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COMPLETE FLOW CHARACTERIZATION FROM SNAPSHOT PIV, FAST PROBES AND PHYSICS-INFORMED NEURAL NETWORKS

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ABSTRACT

The use of machine learning (ML) algorithms to solve complex problems of difficult experimental or theoretical access has been exponentially increasing since the beginning of the 2010's. In the field of fluid mechanics and aerodynamics, current technology allows for limited time-resolved data acquisition, especially in the 3D domain. The most recent novel application based on artificial intelligence to solve physics-based problems is implemented by the so-called physics-informed neural networks (PINNs), developed by Raissi *et al.* (2019) and which are based on a fully-connected neural network model that incorporates governing laws in the loss function. PINNs are therefore a very powerful tool to enforce physical constraints in experimental data.

Concerning the benefit of PINNs, their use comes in handy when applied to the data reconstruction from fluid flow experiments, since their behaviour is accurately modeled by the well-known Navier-Stokes equations:

$$\begin{cases} \frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0 \\ \frac{\partial u}{\partial t}+\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}\right)=-\frac{\partial p}{\partial x}+\frac{1}{Re}\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right) \\ \frac{\partial v}{\partial t}+\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}\right)=-\frac{\partial p}{\partial y}+\frac{1}{Re}\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2}\right) \\ \frac{\partial w}{\partial t}+\left(u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}\right)=-\frac{\partial p}{\partial z}+\frac{1}{Re}\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2 w}{\partial z^2}\right) \end{cases}, \tag{1}$$

where (u,v,w) is the dimensionless velocity vector, p is the reduced pressure and Re is Reynolds number.

In the implementation of Raissi *et al.* (2019), PINNs receive as input the spatial grid over which the most relevant fluid properties want to be computed and the time-resolved distribution of one flow quantity over a 3D domain, and make use of automatic differentiation to calculate the corresponding gradients. During the training process, the weights of the dense layers are updated to reduce the residual of Eq. (1).

One of the main limitations of PINNs is that they require time-resolved data to be able to properly calculate time derivatives. While to date volumetric velocimetry offers full 3D description of the three velocity components, often the time resolution is not available due to hardware limitation, especially at moderate-to-high Reynolds number. Therefore, even though 3D PIV data have great potential to disclose a full flow description (also including pressure fields), the lack of time resolution impedes the proper application of PINNs.

The objective of this work is to enable the use of PINNs to characterize in full the fluid behavior exploiting simultaneously time-resolved measurements from few point probes and non-time-resolved PIV measurements.

Proposed novel architecture

We propose incorporating in the PINN architecture a multi-time-delay strategy, based on fast point probes, to achieve the desired time resolution of spatially resolved fields. The following methodology includes time history by introducing as input of the network the data collected by the probes over a certain timespan, rather than a single instant. We aim to exploit proper orthogonal decomposition (POD) as an encoder for the flow field information. The network is trained to receive as input time segments recorded by the probes, and estimate the corresponding POD time coefficients of velocity fields. The output flow fields are regularized with PINNs, which further augment the data disclosing additional quantities, such as the pressure field. In Figure 1 a sketch of the method is reported.

Multi-time delay MLP for estimation of POD temporal modes

Consider a snapshot matrix $\mathscr{U}$ built from PIV data, with dimensions $T \times dN$ (T being the number of snapshots and N the number of points in the grid. The factor d indicates the number of components of the velocity vector, i.e. 2 for 2D and 3 for 3D configuration, respectively). A discrete form of POD can be obtained through Single Value Decomposition (SVD):

$$\mathscr{U} = \Psi\Sigma\Phi^T, \tag{2}$$

where Ψ contains the temporal modes, Σ is a diagonal matrix containing the singular values (which are representative of the energy associated to each mode), and Φ^T contains the spatial modes. The product $\Psi\Sigma$ can be collapsed into a matrix A. It is well known that the matrix product of Eq. (2) can be truncated at rank R by retaining the most energetic modes, and that this would provide the best rank-R estimation of the data.

