

Modeling of Dual Active Half Bridge (DAHB) converter based on discrete time techniques

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Abstract—The Dual-Active-Half-Bridge (DAHB) converter is being explored as a candidate for applications requiring low cost, reliable and power dense power conversion. This paper describes the discrete modelling of a DAHB converter, applicable to different modulation strategies such as Single Phase Shift (SPS), Dual-phase-shift (DPS) or Triple-Phase Shift (TPS). Discrete modelling allows a very accurate description of high frequency effects, phenomena that cannot be effectively predicted with averaged methods. This is especially useful if the half-bridge capacitance is small and the resonant frequency is comparable to the switching frequency. The modelling approach generates an accurate dynamic model, as well as a steady-state model predicting very precisely the current and voltage wave-forms, which in turn allows to make an optimal design based on numerical methods. The model has been validated with time domain simulations in both dynamic and static operation.

Index Terms—Discrete modelling, bidirectional converters, DAHB.

I. INTRODUCTION

The need to reduce our energy footprint makes electrification a key factor in many applications such as the integration of renewable sources, electrical mobility [1] [2], DC microgrids [3], on board electrical systems [4], UPS [5] and energy storage system [6]. Bidirectional DC-DC power converters become a central element in many of these applications that are in the focal point of energy processing systems where Dual-active-bridge (DAB) and Dual-active-half-bridge (DAHB, Fig. 1) converters are very suitable options. These topologies offer high power density, low switching losses in both directions and galvanic isolation.

At low power level or high voltage applications, DAHB is preferred to its full-bridge counterpart because of its low device count, low cost, high power density, simplicity, and natural capability to avoid transformer saturation. Designs based on high switching frequencies (e.g. $>500\text{kHz}$) where a single small bridge capacitor can be used (e.g. $\sim 100\text{nF}$) are a promising option in terms of power density and cost.

The DAB converter has been widely studied [7], but less attention has been paid to the DAHB. Especially if we consider a low value capacitor, its voltage ripple can no longer be considered zero and normally the resonant frequency of the converter is close to the switching frequency. Besides, the

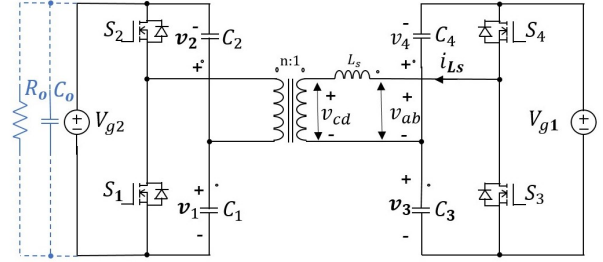


Fig. 1: DAHB converter topology

voltage applied to the inductor is no longer constant during each state of the circuit and conventional averaged models have failed to approximate DAHB dynamics [8]. Moreover, the waveforms in steady state are also difficult to calculate accurately, turning harder the design of the converter, especially if we use design methods based on exploring a wide design space with massive number of alternatives. This paper is based on the discrete time model explained in [9], since it allows the prediction of the quasi-exact value of currents and voltages at the sampling instant. On the one hand, this approach is used to derive a very accurate dynamic model of the DAHB considering any design combination, i.e., without considering any constraint about the component values to ensure a given condition (e.g. constant voltages across capacitors due to large enough values of capacitors). On the other hand, the variation of the sampling instant allows the calculation of the waveform of any electrical quantity in the circuit. As the formulation of the model is general and based on the stated space description of the converter at each switching state, the application of a different modulation technique is easy, since only the sequence of states along a switching cycle has to be considered. The results are an accurate dynamic model accounting for high frequency effects, and a static model to accurately reproduce the waveforms of the electrical quantities of the power converter.

II. DUAL ACTIVE HALF BRIDGE CONVERTER

As shown in Fig. 1 the DAHB converter comprises two half bridge circuits (input bridge S_1, S_2, C_1, C_2 , and output bridge S_3, S_4, C_3, C_4), a high frequency transformer and a linking power inductor L_s . In comparison to DAB, each

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transistor of one of the bridge legs is replaced by a capacitor, so the circuit design is simplified. In contrast, in the DAHB the split dc-link capacitors have to handle the full load current. Every half-bridge is driven at the same switching frequency. The converter can be controlled changing the duty cycle of each half bridge and the phase shift between both half bridge control signals. Therefore, one can apply similar modulation techniques than for the DAB: Single Phase Shift (SPS), Dual Phase Shift (DPS) and Triple Phase Shift (TPS), or 1-D, 2-D or 3-D control methods described in [10].

