Day #6 Notes: More Consequences of Completeness

January 28, 2018

Contents

Theorem 1 *If $x, y \in \mathbb{R}$ and $x < y$, then $\exists r \in \mathbb{Q}$ so that $x < r < y$.*

Proof:

What about the irrationals?

1   Existence of Roots

Theorem 2 *There exists a real number whose square is* 2.

Proof:

2   Cardinality

Complete the worksheet in class and we will go over it together:

Definition 1 *Two sets A and B have the same cardinality if there exists a function $f : A \to B$ that is one-to-one and onto. In this case, we write $A \sim B$.*

Definition 2 *We say a set has cardinality n if $A \sim \{1, \ldots, n\}$. If $\exists n \in \mathbb{N}$ so that A has cardinality n, we say that A is finite. We say a set A is countable if $A \sim \mathbb{N}$. If A is neither finite nor countable, then we say A is uncountable.*

1. [T/F] If $A = \{1, 2, 3\}$ and $B = \{e, \pi, \sqrt{2}\}$ then $A \sim B$.

2. [T/F] The even integers $2\mathbb{Z}$ have the same cardinality as the integers; that is, $2\mathbb{Z} \sim \mathbb{Z}$.

3. [T/F] $\mathbb{Z} \sim \mathbb{N}$; that is, the integers are countable.

4. [T/F] $\mathbb{Q}$ is countable.

5. [T/F] $\mathbb{R}$ is countable.

3 Conclusions

Today we learned about:

1. Implications of the Axiom of Completeness, including:

    (a) The Density of the Rationals in the Reals.

    (b) The existence of roots.

    (c) The cardinality of $\mathbb{R}$.

Monday we will learn about:

1. More on cardinality

2. A start on sequences and series

Upcoming Deadlines:

- Monday, September 11: Add Date.

- Wednesday, September 13: Homework #2.

Questions?