Name: Final Review quiz December 9, 2016

I) Find the following limits:

a) $\lim_{x\to\infty} \arctan\big(-x^2+\ln(\frac{1}{x})\big) =$

b) $\lim_{x\to\infty} \dfrac{e^{2x}+e^x}{3e^{2x}-e^x} =$

c) $\lim_{x\to 1^-} \dfrac{x^2}{\ln(x)} =$

d) $\lim_{x\to 3^+} \dfrac{9-x^2}{|3-x|} =$

e) $\lim_{x\to 0} \dfrac{\ln(x+1)}{\sin(x)} =$

f) $\lim_{x\to 3^+} \dfrac{x^2+12}{x-3} =$

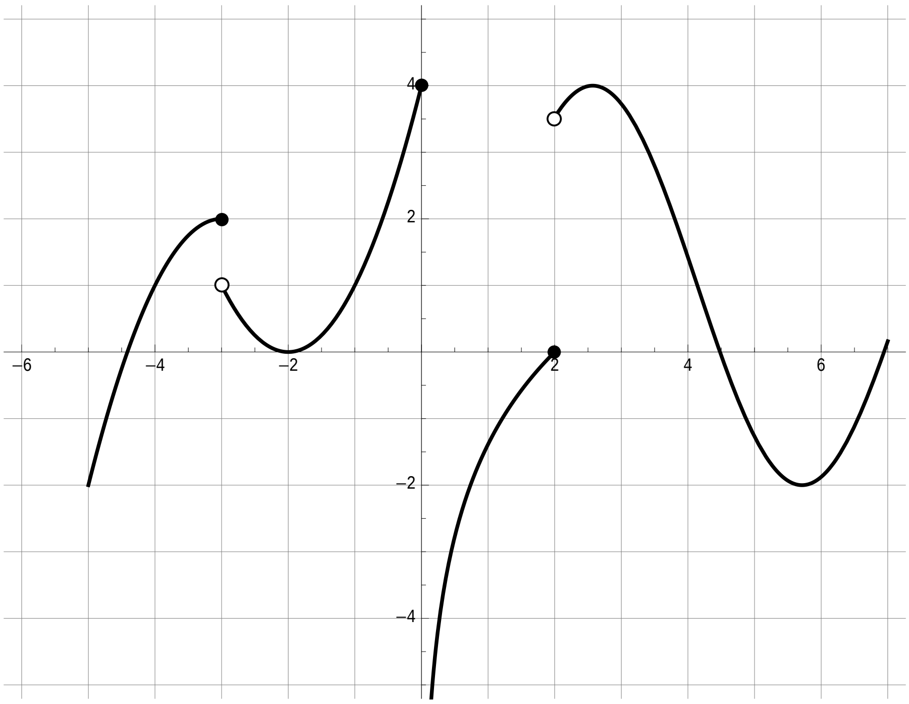
g) $\lim_{x\to 3} \dfrac{\sqrt{h+6}-3}{h-3} =$

II) Draw a graph of a function f on $[-5, 5]$ such that the following hold:

- f has a removable discontinuity on -1, and $f(-1) = 1$.
- f is increasing on $(-5, -3)$.
- f has a jump discontinuity at 2.
- f is continuous but not differentiable at 0.
- $f'(x) < 0$ on (3,5).
- $f''(x) > 0$ on (4,5).

III) Redefine the function: $f(x) = \dfrac{x^2 - 16}{x + 4}$ to make it continuous everywhere.

IV) Use the given graph of f to answer the following questions:



a) find the following limits:

$\lim_{x\to -4^+} f(x) =$ $\qquad\qquad$ $\lim_{x\to 2^-} f(x) =$

$\lim_{x\to 0^+} f(x) =$ $\qquad\qquad$ $\lim_{x\to -4^-} \left(f(x) + (x^2 - 1) - \sqrt{f(x) + 2}\right) =$

b) State the vertical asymptote for $f(x)$.

c) State the values x at which f has a local Minimum.

V) Find the horizontal and vertical asymptotes of the function: $f(x) = \dfrac{3e^x + x}{e^x - 2}$.

VI) i) Use the definition of derivatives to find the derivative of the function $f(x) = \sqrt{x+1}$ at $a = 3$.

ii) Use part (i) to find the equation of the tangent line to the graph of $f(x)$ at $a = 3$.

