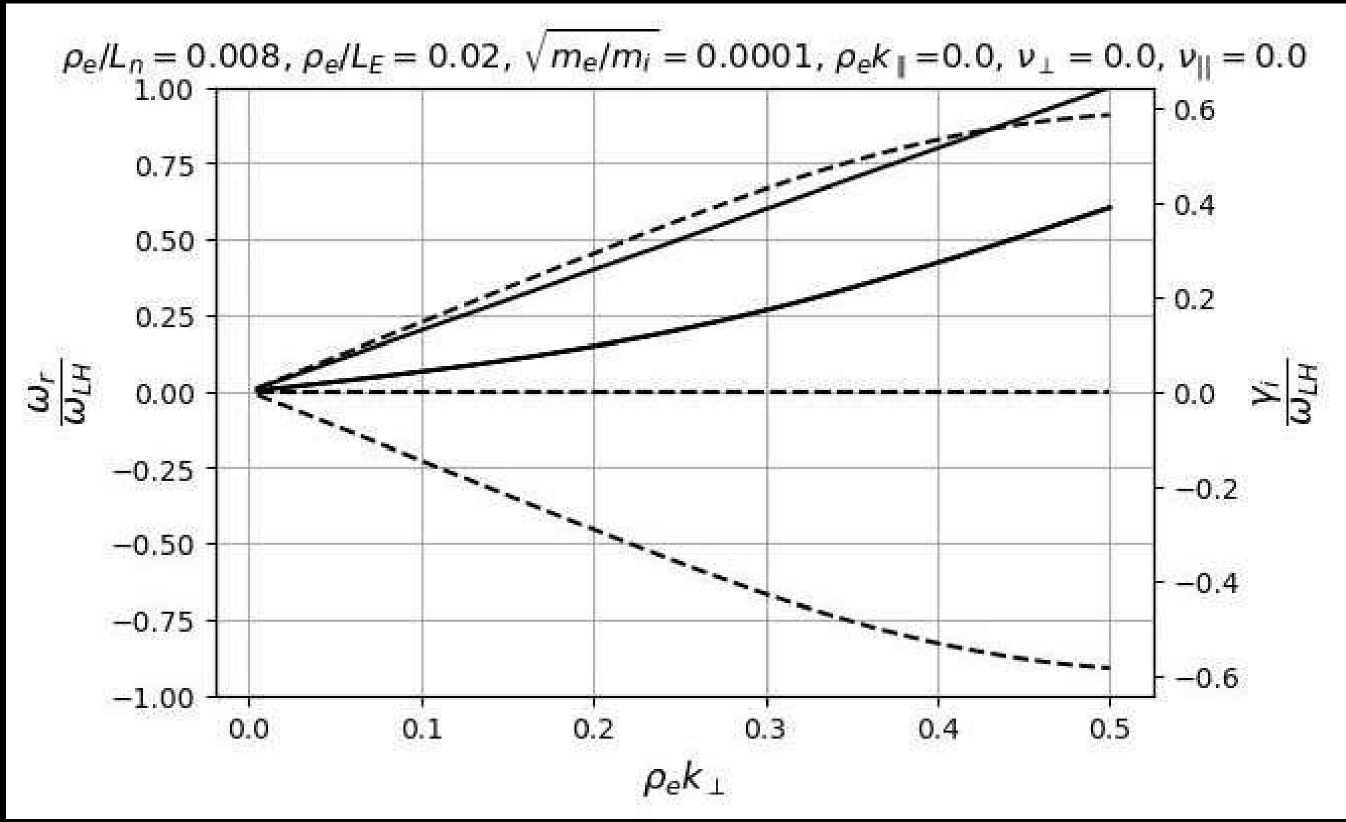
- Estimated gradient lenghts
  - $\ln n \rightarrow$ 1 cm
  - $e\phi/T_e \rightarrow$ 1 cm
  - $\ln B \rightarrow$ 10 cm
- Estimated drift velocities
  - $u_D, u_E \simeq 10^4 \div 10^5$ m/s



Linear Local Stability Analysis

- Collisionless instabilities arising from interaction between branches

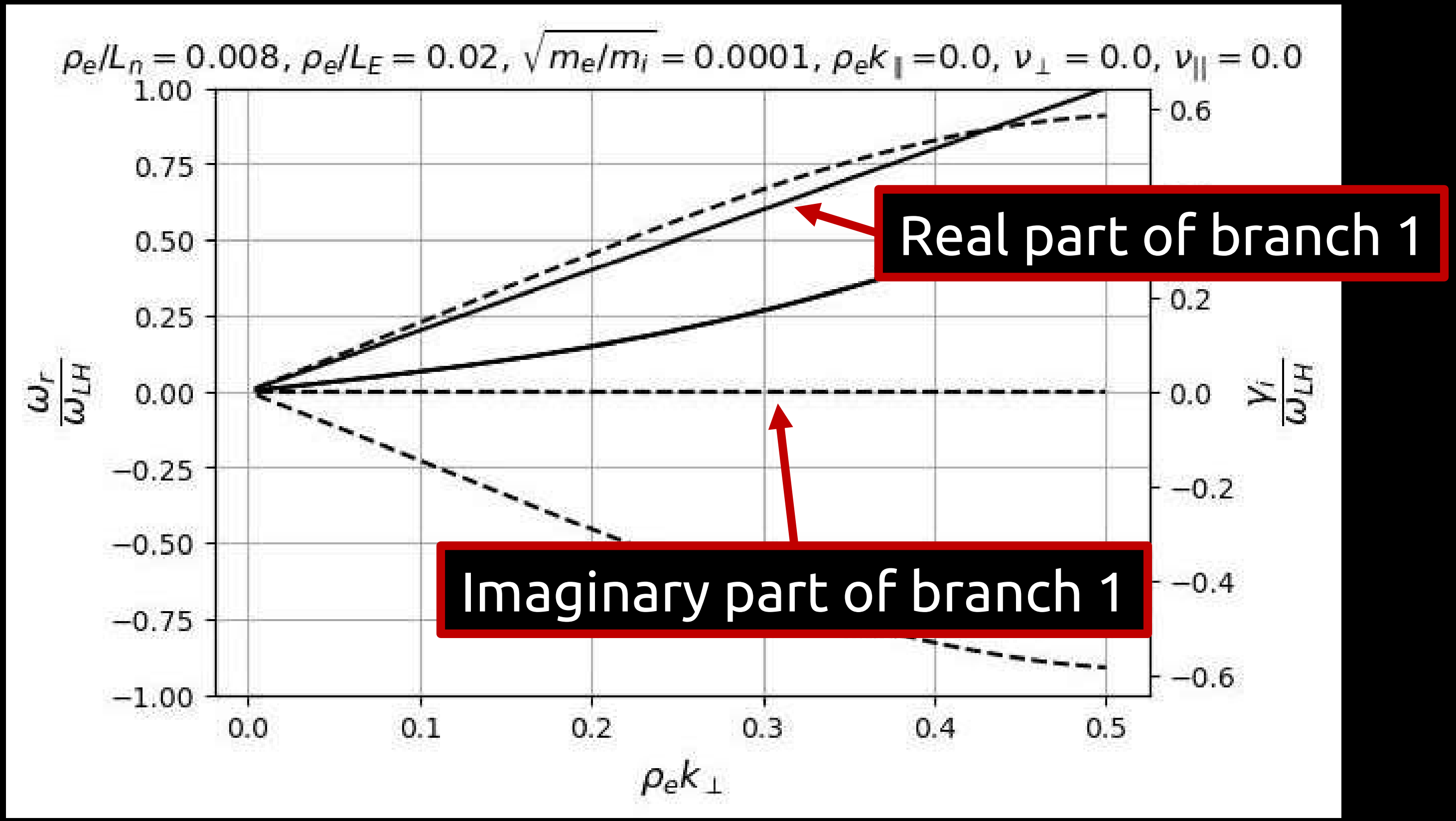
- No parallel propagation:



Linear Local Stability Analysis

- Collisionless instabilities arising from interaction between branches
  - Branch 1: non-trivial solution of parallel component of momentum ($\omega_{\parallel} = 0$)
    - Does not interact with other branches

- No parallel propagation:



Linear Local Stability Analysis

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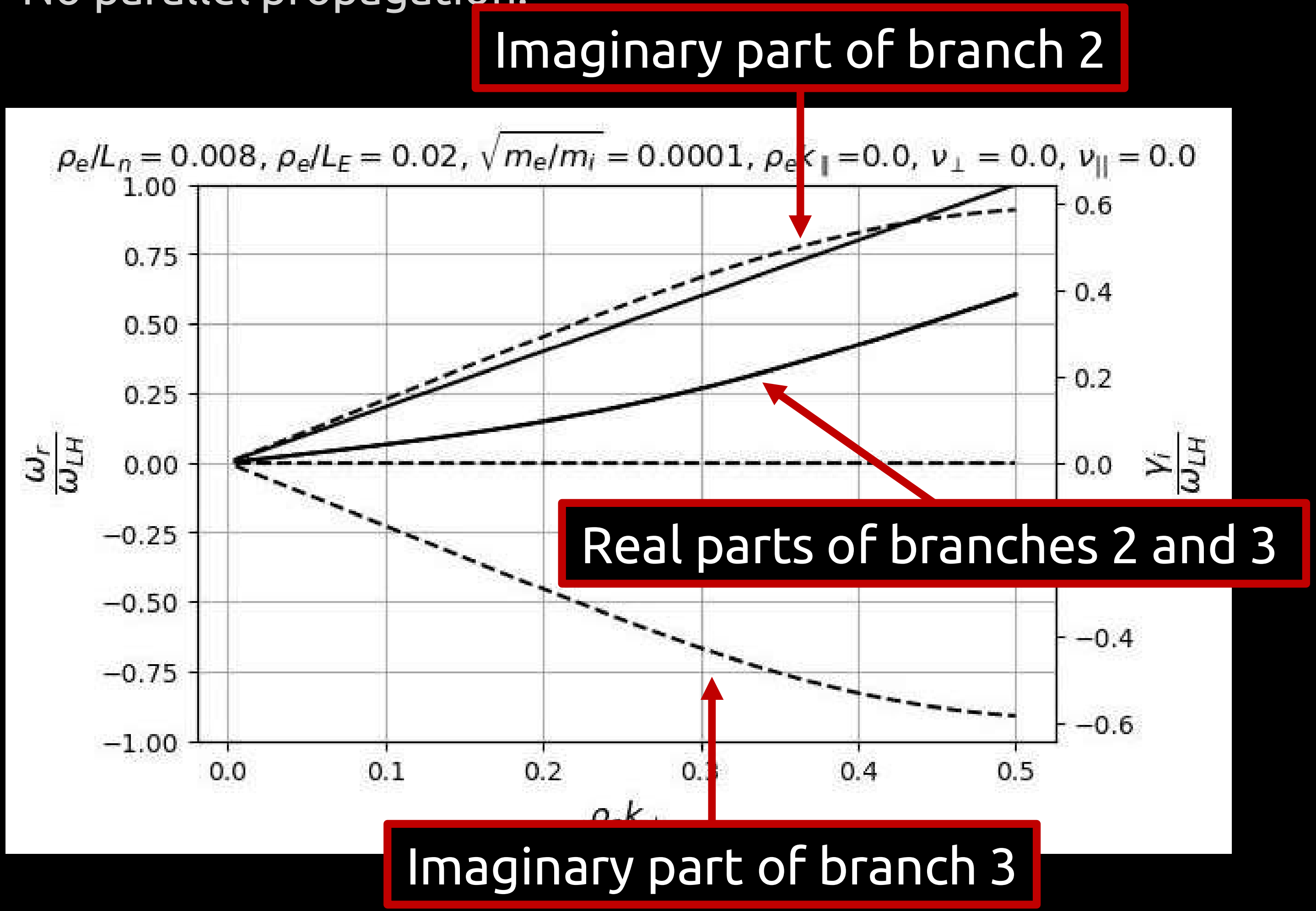
  - Branch 1: non-trivial solution of parallel component of momentum ($\omega_{\parallel} = 0$)
    - Does not interact with other branches
  - Branches 2 and 3: destabilization of «anti-drift wave» (modified SHI, with inertial and FLR effects)

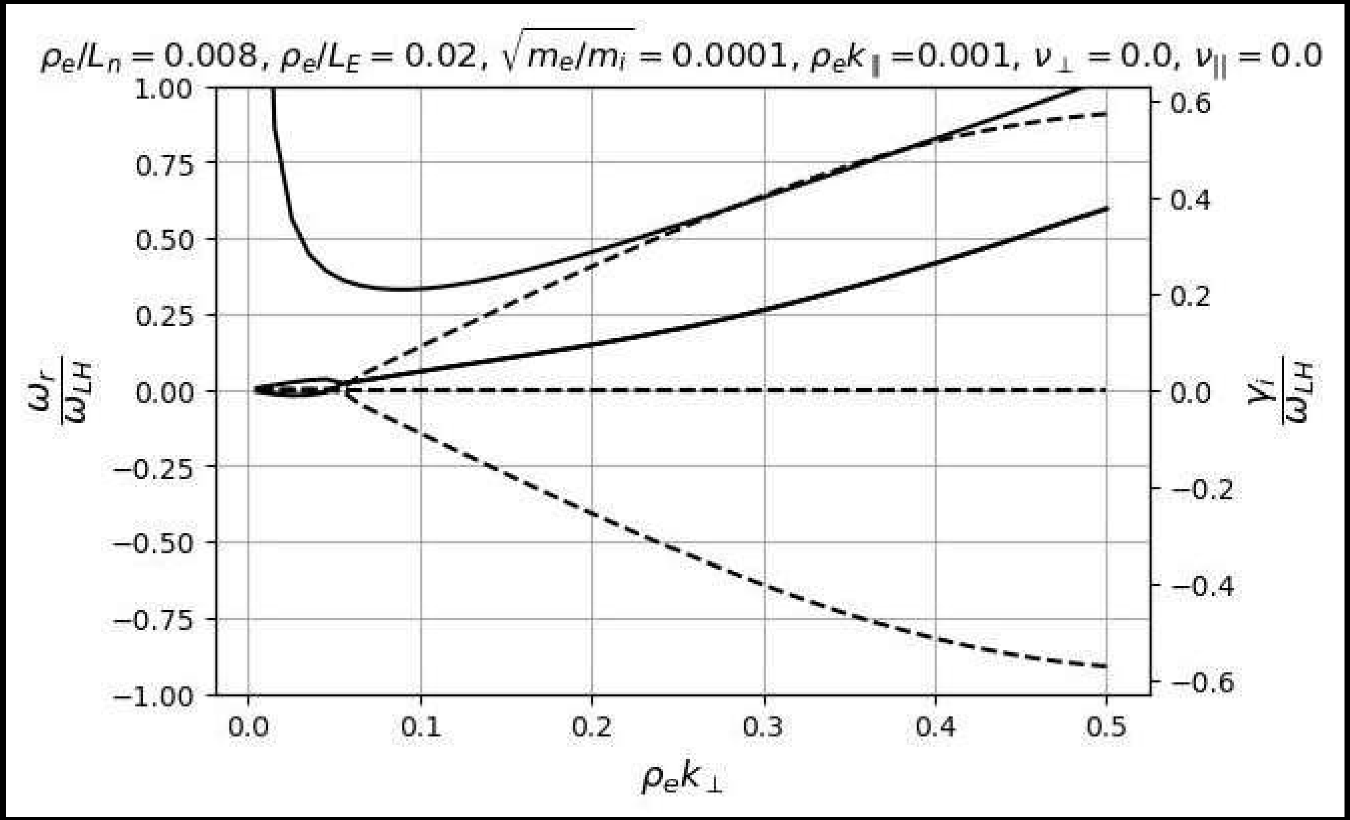
- No parallel propagation:



Linear Local Stability Analysis

- Collisionless instabilities arising from interaction between branches
- Including parallel propagation introduces new branches (and new ways for destabilizations to take place)

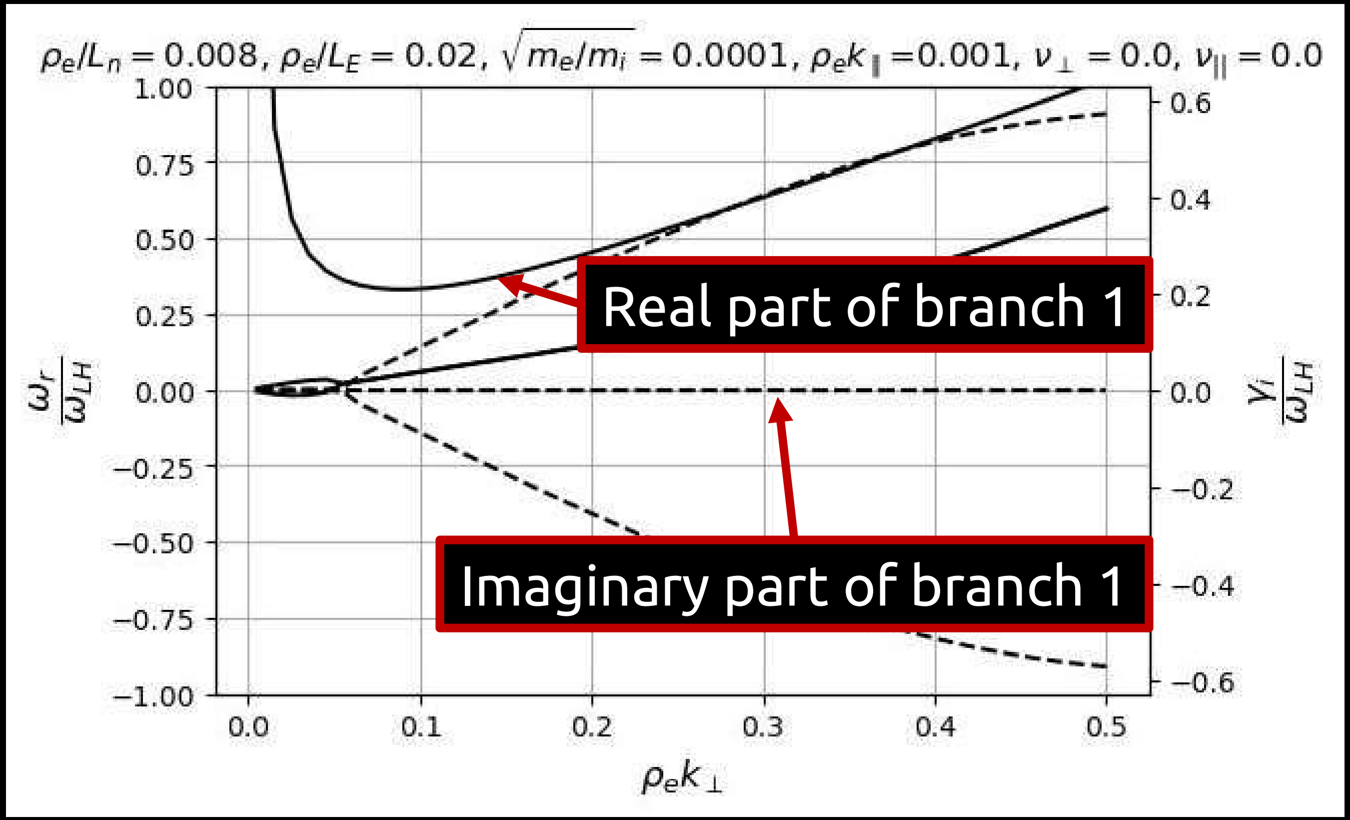
- With parallel propagation:



Linear Local Stability Analysis

- Collisionless instabilities arising from interaction between branches
- Including parallel propagation introduces new branches (and new ways for destabilizations to take place)
    - Branch 1 now «sees» other branches
        - Its solution is no longer decoupled from the rest of the problem

- With parallel propagation:



Linear Local Stability Analysis

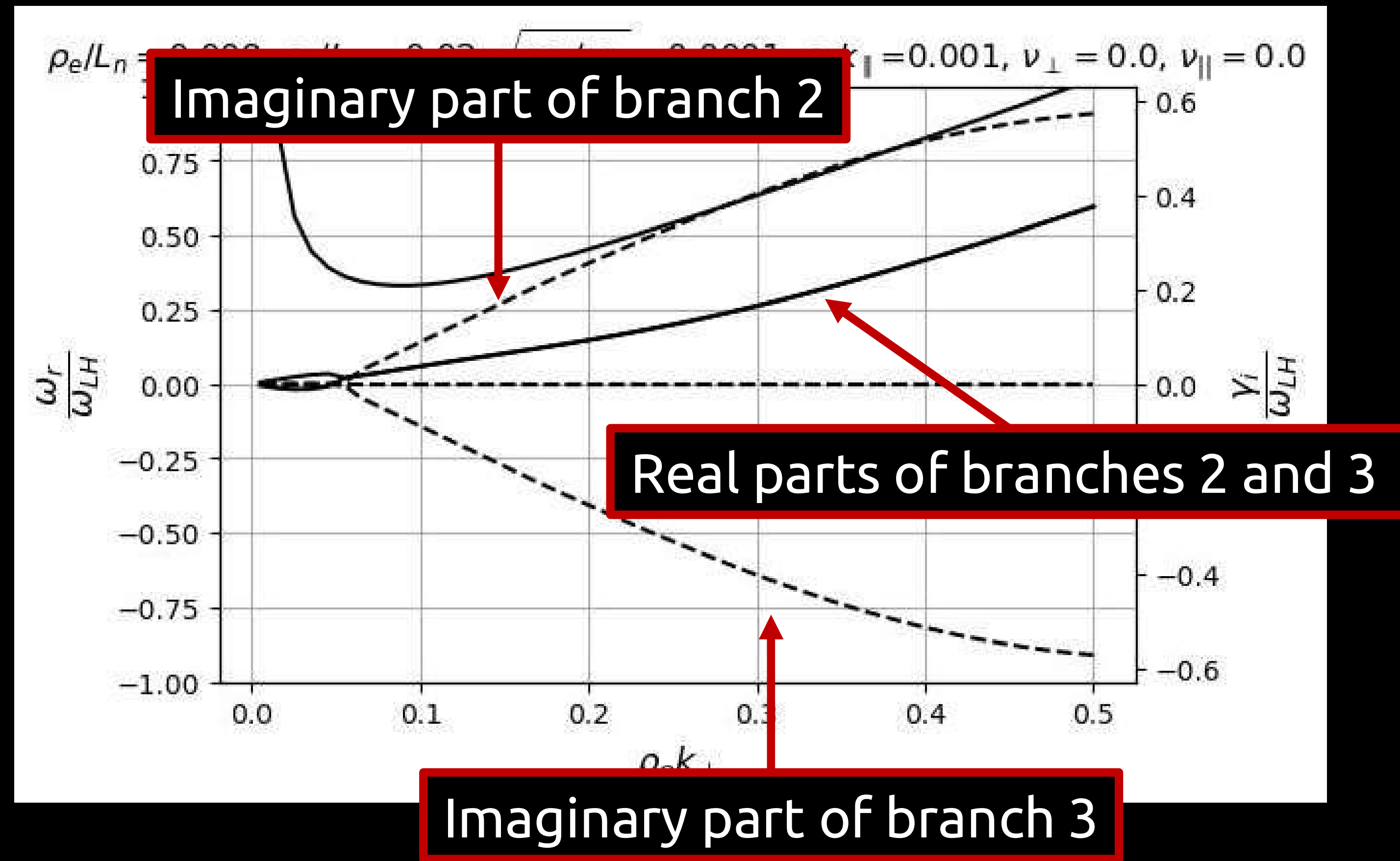
- Collisionless instabilities arising from interaction between branches