In these conditions, voltages v_{c1} , v_{c2} , v_{c3} and v_{c4} , can be considered square or quasi-square waves when the capacitance of C_1 , C_2 , C_3 , and C_4 are large enough to have a small ripple in the capacitor voltages. Fig. 2 shows the main waveforms of a DAHB converter operating with DPS (duty cycle of each half-bridge D_1 and D_2 are different). It can be seen that they are similar to those of the standard DAB converter.

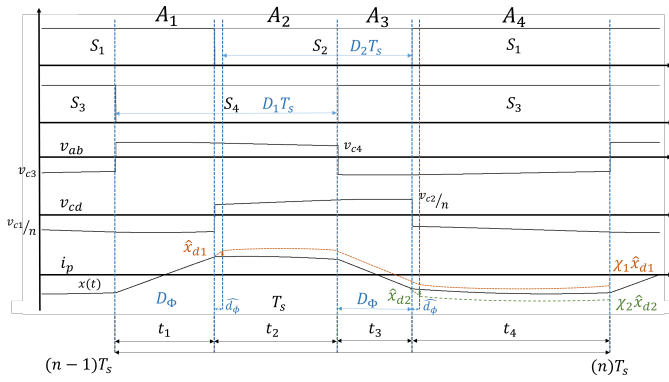


Fig. 2: DAHB waveforms and perturbation propagation

In particular, one can think on exploring the reduction of the capacitance of C_1 , ..., C_4 . By increasing the switching frequency and making these capacitors small, results are a very attractive option to improve power density and cost. Moreover, note that low value capacitors with low ESR and very high ripple currents can be found easily in the market.

The simplest way to regulate the power flow is using SPS, i.e., $D_1 = D_2 = 0.5$ and vary the phase shift between voltages v_{ab} and v_{cd} , called bridge voltages (1), resulting in the variation of the voltage applied across the inductor. The main advantage of this modulation is its simplicity. However, it suffers from higher level of reactive current that leads to higher conduction losses and possible lose of ZVS for light load or when the conversion ratio between the bridges is different to unity. As a consequence, the DAHB converter has several operating modes to drive the bridges. Non 0.5 duty control methods have been proposed for the DAHB converter to reduce the current stress and to extend the ZVS range [10], and the modelling method presented in this paper can be applied to such cases.

As the option of reducing capacitances is revealed as potentially advantageous for more efficient and less costly designs, models to predict accurately the dynamic and static behaviour of the converter are needed. When studying potential models

for such design conditions, different problems arise: a) the resonance between the capacitors C_1 , ..., C_4 and the link inductor L_s can modify the dynamic behaviour significantly, as shown for the DAB series resonant converter [11]; b) the steady state waveforms cannot be derived from square-wave voltages by using Reduced-Order Model; c) Generalized Average Model fails to predict the model accurately because only the first harmonic is included, and thus this model is accurate only if the resonance frequency is close to the switching frequency; d) including more harmonics to obtain more accurate models complicates the calculations as the number of state variables is high, especially if we consider higher degrees of freedom as TPS; e) an accurate prediction of ZVS or calculation of *rms* currents to optimize the design is more difficult.

III. DISCRETE TIME MODELING OF DAHB CONVERTER

Discrete time modelling [9] is an accurate, compact and design-oriented method that calculates the value of state variables of a DAHB converter at the sampling instant. Model can include high-frequency dynamics even when currents and voltages have a significant ripple, e.g., with small capacitance in C_1 , ..., C_4 . To proceed with the derivation of the discrete-time model, the continuous state-space equations for each sub-interval is derived in the form:

$$\dot{x}(t) = A_i x(t) + B_i V_g \quad (1)$$

We define state variable vector as:

$$x(t) = [i_{Ls} v_{C1} v_{C2} v_{C3} v_{C4} v_{Co}]^T \quad (2)$$

In this analysis and for the shake of simplicity, the following assumptions are made:

- all switches are ideal;
- the input voltage in Fig. 1 is constant $V_{g1} = V_g$;
- dead times are ignored which is a good approach justified in the literature when working at moderate switching frequencies and the energy of the series inductor is enough to make ZVS fast [12];
- SPS is considered being the control variable d_ϕ the phase-shift between both bridges and
- V_{g2} in Fig. 1 is replaced by a resistor R_o in parallel with capacitor C_o , being the voltage across these two paralleled elements v_{Co} , with the same polarity than V_{g2} in Fig. 1.

To illustrate the application of the discrete model, we focus on the control-to-output transfer function,

$$G_{vd}(z) = \frac{\hat{v}_{Co}}{\hat{d}_\phi}. \quad (3)$$

For simplicity, SPS modulation has been chosen to describe the transition matrices, but the discrete modeling method can be easily exportable to other types of modulation by studying its equivalent circuits and the order of its matrices. As shown in Fig. 2 the duty on both bridges is 0.5 so four switching states appear. The equivalent circuit at each sub-interval are shown in Fig. 3 and the state transition matrices describing

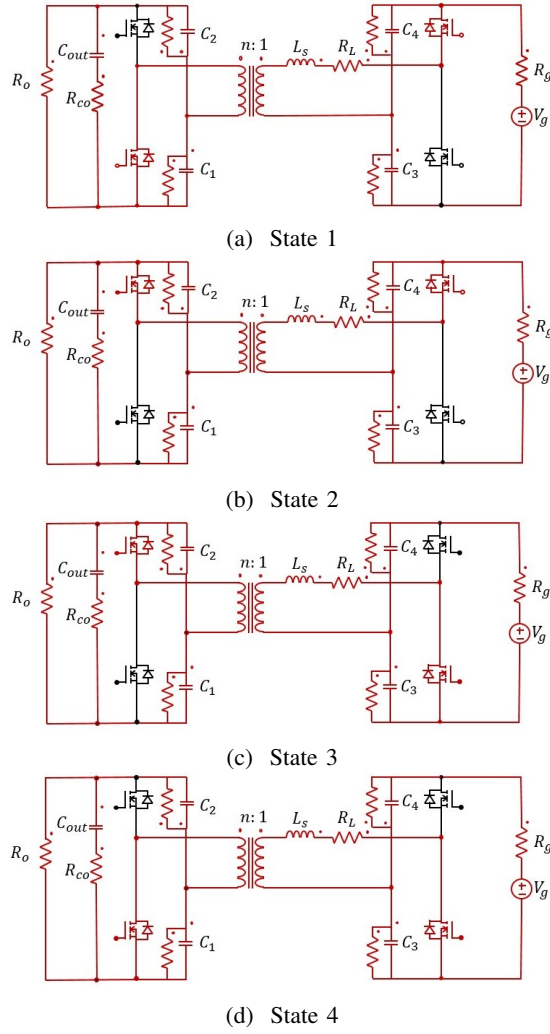


Fig. 3: DAHB Equivalent circuit for each sub-interval

the DAHB converter are shown in (4) - (11) where R_L in the parasitic resistance of L_s , R_i is the parallel parasitic resistance of C_i , R_o is the load resistor, R_g is the output impedance of voltage source V_{g1} and R_{Co} is the equivalent series resistance of C_{out} .

A. Dynamic model of DHAB converter

To obtain the small signal frequency response the state space equation must be in the form:

$$\begin{aligned}\hat{\mathbf{x}}[k] &= \Phi \hat{\mathbf{x}}[k-1] + \gamma \hat{\mathbf{u}}[k-1] \\ \hat{\mathbf{y}}[k-1] &= \delta \hat{\mathbf{x}}[k-1]\end{aligned}\quad (12)$$

Equation (12) includes the two fundamental matrices in the discrete model:

- the small-signal propagation state matrix Φ that can be deduced from the sequence of the states, as stated in (13);
- the input matrix γ that is obtained from Fig. 2 as the propagation of the perturbation through the states, as stated in (15).

Note that the control variable is $\mathbf{u} = d_\phi$

$$\Phi = e^{A_4 t_4} e^{A_3 t_3} e^{A_2 t_2} e^{A_1 t_1} \quad (13)$$

$$\begin{aligned}\chi_1 &= e^{A_4 t_4} e^{A_3 t_3} e^{A_2 t_2} \\ \chi_2 &= e^{A_4 t_4}\end{aligned}\quad (14)$$

$$\begin{aligned}\hat{x}_{d1} &= (A_1 - A_2)X_{Down2} \hat{d}_\phi T_s + (B_1 - B_2)V_g \hat{d}_\phi T_s \\ \hat{x}_{d2} &= (A_3 - A_4)X_{Down4} \hat{d}_\phi T_s + (B_3 - B_4)V_g \hat{d}_\phi T_s \\ \gamma &= (e^{A_4 t_4} e^{A_3 t_3} e^{A_2 t_2} ((A_1 - A_2)X_{Down2} + (B_1 - B_2)V_g) + \\ &\quad + e^{A_4 t_4} ((A_3 - A_4)X_{Down4} + (B_3 - B_4)V_g))T_s\end{aligned}\quad (15)$$

To obtain the small-signal control-to-output transfer matrix, the matrix δ must be also considered as the output voltage is the sum of the two capacitors C_1 and C_2 . We define δ as (16):

$$\delta = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad (16)$$

Combining equations (13) and (15), the control-to-output transfer function is in the form:

$$\mathbf{W}(z) = \delta(zI - \Phi)^{-1} \gamma \quad (17)$$

The extension to other modulation approaches, such as DPS or TPS (or 2-D and 3-D in [10]) can be easily obtained by recalculating Φ and γ matrices considering the corresponding state sequence.

B. Steady state model of DHAB converter: exact waveforms calculation

As shown in [13], it is possible to obtain the static value of any state variable at the sampling instant. Based on the state equation (1) and assuming that all independent inputs are constant any switching interval can be obtained (18). By solving (18) over the set of n linear-equivalent circuits within one switching period, the periodic steady-state solution ($x(0) = x(T_s)$) is given in (19). Adding a new state representing a small time-step, we can obtain the static value of any state variable at any time instant.

However, there are some limitations of discrete time state-space modeling via (19). That is, all inputs have to be constant during each switching interval, the matrix A_i must be invertible during each switching interval and the duration of each switching interval must be known. In DAHB matrices A_i are not invertible, so high ohmic resistors R_i are placed in parallel to make the result converge and obtain an invertible matrix. This method of obtaining the time variables is especially useful when obtaining the current and voltage expressions, which is complicated by the appearance of sub-oscillations due to converter resonances.

$$a_1 = \begin{bmatrix} -R_L & 1/n & 0 & 0 & -1 & 0 \\ -1/n & -1/R_1 & -1/R_2 & 0 & 0 & 0 \\ 0 & 0 & 1/R_2 & 0 & 0 & -1/R_o \\ 1 & 0 & 0 & -1/R_3 & -1/R_4 & 0 \\ R_g & 0 & 0 & 1 & -1-R_g/R_4 & 0 \\ 0 & 1 & -1 & 0 & 0 & -1 \end{bmatrix}; a_{01} = \begin{bmatrix} L_{lk} & 0 & 0 & 0 & 0 & 0 \\ 0 & C_1 & C_2 & 0 & 0 & 0 \\ 0 & 0 & -C_2 & 0 & 0 & C_{out}*(1+\frac{R_{Co}}{R_o}) \\ 0 & 0 & 0 & C_3 & C_4 & 0 \\ 0 & 0 & 0 & 0 & R_g*C_4 & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{out}*R_{Co} \end{bmatrix} \quad (4)$$

$$b_1 = [0 \ 0 \ 0 \ 0 \ -1 \ 0]^T; A_1 = a_{01}^{-1}a_1; B_1 = a_{01}^{-1}b_1 \quad (5)$$

$$a_2 = \begin{bmatrix} -R_L & 0 & 1/n & 0 & -1 & 0 \\ -1/n & -1/R_1 & -1/R_2 & 0 & 0 & 0 \\ 1/n & 0 & 1/R_2 & 0 & 0 & -1/R_o \\ 1 & 0 & 0 & -1/R_3 & -1/R_4 & 0 \\ R_g & 0 & 0 & 1 & -1-R_g/R_4 & 0 \\ 0 & 1 & -1 & 0 & 0 & -1 \end{bmatrix}; a_{02} = \begin{bmatrix} L_{lk} & 0 & 0 & 0 & 0 & 0 \\ 0 & C_1 & C_2 & 0 & 0 & 0 \\ 0 & 0 & -C_2 & 0 & 0 & C_{out}*(1+\frac{R_{Co}}{R_o}) \\ 0 & 0 & 0 & C_3 & C_4 & 0 \\ 0 & 0 & 0 & 0 & R_g*C_4 & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{out}*R_{Co} \end{bmatrix} \quad (6)$$

$$b_2 = [0 \ 0 \ 0 \ 0 \ -1 \ 0]^T; A_2 = a_{02}^{-1}a_2; B_2 = a_{02}^{-1}b_2 \quad (7)$$

$$a_3 = \begin{bmatrix} -R_L & 0 & 1/n & -1 & 0 & 0 \\ -1/n & -1/R_1 & -1/R_2 & 0 & 0 & 0 \\ 1/n & 0 & 1/R_2 & 0 & 0 & -1/R_o \\ 1 & 0 & 0 & -1/R_3 & -1/R_4 & 0 \\ 0 & 0 & 0 & 1 & -1-R_g/R_4 & 0 \\ 0 & 1 & -1 & 0 & 0 & -1 \end{bmatrix}; a_{03} = \begin{bmatrix} L_{lk} & 0 & 0 & 0 & 0 & 0 \\ 0 & C_1 & C_2 & 0 & 0 & 0 \\ 0 & 0 & -C_2 & 0 & 0 & C_{out}*(1+\frac{R_{Co}}{R_o}) \\ 0 & 0 & 0 & C_3 & C_4 & 0 \\ 0 & 0 & 0 & 0 & R_g*C_4 & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{out}*R_{Co} \end{bmatrix} \quad (8)$$

$$b_3 = [0 \ 0 \ 0 \ 0 \ -1 \ 0]^T; A_3 = a_{03}^{-1}a_3; B_3 = a_{03}^{-1}b_3 \quad (9)$$

$$a_4 = \begin{bmatrix} -R_L & 1/n & 0 & -1 & 0 & 0 \\ -1/n & -1/R_1 & -1/R_2 & 0 & 0 & 0 \\ 0 & 0 & 1/R_2 & 0 & 0 & -1/R_o \\ 1 & 0 & 0 & -1/R_3 & -1/R_4 & 0 \\ 0 & 0 & 0 & 1 & -1-R_g/R_4 & 0 \\ 0 & 1 & -1 & 0 & 0 & -1 \end{bmatrix}; a_{04} = \begin{bmatrix} L_{lk} & 0 & 0 & 0 & 0 & 0 \\ 0 & C_1 & C_2 & 0 & 0 & 0 \\ 0 & 0 & -C_2 & 0 & 0 & C_{out}*(1+\frac{R_{Co}}{R_o}) \\ 0 & 0 & 0 & C_3 & C_4 & 0 \\ 0 & 0 & 0 & 0 & R_g*C_4 & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{out}*R_{Co} \end{bmatrix} \quad (10)$$

$$b_4 = [0 \ 0 \ 0 \ 0 \ -1 \ 0]^T; A_4 = a_{04}^{-1}a_4; B_4 = a_{04}^{-1}b_4 \quad (11)$$

The described straightforward calculation method of steady state waveforms can be potentially applied to design methods based on the exploration of a wide design space. RMS currents, maximum voltages, and other singular values of the electrical quantities in every element of the converter can be numerically calculated applying (19). The number of points per period should be considered carefully to avoid a heavy computational load.

F_s	V_g	V_{out}	P_{out}	n	C_{1-2}	C_{3-4}	R_{Co}
500kHz	540V	270V	3100W	1:2	100nF	220nF	3mΩ
R_L	C_{out}	$ESR_{C_{out}}$	R_g	R_{1-4}	$Duty_\phi$	$Duty_1$	$Duty_2$
50mΩ	10μF	50mΩ	100mΩ	100kΩ	0.2	0.45	0.45

TABLE I: Converter parameters

$$x(t) = e^{A_i t} x(0) + A_i^{-1} [e^{A_i t} - \mathbf{I}] B_i u \quad (18)$$

$$\mathbf{X}_{ss} = \left[\mathbf{I} - \prod_{i=1}^n e^{A_i t_i} \right]^{-1} \sum_{i=1}^n \left[\left(\prod_{k=i+1}^n e^{A_k t_k} \right) A_i^{-1} [e^{A_i t_i} - \mathbf{I}] B_i u \right] \quad (19)$$

IV. MODEL VALIDATION

The result of the obtained discrete model, the comparison of the averaged model and the verification through the PSIM simulation tool are shown in Fig. 4a with the converter parameters of table I. It can be seen how the average reduced order methods do not predict well the resonance that appears, nor the gain at low frequencies.

In Fig. 4b the frequency response is shown keeping the same parameters as in the Table I, but the capacitance of $C_1 - C_4$ is increased so the bridge voltages can be considered square. It can be seen how at low frequencies the gain is correctly predicted, but we still do not observe the resonance that could generate stability problems due to the phase shift it introduces.

Fig. 5 shows a load step from 2400 W to 2700 W in the considered converter working in closed loop (cross over frequency $f_c = 2.2\text{kHz}$, phase margin $PM = 125^\circ$), using as the plant the transfer function $G_{vd}(z) = \frac{v_{\hat{C}_o}}{\hat{d}}$ calculated with the proposed method. Results have been obtained with PSIM, matching the expected behaviour for the closed loop system.

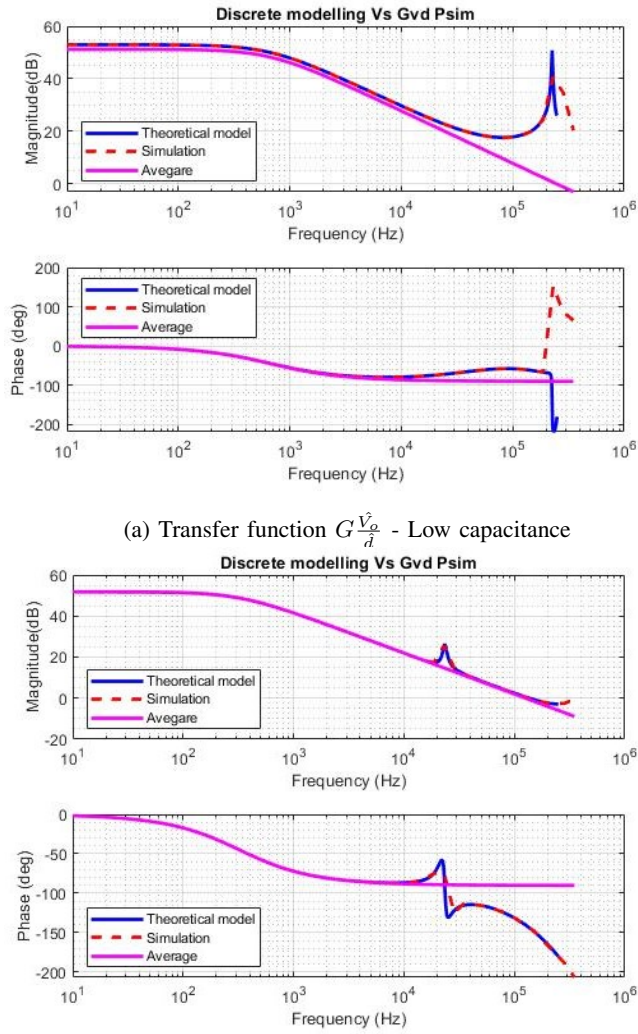


Fig. 4: Transfer function comparison between theory and simulation

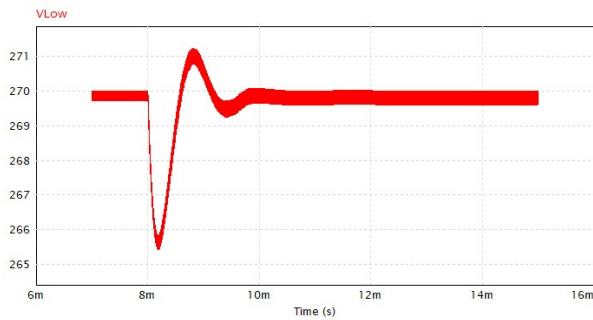


Fig. 5: DAHB Load step

The steady state value of the state variables in the time domain and electrical quantities in the rest of the circuit components are calculated through the exact waveforms calculation method. Fig. 6 and Fig. 7 show these variables calculated and