Our purpose is to reconstruct the fields by estimating the first R temporal coefficients through a set of probes. The idea is that, if data are captured simultaneously by a set of probes with high temporal resolution and by field measurements with high spatial resolution, we can establish the relationship between probe and field data. The principle we aim to exploit is similar to the Extended POD estimation by Tinney *et al.* (2008). In our approach, the relationship between probe and field data is established using a Multi-Layer Perceptron (MLP). In order to augment the input of the network, the MLP is fed with probe data from a time segment. In particular, since in advection-dominated flows it is reasonable to locate the probes at the downstream edge of the domain, the data collected by the probes at instants prior to the moment where a snapshot field is available are actually representative of information present in the field at that time instant. This principles was also leveraged in different forms by Hosseini *et al.* (2015); Discetti *et al.* (2018), among others.

Regularization of time-resolved velocity field

The time-resolved velocity fields obtained from estimation with the probes contain errors due to noise, poor correlation and truncation to the adopted rank for the POD reconstruction. To this purpose, a PINN is included in the process to regularize the data, enforcing the validity of Eq. (1) to reduce noise and truncation errors. Furthermore, the PINN augments the measurements giving access to additional quantities, such as the pressure distribution. It must be remarked here that the grid onto which the final data will be available is not constrained to be the same of the PIV data, and actually it is normally refined to allow accurate computation of the spatial derivatives.

The loss function adopted for training consists of three different contributions:

- residual of the Navier-Stokes (NS) equations,
- compliance with the velocity in the nodes where PIV data are available,
- and a boundary condition on pressure.

Each of these contributions has an independent role in terms of optimizing the weights of PINNs. First, it has been observed that enforcing the validity of the NS equations tends to homogenize the field. Secondly, the reference velocities force that the solution complies the experimentally-accessible data. It is clear, however, that a perfect matching in this case is not desirable, since the original data are inevitably corrupted by noise. The two previous contributions together ensure the model provides a solution which matches the velocity profile in accessible places while complying with the NS equations. The third condition has to be set since, strictly speaking, PINNs provide the distribution of the pressure gradient. In order to obtain the pressure field, one value of the pressure should be defined at an arbitrary point of the domain.

The different error contributions are reduced differently as the model is being trained through the multiple epochs. In this work we implemented an adaptive loss function. Our modified loss function weights the different contributions of the total loss function in a dynamic way in which the component with bigger error Ł receives a proportional higher weight:

$$\mathscr{L} = \sum_{i=NS,vel,inlet} w_i Ł_i \text{ where } w_i = \frac{Ł_i}{\sum Ł_i}. \tag{3}$$

The benefit of this formulation is that it permits a faster convergence to the real solution while keeping a very high accuracy on the final results.

Validation

For a PIV experiment, even though time resolution cannot be fully achieved due to the limitations of the experimental set-up, the matrix A_{PIV} and Φ_{PIV} may be obtained thanks to Eq. (2) after being projected onto a refined grid. A tentative prediction of the time-resolved ground truth $\tilde{A}_{TR}$ may be then obtained by using our multi-time delay MLP. To reconstruct the approximate time-resolved velocity field, we return again to Eq. (2) to obtain $\tilde{\mathscr{U}}_{TR} = \tilde{A}_{TR}\Phi_{PIV}^T$. This first-order approximation will therefore contain truncation errors as well as corrupted data coming from PIV, errors which will be subsequently regularized using PINNs.

By the time of submission of this abstract, our real-life situation is modeled by a Direct Numerical Simulation (DNS) from which we also extract PIV synthetic data (we aim to have a fully-validated method with real PIV experiments by the time of the conference). The synthesized PIV is computed by projecting the DNS field onto a regular grid of different pixel resolution (from 2048x2048 to 256x256) and calculating the average velocity within a window of size 32x32 pixels and grid distance of 16 pixels. Additionally, we add artificial noise to the data following a random normal distribution. The u component of the velocity from the PIV corrupted data as well as the final reconstruction using PINNs is shown in Figure 2a. The three contributions to the loss function are represented in Figure 2b, together with the mean squared error of snapshot 50 referred to the DNS data as a function of the PIV resolution. The improvement of the PINN reconstruction with respect to the original PIV corrupted data is more than evident and shows that an increase in accuracy can be obtained up to 2 orders of magnitude. The use of PINNs as a regularizer becomes thus a significant improvement of experimental data acquisition by PIV.