VII) The following limit represent the derivative of a function f at a value a. State what function f and value a.

$$\lim_{h \to 0} \frac{e^{-5+h} - e^{-5}}{h}$$

VIII) Find the derivatives of the following functions:

i) $f(t) = \dfrac{t^3 + 3t^2 + t - 1}{\sqrt[3]{t}}$

$f'(t) =$

ii) $g(x) = \dfrac{\sin(x) + x^2}{x + 1}$

$g'(x) =$

iii) $h(\theta) = \tan^2(\theta)$
$h'(\theta) =$

iv) $l(r) = e^{r\cos(r)}$
$l'(r) =$

v) $k(s) = s^2 \arcsin(s^2 + 1)$
$k'(r) =$

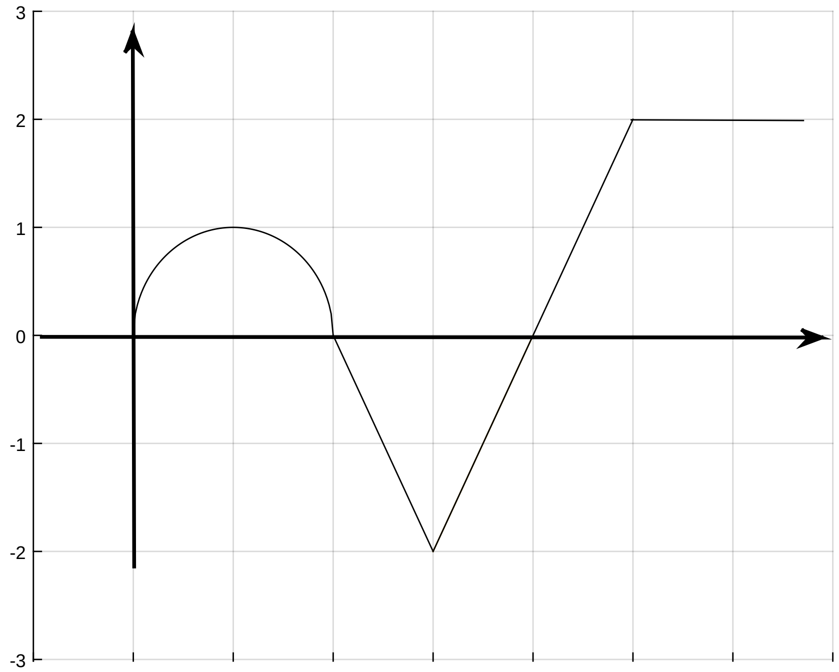
IX) Prove that the function $f(x) = 2e^{x^2-4} + 1$ satisfies the conditions of Rolle's theorem on the interval $[-2, 2]$, then find the value c in the conclusion of the theorem.

X) Find the equation of the tangent line of the function $f(x) = \ln(2x^2 + x - 1)$ at the value $a = 2$.

XI) A box with a square base and open top must have a volume of 32,000 cm^3. Find the dimensions of the box that minimize the amount of material used.

XII) Find f if $f'(x) = x^2 + 3\sqrt{x}$ and $f(0) = 1$.

XIII) Use the following graph to calculate the given integrals:



i) $\displaystyle\int_0^1 f(x)dx =$

ii) $\displaystyle\int_0^3 f(x)dx =$

iii) $\displaystyle\int_2^6 f(x)dx =$

iv) $\displaystyle\int_5^6 \left(f(x) - x^2 + 3\right) dx$

XIV) Find the area enclosed by:

$y = \sin(x) + 1 \quad , \quad y = 0 \quad$ and $\quad$ for x between 0 and $\frac{\pi}{2}$.

XV) Find the following integrals:

a) $\displaystyle\int \cos^5(\theta)\sin(\theta)d\theta =$

b) $\displaystyle\int \frac{e^x}{1+e^{2x}}dx$

c) $\displaystyle\int_0^1 e^{e^x}e^x dx$

d) $\displaystyle\int \frac{(\arctan x)^2}{1+x^2}dx$

e) $\int_{\frac{-1}{2}}^{7} \frac{1}{\sqrt[4]{2+2t}} dt$

XVI) Use the following graph of the **derivative** of f to answer the following questions:

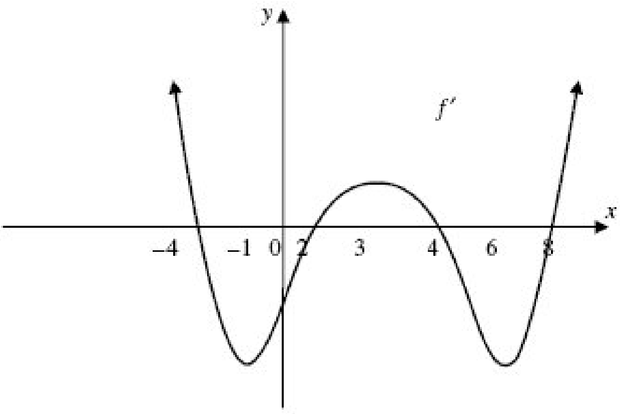


Figure 7.4-4

a) For what values of x is the function f increasing.

b) For what values of x is the fuction f concave down.

c) Where does f has local maximums.