- Including parallel propagation introduces new branches (and new ways for destabilizations to take place)

  - Branch 1 now «sees» other branches

  - Branches 2 and 3 might still interact and destabilize under simil-SHI conditions

- With parallel propagation:



Linear Local Stability Analysis

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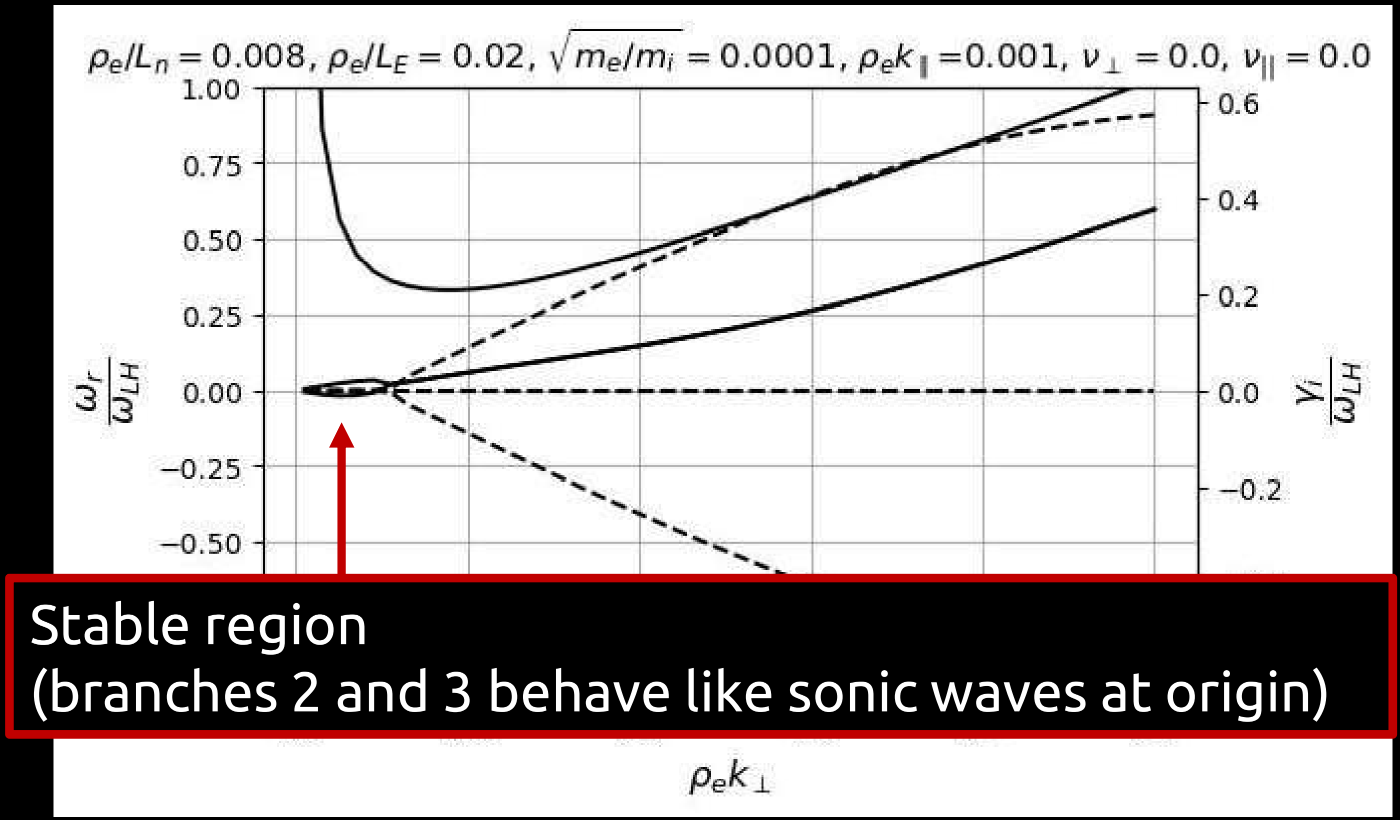
    - Stable close to the origin

- With parallel propagation:



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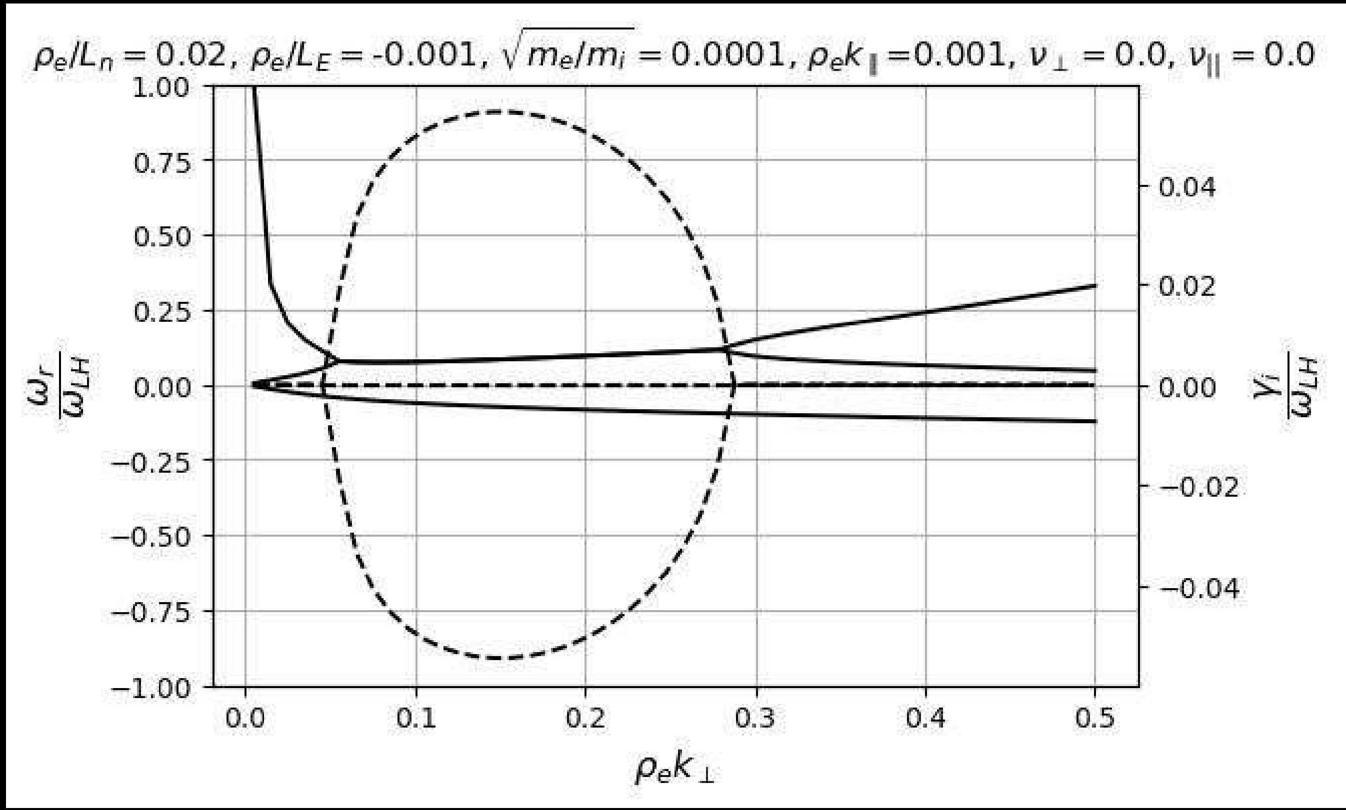
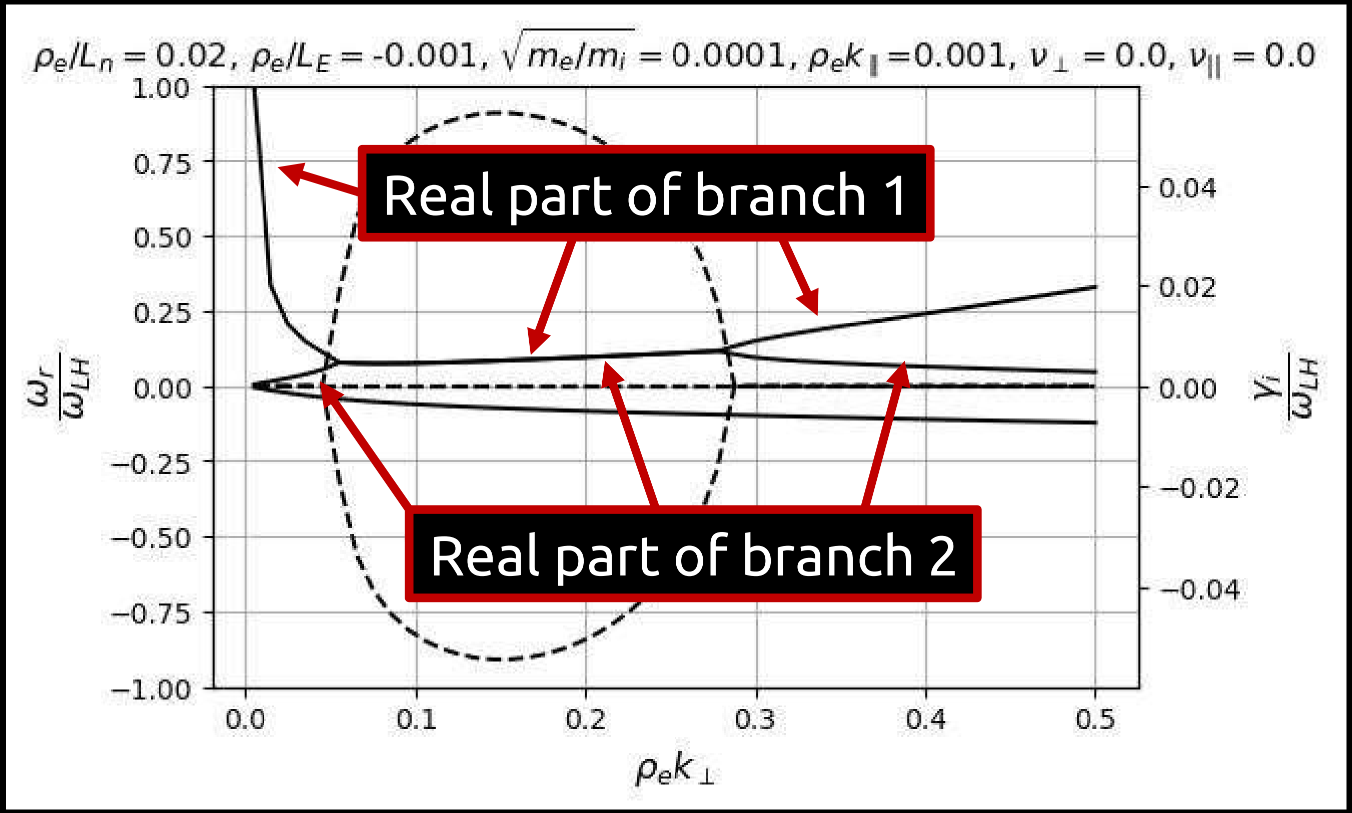


Linear Local Stability Analysis

- Collisionless instabilities arising from interaction between branches
- Including parallel propagation introduces new branches (and new ways for destabilizations to take place)
  - Depending on the choice of parameters, branch 1 might interact with one of the two sonic branches

- With parallel propagation:



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    - New conditions for instability