Conclusions

In this research, we have designed a new architecture which allows us to extract and reconstruct from non-time-resolved corrupted data (PIV experiments) a full time-resolved domain, not only of the velocity field but of the pressure as well. We have updated the architecture of a MLP to act as a multi-time delay module in which non-time-resolved PIV information and local time-resolved data measured by point probes are used to reconstruct and predict the temporal modes of an approximate time-resolved ground truth velocity matrix. Subsequently, we are able to correct and regularize for the spurious errors and instabilities that may have been incorporated in the system with PINNs. In conclusion, our new architecture is able to combine the strengths of a MLP in order to reconstruct data from non-time resolved experiments to time-resolved and regularize using PINNs. The combined modules are able to correct and reconstruct from faded or corrupted information originating from experimental noise during data acquisition by fast probes and / or PIV, POD truncation and prediction of temporal modes by the multi-time delay MLP. The robust adaptative loss function by which our PINN is governed facilitates a fast convergence. Our methodology stands for a novel manner of combining two different architectures to bridge non-time resolution experimental data corrupted with noise with accurate time-resolved full flow description, and therefore, artificially overcome the limitations given by experimental techniques.

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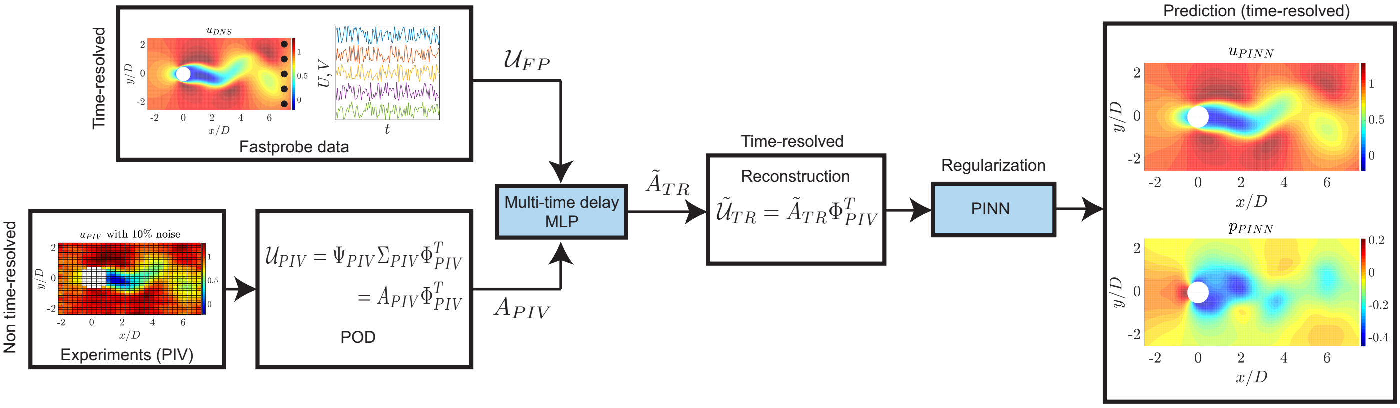

Figure 1: (a) Model architecture combining a multi-time delay MLP with PINNs. The MLP learns from non-time-resolved PIV noisy experimental data and receives as input the time-resolved history data measured by fast probes. The outcome of the module is a time-resolved approximation of the temporal modes of the approximate velocity matrix. The estimated time-resolved 'ground truth' velocity field is then regularized thanks to PINNs enforcing Navier-Stokes's equations within a robust adaptative loss function. A very accurate reconstruction of the time-resolved velocity and pressure fields may be then obtained.

