Name:Solution Manual Class quiz 4

I) Find all Horizontal Asymptotes of the following functions.

a- $f(x) = \dfrac{4x^3 + 3x^2 + 15}{x^3 - 1}$

$$\lim_{x\to\pm\infty} f(x) = \lim_{x\to\pm\infty} \frac{\frac{4x^3}{x^3} + \frac{3x^2}{x^3} + \frac{15}{x^3}}{\frac{x^3}{x^3} - \frac{1}{x^3}} = \lim_{x\to\pm\infty} \frac{4 + \frac{3}{x} + \frac{15}{x^3}}{1 - \frac{1}{x^3}} = 4$$

So $y = 4$ is a Horizontal Asy. at $\pm\infty$.

b- $f(x) = \dfrac{e^{-x} - 5}{3e^{-x} + 2}$

$$\lim_{x\to-\infty} f(x) = \lim_{x\to-\infty} \frac{e^{-x} - 5}{3e^{-x} + 2} = \lim_{x\to-\infty} \frac{e^{-x}(1 - \frac{5}{e^{-x}})}{e^{-x}(3 + \frac{2}{e^{-x}})} = \lim_{x\to-\infty} \frac{(1 - \frac{5}{e^{-x}})}{(3 + \frac{2}{e^{-x}})} = \frac{1}{3}$$

So: $y = \frac{1}{3}$ is Horizontal Asy at $-\infty$

$$\lim_{x\to+\infty} f = \lim_{x\to+\infty} \frac{e^{-x} - 5}{3e^{-x} + 2} = \frac{0 - 5}{3 \cdot 0 + 2} = \frac{-5}{2}$$

So: $y = \frac{-5}{2}$ is Horizontal Asy at $+\infty$

II) Consider the following function:

$$f = \begin{cases} x^2 - 9 & if\ x < 0 \\ 14 & if\ x = 0 \\ -9 \cdot \cos(x) & if\ x > 0 \end{cases}$$

a- What kind of discontinuity does f has at zero. Justify your answer.

We have a removable discontinuity at zero Because the limit from the left and the limit from the right at zero are equal indeed :

$$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} x^2 - 9 = -9$$
$$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} -9cos(x) = -9$$

b- Can you redefine f to be continuous at zero.

Yes we can remove the dis-continuity by redefining f at zero to be -9 as follows:

$$f = \begin{cases} x^2 - 9 & if\ x < 0 \\ -9 & if\ x = 0 \\ -9 \cdot \cos(x) & if\ x > 0 \end{cases}$$

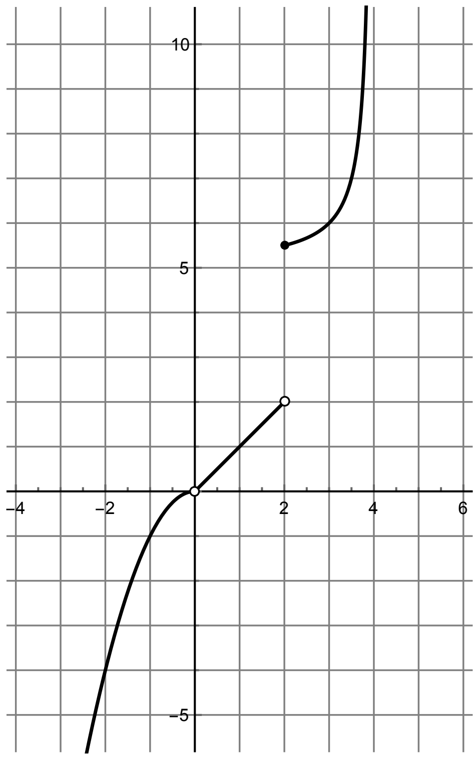
III) Draw a graph of a function f on [-2,4) that have:

a- Jump discontinuity at 2, and $\lim_{x\to 2^-} f = 2$.

b- Infinite discontinuity at 4.

c- Removable discontinuity at 0, and $\lim_{x\to 0^+} f = 0$.

There is no one right answer, but here is an example



IV) Consider the following function: $f(x) = 3x^2 + 2x$. Use the Intermediate value theorem to prove that $f(x) = \pi$ has at least one solution. (Show all your work).

We have to find two values for x , x_1 and x_2 such that $f(x_1) < \pi$ and $f(x_2) > \pi$.

So we choose $x_1 = 0$ then $f(x_1) = 0 < \pi$

And choose $x_2 = 1$ then $f(x_1) = 5 > \pi \approx 3.14$

So by the intermediate value theorem since $0 = f(0) < \pi < f(1) = 5$ and since f is continuous on $(0,1)$ then there exist c in the interval $(0,1)$ such that $f(c) = \pi$