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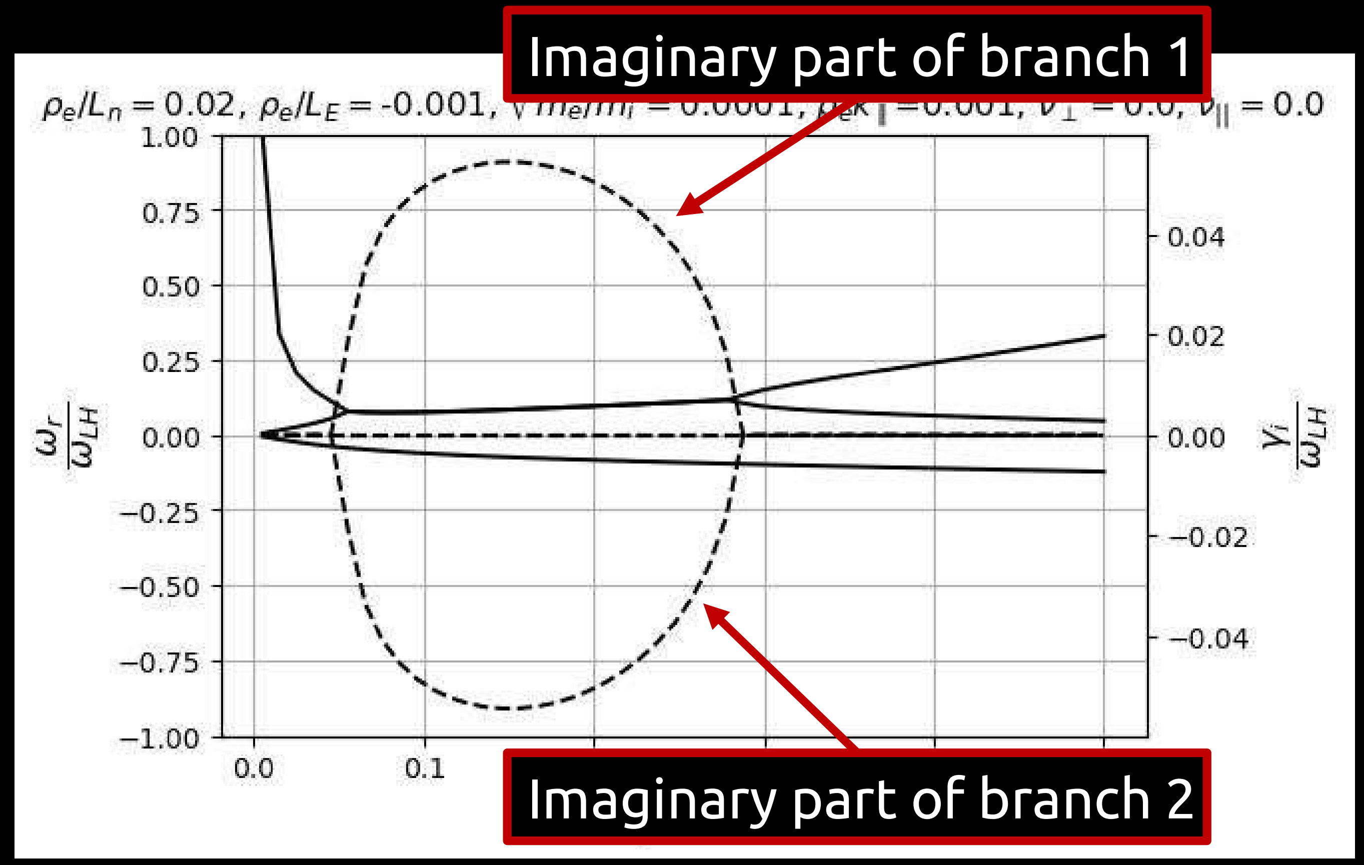

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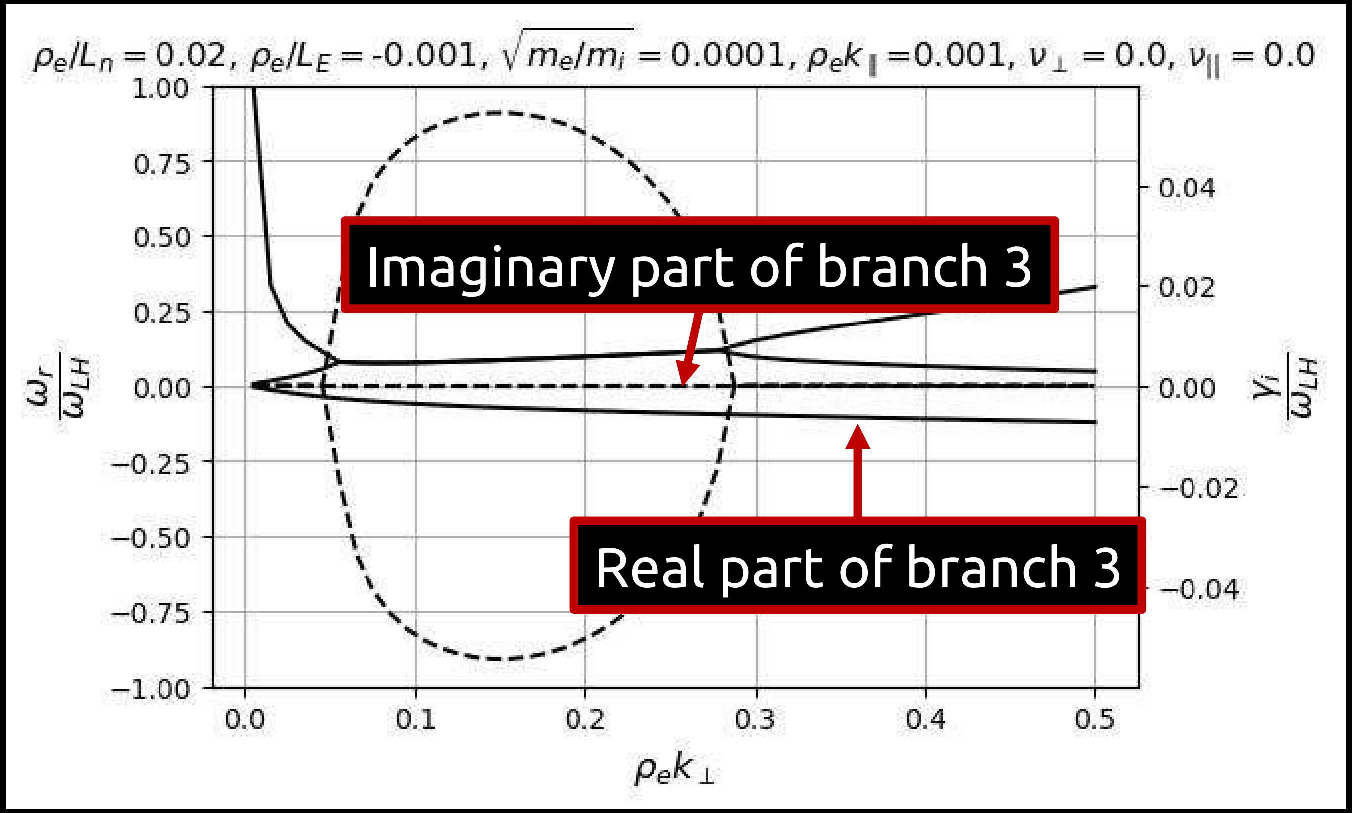