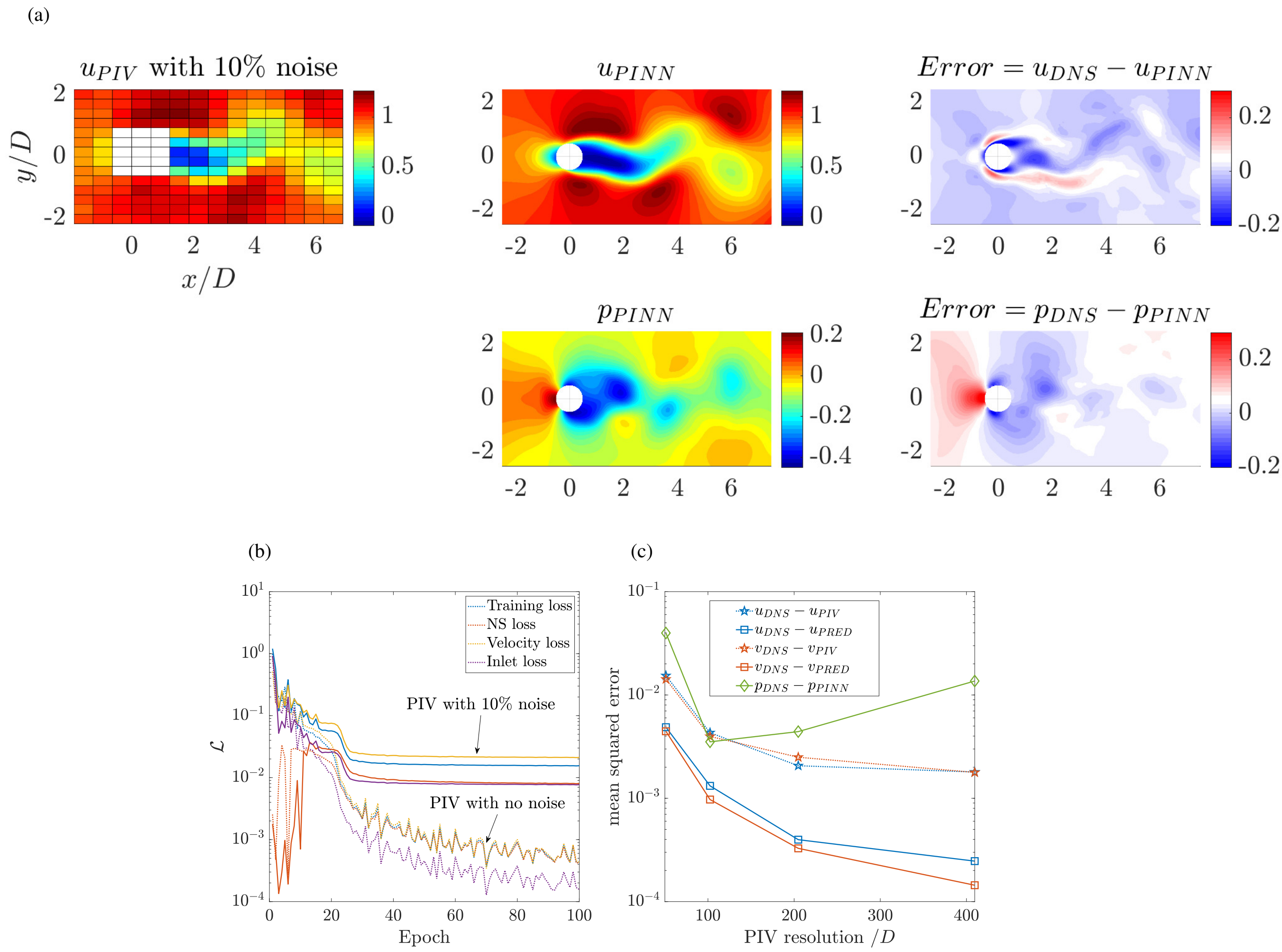

Figure 2: a) Dimensionless velocity (first row) and pressure (second row) field reconstruction using PINNs. The reconstruction starts from PIV data, which has very small pixel resolution (256x256) and a 10% noise contribution. Our model is able to fully reconstruct with precision the pressure field and improve the velocity domain. (b) Training loss and contribution of the three different components as a function of the epoch number for a PIV configuration with and without noise. The robust adaptative loss function is able to account for the existing noise in the reference data. (c) Mean squared error of original PIV corrupted data and PINN-reconstructed field with respect to DNS. The improvement up to 2 orders of magnitude shows the benefit from the application of our model architecture.