Math 311
Spring 2018

Dr. Hussein Awala

Day #3 Notes: The Axiom of Completeness

January 24, 2018

Contents

1 The Triangle Inequality

Complete today's worksheet in groups and we will go over the answers.

1. [T/F] For every pair of real numbers a and b, $|a+b| \leq |a| + |b|$.

2. [T/F] For every pair of real numbers a and b, $|a+b| \leq |a| - |b|$.

3. [T/F] For every pair of real numbers a and b, $|a-b| \leq |a| + |b|$.

4. [T/F] For every pair of real numbers a and b, $|a-b| \leq |a| - |b|$.

5. [T/F] Given $a, b \in \mathbb{R}$, we have $a = b$ if for every $\epsilon > 0$, it follows that $|a - b| < \epsilon$.

6. [T/F] Given $a, b \in \mathbb{R}$, we have $a = b$ only if for every $\epsilon > 0$, it follows that $|a - b| < \epsilon$.

7. [T/F] Given $a, b \in \mathbb{R}$, we have $a = b$ if for some $\epsilon > 0$, it follows that $|a - b| < \epsilon$.

8. [T/F] Given $a, b \in \mathbb{R}$, we have $a = b$ only if for some $\epsilon > 0$, it follows that $|a - b| < \epsilon$.

2 Induction

Proposition 1 *Let* $y_1 = 3$ *and* $y_n = \frac{3y_n+4}{6}$. *Show that* y_n *is decreasing and that* $y_n \geq \frac{4}{3}$ *for all* $n \in \mathbb{N}$.

3 Statement and Some Definitions

How shall we distinguish $\mathbb{R}$ from $\mathbb{Q}$?

What does it mean to say that $\mathbb{R}$ is complete?

Axiom 1 *Every nonempty subset of* $\mathbb{R}$ *that is bounded above has a least upper bound.*

What does this mean?

Definition 1 *For a set* $A \subset \mathbb{R}$*, we say*

- *A is bounded above if there exists a number $b \in \mathbb{R}$ such that $a \leq b$ for all $a \in A$. The number b is called an upper bound for A.*
- *$a_0 = \max(A)$ if $a_0 \in A$ and $a_0 \geq a$ for all $a \in A$ We might also say a_0 is the maximum of A.*
- *A real number s is the least upper bound (or sometimes, we use the Latin word supremum) for a set $A \subset \mathbb{R}$ if s is an upper bound for A and if for every upper bound b of A, we have $s \leq b$. If s exists, we use the notation $s = \sup(A)$.*

Here are some questions to orient us to the idea:

1. If $A = \{\frac{n}{n+1} \mid n \in \mathbb{N}\}$, then which if any of these numbers are an upper bound for A: $\frac{1}{2}$, 1, 5?

2. [T/F] An upper bound for $A \subset \mathbb{R}$ is necessarily an element of A.

3. [T/F] A least upper bound for $A \subset \mathbb{R}$ is necessarily an element of A.

4. [T/F] A set $A \subset \mathbb{R}$ has at least one maximum.

5. [T/F] A set $A \subset \mathbb{R}$ has at most one maximum.

6. [T/F] A set $A \subset \mathbb{R}$ has at least one upper bound.

7. [T/F] A set $A \subset \mathbb{R}$ has at most one upper bound.

8. [T/F] A set $A \subset \mathbb{R}$ has at least one least upper bound.

9. [T/F] A set $A \subset \mathbb{R}$ has at most one least upper bound.

10. When does $\max(A) \neq \sup(A)$?

4 Conclusions

Today we learned about:

1. Triangle Inequality
2. Induction
3. The Statement of the Axiom of Completeness
4. Some related definitions and properties

Friday we will learn about:

1. Some Consequences of Completeness

Upcoming Deadlines:

- Wednesday January 31, 2018: Homework 1

Questions?