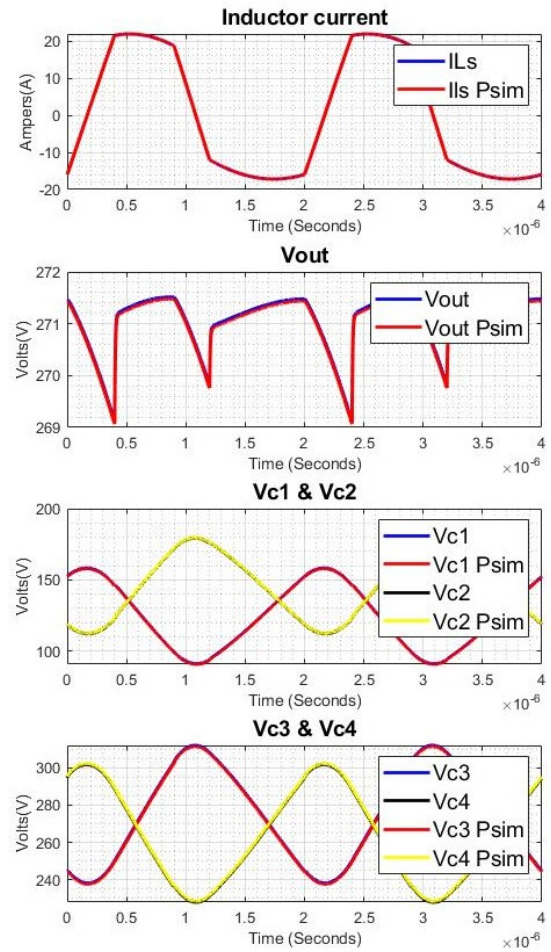


Fig. 6: Steady State waveforms comparison propagation

simulated with PSIM. As a result of this method, the RMS currents through transistors and capacitors are calculated in order to predict losses; the volt-time relationships applied to the transformer and inductor are calculated as well, in order to design the magnetic components. This method also brings the possibility to determine the ZVS limits. Therefore this method is a design-oriented modelling technique suitable to make optimal design of the converter.

V. CONCLUSION

In this work, the discrete time model of DAHB has been obtained including parasitic resistances, focusing on low capacitance values.

The dynamic model predicts accurately the effect of the peak resonance between the bridge capacitors and the series inductance. It results in an increase in magnitude and a phase lag, which is important for optimal control of the converter.

Moreover, the modelling method presented allows obtaining the converter waveforms and their static values. In this way it is possible to make an optimal design of the converter also in steady state. The results have been verified by simulation and compared with those obtained by averaging methods.

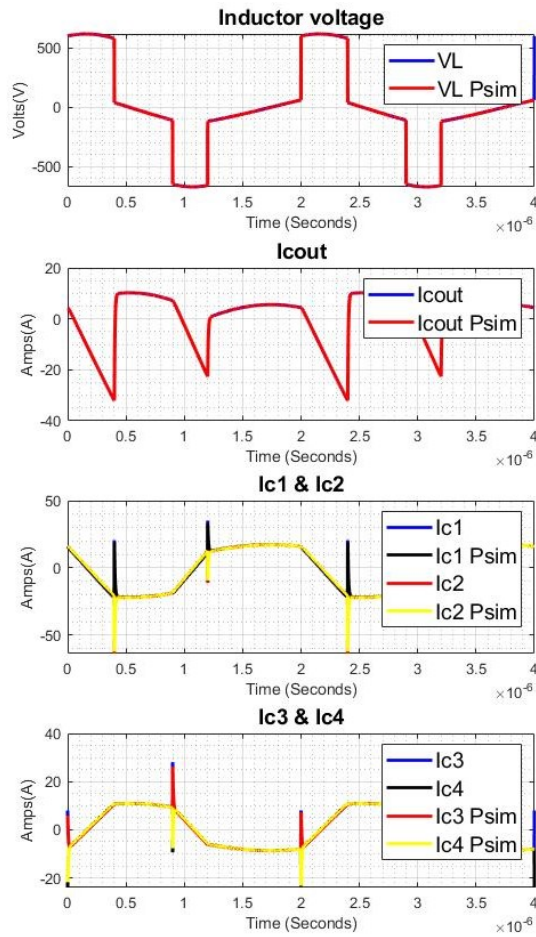


Fig. 7: Steady State waveforms comparison propagation (II)

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