Math 311
Spring 2018
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Day #5 Notes: Consequences of Completeness

January 29, 2018

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1 Consequences of Completeness

Theorem 1 (Nested Interval Property) *Suppose that* $(I_n)_{n\in\mathbb{N}}$ *is a nested sequence of closed intervals. That is,* $\forall n \in \mathbb{N}$, $I_n = [a_n, b_n]$ *and* $\forall n \in \mathbb{N}$, $a_n \leq a_{n+1} \leq b_{n+1} \leq b_n$*. Then* $\exists x \in \mathbb{R}$ *so that* $x \in \cap_{n=1}^{\infty} I_n$.

Proof:

(continued)

Theorem 2 (Archimedean Property) $\forall x \in \mathbb{R}, \exists n \in \mathbb{N}$ *so that* $n > x$, *and* $\forall y > 0$, $\exists m \in \mathbb{N}$ *so that* $y > \frac{1}{m}$.

Proof:

What does this mean? Why is it useful?

Density of $\mathbb{Q}$ in $\mathbb{R}$:

Theorem 3 *If* $x, y \in \mathbb{R}$ *and* $x < y$*, then* $\exists r \in \mathbb{Q}$ *so that* $x < r < y$*.*

Proof:

What about the irrationals?

2 Conclusions

Today we learned about:

1. Implications of the Axiom of Completeness, including:
   (a) The Nested Interval Property
   (b) The Archimedean Property
   (c) The Density of the Rationals in the Reals.

Wednesday we will learn about:

1. More Consequences of Completeness
2. Cardinality

Upcoming Deadlines:

- Wednesday, January 31, 2018: Homework #1.