Name: Solution Manual Class quiz 3

I) Define the following statements:

a- The function f is continuous at the point $x = a$.

The function f is continuous on a point a, if : $\lim_{x\to a^+} f(x) = \lim_{x\to a^-} f(x) = f(a)$

b- The limit of the function g exists at the point $x = b$.

The function f has a limit at a point a, if : $\lim_{x\to a^+} f(x) = \lim_{x\to a^-} f(x)$

II) Find the following limits: (show all your work)

a- $\lim_{x\to 2} \dfrac{x^2-4}{x-2}$

$$\lim_{x\to 2} \frac{x^2-4}{x-2} = \lim_{x\to 2} \frac{(x-2)(x+2)}{x-2} = \lim_{x\to 2} \frac{x+2}{1} = 4$$

b- $\lim_{x\to 3^+} \left(\dfrac{15}{|x-3|} - \dfrac{15}{x-3}\right)$

$$\lim_{x\to 3^+} \left(\frac{15}{|x-3|} - \frac{15}{x-3}\right) = \lim_{x\to 3^+} \left(\frac{15}{x-3} - \frac{15}{x-3}\right) = \lim_{x\to 3^+} \quad 0 = 0$$

c- $\lim_{x\to -1} \dfrac{\sqrt{x^2-3x}+2x}{x+1}$

$$\lim_{x\to -1} \frac{\sqrt{x^2-3x}+2x}{x+1} = \lim_{x\to -1} \frac{\sqrt{x^2-3x}+2x}{x+1} \cdot \frac{\sqrt{x^2-3x}-2x}{\sqrt{x^2-3x}-2x} = \lim_{x\to -1} \frac{x^2-3x-4x^2}{(x+1)(\sqrt{x^2-3x}-2x)} =$$
$$\lim_{x\to -1} \frac{-3x^2-3x}{(x+1)(\sqrt{x^2-3x}-2x)} = \lim_{x\to -1} \frac{-3x(x+1)}{(x+1)(\sqrt{x^2-3x}-2x)} = \lim_{x\to -1} \frac{-3x}{\sqrt{x^2-3x}-2x} = \frac{3}{4}$$

III) Consider the following function:

$$f = \begin{cases} x^2-9 & if\ x<0 \\ 14 & if\ x=0 \\ -9\cdot\cos(x) & if\ x>0. \end{cases}$$

a- Find the limit: $\lim_{x\to 0^-} f(x)$

$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} x^2-9 = -9$

b- Find the limit: $\lim_{x\to 0^+} f(x)$

$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} -9\cdot\cos(x) = -9$

c- Find: $f(0) =$

$f(0) = 14$.

d- does the limit at 0 exist? Explain why.

The limit at 0 exists because the limit from the left and the right are equal.

e- Is f continuous at 0? Explain why.

The function f is not continuous at 0, because the $\lim_{x\to 0} f(x) \neq f(0)$