Linear Local Stability Analysis

- Collisions widen parametric range favourable for instabilities to arise

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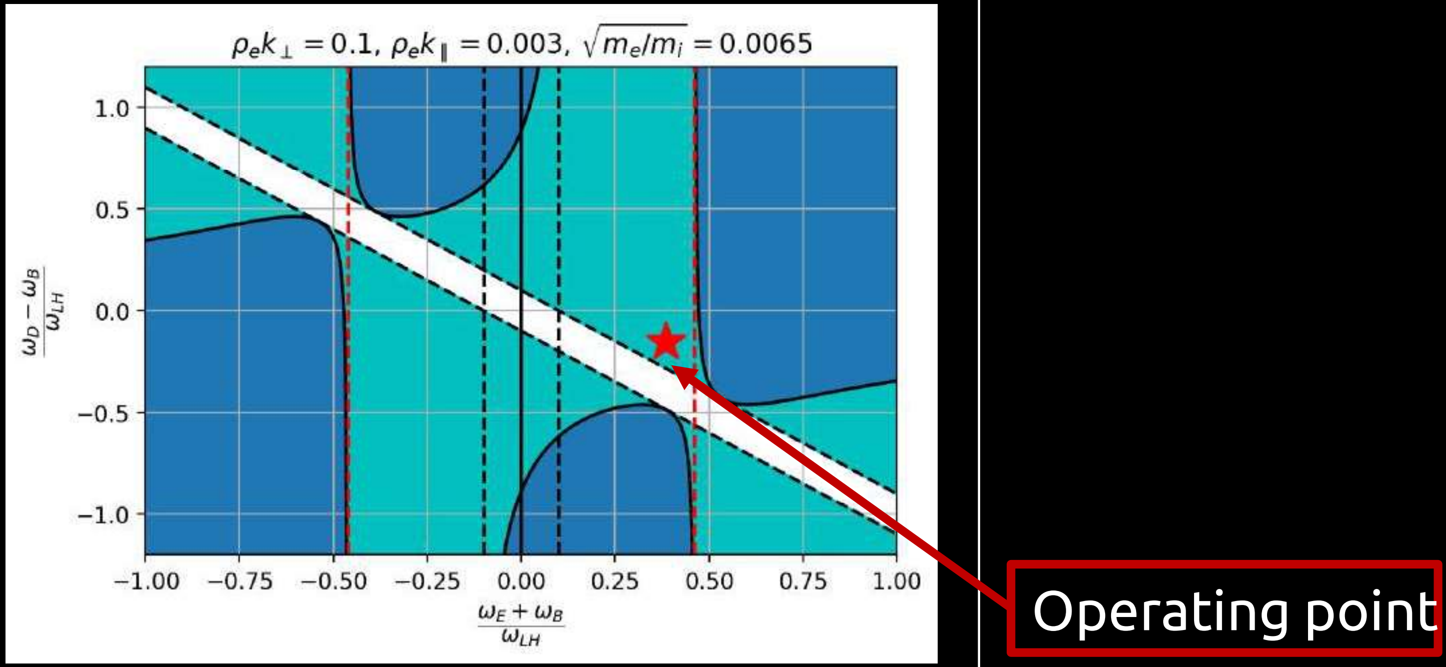
- Consider a point where no collisionless instability takes place:



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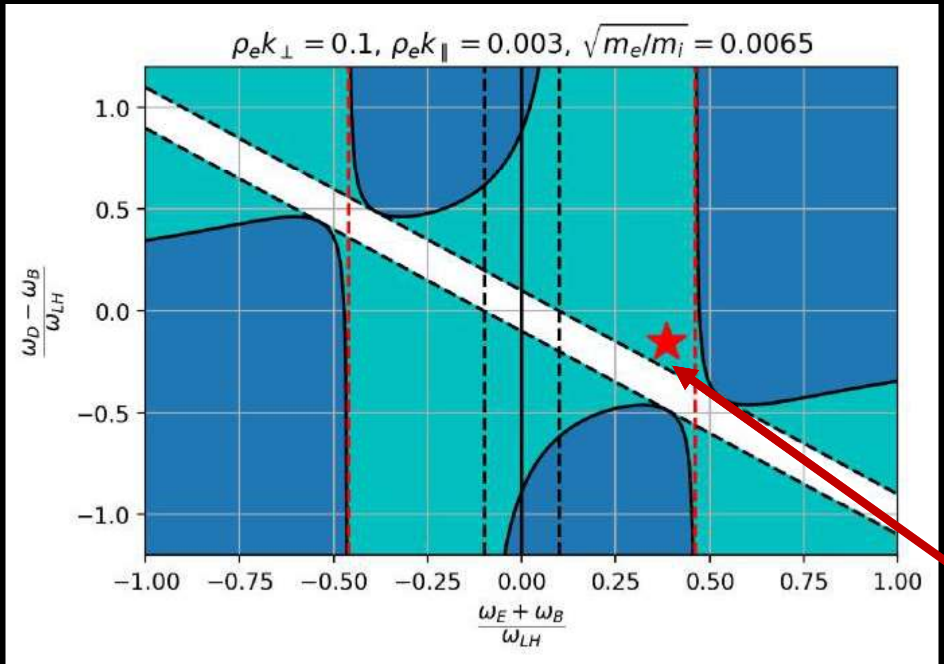
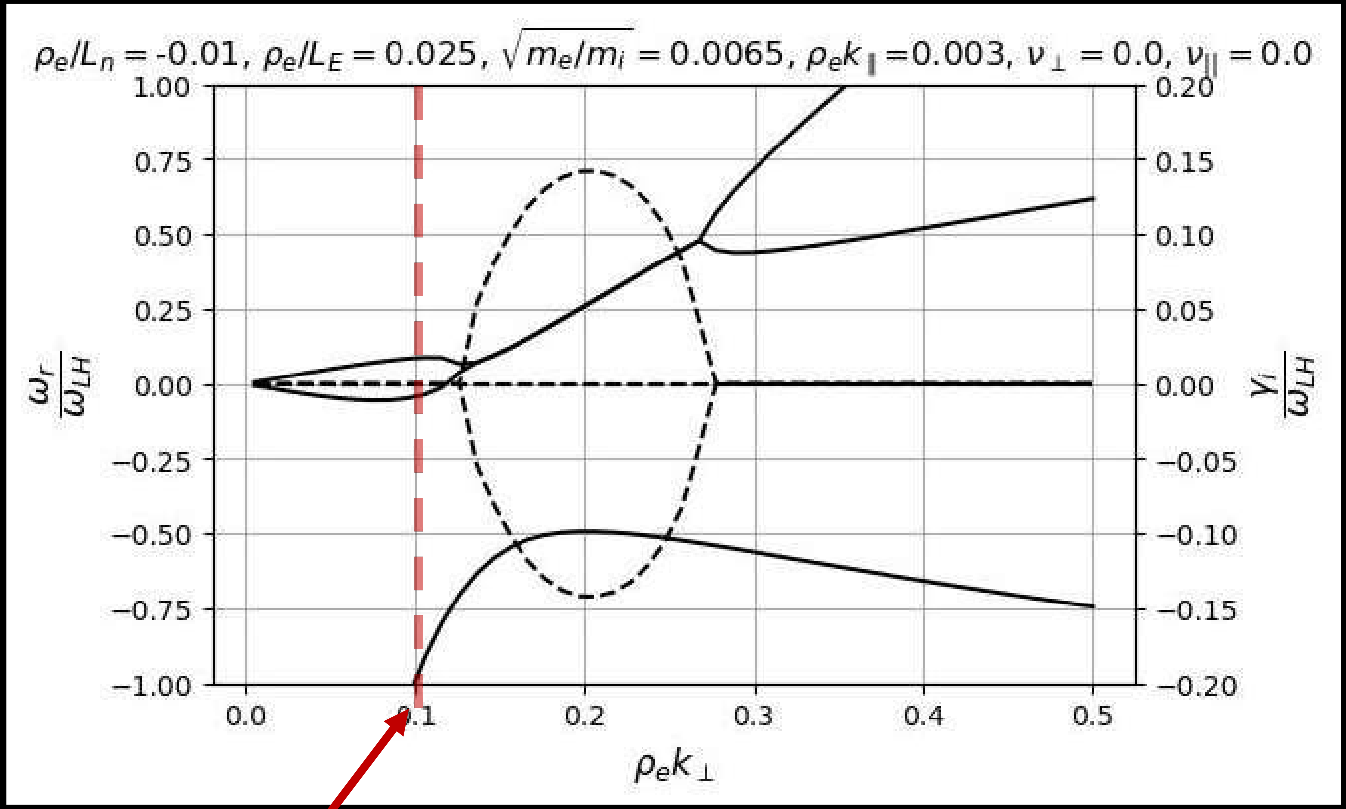


- Without collisions:



Operating point

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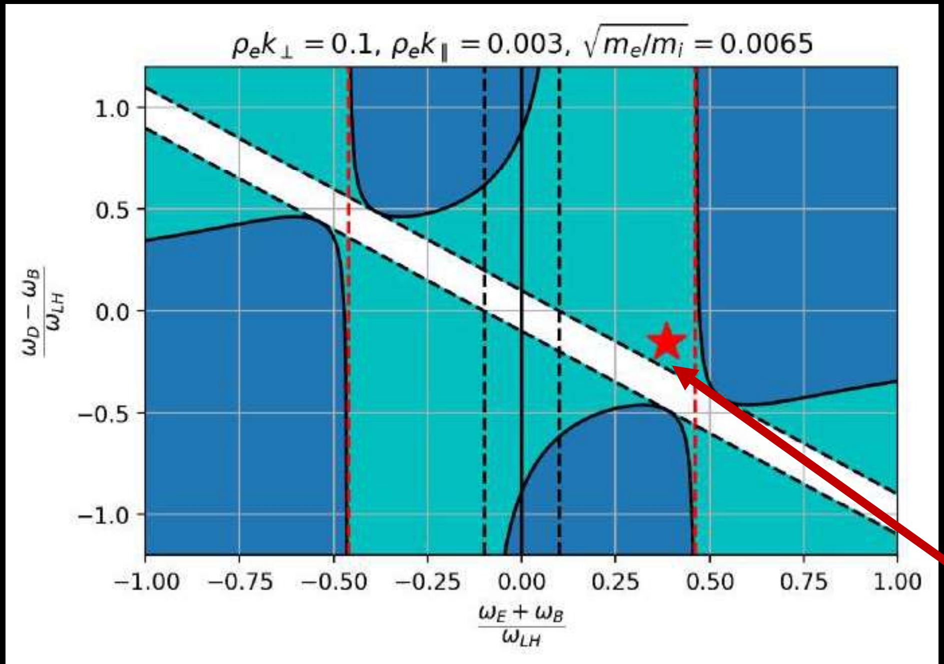
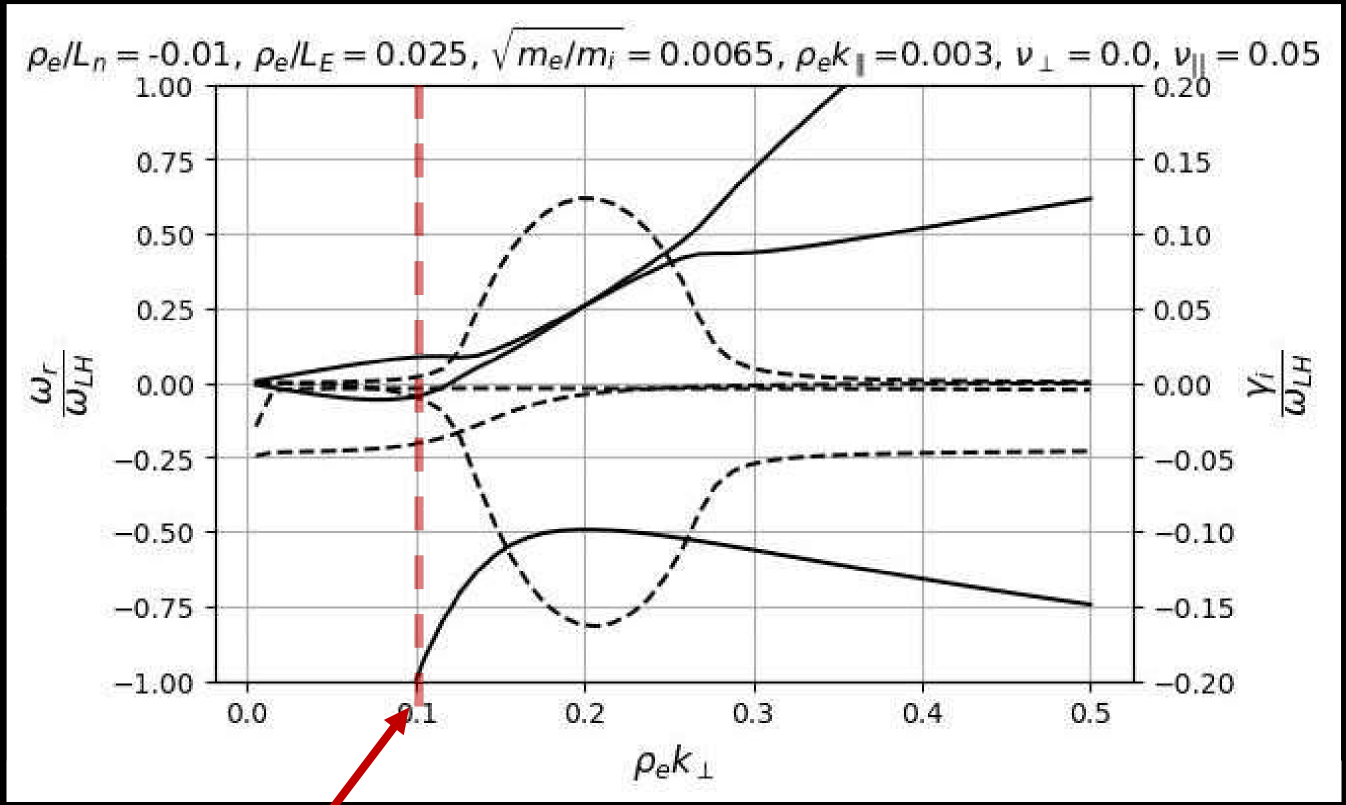


- With collisions:



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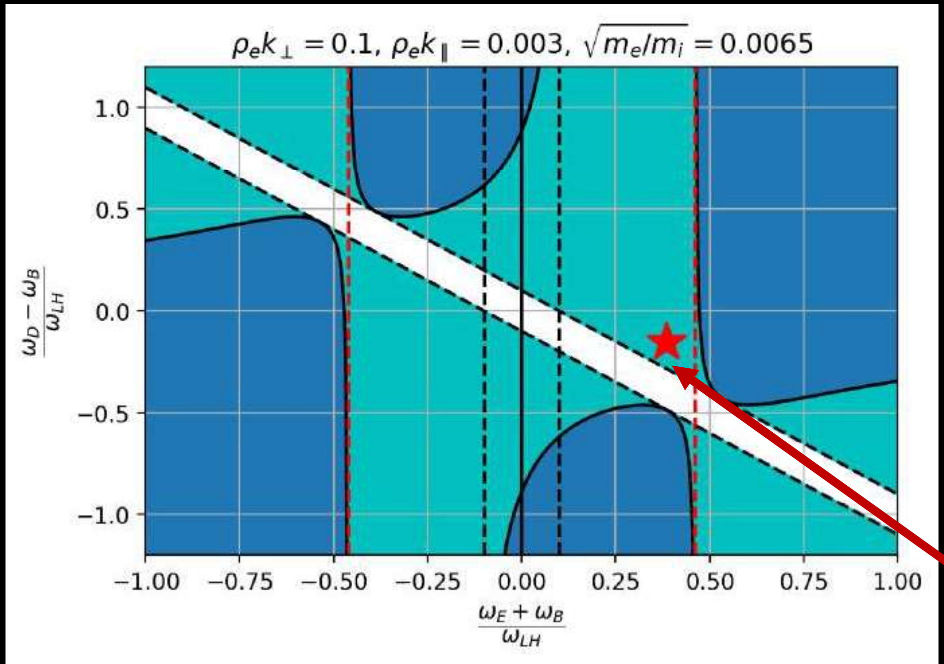

- With collisions: