Name: Class quiz 8 October 31, 2016

I) Find the derivatives of the following functions:

a- $f(x) = \ln(\sin(\ln(x)))$

$$f'(x) = \frac{\cos(\ln x)\left(\frac{1}{x}\right)}{\sin(\ln x)}$$

b- $y = (x+1)^{\tan(x)}$ (Use logarithmic differentiation)

Take logarithm of both sides to get: $\ln y = \tan(x) \ln\left(x+1\right)$

Then we differentiate both sides and get: $\frac{y'}{y} = \sec^2(x) \ln\left(x+1\right) + \frac{1}{x+1} \tan(x)$

Finally : $y' = y\Big(\sec^2(x) \ln\left(x+1\right) + \frac{1}{x+1} \tan(x)\Big) = (x+1)^{\tan(x)}\Big(\sec^2(x) \ln\left(x+1\right) + \frac{1}{x+1} \tan(x)\Big)$

II) Find $\frac{d}{dx}y$.

$y + 3\ln(x^2+5) = \cos(xy^2)$

We start by differentiating both sides to get: $y' + 3\frac{2x}{x^2+5} = -\sin(xy^2)\left(y^2 + 2yy'x\right)$

Then we try to move y' into one side: $y' + \sin(xy^2)2yy'x = -3\frac{2x}{x^2+5} - \sin(xy^2)y^2$

factor out y': $y'\left(1 + \sin(xy^2)2yx\right) = -3\frac{2x}{x^2+5} - \sin(xy^2)y^2$

Finally $y' = \dfrac{-3\frac{2x}{x^2+5} - \sin(xy^2)y^2}{1 + \sin(xy^2)2yx}$

IV) Consider the following equation of motion of a particle (P) : $f(t) = t^3 - 6t^2 + 9t + 1$ in feet per seconds.

a- Find the velocity $v(t)$

$v(t) = 3t^2 - 12t + 9 = 3(t^2 - 4t + 3)$.

b- Find $v(2)$

v(2)=3(4-8+3)=-3 ft/sec.

c- Find the acceleration $a(t)$

a(t)=6t-12.

d- When is the particle at rest.

The particle is at rest when $v(t) = 0$:

$3(t^2 - 4t + 3) = 3(t-3)(t-1) = 0$

So $t = 1$ or $t = 3$

e- When is the particle moving forward.

The particle is moving forward when the velocity $(v(t))$ is positive:

you draw the table and you will get : $v(t)$ is positive when $0 < t < 1$ or $t > 3$'.

In integral notation this would be: $(0, 1) \cup (3, \infty)$.

f- Is the particle speeding up or slowing down at $t = 2$ sec.

$v(2) = -3$ but $a(2) = 0$. So the particle is neither slowing down nor speeding up at $t = 2$.

g- What is the total distance travelled between $t = 0$ and $t = 4$.

The total distance travelled between $t = 0$ and $t = 4$ is:

$|f(1) - f(0)| + |f(3) - f(1)| + |f(4) - f(3)| = \Big(f(1) - f(0)\Big) + \Big(f(1) - f(3)\Big) + \Big(f(4) - f(3)\Big)$

You have to complete this calculation!!