Analytical Hierarchy Process

Introduction

The Analytic Hierarchy Process (AHP) is a multi-criteria decision-making approach in problem solving. The AHP process provides a logical framework to determine the benefits of different alternatives in a decision-making process. In simpler terms, it helps showcase the advantages of each of the choices on a menu. This approach can be quite useful in making business and corporate decisions, government decisions, and even decisions while creating a mathematical model.

How It Works

AHP problems are decomposed into a hierarchy of criteria and alternatives.

To more easily understand AHP, here's a real-world example:

You want to purchase a new car, but you don't know which brand or model of car you want. You'd probably list out all the factors that you'd consider in making your final pick of a car, and rank these factors by their importance to you.

Theres 3 steps that are essential in the AHP process:

- State the objective
    - In this case: Select a new car
- Define the Criteria
    - In this case, say you have these criteria as your top 3 most important: appearance, endurance, and fuel economy
- List the options
    - In this case: You have a few options: Toyota Prius, Honda Accord, Ford Escape, and Nissan Leaf

The next step, is probably one of the most crucial parts of the AHP. You have to use your judgement to determine the ranking of the criteria.

- Suppose this is how you ranked your criteria (hypothetical)
    - Endurance is 2 times as important as looks
    - Looks is 3 times as important as fuel economy
    - Endurance is 4 times as important as fuel economy

Make a matrix with the ratios of the rankings in importance:

MATRIX 1

		Looks	*Endurance*	*F.Economy*	
Looks	[	1/1	1/2	3/1	]
Endurance	[	2/1	1/1	4/1	]
F. Economy	[	1/3	1/4	1/1	]

Convert these ratios to decimal values.

MATRIX 2

		Looks	*Endurance*	*F. conomy*	
Looks	[	1.0000	0.5000	3.0000	]
Endurance	[	2.0000	1.0000	4.0000	]
F. Economy	[	0.3333	0.2500	1.0000	]

Now begins the process of **normalization.**

Normalization consists of finding the sum of each row, then dividing each sum by the sum of all the rows.

You have to square the matrix (matrix multiplication) to get :

MATRIX 3

		Looks	*Endurance*	*F. Economy*		*Row Sum*
Looks	[	3.0000	1.7500	8.0000	]	12.7500
Endurance	[	5.3332	3.0000	14.0000	]	22.3332
F. Economy	[	1.1666	0.6667	3.0000	]	4.8333

Row Total **39.9165**

Next, we have to ***normalize each row sum*** by dividing it by the row total to get :

12.7500 / 39.9165 = 0.3194

22.3332 / 39.9165 = 0.5595

4.8333 / 39.9165 = 0.1211

Normalized Sum **1.0000**

This process must be repeated until finally the normalized values are very close to the previous iterations.

We will normalize the matrix again.

Squaring MATRIX 3 (*the matrix after the first normalization)* then normalizing it again gives us the following normalized row sums:

[0.3196]

[0.5584] Since the normalized matrix in this iteration is extremely similar to the

[0.1220] normalized matrix in the previous iteration, we can determine the

relative ranking of the criterion.

[1.0000]

Now, associate each matrix row back to its respective criteria.

Looks → [0.3196] Endurance → [0.5584] Fuel Efficiency → [0.1220]

From this we can determine that Endurance is #1, Looks is #2, and Fuel Efficiency is #3.

Then we can just determine which car we want based on our most important criteria.

In terms of each of our criteria, we need to make pairwise comparisons

Looks :

	Prius	Accord	Escape	Leaf
Prius	[1/1	1/4	4/1	1/6]
Accord	[4/1	1/1	4/1	1/4]
Escape	[1/4	1/4	1/1	1/5]
Leaf	[6/1	4/1	5/1	1/1]

Endurance:

	Prius	Accord	Escape	Leaf
Prius	[1/1	2/1	5/1	1/1]
Accord	[1/2	1/1	3/1	2/1]
Escape	[1/5	1/3	1/1	1/4]
Leaf	[1/1	1/2	4/1	1/1]

Fuel Economy:

	Prius	Accord	Escape	Leaf
Prius	[1/1	2/1	5/1	1/1]
Accord	[1/2	1/1	3/1	2/1]
Escape	[1/5	1/3	1/1	1/4]
Leaf	[1/1	1/2	4/1	1/1]

Then, take the normalized matrix sums:

Looks		Endurance	
3	Prius [.1160]	1	Prius [.3790]
2	Accord [.2470]	2	Accord [.2900]
4	Escape [.0600]	4	Escape [.0740]
1	Leaf [.5770]	3	Leaf [.2570]

For fuel economy, we'll use online data and normalize that.

Prius	34 mpg	: 34/113	= .3010
Accord	27 mpg	: 27/113	= .2390
Escape	24 mpg	: 24/113	= .2120
Leaf	28 mpg	: 28/113	= .2480
	113		1.000

To Obtain The Answer

Multiply:

	CAR/CRITERIA MATRIX			×	**CRITERIA RANKINGS**	
	Looks	Endurance	Fuel Economy		Criteria Ranking	
Prius	[.1160	.3790	.3010]		[0.3196]	Looks
Accord	[.2470	.2900	.2390]	•	[0.5584]	Endurance
Escape	[.0600	.2120	.2120]		[0.1220]	F. Economy
Leaf	[.5770	.2480	.2480]			

=

Prius [.3060]

Accord [.2720]

Escape [.0940]

Leaf [.3280] -----> **HIGHEST RANKED car is the <u>LEAF</u>**

Further Applications

AHP is applicable to all sorts of inquiries and problems. From small scale things such as deciding which car to buy (as demonstrated above), choosing a college to study into preparing for natural disasters, AHP is versatile and flexible to meet the needs of any type of problem.

Sample Application #1 -- Earthquakes

We need to figure out how to minimize the impact of an earthquake on populated areas. Factors to consider in the AHP would include city prioritization, population density, gross domestic product, and of course, proximity to the earthquake's epicenter. By obtaining pairwise comparison index from experts of this field, we would be able to determine what cities should be prioritized, and thus, determine the distribution of resources.

Sample Application #2 -- University Faculty

Adapted from Bloomberg UPenn

We need to know how to select university faculty. Factors to consider in the AHP would include the graduate school each faculty came from, their numbers (GPA, entrance exam scores), research conducted, appropriate internships held, volunteer service, etc.

Sample Application #3 -- Selecting Offshore Manufacturing Plants

Adapted from University of Cambridge

We need to know where to locate offshore manufacturing plants. Factors to consider in the AHP would include how many manufacturing plants there are, how to evenly distribute them so that every location nearby has near access to the manufacturing plant, etc.

Sample Application #4: Choosing a Car

Adapted from Wikipedia

Some factors that we would want to consider when choosing a car, initially: cost, safety, analysis and capacity. The above are called **features.** However, we must dig a bit deeper because we cannot just look at the features above and make a choice. We must now look at **subfeatures** under each of the four features. More specifically, under cost, we must look at the purchase price, fuel costs, maintenance costs, resale value, etc. Under safety and style there are not many subfeatures there are no real subfeatures we can establish. Most of the work for determining these two categories will be based more on subjectivity vs. objectivity (e.g. one car may be safe because it has an airbag specific for infants in a car crash while one car may have some other feature which makes it safe, but how do we get past a point where we can decide which one is better?).